

# Java Source and Bytecode Formalizations in Isabelle: Bali

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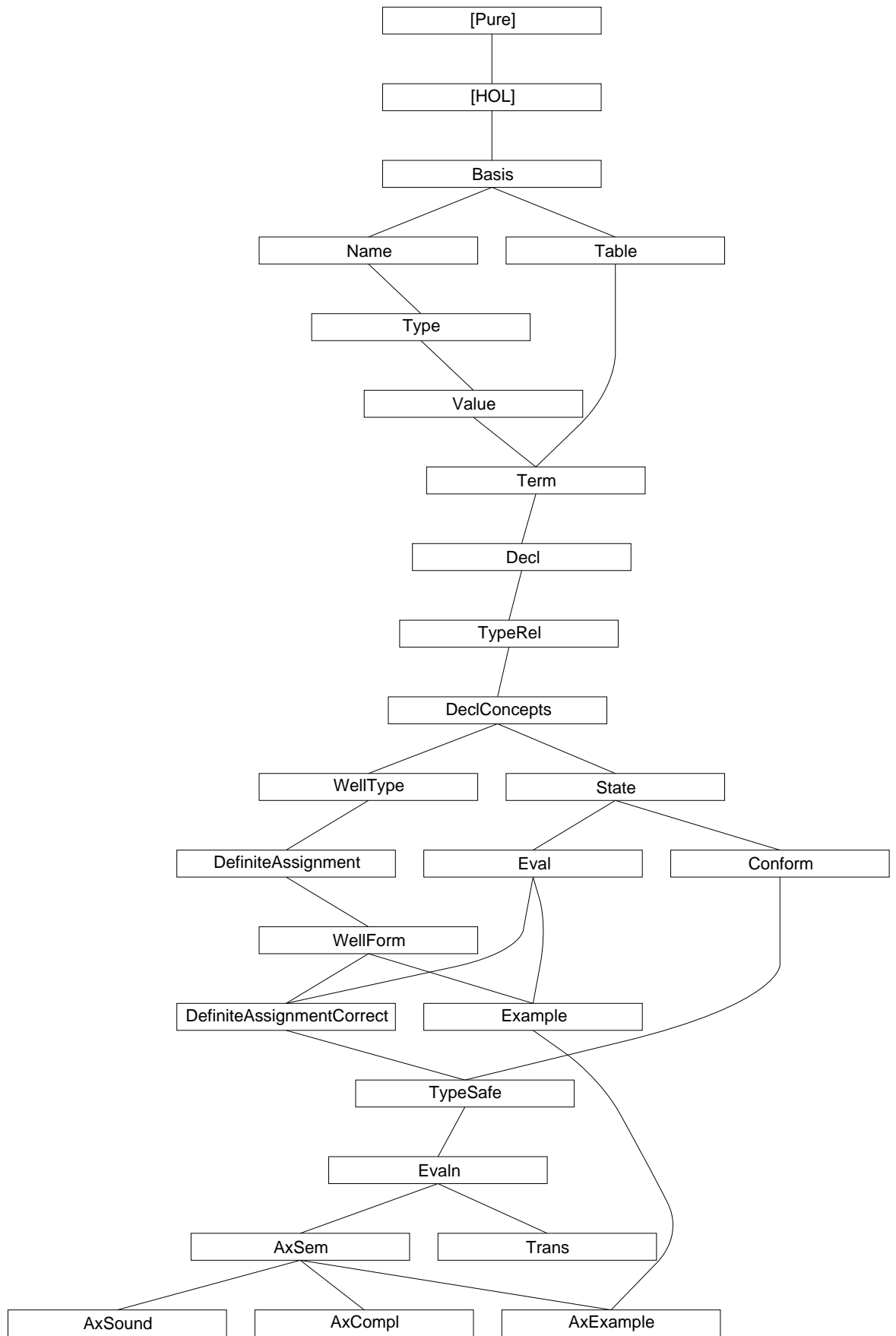
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# Chapter 1

## Overview

These theories, called Bali, model and analyse different aspects of the JavaCard **source language**. The basis is an abstract model of the JavaCard source language. On it, a type system, an operational semantics and an axiomatic semantics (Hoare logic) are built. The execution of a wellformed program (with respect to the type system) according to the operational semantics is proved to be typesafe. The axiomatic semantics is proved to be sound and relative complete with respect to the operational semantics.

We have modelled large parts of the original JavaCard source language. It models features such as:

- The basic “primitive types” of Java
- Classes and related concepts
- Class fields and methods
- Instance fields and methods
- Interfaces and related concepts
- Arrays
- Static initialisation
- Static overloading of fields and methods
- Inheritance, overriding and hiding of methods, dynamic binding
- All cases of abrupt termination
  - Exception throwing and handling
  - `break`, `continue` and `return`
- Packages
- Access Modifiers (`private`, `protected`, `public`)
- A “definite assignment” check

The following features are missing in Bali wrt. JavaCard:

- Some primitive types (`byte`, `short`)
- Syntactic variants of statements (`do-loop`, `for-loop`)
- Interface fields

- Inner Classes

In addition, features are missing that are not part of the JavaCard language, such as multithreading and garbage collection. No attempt has been made to model peculiarities of JavaCard such as the applet firewall or the transaction mechanism.

Overview of the theories:

**Basis** Some basic definitions and settings not specific to JavaCard but missing in HOL.

**Table** Definition and some properties of a lookup table to map various names (like class names or method names) to some content (like classes or methods).

**Name** Definition of various names (class names, variable names, package names,...)

**Value** JavaCard expression values (Boolean, Integer, Addresses,...)

**Type** JavaCard types. Primitive types (Boolean, Integer,...) and reference types (Classes, Interfaces, Arrays,...)

**Term** JavaCard terms. Variables, expressions and statements.

**Decl** Class, interface and program declarations. Recursion operators for the class and the interface hierarchy.

**TypeRel** Various relations on types like the subclass-, subinterface-, widening-, narrowing- and casting-relation.

**DeclConcepts** Advanced concepts on the class and interface hierarchy like inheritance, overriding, hiding, accessibility of types and members according to the access modifiers, method lookup.

**WellType** Typesystem on the JavaCard term level.

**DefiniteAssignment** The definite assignment analysis on the JavaCard term level.

**WellForm** Typesystem on the JavaCard class, interface and program level.

**State** The program state (like object store) for the execution of JavaCard. Abrupt completion (exceptions, break, continue, return) is modelled as flag inside the state.

**Eval** Operational (big step) semantics for JavaCard.

**Example** An concrete example of a JavaCard program to validate the typesystem and the operational semantics.

**Conform** Conformance predicate for states. When does an execution state conform to the static types of the program given by the typesystem.

**DefiniteAssignmentCorrect** Correctness of the definite assignment analysis. If the analysis regards a variable as definitely assigned at a certain program point, the variable will actually be assigned there during execution.

**TypeSafe** Typesafety proof of the execution of JavaCard. "Welltyped programs don't go wrong" or more technical: The execution of a welltyped JavaCard program preserves the conformance of execution states.

**Evaln** Copy of the operational semantics given in theory Eval expanded with an annotation for the maximal recursive depth. The semantics is not altered. The annotation is needed for the soundness proof of the axiomatic semantics.



**Trans** A smallstep operational semantics for JavaCard.

**AxSem** An axiomatic semantics (Hoare logic) for JavaCard.

**AxSound** The soundness proof of the axiomatic semantics with respect to the operational semantics.

**AxComple** The proof of (relative) completeness of the axiomatic semantics with respect to the operational semantics.

**AxExample** An concrete example of the axiomatic semantics at work, applied to prove some properties of the JavaCard example given in theory Example.



## Chapter 2

# Basis

# 1 Definitions extending HOL as logical basis of Bali

theory *Basis* imports *Main* begin

declare  $[[unify-search-bound = 40, unify-trace-bound = 40]]$

misc

declare *same-fstI* [intro!]

declare *split-if-asm* [split] *option.split* [split] *option.split-asm* [split]  
 $\langle ML \rangle$

declare *if-weak-cong* [cong del] *option.weak-case-cong* [cong del]

declare *length-Suc-conv* [iff]

lemma *Collect-split-eq*:  $\{p. P (split\ f\ p)\} = \{(a,b). P (f\ a\ b)\}$   
 $\langle proof \rangle$

lemma *subset-insertD*:

$A \leq insert\ x\ B \implies A \leq B \ \& \ x \sim: A \mid (EX\ B'. A = insert\ x\ B' \ \& \ B' \leq B)$   
 $\langle proof \rangle$

syntax

$3 :: nat \quad (3)$

$4 :: nat \quad (4)$

translations

$3 == Suc\ 2$

$4 == Suc\ 3$

lemma *range-bool-domain*:  $range\ f = \{f\ True, f\ False\}$   
 $\langle proof \rangle$

lemma *irrefl-tranclI'*:  $r^{\wedge}-1\ Int\ r^{\wedge}+ = \{\} \implies !x. (x, x) \sim: r^{\wedge}+$   
 $\langle proof \rangle$

lemma *trancl-rtrancl-trancl*:

$[(x,y) \in r^{\wedge}+; (y,z) \in r^{\wedge}*] \implies (x,z) \in r^{\wedge}+$   
 $\langle proof \rangle$

lemma *rtrancl-into-trancl3*:

$[(a,b) \in r^{\wedge}*; a \neq b] \implies (a,b) \in r^{\wedge}+$   
 $\langle proof \rangle$

lemma *rtrancl-into-rtrancl2*:

$[(a, b) \in r; (b, c) \in r^{\wedge}*] \implies (a, c) \in r^{\wedge}*$   
 $\langle proof \rangle$

lemma *triangle-lemma*:

$$\llbracket \bigwedge a\ b\ c.\ \llbracket (a,b) \in r; (a,c) \in r \rrbracket \implies b=c; (a,x) \in r^*; (a,y) \in r^* \rrbracket$$

$$\implies (x,y) \in r^* \vee (y,x) \in r^*$$

$$\langle \text{proof} \rangle$$

**lemma** *rtrancl-cases* [consumes 1, case-names *Refl Trancl*]:  

$$\llbracket (a,b) \in r^*; a = b \implies P; (a,b) \in r^+ \implies P \rrbracket \implies P$$

$$\langle \text{proof} \rangle$$

**theorems** *converse-rtrancl-induct*  

$$= \text{converse-rtrancl-induct} \text{ [consumes 1, case-names } Id \text{ Step]}$$

**theorems** *converse-trancl-induct*  

$$= \text{converse-trancl-induct} \text{ [consumes 1, case-names } Single \text{ Step]}$$

**lemma** *Ball-weaken*: 
$$\llbracket Ball\ s\ P; \bigwedge x.\ P\ x \longrightarrow Q\ x \rrbracket \implies Ball\ s\ Q$$

$$\langle \text{proof} \rangle$$

**lemma** *finite-SetCompr2*: 
$$\llbracket finite\ (Collect\ P); !y.\ P\ y \longrightarrow finite\ (range\ (f\ y)) \rrbracket \implies$$
  

$$finite\ \{f\ y\ x \mid x\ y.\ P\ y\}$$

$$\langle \text{proof} \rangle$$

**lemma** *list-all2-trans*: 
$$\forall\ a\ b\ c.\ P1\ a\ b \longrightarrow P2\ b\ c \longrightarrow P3\ a\ c \implies$$
  

$$\forall\ xs2\ xs3.\ list\text{-all2}\ P1\ xs1\ xs2 \longrightarrow list\text{-all2}\ P2\ xs2\ xs3 \longrightarrow list\text{-all2}\ P3\ xs1\ xs3$$

$$\langle \text{proof} \rangle$$

## pairs

**lemma** *surjective-pairing5*: 
$$p = (fst\ p,\ fst\ (snd\ p),\ fst\ (snd\ (snd\ p)),\ fst\ (snd\ (snd\ (snd\ p))),$$
  

$$snd\ (snd\ (snd\ (snd\ p))))$$

$$\langle \text{proof} \rangle$$

**lemma** *fst-splitE* [elim!]:  

$$\llbracket fst\ s' = x'; !!x\ s.\ \llbracket s' = (x,s); x = x' \rrbracket \implies Q \rrbracket \implies Q$$

$$\langle \text{proof} \rangle$$

**lemma** *fst-in-set-lemma* [rule-format (no-asm)]: 
$$(x,\ y) : set\ l \longrightarrow x : fst\ 'set\ l$$

$$\langle \text{proof} \rangle$$

## quantifiers

**lemma** *All-Ex-refl-eq2* [simp]:  

$$(!x.\ (?\ b.\ x = f\ b \ \&\ Q\ b) \longrightarrow P\ x) = (!b.\ Q\ b \longrightarrow P\ (f\ b))$$

$$\langle \text{proof} \rangle$$

**lemma** *ex-ex-miniscope1* [simp]:  

$$(EX\ w\ v.\ P\ w\ v \ \&\ Q\ v) = (EX\ v.\ (EX\ w.\ P\ w\ v) \ \&\ Q\ v)$$

$\langle \text{proof} \rangle$

**lemma** *ex-miniscope2* [simp]:

$$(EX\ v.\ P\ v \ \&\ Q \ \&\ R\ v) = (Q \ \&\ (EX\ v.\ P\ v \ \&\ R\ v))$$

$\langle \text{proof} \rangle$

**lemma** *ex-reorder31*:  $(\exists\ z\ x\ y.\ P\ x\ y\ z) = (\exists\ x\ y\ z.\ P\ x\ y\ z)$

$\langle \text{proof} \rangle$

**lemma** *All-Ex-refl-eq1* [simp]:  $(!x.\ (?\ b.\ x = f\ b) \dashrightarrow P\ x) = (!b.\ P\ (f\ b))$

$\langle \text{proof} \rangle$

**sums**

**hide** *const In0 In1*

**syntax**

$$\text{fun-sum} :: ('a \Rightarrow 'c) \Rightarrow ('b \Rightarrow 'c) \Rightarrow (('a + 'b) \Rightarrow 'c) \ (\text{infixr } '(+)80)$$

**translations**

$$\text{fun-sum} == \text{CONST sum-case}$$

**consts** *the-Inl* ::  $'a + 'b \Rightarrow 'a$

*the-Inr* ::  $'a + 'b \Rightarrow 'b$

**primrec** *the-Inl* (*Inl* *a*) = *a*

**primrec** *the-Inr* (*Inr* *b*) = *b*

**datatype**  $('a, 'b, 'c)\ \text{sum3} = \text{In1}\ 'a \mid \text{In2}\ 'b \mid \text{In3}\ 'c$

**consts** *the-In1* ::  $('a, 'b, 'c)\ \text{sum3} \Rightarrow 'a$

*the-In2* ::  $('a, 'b, 'c)\ \text{sum3} \Rightarrow 'b$

*the-In3* ::  $('a, 'b, 'c)\ \text{sum3} \Rightarrow 'c$

**primrec** *the-In1* (*In1* *a*) = *a*

**primrec** *the-In2* (*In2* *b*) = *b*

**primrec** *the-In3* (*In3* *c*) = *c*

**syntax**

*In1l* ::  $'al \Rightarrow ('al + 'ar, 'b, 'c)\ \text{sum3}$

*In1r* ::  $'ar \Rightarrow ('al + 'ar, 'b, 'c)\ \text{sum3}$

**translations**

$$\text{In1l}\ e == \text{In1}\ (\text{Inl}\ e)$$

$$\text{In1r}\ c == \text{In1}\ (\text{Inr}\ c)$$

**syntax** *the-In1l* ::  $('al + 'ar, 'b, 'c)\ \text{sum3} \Rightarrow 'al$

*the-In1r* ::  $('al + 'ar, 'b, 'c)\ \text{sum3} \Rightarrow 'ar$

**translations**

$$\text{the-In1l} == \text{the-Inl} \circ \text{the-In1}$$

$$\text{the-In1r} == \text{the-Inr} \circ \text{the-In1}$$

$\langle \text{ML} \rangle$

**translations**

$$\text{option} <= (\text{type})\ \text{Datatype.option}$$

$$\text{list} <= (\text{type})\ \text{List.list}$$

$$\text{sum3} <= (\text{type})\ \text{Basis.sum3}$$

**quantifiers for option type****syntax**

$Oall :: [pttrn, 'a\ option, bool] \Rightarrow bool \quad ((\exists! \text{--}::/ \text{--}) [0,0,10] 10)$   
 $Oex :: [pttrn, 'a\ option, bool] \Rightarrow bool \quad ((\exists? \text{--}::/ \text{--}) [0,0,10] 10)$

**syntax (symbols)**

$Oall :: [pttrn, 'a\ option, bool] \Rightarrow bool \quad ((\forall \text{--}::/ \text{--}) [0,0,10] 10)$   
 $Oex :: [pttrn, 'a\ option, bool] \Rightarrow bool \quad ((\exists \text{--}::/ \text{--}) [0,0,10] 10)$

**translations**

$! x:A: P \quad == \quad ! x:option A. P$   
 $? x:A: P \quad == \quad ? x:option A. P$

**Special map update**

Deemed too special for theory Map.

**constdefs**

$chg\text{-}map :: ('b \Rightarrow 'b) \Rightarrow 'a \Rightarrow ('a \rightsquigarrow \Rightarrow 'b) \Rightarrow ('a \rightsquigarrow \Rightarrow 'b)$   
 $chg\text{-}map\ f\ a\ m == \text{case } m\ a\ \text{of } None \Rightarrow m \mid Some\ b \Rightarrow m(a|->f\ b)$

**lemma**  $chg\text{-}map\text{-}new[simp]: m\ a = None \implies chg\text{-}map\ f\ a\ m = m$   
 $\langle proof \rangle$

**lemma**  $chg\text{-}map\text{-}upd[simp]: m\ a = Some\ b \implies chg\text{-}map\ f\ a\ m = m(a|->f\ b)$   
 $\langle proof \rangle$

**lemma**  $chg\text{-}map\text{-}other\ [simp]: a \neq b \implies chg\text{-}map\ f\ a\ m\ b = m\ b$   
 $\langle proof \rangle$

**unique association lists****constdefs**

$unique :: ('a \times 'b)\ list \Rightarrow bool$   
 $unique \equiv distinct \circ map\ fst$

**lemma**  $uniqueD\ [rule\text{-}format\ (no\text{-}asm)]:$   
 $unique\ l \longrightarrow (!x\ y. (x,y):set\ l \longrightarrow (!x'\ y'. (x',y'):set\ l \longrightarrow x=x' \longrightarrow y=y'))$   
 $\langle proof \rangle$

**lemma**  $unique\text{-}Nil\ [simp]: unique\ []$   
 $\langle proof \rangle$

**lemma**  $unique\text{-}Cons\ [simp]: unique\ ((x,y)\#l) = (unique\ l \ \&\ (!y. (x,y) \rightsquigarrow:: set\ l))$   
 $\langle proof \rangle$

**lemmas**  $unique\text{-}ConsI = conjI\ [THEN\ unique\text{-}Cons\ [THEN\ iffD2],\ standard]$

**lemma**  $unique\text{-}single\ [simp]: !!p. unique\ [p]$   
 $\langle proof \rangle$

**lemma** *unique-ConsD*: *unique (x#xs) ==> unique xs*  
 ⟨*proof*⟩

**lemma** *unique-append* [*rule-format (no-asm)*]: *unique l' ==> unique l -->*  
*(!(x,y):set l. !(x',y'):set l'. x' ~ x) --> unique (l @ l')*  
 ⟨*proof*⟩

**lemma** *unique-map-inj* [*rule-format (no-asm)*]: *unique l --> inj f --> unique (map (%(k,x). (f k, g k x)) l)*  
 ⟨*proof*⟩

**lemma** *map-of-SomeI* [*rule-format (no-asm)*]: *unique l --> (k, x) : set l --> map-of l k = Some x*  
 ⟨*proof*⟩

## list patterns

### consts

*lsplit* :: [*'a, 'a list*] => *'b, 'a list*] => *'b*

### defs

*lsplit-def*: *lsplit == %f l. f (hd l) (tl l)*

### syntax

*-lpttrn* :: [*pttrn, pttrn*] => *pttrn* (-#/- [901,900] 900)

### translations

*%y#x#xs. b == lsplit (%y x#xs. b)*

*%x#xs . b == lsplit (%x xs . b)*

**lemma** *lsplit* [*simp*]: *lsplit c (x#xs) = c x xs*  
 ⟨*proof*⟩

**lemma** *lsplit2* [*simp*]: *lsplit P (x#xs) y z = P x xs y z*  
 ⟨*proof*⟩

**end**



## Chapter 3

## Table

## 2 Abstract tables and their implementation as lists

**theory** *Table* **imports** *Basis* **begin**

design issues:

- definition of table: infinite map vs. list vs. finite set list chosen, because:
  - + a priori finite
  - + lookup is more operational than for finite set
    - not very abstract, but function table converts it to abstract mapping
- coding of lookup result: Some/None vs. value/arbitrary Some/None chosen, because:
  - ++ makes definedness check possible (applies also to finite set), which is important for the type standard, hiding/overriding, etc. (though it may perhaps be possible at least for the operational semantics to treat programs as infinite, i.e. where classes, fields, methods etc. of any name are considered to be defined)
  - sometimes awkward case distinctions, alleviated by operator 'the'

**types**  $('a, 'b)$  *table* — table with key type 'a and contents type 'b  
 $= 'a \multimap 'b$   
 $('a, 'b)$  *tables* — non-unique table with key 'a and contents 'b  
 $= 'a \Rightarrow 'b$  *set*

**map of / table of**

**syntax**

*table-of* ::  $('a \times 'b)$  *list*  $\Rightarrow ('a, 'b)$  *table* — concrete table

**translations**

*table-of* == *map-of*

$(type)'a \multimap 'b \leq (type)'a \Rightarrow 'b$  *Datatype.option*  
 $(type)('a, 'b)$  *table*  $\leq (type)'a \multimap 'b$

**lemma** *map-add-find-left[simp]*:

$n\ k = \text{None} \implies (m\ ++\ n)\ k = m\ k$

*<proof>*

### Conditional Override

**constdefs**

*cond-override*::

$('b \Rightarrow 'b \Rightarrow \text{bool}) \Rightarrow ('a, 'b)$  *table*  $\Rightarrow ('a, 'b)$  *table*  $\Rightarrow ('a, 'b)$  *table*

— when merging tables old and new, only override an entry of table old when the condition cond holds

*cond-override cond old new*  $\equiv$

$\lambda k.$

(*case new k of*  
   *None*  $\Rightarrow$  *old k*  
   | *Some new-val*  $\Rightarrow$  (*case old k of*  
     *None*  $\Rightarrow$  *Some new-val*  
     | *Some old-val*  $\Rightarrow$  (*if cond new-val old-val*  
       *then Some new-val*  
       else *Some old-val*)))

**lemma** *cond-override-empty1*[simp]: *cond-override c empty t = t*  
 ⟨proof⟩

**lemma** *cond-override-empty2*[simp]: *cond-override c t empty = t*  
 ⟨proof⟩

**lemma** *cond-override-None*[simp]:  
*old k = None  $\implies$  (cond-override c old new) k = new k*  
 ⟨proof⟩

**lemma** *cond-override-override*:  
 $\llbracket \text{old } k = \text{Some } ov; \text{new } k = \text{Some } nv; C \text{ nv } ov \rrbracket$   
 $\implies (\text{cond-override } C \text{ old new}) k = \text{Some } nv$   
 ⟨proof⟩

**lemma** *cond-override-noOverride*:  
 $\llbracket \text{old } k = \text{Some } ov; \text{new } k = \text{Some } nv; \neg (C \text{ nv } ov) \rrbracket$   
 $\implies (\text{cond-override } C \text{ old new}) k = \text{Some } ov$   
 ⟨proof⟩

**lemma** *dom-cond-override*: *dom (cond-override C s t)  $\subseteq$  dom s  $\cup$  dom t*  
 ⟨proof⟩

**lemma** *finite-dom-cond-override*:  
 $\llbracket \text{finite } (\text{dom } s); \text{finite } (\text{dom } t) \rrbracket \implies \text{finite } (\text{dom } (\text{cond-override } C \text{ s } t))$   
 ⟨proof⟩

## Filter on Tables

**constdefs**  
*filter-tab*:: ('a  $\Rightarrow$  'b  $\Rightarrow$  bool)  $\Rightarrow$  ('a, 'b) table  $\Rightarrow$  ('a, 'b) table  
*filter-tab c t*  $\equiv$   $\lambda k.$  (case t k of  
     None  $\Rightarrow$  None  
     | Some x  $\Rightarrow$  if c k x then Some x else None)

**lemma** *filter-tab-empty*[simp]: *filter-tab c empty = empty*  
 ⟨proof⟩

**lemma** *filter-tab-True*[simp]: *filter-tab ( $\lambda x y.$  True) t = t*  
 ⟨proof⟩

**lemma** *filter-tab-False*[simp]: *filter-tab ( $\lambda x y.$  False) t = empty*  
 ⟨proof⟩

**lemma** *filter-tab-ran-subset*: *ran (filter-tab c t)  $\subseteq$  ran t*  
 ⟨proof⟩

**lemma** *filter-tab-range-subset*:  $\text{range } (\text{filter-tab } c \ t) \subseteq \text{range } t \cup \{\text{None}\}$   
 $\langle \text{proof} \rangle$

**lemma** *finite-range-filter-tab*:  
 $\text{finite } (\text{range } t) \implies \text{finite } (\text{range } (\text{filter-tab } c \ t))$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-SomeD[dest!]*:  
 $\text{filter-tab } c \ t \ k = \text{Some } x \implies (t \ k = \text{Some } x) \wedge c \ k \ x$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-SomeI*:  $\llbracket t \ k = \text{Some } x; C \ k \ x \rrbracket \implies \text{filter-tab } C \ t \ k = \text{Some } x$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-all-True*:  
 $\forall k \ y. \ t \ k = \text{Some } y \longrightarrow p \ k \ y \implies \text{filter-tab } p \ t = t$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-all-True-Some*:  
 $\llbracket \forall k \ y. \ t \ k = \text{Some } y \longrightarrow p \ k \ y; t \ k = \text{Some } v \rrbracket \implies \text{filter-tab } p \ t \ k = \text{Some } v$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-all-False*:  
 $\forall k \ y. \ t \ k = \text{Some } y \longrightarrow \neg p \ k \ y \implies \text{filter-tab } p \ t = \text{empty}$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-None*:  $t \ k = \text{None} \implies \text{filter-tab } p \ t \ k = \text{None}$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-dom-subset*:  $\text{dom } (\text{filter-tab } C \ t) \subseteq \text{dom } t$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-eq*:  $\llbracket a=b \rrbracket \implies \text{filter-tab } C \ a = \text{filter-tab } C \ b$   
 $\langle \text{proof} \rangle$

**lemma** *finite-dom-filter-tab*:  
 $\text{finite } (\text{dom } t) \implies \text{finite } (\text{dom } (\text{filter-tab } C \ t))$   
 $\langle \text{proof} \rangle$

**lemma** *filter-tab-weaken*:  
 $\llbracket \forall a \in t \ k. \exists b \in s \ k. P \ a \ b; \bigwedge k \ x \ y. \llbracket t \ k = \text{Some } x; s \ k = \text{Some } y \rrbracket \implies \text{cond } k \ x \longrightarrow \text{cond } k \ y \rrbracket \implies \forall a \in \text{filter-tab } \text{cond } t \ k. \exists b \in \text{filter-tab } \text{cond } s \ k. P \ a \ b$   
 $\langle \text{proof} \rangle$

**lemma** *cond-override-filter*:

$$\begin{aligned} & \llbracket \bigwedge k \text{ old new. } \llbracket s \text{ } k = \text{Some new}; t \text{ } k = \text{Some old} \rrbracket \\ & \implies (\neg \text{overC new old} \longrightarrow \neg \text{filterC } k \text{ new}) \wedge \\ & \quad (\text{overC new old} \longrightarrow \text{filterC } k \text{ old} \longrightarrow \text{filterC } k \text{ new}) \\ & \rrbracket \implies \\ & \quad \text{cond-override overC (filter-tab filterC } t) \text{ (filter-tab filterC } s) \\ & \quad = \text{filter-tab filterC (cond-override overC } t \text{ } s) \\ & \langle \text{proof} \rangle \end{aligned}$$

**Misc.**

**lemma** *Ball-set-table*:  $(\forall (x,y) \in \text{set } l. P \text{ } x \text{ } y) \implies \forall x. \forall y \in \text{map-of } l \text{ } x: P \text{ } x \text{ } y$   
 $\langle \text{proof} \rangle$

**lemma** *Ball-set-tableD*:

$$\llbracket (\forall (x,y) \in \text{set } l. P \text{ } x \text{ } y); x \in \text{o2s (table-of } l \text{ } xa) \rrbracket \implies P \text{ } xa \text{ } x$$
 $\langle \text{proof} \rangle$ 

**declare** *map-of-SomeD* [elim]

**lemma** *table-of-Some-in-set*:

$$\text{table-of } l \text{ } k = \text{Some } x \implies (k, x) \in \text{set } l$$
 $\langle \text{proof} \rangle$ 

**lemma** *set-get-eq*:

$$\text{unique } l \implies (k, \text{the (table-of } l \text{ } k)) \in \text{set } l = (\text{table-of } l \text{ } k \neq \text{None})$$
 $\langle \text{proof} \rangle$ 

**lemma** *inj-Pair-const2*:  $\text{inj } (\lambda k. (k, C))$

$\langle \text{proof} \rangle$

**lemma** *table-of-mapconst-SomeI*:

$$\begin{aligned} & \llbracket \text{table-of } t \text{ } k = \text{Some } y'; \text{snd } y=y'; \text{fst } y=c \rrbracket \implies \\ & \quad \text{table-of (map } (\lambda(k,x). (k,c,x)) \text{ } t) \text{ } k = \text{Some } y \\ & \langle \text{proof} \rangle \end{aligned}$$

**lemma** *table-of-mapconst-NoneI*:

$$\begin{aligned} & \llbracket \text{table-of } t \text{ } k = \text{None} \rrbracket \implies \\ & \quad \text{table-of (map } (\lambda(k,x). (k,c,x)) \text{ } t) \text{ } k = \text{None} \\ & \langle \text{proof} \rangle \end{aligned}$$

**lemmas** *table-of-map2-SomeI* = *inj-Pair-const2* [THEN *map-of-mapk-SomeI*, standard]

**lemma** *table-of-map-SomeI* [rule-format (no-asm)]:  $\text{table-of } t \text{ } k = \text{Some } x \longrightarrow$

$$\text{table-of (map } (\lambda(k,x). (k, f \text{ } x)) \text{ } t) \text{ } k = \text{Some (f } x)$$
 $\langle \text{proof} \rangle$ 

**lemma** *table-of-remap-SomeD* [rule-format (no-asm)]:

$$\begin{aligned} & \text{table-of (map } (\lambda((k,k'),x). (k,(k',x))) \text{ } t) \text{ } k = \text{Some (k',x)} \longrightarrow \\ & \quad \text{table-of } t \text{ } (k, k') = \text{Some } x \\ & \langle \text{proof} \rangle \end{aligned}$$

**lemma** *table-of-mapf-Some* [rule-format (no-asm)]:  $\forall x y. f x = f y \longrightarrow x = y \implies$   
 $\text{table-of } (\text{map } (\lambda(k,x). (k, f x)) t) k = \text{Some } (f x) \longrightarrow \text{table-of } t k = \text{Some } x$   
 <proof>

**lemma** *table-of-mapf-SomeD* [rule-format (no-asm), dest!]:  
 $\text{table-of } (\text{map } (\lambda(k,x). (k, f x)) t) k = \text{Some } z \longrightarrow (\exists y \in \text{table-of } t k. z = f y)$   
 <proof>

**lemma** *table-of-mapf-NoneD* [rule-format (no-asm), dest!]:  
 $\text{table-of } (\text{map } (\lambda(k,x). (k, f x)) t) k = \text{None} \longrightarrow (\text{table-of } t k = \text{None})$   
 <proof>

**lemma** *table-of-mapkey-SomeD* [rule-format (no-asm), dest!]:  
 $\text{table-of } (\text{map } (\lambda(k,x). ((k,C),x)) t) (k,D) = \text{Some } x \longrightarrow C = D \wedge \text{table-of } t k = \text{Some } x$   
 <proof>

**lemma** *table-of-mapkey-SomeD2* [rule-format (no-asm), dest!]:  
 $\text{table-of } (\text{map } (\lambda(k,x). ((k,C),x)) t) ek = \text{Some } x$   
 $\longrightarrow C = \text{snd } ek \wedge \text{table-of } t (\text{fst } ek) = \text{Some } x$   
 <proof>

**lemma** *table-append-Some-iff*:  $\text{table-of } (xs @ ys) k = \text{Some } z =$   
 $(\text{table-of } xs k = \text{Some } z \vee (\text{table-of } xs k = \text{None} \wedge \text{table-of } ys k = \text{Some } z))$   
 <proof>

**lemma** *table-of-filter-unique-SomeD* [rule-format (no-asm)]:  
 $\text{table-of } (\text{filter } P xs) k = \text{Some } z \implies \text{unique } xs \longrightarrow \text{table-of } xs k = \text{Some } z$   
 <proof>

### consts

*Un-tables* :: ('a, 'b) tables set  $\Rightarrow$  ('a, 'b) tables  
*overrides-t* :: ('a, 'b) tables  $\Rightarrow$  ('a, 'b) tables  $\Rightarrow$   
 ('a, 'b) tables (infixl  $\oplus \oplus$  100)  
*hidings-entails*:: ('a, 'b) tables  $\Rightarrow$  ('a, 'c) tables  $\Rightarrow$   
 ('b  $\Rightarrow$  'c  $\Rightarrow$  bool)  $\Rightarrow$  bool (- hidings - entails - 20)  
 — variant for unique table:  
*hiding-entails* :: ('a, 'b) table  $\Rightarrow$  ('a, 'c) table  $\Rightarrow$   
 ('b  $\Rightarrow$  'c  $\Rightarrow$  bool)  $\Rightarrow$  bool (- hiding - entails - 20)  
 — variant for a unique table and conditional overriding:  
*cond-hiding-entails* :: ('a, 'b) table  $\Rightarrow$  ('a, 'c) table  
 $\Rightarrow$  ('b  $\Rightarrow$  'c  $\Rightarrow$  bool)  $\Rightarrow$  ('b  $\Rightarrow$  'c  $\Rightarrow$  bool)  $\Rightarrow$  bool  
 (- hiding - under - entails - 20)

### defs

*Un-tables-def*:  $\text{Un-tables } ts \equiv \lambda k. \bigcup t \in ts. t k$   
*overrides-t-def*:  $s \oplus \oplus t \equiv \lambda k. \text{if } t k = \{\} \text{ then } s k \text{ else } t k$   
*hidings-entails-def*:  $t \text{ hidings } s \text{ entails } R \equiv \forall k. \forall x \in t k. \forall y \in s k. R x y$   
*hiding-entails-def*:  $t \text{ hiding } s \text{ entails } R \equiv \forall k. \forall x \in t k: \forall y \in s k: R x y$   
*cond-hiding-entails-def*:  $t \text{ hiding } s \text{ under } C \text{ entails } R$   
 $\equiv \forall k. \forall x \in t k: \forall y \in s k: C x y \longrightarrow R x y$

**Untables**

**lemma** *Un-tablesI* [intro]:  $\bigwedge x. \llbracket t \in ts; x \in t \ k \rrbracket \implies x \in \text{Un-tables } ts \ k$   
 ⟨proof⟩

**lemma** *Un-tablesD* [dest!]:  $\bigwedge x. x \in \text{Un-tables } ts \ k \implies \exists t. t \in ts \wedge x \in t \ k$   
 ⟨proof⟩

**lemma** *Un-tables-empty* [simp]:  $\text{Un-tables } \{\} = (\lambda k. \{\})$   
 ⟨proof⟩

**overrides**

**lemma** *empty-overrides-t* [simp]:  $(\lambda k. \{\}) \oplus \oplus m = m$   
 ⟨proof⟩

**lemma** *overrides-empty-t* [simp]:  $m \oplus \oplus (\lambda k. \{\}) = m$   
 ⟨proof⟩

**lemma** *overrides-t-Some-iff*:  
 $(x \in (s \oplus \oplus t) \ k) = (x \in t \ k \vee t \ k = \{\} \wedge x \in s \ k)$   
 ⟨proof⟩

**lemmas** *overrides-t-SomeD* = *overrides-t-Some-iff* [THEN iffD1, dest!]

**lemma** *overrides-t-right-empty* [simp]:  $n \ k = \{\} \implies (m \oplus \oplus n) \ k = m \ k$   
 ⟨proof⟩

**lemma** *overrides-t-find-right* [simp]:  $n \ k \neq \{\} \implies (m \oplus \oplus n) \ k = n \ k$   
 ⟨proof⟩

**hiding entails**

**lemma** *hiding-entailsD*:  
 $\llbracket t \text{ hiding } s \text{ entails } R; t \ k = \text{Some } x; s \ k = \text{Some } y \rrbracket \implies R \ x \ y$   
 ⟨proof⟩

**lemma** *empty-hiding-entails*: *empty hiding s entails R*  
 ⟨proof⟩

**lemma** *hiding-empty-entails*: *t hiding empty entails R*  
 ⟨proof⟩

**declare** *empty-hiding-entails* [simp] *hiding-empty-entails* [simp]

**cond hiding entails**

**lemma** *cond-hiding-entailsD*:  
 $\llbracket t \text{ hiding } s \text{ under } C \text{ entails } R; t \ k = \text{Some } x; s \ k = \text{Some } y; C \ x \ y \rrbracket \implies R \ x \ y$   
 ⟨proof⟩

**lemma** *empty-cond-hiding-entails*[simp]: *empty hiding s under C entails R*  
 ⟨proof⟩

**lemma** *cond-hiding-empty-entails*[simp]: *t hiding empty under C entails R*  
 ⟨proof⟩

**lemma** *hidings-entailsD*:  $\llbracket t \text{ hidings } s \text{ entails } R; x \in t \ k; y \in s \ k \rrbracket \implies R \ x \ y$   
 ⟨proof⟩

**lemma** *hidings-empty-entails*: *t hidings* ( $\lambda k. \{\}$ ) *entails R*  
 ⟨proof⟩

**lemma** *empty-hidings-entails*:  
 ( $\lambda k. \{\}$ ) *hidings s entails R*⟨proof⟩  
**declare** *empty-hidings-entails* [intro!] *hidings-empty-entails* [intro!]

**consts**  
*atleast-free* :: ('a  $\sim \Rightarrow$  'b)  $\Rightarrow$  nat  $\Rightarrow$  bool  
**primrec**  
*atleast-free* *m* 0 = True  
*atleast-free-Suc*:  
*atleast-free* *m* (Suc *n*) = (? *a*. *m* *a* = None & (!*b*. *atleast-free* (*m*(*a*| $\rightarrow$ *b*)) *n*))

**lemma** *atleast-free-weaken* [rule-format (no-asm)]:  
 !*m*. *atleast-free* *m* (Suc *n*)  $\longrightarrow$  *atleast-free* *m* *n*  
 ⟨proof⟩

**lemma** *atleast-free-SucI*:  
 [| *h* *a* = None; !*obj*. *atleast-free* (*h*(*a*| $\rightarrow$ *obj*)) *n* |]  $\implies$  *atleast-free* *h* (Suc *n*)  
 ⟨proof⟩

**declare** *fun-upd-apply* [simp del]

**lemma** *atleast-free-SucD-lemma* [rule-format (no-asm)]:  
 !*m* *a*. *m* *a* = None  $\longrightarrow$  (!*c*. *atleast-free* (*m*(*a*| $\rightarrow$ *c*)) *n*)  $\longrightarrow$   
 (!*b* *d*. *a*  $\sim$  *b*  $\longrightarrow$  *atleast-free* (*m*(*b*| $\rightarrow$ *d*)) *n*)  
 ⟨proof⟩  
**declare** *fun-upd-apply* [simp]

**lemma** *atleast-free-SucD* [rule-format (no-asm)]: *atleast-free* *h* (Suc *n*)  $\implies$  *atleast-free* (*h*(*a*| $\rightarrow$ *b*)) *n*  
 ⟨proof⟩

**declare** *atleast-free-Suc* [simp del]  
**end**



## Chapter 4

Name

### 3 Java names

**theory** *Name* **imports** *Basis* **begin**

**typeddecl** *tnam* — ordinary type name, i.e. class or interface name

**typeddecl** *pname* — package name

**typeddecl** *mname* — method name

**typeddecl** *vname* — variable or field name

**typeddecl** *label* — label as destination of break or continue

**datatype** *ename* — expression name

= *VName vname*

| *Res* — special name to model the return value of methods

**datatype** *lname* — names for local variables and the This pointer

= *ENAME ename*

| *This*

**syntax**

*VName* :: *vname*  $\Rightarrow$  *lname*

*Result* :: *lname*

**translations**

*VName* *n* == *ENAME* (*VName* *n*)

*Result* == *ENAME* *Res*

**datatype** *xname* — names of standard exceptions

= *Throwable*

| *NullPointerException* | *OutOfMemory* | *ClassCast*

| *NegativeArraySize* | *IndexOutOfBoundsException* | *ArrayStore*

**lemma** *xn-cases*:

*xn* = *Throwable*  $\vee$  *xn* = *NullPointerException*  $\vee$

*xn* = *OutOfMemory*  $\vee$  *xn* = *ClassCast*  $\vee$

*xn* = *NegativeArraySize*  $\vee$  *xn* = *IndexOutOfBoundsException*  $\vee$  *xn* = *ArrayStore*

$\langle$ proof $\rangle$

**datatype** *tname* — type names for standard classes and other type names

= *Object'*

| *SXcpt'* *xname*

| *TName* *tnam*

**record** *qname* = — qualified tname cf. 6.5.3, 6.5.4

*pid* :: *pname*

*tid* :: *tname*

**axclass** *has-pname* < *type*

**consts** *pname*::'*a*::*has-pname*  $\Rightarrow$  *pname*

**instance** *pname*::*has-pname*  $\langle$ proof $\rangle$

**defs** (overloaded)

*pname-pname-def*: *pname* (*p*::*pname*)  $\equiv$  *p*

**axclass** *has-tname* < *type*

**consts** *tname*::'*a*::*has-tname*  $\Rightarrow$  *tname*

**instance** *tname::has-tname*  $\langle \text{proof} \rangle$

**defs** (overloaded)

*tname-tname-def*: *tname* (*t::tname*)  $\equiv t$

**axclass** *has-qtname* < *type*

**consts** *qtname::'a::has-qtname*  $\Rightarrow$  *qtname*

**instance** *qtname-ext-type* :: (*type*) *has-qtname*  $\langle \text{proof} \rangle$

**defs** (overloaded)

*qtname-qtname-def*: *qtname* (*q::qtname*)  $\equiv q$

**translations**

*mname* <= *Name.mname*

*xname* <= *Name.xname*

*tname* <= *Name.tname*

*ename* <= *Name.ename*

*qtname* <= (*type*) ( $\llbracket \text{pid}::\text{pname}, \text{tid}::\text{tname} \rrbracket$ )

(*type*) '*a* *qtname-scheme* <= (*type*) ( $\llbracket \text{pid}::\text{pname}, \text{tid}::\text{tname}, \dots::'a \rrbracket$ )

**axiomatization** *java-lang::pname* — package java.lang

**consts**

*Object* :: *qtname*

*SXcpt* :: *xname*  $\Rightarrow$  *qtname*

**defs**

*Object-def*: *Object*  $\equiv \llbracket \text{pid} = \text{java-lang}, \text{tid} = \text{Object}' \rrbracket$

*SXcpt-def*: *SXcpt*  $\equiv \lambda x. \llbracket \text{pid} = \text{java-lang}, \text{tid} = \text{SXcpt}' x \rrbracket$

**lemma** *Object-neq-SXcpt* [*simp*]: *Object*  $\neq$  *SXcpt* *xn*

$\langle \text{proof} \rangle$

**lemma** *SXcpt-inject* [*simp*]: (*SXcpt* *xn* = *SXcpt* *xm*) = (*xn* = *xm*)

$\langle \text{proof} \rangle$

**end**



## Chapter 5

# Value

## 4 Java values

**theory** *Value* **imports** *Type* **begin**

**typeddecl** *loc* — locations, i.e. abstract references on objects

**datatype** *val*

= *Unit* — dummy result value of void methods  
 | *Bool bool* — Boolean value  
 | *Intg int* — integer value  
 | *Null* — null reference  
 | *Addr loc* — addresses, i.e. locations of objects

**translations** *val* <= (*type*) *Term.val*  
*loc* <= (*type*) *Term.loc*

**consts** *the-Bool* :: *val* ⇒ *bool*

**primrec** *the-Bool* (*Bool b*) = *b*

**consts** *the-Intg* :: *val* ⇒ *int*

**primrec** *the-Intg* (*Intg i*) = *i*

**consts** *the-Addr* :: *val* ⇒ *loc*

**primrec** *the-Addr* (*Addr a*) = *a*

**types** *dyn-ty* = *loc* ⇒ *ty option*

**consts**

*typeof* :: *dyn-ty* ⇒ *val* ⇒ *ty option*

*defpval* :: *prim-ty* ⇒ *val* — default value for primitive types

*default-val* :: *ty* ⇒ *val* — default value for all types

**primrec** *typeof dt Unit* = *Some (PrimT Void)*

*typeof dt (Bool b)* = *Some (PrimT Boolean)*

*typeof dt (Intg i)* = *Some (PrimT Integer)*

*typeof dt Null* = *Some NT*

*typeof dt (Addr a)* = *dt a*

**primrec** *defpval Void* = *Unit*

*defpval Boolean* = *Bool False*

*defpval Integer* = *Intg 0*

**primrec** *default-val (PrimT pt)* = *defpval pt*

*default-val (RefT r)* = *Null*

**end**

## Chapter 6

# Type

## 5 Java types

**theory** *Type* **imports** *Name* **begin**

simplifications:

- only the most important primitive types
- the null type is regarded as reference type

**datatype** *prim-ty* — primitive type, cf. 4.2  
 = *Void* — result type of void methods  
 | *Boolean*  
 | *Integer*

**datatype** *ref-ty* — reference type, cf. 4.3  
 = *NullT* — null type, cf. 4.1  
 | *IfaceT qtname* — interface type  
 | *ClassT qtname* — class type  
 | *ArrayT ty* — array type

**and** *ty* — any type, cf. 4.1  
 = *PrimT prim-ty* — primitive type  
 | *RefT ref-ty* — reference type

**translations**

*prim-ty* <= (*type*) *Type.prim-ty*  
*ref-ty* <= (*type*) *Type.ref-ty*  
*ty* <= (*type*) *Type.ty*

**syntax**

*NT* :: *ty*  
*Iface* :: *qtname*  $\Rightarrow$  *ty*  
*Class* :: *qtname*  $\Rightarrow$  *ty*  
*Array* :: *ty*  $\Rightarrow$  *ty* ( $\cdot$ .[] [90] 90)

**translations**

*NT* == *RefT NullT*  
*Iface I* == *RefT (IfaceT I)*  
*Class C* == *RefT (ClassT C)*  
*T.[]* == *RefT (ArrayT T)*

**constdefs**

*the-Class* :: *ty*  $\Rightarrow$  *qtname*  
*the-Class T*  $\equiv$  *SOME C. T = Class C*

**lemma** *the-Class-eq [simp]: the-Class (Class C) = C*  
 <proof>

**end**



## Chapter 7

## Term

## 6 Java expressions and statements

**theory** *Term* **imports** *Value Table* **begin**

design issues:

- invocation frames for local variables could be reduced to special static objects (one per method). This would reduce redundancy, but yield a rather non-standard execution model more difficult to understand.
  - method bodies separated from calls to handle assumptions in axiomat. semantics NB: Body is intended to be in the environment of the called method.
  - class initialization is regarded as (auxiliary) statement (required for AxSem)
  - result expression of method return is handled by a special result variable result variable is treated uniformly with local variables
- + welltypedness and existence of the result/return expression is ensured without extra effort

simplifications:

- expression statement allowed for any expression
- This is modeled as a special non-assignable local variable
- Super is modeled as a general expression with the same value as This
- access to field x in current class via This.x
- NewA creates only one-dimensional arrays; initialization of further subarrays may be simulated with nested NewAs
- The 'Lit' constructor is allowed to contain a reference value. But this is assumed to be prohibited in the input language, which is enforced by the type-checking rules.
- a call of a static method via a type name may be simulated by a dummy variable
- no nested blocks with inner local variables
- no synchronized statements
- no secondary forms of if, while (e.g. no for) (may be easily simulated)
- no switch (may be simulated with if)
- the *try-catch-finally* statement is divided into the *try-catch* statement and a finally statement, which may be considered as try..finally with empty catch
- the *try-catch* statement has exactly one catch clause; multiple ones can be simulated with instanceof
- the compiler is supposed to add the annotations - during type-checking. This transformation is left out as its result is checked by the type rules anyway

**types** *locals* = (*lname*, *val*) *table* — local variables

**datatype** *jump*  
= *Break label* — break

| *Cont label* — continue  
 | *Ret* — return from method

**datatype** *xcpt* — exception  
 = *Loc loc* — location of allocated execption object  
 | *Std xname* — intermediate standard exception, see Eval.thy

**datatype** *error*  
 = *AccessViolation* — Access to a member that isn't permitted  
 | *CrossMethodJump* — Method exits with a break or continue

**datatype** *abrupt* — abrupt completion  
 = *Xcpt xcpt* — exception  
 | *Jump jump* — break, continue, return  
 | *Error error* — runtime errors, we wan't to detect and proof absent in welltyped programmms

**types**  
*abopt* = *abrupt option*

Local variable store and exception. Anticipation of State.thy used by smallstep semantics. For a method call, we save the local variables of the caller in the term Callee to restore them after method return. Also an exception must be restored after the finally statement

**translations**  
*locals* <= (*type*) (*lname, val*) *table*

**datatype** *inv-mode* — invocation mode for method calls  
 = *Static* — static  
 | *SuperM* — super  
 | *IntVir* — interface or virtual

**record** *sig* = — signature of a method, cf. 8.4.2  
*name* :: *mname* — acutally belongs to Decl.thy  
*parTs* :: *ty list*

**translations**  
*sig* <= (*type*) (*name* :: *mname*, *parTs* :: *ty list*)  
*sig* <= (*type*) (*name* :: *mname*, *parTs* :: *ty list*, ... : '*a*' )

— function codes for unary operations

**datatype** *unop* = *UPlus* — + unary plus  
 | *UMinus* — - unary minus  
 | *UBitNot* — bitwise NOT  
 | *UNot* — ! logical complement

— function codes for binary operations

**datatype** *binop* = *Mul* — \* multiplication  
 | *Div* — / division  
 | *Mod* — % remainder  
 | *Plus* — + addition  
 | *Minus* — - subtraction  
 | *LShift* — << left shift  
 | *RShift* — >> signed right shift  
 | *RShiftU* — >>> unsigned right shift  
 | *Less* — < less than  
 | *Le* — <= less than or equal  
 | *Greater* — > greater than  
 | *Ge* — >= greater than or equal  
 | *Eq* — == equal  
 | *Neq* — != not equal

- | *BitAnd* — & bitwise AND
- | *And* — & boolean AND
- | *BitXor* — ^ bitwise Xor
- | *Xor* — ^ boolean Xor
- | *BitOr* — | bitwise Or
- | *Or* — | boolean Or
- | *CondAnd* — && conditional And
- | *CondOr* — || conditional Or

The boolean operators & and | strictly evaluate both of their arguments. The conditional operators && and || only evaluate the second argument if the value of the whole expression isn't already determined by the first argument. e.g.: `false && e e` is not evaluated; `true || e e` is not evaluated;

#### **datatype** *var*

- = *LVar lname* — local variable (incl. parameters)
- | *FVar qname qname bool expr vname* ( $\{-,-,-\}$ ...[10,10,10,85,99]90)
  - class field
  - {*accC*,*statDeclC*,*stat*}*e*..*fn*
  - *accC*: accessing class (static class were
  - the code is declared. Annotation only needed for
  - evaluation to check accessibility)
  - *statDeclC*: static declaration class of field
  - *stat*: static or instance field?
  - *e*: reference to object
  - *fn*: field name
- | *AVar expr expr* ( $\{-,-,-\}$ [90,10 ]90)
  - array component
  - *e1*[*e2*]: *e1* array reference; *e2* index
- | *InsInitV stmt var*
  - insertion of initialization before evaluation
  - of var (technical term for smallstep semantics.)

#### **and** *expr*

- = *NewC qname* — class instance creation
- | *NewA ty expr* (*New*  $\{-,-,-\}$ [99,10 ]85)
  - array creation
- | *Cast ty expr* — type cast
- | *Inst expr ref-ty* (*InstOf*  $\{-,-,-\}$ [85,99] 85)
  - instanceof
- | *Lit val* — literal value, references not allowed
- | *UnOp unop expr* — unary operation
- | *BinOp binop expr expr* — binary operation
- | *Super* — special Super keyword
- | *Acc var* — variable access
- | *Ass var expr* (*:-=-* [90,85 ]85)
  - variable assign
- | *Cond expr expr expr* (*- ? - :* - [85,85,80]80) — conditional
- | *Call qname ref-ty inv-mode expr mname* (*ty list*) (*expr list*)
  - ( $\{-,-,-\}$ ...( $\{-,-,-\}$ )[10,10,10,85,99,10,10]85)
  - method call
  - {*accC*,*statT*,*mode*}*e*..*mn*( {*pTs*}*args*) "
  - *accC*: accessing class (static class were
  - the call code is declared. Annotation only needed for
  - evaluation to check accessibility)
  - *statT*: static declaration class/interface of
  - method
  - *mode*: invocation mode
  - *e*: reference to object

— *mn*: field name  
 — *pTs*: types of parameters  
 — *args*: the actual parameters/arguments  
 | *Methd qname sig* — (folded) method (see below)  
 | *Body qname stmt* — (unfolded) method body  
 | *InsInitE stmt expr*  
   — insertion of initialization before  
   — evaluation of *expr* (technical term for smallstep sem.)  
 | *Callee locals expr* — save callers locals in callee-Frame  
   — (technical term for smallstep semantics)  
**and** *stmt*  
 = *Skip* — empty statement  
 | *Expr expr* — expression statement  
 | *Lab jump stmt* ( $\rightarrow$  - [ 99,66]66)  
   — labeled statement; handles break  
 | *Comp stmt stmt* (-;; - [ 66,65]65)  
 | *If' expr stmt stmt* (*If'*(-') - *Else* - [ 80,79,79]70)  
 | *Loop label expr stmt* ( $\rightarrow$  *While'*(-') - [ 99,80,79]70)  
 | *Jmp jump* — break, continue, return  
 | *Throw expr*  
 | *TryC stmt qname vname stmt* (*Try* - *Catch'*(- -) - [79,99,80,79]70)  
   — *Try c1 Catch(C vn) c2*  
   — *c1*: block where exception may be thrown  
   — *C*: exception class to catch  
   — *vn*: local name for exception used in *c2*  
   — *c2*: block to execute when exception is caught  
 | *Fin stmt stmt* (- *Finally* - [ 79,79]70)  
 | *FinA abrupt stmt* — Save abrupt of first statement  
   — technical term for smallstep sem.)  
 | *Init qname* — class initialization

The expressions *Methd* and *Body* are artificial program constructs, in the sense that they are not used to define a concrete Bali program. In the operational semantics they are "generated on the fly" to decompose the task to define the behaviour of the *Call* expression. They are crucial for the axiomatic semantics to give a syntactic hook to insert some assertions (cf. *AxSem.thy*, *Eval.thy*). The *Init* statement (to initialize a class on its first use) is inserted in various places by the semantics. *Callee*, *InsInitV*, *InsInitE*, *FinA* are only needed as intermediate steps in the smallstep (transition) semantics (cf. *Trans.thy*). *Callee* is used to save the local variables of the caller for method return. So we avoid modelling a frame stack. The *InsInitV/E* terms are only used by the smallstep semantics to model the intermediate steps of class-initialisation.

**types** *term* = (*expr+stmt,var,expr list*) *sum3*

**translations**

*sig* <= (*type*) *mname*  $\times$  *ty list*  
*var* <= (*type*) *Term.var*  
*expr* <= (*type*) *Term.expr*  
*stmt* <= (*type*) *Term.stmt*  
*term* <= (*type*) (*expr+stmt,var,expr list*) *sum3*

**syntax**

*this* :: *expr*  
*LAcc* :: *vname*  $\Rightarrow$  *expr* (!)  
*LAss* :: *vname*  $\Rightarrow$  *expr*  $\Rightarrow$  *stmt* ( $\rightarrow$  ::= - [90,85] 85)  
*Return* :: *expr*  $\Rightarrow$  *stmt*  
*StatRef* :: *ref-ty*  $\Rightarrow$  *expr*

**translations**

```

this      == Acc (LVar This)
!!v       == Acc (LVar (ENam (VNam v)))
v::=e     == Expr (Ass (LVar (ENam (VNam v))) e)
Return e  == Expr (Ass (LVar (ENam Res)) e);; Jmp Ret
          — Res := e;; Jmp Ret
StatRef rt == Cast (RefT rt) (Lit Null)

```

### constdefs

```

is-stmt :: term ⇒ bool
is-stmt t ≡ ∃ c. t=In1r c

```

⟨ML⟩

**declare** *is-stmt-rews* [simp]

Here is some syntactic stuff to handle the injections of statements, expressions, variables and expression lists into general terms.

### syntax

```

expr-inj-term:: expr ⇒ term (⟨-⟩e 1000)
stmt-inj-term:: stmt ⇒ term (⟨-⟩s 1000)
var-inj-term:: var ⇒ term (⟨-⟩v 1000)
lst-inj-term:: expr list ⇒ term (⟨-⟩l 1000)

```

### translations

```

⟨e⟩e ↦ In1l e
⟨c⟩s ↦ In1r c
⟨v⟩v ↦ In2 v
⟨es⟩l ↦ In3 es

```

It seems to be more elegant to have an overloaded injection like the following.

```

axclass inj-term < type
consts inj-term:: 'a::inj-term ⇒ term (⟨-⟩ 1000)

```

How this overloaded injections work can be seen in the theory *DefiniteAssignment*. Other big inductive relations on terms defined in theories *WellType*, *Eval*, *Evaln* and *AxSem* don't follow this convention right now, but introduce subtle syntactic sugar in the relations themselves to make a distinction on expressions, statements and so on. So unfortunately you will encounter a mixture of dealing with these injections. The translations above are used as bridge between the different conventions.

**instance** *stmt::inj-term* ⟨proof⟩

### defs (overloaded)

*stmt-inj-term-def*: ⟨*c::stmt*⟩ ≡ *In1r c*

**lemma** *stmt-inj-term-simp*: ⟨*c::stmt*⟩ = *In1r c*  
 ⟨proof⟩

**lemma** *stmt-inj-term [iff]*: ⟨*x::stmt*⟩ = ⟨*y*⟩ ≡ *x = y*  
 ⟨proof⟩

**instance** *expr::inj-term* ⟨proof⟩

### defs (overloaded)

*expr-inj-term-def*: ⟨*e::expr*⟩ ≡ *In1l e*

**lemma** *expr-inj-term-simp*:  $\langle e::\text{expr} \rangle = \text{In1 } e$   
 $\langle \text{proof} \rangle$

**lemma** *expr-inj-term [iff]*:  $\langle x::\text{expr} \rangle = \langle y \rangle \equiv x = y$   
 $\langle \text{proof} \rangle$

**instance** *var::inj-term*  $\langle \text{proof} \rangle$

**defs (overloaded)**  
*var-inj-term-def*:  $\langle v::\text{var} \rangle \equiv \text{In2 } v$

**lemma** *var-inj-term-simp*:  $\langle v::\text{var} \rangle = \text{In2 } v$   
 $\langle \text{proof} \rangle$

**lemma** *var-inj-term [iff]*:  $\langle x::\text{var} \rangle = \langle y \rangle \equiv x = y$   
 $\langle \text{proof} \rangle$

**instance** *list::(type) inj-term*  $\langle \text{proof} \rangle$

**defs (overloaded)**  
*expr-list-inj-term-def*:  $\langle es::\text{expr list} \rangle \equiv \text{In3 } es$

**lemma** *expr-list-inj-term-simp*:  $\langle es::\text{expr list} \rangle = \text{In3 } es$   
 $\langle \text{proof} \rangle$

**lemma** *expr-list-inj-term [iff]*:  $\langle x::\text{expr list} \rangle = \langle y \rangle \equiv x = y$   
 $\langle \text{proof} \rangle$

**lemmas** *inj-term-simps* = *stmt-inj-term-simp* *expr-inj-term-simp* *var-inj-term-simp*  
*expr-list-inj-term-simp*

**lemmas** *inj-term-sym-simps* = *stmt-inj-term-simp* [THEN *sym*]  
*expr-inj-term-simp* [THEN *sym*]  
*var-inj-term-simp* [THEN *sym*]  
*expr-list-inj-term-simp* [THEN *sym*]

**lemma** *stmt-expr-inj-term [iff]*:  $\langle t::\text{stmt} \rangle \neq \langle w::\text{expr} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *expr-stmt-inj-term [iff]*:  $\langle t::\text{expr} \rangle \neq \langle w::\text{stmt} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *stmt-var-inj-term [iff]*:  $\langle t::\text{stmt} \rangle \neq \langle w::\text{var} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *var-stmt-inj-term [iff]*:  $\langle t::\text{var} \rangle \neq \langle w::\text{stmt} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *stmt-elist-inj-term [iff]*:  $\langle t::\text{stmt} \rangle \neq \langle w::\text{expr list} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *elist-stmt-inj-term [iff]*:  $\langle t::\text{expr list} \rangle \neq \langle w::\text{stmt} \rangle$

$\langle \text{proof} \rangle$

**lemma** *expr-var-inj-term* [iff]:  $\langle t::\text{expr} \rangle \neq \langle w::\text{var} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *var-expr-inj-term* [iff]:  $\langle t::\text{var} \rangle \neq \langle w::\text{expr} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *expr-elist-inj-term* [iff]:  $\langle t::\text{expr} \rangle \neq \langle w::\text{expr list} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *elist-expr-inj-term* [iff]:  $\langle t::\text{expr list} \rangle \neq \langle w::\text{expr} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *var-elist-inj-term* [iff]:  $\langle t::\text{var} \rangle \neq \langle w::\text{expr list} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *elist-var-inj-term* [iff]:  $\langle t::\text{expr list} \rangle \neq \langle w::\text{var} \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *term-cases*:

$\llbracket \bigwedge v. P \langle v \rangle_v; \bigwedge e. P \langle e \rangle_e; \bigwedge c. P \langle c \rangle_s; \bigwedge l. P \langle l \rangle_l \rrbracket$   
 $\implies P t$   
 $\langle \text{proof} \rangle$

## Evaluation of unary operations

**consts** *eval-unop* :: *unop*  $\Rightarrow$  *val*  $\Rightarrow$  *val*

**primrec**

*eval-unop* *UPlus*  $v = \text{Intg } (\text{the-Intg } v)$

*eval-unop* *UMinus*  $v = \text{Intg } (- (\text{the-Intg } v))$

*eval-unop* *UBitNot*  $v = \text{Intg } 42$  — FIXME: Not yet implemented

*eval-unop* *UNot*  $v = \text{Bool } (\neg \text{the-Bool } v)$

## Evaluation of binary operations

**consts** *eval-binop* :: *binop*  $\Rightarrow$  *val*  $\Rightarrow$  *val*  $\Rightarrow$  *val*

**primrec**

*eval-binop* *Mul*  $v1 \ v2 = \text{Intg } ((\text{the-Intg } v1) * (\text{the-Intg } v2))$

*eval-binop* *Div*  $v1 \ v2 = \text{Intg } ((\text{the-Intg } v1) \text{ div } (\text{the-Intg } v2))$

*eval-binop* *Mod*  $v1 \ v2 = \text{Intg } ((\text{the-Intg } v1) \text{ mod } (\text{the-Intg } v2))$

*eval-binop* *Plus*  $v1 \ v2 = \text{Intg } ((\text{the-Intg } v1) + (\text{the-Intg } v2))$

*eval-binop* *Minus*  $v1 \ v2 = \text{Intg } ((\text{the-Intg } v1) - (\text{the-Intg } v2))$

— Be aware of the explicit coercion of the shift distance to nat

*eval-binop* *LShift*  $v1 \ v2 = \text{Intg } ((\text{the-Intg } v1) * (2^{(\text{nat } (\text{the-Intg } v2))}))$

*eval-binop* *RShift*  $v1 \ v2 = \text{Intg } ((\text{the-Intg } v1) \text{ div } (2^{(\text{nat } (\text{the-Intg } v2))}))$

*eval-binop* *RShiftU*  $v1 \ v2 = \text{Intg } 42$  — FIXME: Not yet implemented

*eval-binop* *Less*  $v1 \ v2 = \text{Bool } ((\text{the-Intg } v1) < (\text{the-Intg } v2))$

*eval-binop* *Le*  $v1 \ v2 = \text{Bool } ((\text{the-Intg } v1) \leq (\text{the-Intg } v2))$

*eval-binop* *Greater*  $v1 \ v2 = \text{Bool } ((\text{the-Intg } v2) < (\text{the-Intg } v1))$

*eval-binop* *Ge*  $v1 \ v2 = \text{Bool } ((\text{the-Intg } v2) \leq (\text{the-Intg } v1))$

*eval-binop* *Eq*  $v1 \ v2 = \text{Bool } (v1=v2)$

*eval-binop* *Neq*  $v1 \ v2 = \text{Bool } (v1 \neq v2)$

*eval-binop* *BitAnd*  $v1 \ v2 = \text{Intg } 42$  — FIXME: Not yet implemented

*eval-binop* *And*  $v1 \ v2 = \text{Bool } ((\text{the-Bool } v1) \wedge (\text{the-Bool } v2))$



*eval-binop BitXor*  $v1\ v2 = \text{Intg } 42$  — FIXME: Not yet implemented  
*eval-binop Xor*  $v1\ v2 = \text{Bool } ((\text{the-Bool } v1) \neq (\text{the-Bool } v2))$   
*eval-binop BitOr*  $v1\ v2 = \text{Intg } 42$  — FIXME: Not yet implemented  
*eval-binop Or*  $v1\ v2 = \text{Bool } ((\text{the-Bool } v1) \vee (\text{the-Bool } v2))$   
*eval-binop CondAnd*  $v1\ v2 = \text{Bool } ((\text{the-Bool } v1) \wedge (\text{the-Bool } v2))$   
*eval-binop CondOr*  $v1\ v2 = \text{Bool } ((\text{the-Bool } v1) \vee (\text{the-Bool } v2))$

**constdefs** *need-second-arg* :: *binop*  $\Rightarrow$  *val*  $\Rightarrow$  *bool*  
*need-second-arg binop v1*  $\equiv \neg ((\text{binop} = \text{CondAnd} \wedge \neg \text{the-Bool } v1) \vee$   
 $(\text{binop} = \text{CondOr} \wedge \text{the-Bool } v1))$

*CondAnd* and *CondOr* only evaluate the second argument if the value isn't already determined by the first argument

**lemma** *need-second-arg-CondAnd* [*simp*]: *need-second-arg CondAnd (Bool b) = b*  
 $\langle \text{proof} \rangle$

**lemma** *need-second-arg-CondOr* [*simp*]: *need-second-arg CondOr (Bool b) = ( $\neg$  b)*  
 $\langle \text{proof} \rangle$

**lemma** *need-second-arg-strict* [*simp*]:  
 $\llbracket \text{binop} \neq \text{CondAnd}; \text{binop} \neq \text{CondOr} \rrbracket \Longrightarrow \text{need-second-arg binop } b$   
 $\langle \text{proof} \rangle$   
**end**



## Chapter 8

### Decl

## 7 Field, method, interface, and class declarations, whole Java programs

**theory** *Decl imports Term Table begin*

improvements:

- clarification and correction of some aspects of the package/access concept (Also submitted as bug report to the Java Bug Database: Bug Id: 4485402 and Bug Id: 4493343 <http://developer.java.s>)

simplifications:

- the only field and method modifiers are static and the access modifiers
- no constructors, which may be simulated by new + suitable methods
- there is just one global initializer per class, which can simulate all others
- no throws clause
- a void method is replaced by one that returns Unit (of dummy type Void)
- no interface fields
- every class has an explicit superclass (unused for Object)
- the (standard) methods of Object and of standard exceptions are not specified
- no main method

## 8 Modifier

### Access modifier

**datatype** *acc-modi*  
 $= Private \mid Package \mid Protected \mid Public$

We can define a linear order for the access modifiers. With Private yielding the most restrictive access and public the most liberal access policy: Private ; Package ; Protected ; Public

**instance** *acc-modi:: ord*  $\langle proof \rangle$

**defs** (overloaded)

*less-acc-def*:

$$\begin{aligned}
 a < (b::acc-modi) \\
 \equiv (case\ a\ of \\
 \quad Private &\Rightarrow (b=Package \vee b=Protected \vee b=Public) \\
 \quad | Package &\Rightarrow (b=Protected \vee b=Public) \\
 \quad | Protected &\Rightarrow (b=Public) \\
 \quad | Public &\Rightarrow False)
 \end{aligned}$$

*le-acc-def*:

$$a \leq (b::acc-modi) \equiv (a = b) \vee (a < b)$$

**instance** *acc-modi:: order*  
 $\langle proof \rangle$

**instance** *acc-modi:: linorder*  
 $\langle proof \rangle$



## 9 Declaration (base "class" for member, interface and class declarations)

**record** *decl* =  
     *access* :: *acc-modi*

### translations

*decl* <= (*type*) ( $\downarrow$  *access*::*acc-modi*)  
*decl* <= (*type*) ( $\downarrow$  *access*::*acc-modi*,...::'*a*)

## 10 Member (field or method)

**record** *member* = *decl* +  
     *static* :: *stat-modi*

### translations

*member* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*)  
*member* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*,...::'*a*)

## 11 Field

**record** *field* = *member* +  
     *type* :: *ty*

### translations

*field* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*, *type*::*ty*)  
*field* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*, *type*::*ty*,...::'*a*)

### types

*fdecl*  
     = *vname*  $\times$  *field*

### translations

*fdecl* <= (*type*) *vname*  $\times$  *field*

## 12 Method

**record** *mhead* = *member* +  
     *pars* :: *vname list*  
     *resT* :: *ty*

**record** *mbody* =  
     *lcls*:: (*vname*  $\times$  *ty*) *list*  
     *stmt*:: *stmt*

**record** *methd* = *mhead* +  
     *mbody*::*mbody*

**types** *mdecl* = *sig*  $\times$  *methd*

### translations

*mhead* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*,  
     *pars*::*vname list*, *resT*::*ty*)  
*mhead* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*,  
     *pars*::*vname list*, *resT*::*ty*,...::'*a*)  
*mbody* <= (*type*) ( $\downarrow$  *lcls*::(*vname*  $\times$  *ty*) *list*, *stmt*::*stmt*)  
*mbody* <= (*type*) ( $\downarrow$  *lcls*::(*vname*  $\times$  *ty*) *list*, *stmt*::*stmt*,...::'*a*)  
*methd* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*,  
     *pars*::*vname list*, *resT*::*ty*, *mbody*::*mbody*)  
*methd* <= (*type*) ( $\downarrow$  *access*::*acc-modi*, *static*::*bool*,

$$mdecl \leq (type) \text{ sig} \times methd$$
**constdefs**

$$mhead::methd \Rightarrow mhead$$

$$mhead \ m \equiv (\text{access} = \text{access } m, \text{static} = \text{static } m, \text{pars} = \text{pars } m, \text{resT} = \text{resT } m)$$

**lemma** *access-mhead* [simp]:  $\text{access } (mhead \ m) = \text{access } m$   
 <proof>

**lemma** *static-mhead* [simp]:  $\text{static } (mhead \ m) = \text{static } m$   
 <proof>

**lemma** *pars-mhead* [simp]:  $\text{pars } (mhead \ m) = \text{pars } m$   
 <proof>

**lemma** *resT-mhead* [simp]:  $\text{resT } (mhead \ m) = \text{resT } m$   
 <proof>

To be able to talk uniformly about field and method declarations we introduce the notion of a member declaration (e.g. useful to define accessibility)

**datatype** *memberdecl* = *fdecl* *fdecl* | *mdecl* *mdecl*

**datatype** *memberid* = *fid* *vname* | *mid* *sig*

**axclass** *has-memberid* < *type*

**consts**

$$memberid :: 'a::has-memberid \Rightarrow memberid$$

**instance** *memberdecl::has-memberid* <proof>

**defs (overloaded)**

*memberdecl-memberid-def*:

$$memberid \ m \equiv (\text{case } m \text{ of} \\
\quad fdecl \ (vn, f) \Rightarrow fid \ vn \\
\quad | \ mdecl \ (sig, m) \Rightarrow mid \ sig)$$

**lemma** *memberid-fdecl-simp*[simp]:  $memberid \ (fdecl \ (vn, f)) = fid \ vn$   
 <proof>

**lemma** *memberid-fdecl-simp1*:  $memberid \ (fdecl \ f) = fid \ (fst \ f)$   
 <proof>

**lemma** *memberid-mdecl-simp*[simp]:  $memberid \ (mdecl \ (sig, m)) = mid \ sig$   
 <proof>

**lemma** *memberid-mdecl-simp1*:  $memberid \ (mdecl \ m) = mid \ (fst \ m)$   
 <proof>

**instance**  $*$  :: (*type*, *has-memberid*) *has-memberid* <proof>

**defs (overloaded)**

*pair-memberid-def*:

$\text{memberid } p \equiv \text{memberid } (\text{snd } p)$

**lemma** *memberid-pair-simp*[*simp*]:  $\text{memberid } (c, m) = \text{memberid } m$   
 $\langle \text{proof} \rangle$

**lemma** *memberid-pair-simp1*:  $\text{memberid } p = \text{memberid } (\text{snd } p)$   
 $\langle \text{proof} \rangle$

**constdefs** *is-field* ::  $qname \times memberdecl \Rightarrow bool$

*is-field*  $m \equiv \exists \text{ decl } C \text{ f. } m = (\text{decl } C, \text{fdecl } f)$

**lemma** *is-fieldD*:  $\text{is-field } m \Longrightarrow \exists \text{ decl } C \text{ f. } m = (\text{decl } C, \text{fdecl } f)$   
 $\langle \text{proof} \rangle$

**lemma** *is-fieldI*:  $\text{is-field } (C, \text{fdecl } f)$   
 $\langle \text{proof} \rangle$

**constdefs** *is-method* ::  $qname \times memberdecl \Rightarrow bool$

*is-method*  $\text{membr} \equiv \exists \text{ decl } C \text{ m. } \text{membr} = (\text{decl } C, \text{mdecl } m)$

**lemma** *is-methodD*:  $\text{is-method } \text{membr} \Longrightarrow \exists \text{ decl } C \text{ m. } \text{membr} = (\text{decl } C, \text{mdecl } m)$   
 $\langle \text{proof} \rangle$

**lemma** *is-methodI*:  $\text{is-method } (C, \text{mdecl } m)$   
 $\langle \text{proof} \rangle$

### 13 Interface

**record** *ibody* = *decl* + — interface body

*imethods* ::  $(sig \times mhead) \text{ list}$  — method heads

**record** *iface* = *ibody* + — interface

*isuperIfs* ::  $qname \text{ list}$  — superinterface list

**types**

*idecl* — interface declaration, cf. 9.1

=  $qname \times \text{iface}$

**translations**

$\text{ibody} \leq (\text{type}) \ (\backslash \text{access} :: \text{acc-modi}, \text{imethods} :: (\text{sig} \times \text{mhead}) \text{ list})$

$\text{ibody} \leq (\text{type}) \ (\backslash \text{access} :: \text{acc-modi}, \text{imethods} :: (\text{sig} \times \text{mhead}) \text{ list}, \dots :: 'a)$

$\text{iface} \leq (\text{type}) \ (\backslash \text{access} :: \text{acc-modi}, \text{imethods} :: (\text{sig} \times \text{mhead}) \text{ list},$   
 $\text{isuperIfs} :: qname \text{ list})$

$\text{iface} \leq (\text{type}) \ (\backslash \text{access} :: \text{acc-modi}, \text{imethods} :: (\text{sig} \times \text{mhead}) \text{ list},$   
 $\text{isuperIfs} :: qname \text{ list}, \dots :: 'a)$

$\text{idecl} \leq (\text{type}) \ qname \times \text{iface}$

**constdefs**

$\text{ibody} :: \text{iface} \Rightarrow \text{ibody}$

$\text{ibody } i \equiv (\backslash \text{access} = \text{access } i, \text{imethods} = \text{imethods } i)$



**lemma** *access-ibody* [simp]: (*access* (*ibody* *i*)) = *access* *i*  
 ⟨*proof*⟩

**lemma** *imethods-ibody* [simp]: (*imethods* (*ibody* *i*)) = *imethods* *i*  
 ⟨*proof*⟩

## 14 Class

**record** *cbody* = *decl* + — class body  
     *cfields*:: *fdecl* list  
     *methods*:: *mdecl* list  
     *init* :: *stmt* — initializer

**record** *class* = *cbody* + — class  
     *super* :: *qtname* — superclass  
     *superIfs*:: *qtname* list — implemented interfaces

**types**  
     *cdecl* — class declaration, cf. 8.1  
     = *qtname* × *class*

### translations

*cbody* <= (*type*) (⌊*access*::*acc-modi*,*cfields*::*fdecl* list,  
                   *methods*::*mdecl* list,*init*::*stmt*⌋)  
*cbody* <= (*type*) (⌊*access*::*acc-modi*,*cfields*::*fdecl* list,  
                   *methods*::*mdecl* list,*init*::*stmt*,...::'*a*'⌋)  
*class* <= (*type*) (⌊*access*::*acc-modi*,*cfields*::*fdecl* list,  
                   *methods*::*mdecl* list,*init*::*stmt*,  
                   *super*::*qtname*,*superIfs*::*qtname* list⌋)  
*class* <= (*type*) (⌊*access*::*acc-modi*,*cfields*::*fdecl* list,  
                   *methods*::*mdecl* list,*init*::*stmt*,  
                   *super*::*qtname*,*superIfs*::*qtname* list,...::'*a*'⌋)  
*cdecl* <= (*type*) *qtname* × *class*

### constdefs

*cbody* :: *class* ⇒ *cbody*  
*cbody* *c* ≡ (⌊*access*=*access* *c*, *cfields*=*cfields* *c*,*methods*=*methods* *c*,*init*=*init* *c*⌋)

**lemma** *access-cbody* [simp]: *access* (*cbody* *c*) = *access* *c*  
 ⟨*proof*⟩

**lemma** *cfields-cbody* [simp]: *cfields* (*cbody* *c*) = *cfields* *c*  
 ⟨*proof*⟩

**lemma** *methods-cbody* [simp]: *methods* (*cbody* *c*) = *methods* *c*  
 ⟨*proof*⟩

**lemma** *init-cbody* [simp]: *init* (*cbody* *c*) = *init* *c*  
 ⟨*proof*⟩

### standard classes

#### consts

*Object-mdecls* :: *mdecl list* — methods of Object  
*SXcpt-mdecls* :: *mdecl list* — methods of SXcpts  
*ObjectC* :: *cdecl* — declaration of root class  
*SXcptC* :: *xname*  $\Rightarrow$  *cdecl* — declarations of throwable classes

## defs

*ObjectC-def: ObjectC*  $\equiv$  (*Object*, ( $\text{access} = \text{Public}$ ,  $\text{cfields} = []$ ,  $\text{methods} = \text{Object-mdecls}$ ,  
 $\text{init} = \text{Skip}$ ,  $\text{super} = \text{arbitrary}$ ,  $\text{superIfs} = []$ ))  
*SXcptC-def: SXcptC xn*  $\equiv$  (*SXcpt xn*, ( $\text{access} = \text{Public}$ ,  $\text{cfields} = []$ ,  $\text{methods} = \text{SXcpt-mdecls}$ ,  
 $\text{init} = \text{Skip}$ ,  
 $\text{super} = \text{if } xn = \text{Throwable then Object}$   
 $\text{else SXcpt Throwable}$ ,  
 $\text{superIfs} = []$ ))

**lemma** *ObjectC-neq-SXcptC* [simp]: *ObjectC*  $\neq$  *SXcptC xn*  
 <proof>

**lemma** *SXcptC-inject* [simp]: (*SXcptC xn* = *SXcptC xm*) = (*xn* = *xm*)  
 <proof>

**constdefs** *standard-classes* :: *cdecl list*  
*standard-classes*  $\equiv$  [*ObjectC*, *SXcptC Throwable*,  
*SXcptC NullPointer*, *SXcptC OutOfMemory*, *SXcptC ClassCast*,  
*SXcptC NegArrSize*, *SXcptC IndOutBound*, *SXcptC ArrStore*]

## programs

**record** *prog* =  
*ifaces* :: *idecl list*  
*classes* :: *cdecl list*

## translations

*prog*  $\leq$  (*type*) ( $\text{ifaces} :: \text{idecl list}$ ,  $\text{classes} :: \text{cdecl list}$ )  
*prog*  $\leq$  (*type*) ( $\text{ifaces} :: \text{idecl list}$ ,  $\text{classes} :: \text{cdecl list}$ , ... : 'a)

## syntax

*iface* :: *prog*  $\Rightarrow$  (*qname*, *iface*) *table*  
*class* :: *prog*  $\Rightarrow$  (*qname*, *class*) *table*  
*is-iface* :: *prog*  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*  
*is-class* :: *prog*  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*

## translations

*iface G I* == *table-of (ifaces G) I*  
*class G C* == *table-of (classes G) C*  
*is-iface G I* == *iface G I*  $\neq$  *None*  
*is-class G C* == *class G C*  $\neq$  *None*

## is type

### consts

*is-type* :: *prog*  $\Rightarrow$  *ty*  $\Rightarrow$  *bool*  
*isrtype* :: *prog*  $\Rightarrow$  *ref-ty*  $\Rightarrow$  *bool*

**primrec** *is-type G (PrimT pt)* = *True*  
*is-type G (RefT rt)* = *isrtype G rt*

```

isrtype G (NullT _) = True
isrtype G (IfaceT tn) = is-iface G tn
isrtype G (ClassT tn) = is-class G tn
isrtype G (ArrayT T) = is-type G T

```

**lemma** *type-is-iface*:  $is\text{-}type\ G\ (Iface\ I) \implies is\text{-}iface\ G\ I$   
 <proof>

**lemma** *type-is-class*:  $is\text{-}type\ G\ (Class\ C) \implies is\text{-}class\ G\ C$   
 <proof>

### subinterface and subclass relation, in anticipation of TypeRel.thy

#### consts

```

subint1 :: prog => (qname × qname) set — direct subinterface
subcls1 :: prog => (qname × qname) set — direct subclass

```

#### defs

```

subint1-def: subint1 G ≡ {(I,J). ∃ i∈iface G I: J∈set (isuperIfs i)}
subcls1-def: subcls1 G ≡ {(C,D). C≠Object ∧ (∃ c∈class G C: super c = D)}

```

#### syntax

```

-subcls1 :: prog => [qname, qname] => bool (|-<:C1- [71,71,71] 70)
-subclseq:: prog => [qname, qname] => bool (|-<=:C-[71,71,71] 70)
-subcls  :: prog => [qname, qname] => bool (|-<:C-[71,71,71] 70)

```

#### syntax (*xsymbols*)

```

-subcls1 :: prog => [qname, qname] => bool (⊢-<_{C1}- [71,71,71] 70)
-subclseq:: prog => [qname, qname] => bool (⊢-<_C - [71,71,71] 70)
-subcls  :: prog => [qname, qname] => bool (⊢-<_C - [71,71,71] 70)

```

#### translations

```

G⊢C <_{C1} D == (C,D) ∈ subcls1 G
G⊢C ⊆_C D == (C,D) ∈ (subcls1 G) ^*
G⊢C <_C D == (C,D) ∈ (subcls1 G) ^+

```

**lemma** *subint1I*:  $\llbracket iface\ G\ I = Some\ i;\ J \in set\ (isuperIfs\ i) \rrbracket$   
 $\implies (I,J) \in subint1\ G$   
 <proof>

**lemma** *subcls1I*:  $\llbracket class\ G\ C = Some\ c;\ C \neq Object \rrbracket \implies (C,(super\ c)) \in subcls1\ G$   
 <proof>

**lemma** *subint1D*:  $(I,J) \in subint1\ G \implies \exists i \in iface\ G\ I: J \in set\ (isuperIfs\ i)$   
 <proof>

**lemma** *subcls1D*:  
 $(C,D) \in subcls1\ G \implies C \neq Object \wedge (\exists c. class\ G\ C = Some\ c \wedge (super\ c = D))$   
 <proof>

**lemma** *subint1-def2*:

$\text{subint1 } G = (\text{SIGMA } I: \{I. \text{is-iface } G \ I\}. \text{set } (\text{isuperIfs } (\text{the } (\text{iface } G \ I))))$   
 $\langle \text{proof} \rangle$

**lemma** *subcls1-def2*:

$\text{subcls1 } G =$   
 $(\text{SIGMA } C: \{C. \text{is-class } G \ C\}. \{D. C \neq \text{Object} \wedge \text{super } (\text{the } (\text{class } G \ C)) = D\})$   
 $\langle \text{proof} \rangle$

**lemma** *subcls-is-class*:

$\llbracket G \vdash C \prec_C D \rrbracket \implies \exists c. \text{class } G \ C = \text{Some } c$   
 $\langle \text{proof} \rangle$

**lemma** *no-subcls1-Object*:  $G \vdash \text{Object} \prec_{C1} D \implies P$

$\langle \text{proof} \rangle$

**lemma** *no-subcls-Object*:  $G \vdash \text{Object} \prec_C D \implies P$

$\langle \text{proof} \rangle$

## well-structured programs

**constdefs**

$\text{ws-idecl} :: \text{prog} \Rightarrow \text{qname} \Rightarrow \text{qname list} \Rightarrow \text{bool}$   
 $\text{ws-idecl } G \ I \ si \equiv \forall J \in \text{set } si. \text{is-iface } G \ J \ \wedge \ (J, I) \notin (\text{subint1 } G)^\wedge +$

$\text{ws-cdecl} :: \text{prog} \Rightarrow \text{qname} \Rightarrow \text{qname} \Rightarrow \text{bool}$   
 $\text{ws-cdecl } G \ C \ sc \equiv C \neq \text{Object} \longrightarrow \text{is-class } G \ sc \wedge (sc, C) \notin (\text{subcls1 } G)^\wedge +$

$\text{ws-prog} :: \text{prog} \Rightarrow \text{bool}$   
 $\text{ws-prog } G \equiv (\forall (I, i) \in \text{set } (\text{ifaces } G). \text{ws-idecl } G \ I \ (\text{isuperIfs } i)) \wedge$   
 $(\forall (C, c) \in \text{set } (\text{classes } G). \text{ws-cdecl } G \ C \ (\text{super } c))$

**lemma** *ws-progI*:

$\llbracket \forall (I, i) \in \text{set } (\text{ifaces } G). \forall J \in \text{set } (\text{isuperIfs } i). \text{is-iface } G \ J \wedge$   
 $(J, I) \notin (\text{subint1 } G)^\wedge +;$   
 $\forall (C, c) \in \text{set } (\text{classes } G). C \neq \text{Object} \longrightarrow \text{is-class } G \ (\text{super } c) \wedge$   
 $((\text{super } c), C) \notin (\text{subcls1 } G)^\wedge +$   
 $\rrbracket \implies \text{ws-prog } G$   
 $\langle \text{proof} \rangle$

**lemma** *ws-prog-ideclD*:

$\llbracket \text{iface } G \ I = \text{Some } i; J \in \text{set } (\text{isuperIfs } i); \text{ws-prog } G \rrbracket \implies$   
 $\text{is-iface } G \ J \wedge (J, I) \notin (\text{subint1 } G)^\wedge +$   
 $\langle \text{proof} \rangle$

**lemma** *ws-prog-cdeclD*:

$\llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object}; \text{ws-prog } G \rrbracket \implies$   
 $\text{is-class } G \ (\text{super } c) \wedge (\text{super } c, C) \notin (\text{subcls1 } G)^\wedge +$   
 $\langle \text{proof} \rangle$

**well-foundedness**

**lemma** *finite-is-iface*: *finite*  $\{I. \text{is-iface } G \ I\}$   
 $\langle \text{proof} \rangle$

**lemma** *finite-is-class*: *finite*  $\{C. \text{is-class } G \ C\}$   
 $\langle \text{proof} \rangle$

**lemma** *finite-subint1*: *finite* (*subint1*  $G$ )  
 $\langle \text{proof} \rangle$

**lemma** *finite-subcls1*: *finite* (*subcls1*  $G$ )  
 $\langle \text{proof} \rangle$

**lemma** *subint1-irrefl-lemma1*:  
 $\text{ws-prog } G \implies (\text{subint1 } G)^{-1} \cap (\text{subint1 } G)^+ = \{\}$   
 $\langle \text{proof} \rangle$

**lemma** *subcls1-irrefl-lemma1*:  
 $\text{ws-prog } G \implies (\text{subcls1 } G)^{-1} \cap (\text{subcls1 } G)^+ = \{\}$   
 $\langle \text{proof} \rangle$

**lemmas** *subint1-irrefl-lemma2* = *subint1-irrefl-lemma1*  $[ \text{THEN irrefl-tranclI} ]$   
**lemmas** *subcls1-irrefl-lemma2* = *subcls1-irrefl-lemma1*  $[ \text{THEN irrefl-tranclI} ]$

**lemma** *subint1-irrefl*:  $\llbracket (x, y) \in \text{subint1 } G; \text{ws-prog } G \rrbracket \implies x \neq y$   
 $\langle \text{proof} \rangle$

**lemma** *subcls1-irrefl*:  $\llbracket (x, y) \in \text{subcls1 } G; \text{ws-prog } G \rrbracket \implies x \neq y$   
 $\langle \text{proof} \rangle$

**lemmas** *subint1-acyclic* = *subint1-irrefl-lemma2*  $[ \text{THEN acyclicI, standard} ]$   
**lemmas** *subcls1-acyclic* = *subcls1-irrefl-lemma2*  $[ \text{THEN acyclicI, standard} ]$

**lemma** *wf-subint1*:  $\text{ws-prog } G \implies \text{wf } ((\text{subint1 } G)^{-1})$   
 $\langle \text{proof} \rangle$

**lemma** *wf-subcls1*:  $\text{ws-prog } G \implies \text{wf } ((\text{subcls1 } G)^{-1})$   
 $\langle \text{proof} \rangle$

**lemma** *subint1-induct*:  
 $\llbracket \text{ws-prog } G; \bigwedge x. \forall y. (x, y) \in \text{subint1 } G \longrightarrow P \ y \implies P \ x \rrbracket \implies P \ a$   
 $\langle \text{proof} \rangle$

**lemma** *subcls1-induct*  $[ \text{consumes } 1 ]$ :  
 $\llbracket \text{ws-prog } G; \bigwedge x. \forall y. (x, y) \in \text{subcls1 } G \longrightarrow P \ y \implies P \ x \rrbracket \implies P \ a$

$\langle \text{proof} \rangle$

**lemma** *ws-subint1-induct*:

$\llbracket \text{is-iface } G \ I; \text{ws-prog } G; \bigwedge I \ i. \llbracket \text{iface } G \ I = \text{Some } i \wedge$   
 $(\forall J \in \text{set } (\text{isuperIfs } i). (I,J) \in \text{subint1 } G \wedge P \ J \wedge \text{is-iface } G \ J) \rrbracket \implies P \ I$   
 $\rrbracket \implies P \ I$   
 $\langle \text{proof} \rangle$

**lemma** *ws-subcls1-induct*:  $\llbracket \text{is-class } G \ C; \text{ws-prog } G;$

$\bigwedge C \ c. \llbracket \text{class } G \ C = \text{Some } c;$   
 $(C \neq \text{Object} \longrightarrow (C, (\text{super } c)) \in \text{subcls1 } G \wedge$   
 $P (\text{super } c) \wedge \text{is-class } G (\text{super } c)) \rrbracket \implies P \ C$   
 $\rrbracket \implies P \ C$   
 $\langle \text{proof} \rangle$

**lemma** *ws-class-induct* [consumes 2, case-names Object Subcls]:

$\llbracket \text{class } G \ C = \text{Some } c; \text{ws-prog } G;$   
 $\bigwedge co. \text{class } G \ \text{Object} = \text{Some } co \implies P \ \text{Object};$   
 $\bigwedge C \ c. \llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object}; P (\text{super } c) \rrbracket \implies P \ C$   
 $\rrbracket \implies P \ C$   
 $\langle \text{proof} \rangle$

**lemma** *ws-class-induct'* [consumes 2, case-names Object Subcls]:

$\llbracket \text{is-class } G \ C; \text{ws-prog } G;$   
 $\bigwedge co. \text{class } G \ \text{Object} = \text{Some } co \implies P \ \text{Object};$   
 $\bigwedge C \ c. \llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object}; P (\text{super } c) \rrbracket \implies P \ C$   
 $\rrbracket \implies P \ C$   
 $\langle \text{proof} \rangle$

**lemma** *ws-class-induct''* [consumes 2, case-names Object Subcls]:

$\llbracket \text{class } G \ C = \text{Some } c; \text{ws-prog } G;$   
 $\bigwedge co. \text{class } G \ \text{Object} = \text{Some } co \implies P \ \text{Object } co;$   
 $\bigwedge C \ c \ sc. \llbracket \text{class } G \ C = \text{Some } c; \text{class } G (\text{super } c) = \text{Some } sc;$   
 $C \neq \text{Object}; P (\text{super } c) \ sc \rrbracket \implies P \ C \ c$   
 $\rrbracket \implies P \ C \ c$   
 $\langle \text{proof} \rangle$

**lemma** *ws-interface-induct* [consumes 2, case-names Step]:

**assumes** *is-if-I*: *is-iface*  $G \ I$  **and**

*ws*: *ws-prog*  $G$  **and**

*hyp-sub*:  $\bigwedge I \ i. \llbracket \text{iface } G \ I = \text{Some } i;$

$\forall J \in \text{set } (\text{isuperIfs } i).$

$(I,J) \in \text{subint1 } G \wedge P \ J \wedge \text{is-iface } G \ J \rrbracket \implies P \ I$

**shows**  $P \ I$

$\langle \text{proof} \rangle$

**general recursion operators for the interface and class hierarchies**

**consts**

*iface-rec* ::  $\text{prog} \times \text{qtname} \Rightarrow (\text{qtname} \Rightarrow \text{iface} \Rightarrow 'a \ \text{set} \Rightarrow 'a) \Rightarrow 'a$

*class-rec* ::  $\text{prog} \times \text{qtname} \Rightarrow 'a \Rightarrow (\text{qtname} \Rightarrow \text{class} \Rightarrow 'a \Rightarrow 'a) \Rightarrow 'a$

**recdef** *iface-rec same-fst ws-prog* ( $\lambda G. (subint1\ G)^{-1}$ )  
*iface-rec* ( $G, I$ ) =  
 ( $\lambda f. case\ iface\ G\ I\ of$   
    $None \Rightarrow arbitrary$   
    $| Some\ i \Rightarrow if\ ws-prog\ G$   
      $then\ f\ I\ i$   
      $((\lambda J. iface-rec\ (G, J)\ f)\ 'set\ (isuperIfs\ i))$   
      $else\ arbitrary$ )  
 (**hints** *recdef-wf: wf-subint1 intro: subint1I*)  
**declare** *iface-rec.simps* [*simp del*]

**lemma** *iface-rec*:  
 $\llbracket iface\ G\ I = Some\ i; ws-prog\ G \rrbracket \implies$   
 $iface-rec\ (G, I)\ f = f\ I\ i\ ((\lambda J. iface-rec\ (G, J)\ f)\ 'set\ (isuperIfs\ i))$   
 $\langle proof \rangle$

**recdef** *class-rec same-fst ws-prog* ( $\lambda G. (subcls1\ G)^{-1}$ )  
*class-rec*( $G, C$ ) =  
 ( $\lambda t f. case\ class\ G\ C\ of$   
    $None \Rightarrow arbitrary$   
    $| Some\ c \Rightarrow if\ ws-prog\ G$   
      $then\ f\ C\ c$   
      $(if\ C = Object\ then\ t$   
        $else\ class-rec\ (G, super\ c)\ t\ f)$   
      $else\ arbitrary$ )  
 (**hints** *recdef-wf: wf-subcls1 intro: subcls1I*)  
**declare** *class-rec.simps* [*simp del*]

**lemma** *class-rec*:  $\llbracket class\ G\ C = Some\ c; ws-prog\ G \rrbracket \implies$   
 $class-rec\ (G, C)\ t\ f =$   
 $f\ C\ c\ (if\ C = Object\ then\ t\ else\ class-rec\ (G, super\ c)\ t\ f)$   
 $\langle proof \rangle$

**constdefs**  
*imethds:: prog*  $\Rightarrow$  *qtname*  $\Rightarrow$  (*sig, qtname*  $\times$  *mhead*) *tables*  
 — methods of an interface, with overriding and inheritance, cf. 9.2  
*imethds*  $G\ I$   
 $\equiv iface-rec\ (G, I)$   
 $(\lambda I\ i\ ts. (Un-tables\ ts) \oplus \oplus$   
    $(o2s \circ table-of\ (map\ (\lambda(s, m). (s, I, m))\ (imethds\ i))))$

**end**





## Chapter 9

# TypeRel

## 15 The relations between Java types

**theory** *TypeRel* **imports** *Decl* **begin**

simplifications:

- subinterface, subclass and widening relation includes identity

improvements over Java Specification 1.0:

- narrowing reference conversion also in cases where the return types of a pair of methods common to both types are in widening (rather identity) relation
- one could add similar constraints also for other cases

design issues:

- the type relations do not require *is-type* for their arguments
- the *subint1* and *subcls1* relations imply *is-iface/is-class* for their first arguments, which is required for their finiteness

**consts**

*implmt1* :: *prog*  $\Rightarrow$  (*qname*  $\times$  *qname*) *set* — direct implementation

**syntax**

*-subint1* :: *prog*  $\Rightarrow$  [*qname*, *qname*]  $\Rightarrow$  *bool* ( $\neg$   $\neg$   $\neg$   $\neg$  *I1* - [71,71,71] 70)

*-subint* :: *prog*  $\Rightarrow$  [*qname*, *qname*]  $\Rightarrow$  *bool* ( $\neg$   $\neg$   $\neg$   $\neg$   $\neg$  *I* - [71,71,71] 70)

*@implmt1* :: *prog*  $\Rightarrow$  [*qname*, *qname*]  $\Rightarrow$  *bool* ( $\neg$   $\neg$   $\neg$   $\neg$   $\neg$   $\neg$  *I* - [71,71,71] 70)

**syntax** (*xsymbols*)

*-subint1* :: *prog*  $\Rightarrow$  [*qname*, *qname*]  $\Rightarrow$  *bool* ( $\neg$   $\neg$   $\neg$   $\neg$  *I1* - [71,71,71] 70)

*-subint* :: *prog*  $\Rightarrow$  [*qname*, *qname*]  $\Rightarrow$  *bool* ( $\neg$   $\neg$   $\neg$   $\neg$   $\neg$  *I* - [71,71,71] 70)

*-implmt1* :: *prog*  $\Rightarrow$  [*qname*, *qname*]  $\Rightarrow$  *bool* ( $\neg$   $\neg$   $\neg$   $\neg$   $\neg$   $\neg$  *I* - [71,71,71] 70)

**translations**

$G \vdash I \prec I1 J == (I, J) \in \text{subint1 } G$

$G \vdash I \preceq I J == (I, J) \in (\text{subint1 } G)^* \text{ — cf. 9.1.3}$

$G \vdash C \leadsto I I == (C, I) \in \text{implmt1 } G$

**subclass and subinterface relations**

**lemmas** *subcls-direct* = *subcls1I* [*THEN* *r-into-rtrancl*, *standard*]

**lemma** *subcls-direct1*:

$\llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object}; D = \text{super } c \rrbracket \implies G \vdash C \preceq_C D$

$\langle \text{proof} \rangle$

**lemma** *subcls1I1*:

$\llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object}; D = \text{super } c \rrbracket \implies G \vdash C \prec_{C1} D$   
 $\langle \text{proof} \rangle$

**lemma** *subcls-direct2*:

$\llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object}; D = \text{super } c \rrbracket \implies G \vdash C \prec_C D$   
 $\langle \text{proof} \rangle$

**lemma** *subclseq-trans*:  $\llbracket G \vdash A \preceq_C B; G \vdash B \preceq_C C \rrbracket \implies G \vdash A \preceq_C C$

$\langle \text{proof} \rangle$

**lemma** *subcls-trans*:  $\llbracket G \vdash A \prec_C B; G \vdash B \prec_C C \rrbracket \implies G \vdash A \prec_C C$

$\langle \text{proof} \rangle$

**lemma** *SXcpt-subcls-Throwable-lemma*:

$\llbracket \text{class } G \ (\text{SXcpt } xn) = \text{Some } xc;$   
 $\text{super } xc = (\text{if } xn = \text{Throwable} \text{ then } \text{Object} \text{ else } \text{SXcpt } \text{Throwable}) \rrbracket$   
 $\implies G \vdash \text{SXcpt } xn \preceq_C \text{SXcpt } \text{Throwable}$   
 $\langle \text{proof} \rangle$

**lemma** *subcls-ObjectI*:  $\llbracket \text{is-class } G \ C; \text{ws-prog } G \rrbracket \implies G \vdash C \preceq_C \text{Object}$

$\langle \text{proof} \rangle$

**lemma** *subclseq-ObjectD*  $[\text{dest!}]$ :  $G \vdash \text{Object} \preceq_C C \implies C = \text{Object}$

$\langle \text{proof} \rangle$

**lemma** *subcls-ObjectD*  $[\text{dest!}]$ :  $G \vdash \text{Object} \prec_C C \implies \text{False}$

$\langle \text{proof} \rangle$

**lemma** *subcls-ObjectI1*  $[\text{intro!}]$ :

$\llbracket C \neq \text{Object}; \text{is-class } G \ C; \text{ws-prog } G \rrbracket \implies G \vdash C \prec_C \text{Object}$   
 $\langle \text{proof} \rangle$

**lemma** *subcls-is-class*:  $(C, D) \in (\text{subcls1 } G)^+ \implies \text{is-class } G \ C$

$\langle \text{proof} \rangle$

**lemma** *subcls-is-class2*  $[\text{rule-format } (\text{no-asm})]$ :

$G \vdash C \preceq_C D \implies \text{is-class } G \ D \longrightarrow \text{is-class } G \ C$   
 $\langle \text{proof} \rangle$

**lemma** *single-inheritance*:

$\llbracket G \vdash A \prec_{C1} B; G \vdash A \prec_{C1} C \rrbracket \implies B = C$   
 $\langle \text{proof} \rangle$

**lemma** *subcls-compareable*:

$\llbracket G \vdash A \preceq_C X; G \vdash A \preceq_C Y \rrbracket \implies G \vdash X \preceq_C Y \vee G \vdash Y \preceq_C X$   
 $\langle \text{proof} \rangle$

**lemma** *subcls1-irrefl*:  $\llbracket G \vdash C \prec_{C1} D; \text{ws-prog } G \rrbracket$

$\implies C \neq D$   
 $\langle \text{proof} \rangle$

**lemma** *no-subcls-Object*:  $G \vdash C \prec_C D \implies C \neq \text{Object}$

$\langle \text{proof} \rangle$

**lemma** *subcls-acyclic*:  $\llbracket G \vdash C \prec_C D; \text{ws-prog } G \rrbracket \implies \neg G \vdash D \prec_C C$

$\langle \text{proof} \rangle$

**lemma** *subclseq-cases* [*consumes 1, case-names Eq Subcls*]:

$\llbracket G \vdash C \preceq_C D; C = D \implies P; G \vdash C \prec_C D \implies P \rrbracket \implies P$   
 $\langle \text{proof} \rangle$

**lemma** *subclseq-acyclic*:

$\llbracket G \vdash C \preceq_C D; G \vdash D \preceq_C C; \text{ws-prog } G \rrbracket \implies C = D$   
 $\langle \text{proof} \rangle$

**lemma** *subcls-irrefl*:  $\llbracket G \vdash C \prec_C D; \text{ws-prog } G \rrbracket$

$\implies C \neq D$   
 $\langle \text{proof} \rangle$

**lemma** *invert-subclseq*:

$\llbracket G \vdash C \preceq_C D; \text{ws-prog } G \rrbracket$   
 $\implies \neg G \vdash D \prec_C C$   
 $\langle \text{proof} \rangle$

**lemma** *invert-subcls*:

$\llbracket G \vdash C \prec_C D; \text{ws-prog } G \rrbracket$   
 $\implies \neg G \vdash D \preceq_C C$   
 $\langle \text{proof} \rangle$

**lemma** *subcls-superD*:

$\llbracket G \vdash C \prec_C D; \text{class } G \ C = \text{Some } c \rrbracket \implies G \vdash (\text{super } c) \preceq_C D$   
 $\langle \text{proof} \rangle$

**lemma** *subclseq-superD*:

$\llbracket G \vdash C \preceq_C D; C \neq D; \text{class } G \ C = \text{Some } c \rrbracket \implies G \vdash (\text{super } c) \preceq_C D$   
 $\langle \text{proof} \rangle$

**implementation relation**

**defs**

— direct implementation, cf. 8.1.3

*implmt1-def:implmt1*  $G \equiv \{(C, I). C \neq \text{Object} \wedge (\exists c \in \text{class } G \ C: I \in \text{set } (\text{superIfs } c))\}$

**lemma** *implmt1D*:  $G \vdash C \rightsquigarrow 1I \implies C \neq \text{Object} \wedge (\exists c \in \text{class } G \ C: I \in \text{set } (\text{superIfs } c))$   
 ⟨proof⟩

**inductive** — implementation, cf. 8.1.4

*implmt* :: *prog*  $\Rightarrow$  *qtname*  $\Rightarrow$  *qtname*  $\Rightarrow$  *bool* ( $\vdash \rightsquigarrow -$  [71,71,71] 70)

**for** *G* :: *prog*

**where**

*direct*:  $G \vdash C \rightsquigarrow 1J \implies G \vdash C \rightsquigarrow J$   
 | *subint*:  $\llbracket G \vdash C \rightsquigarrow 1I; G \vdash I \preceq I \ J \rrbracket \implies G \vdash C \rightsquigarrow J$   
 | *subcls1*:  $\llbracket G \vdash C \prec_{C1} D; G \vdash D \rightsquigarrow J \rrbracket \implies G \vdash C \rightsquigarrow J$

**lemma** *implmtD*:  $G \vdash C \rightsquigarrow J \implies (\exists I. G \vdash C \rightsquigarrow 1I \wedge G \vdash I \preceq I \ J) \vee (\exists D. G \vdash C \prec_{C1} D \wedge G \vdash D \rightsquigarrow J)$   
 ⟨proof⟩

**lemma** *implmt-ObjectE* [*elim!*]:  $G \vdash \text{Object} \rightsquigarrow I \implies R$   
 ⟨proof⟩

**lemma** *subcls-implmt* [*rule-format (no-asm)*]:  $G \vdash A \preceq_C B \implies G \vdash B \rightsquigarrow K \longrightarrow G \vdash A \rightsquigarrow K$   
 ⟨proof⟩

**lemma** *implmt-subint2*:  $\llbracket G \vdash A \rightsquigarrow J; G \vdash J \preceq I \ K \rrbracket \implies G \vdash A \rightsquigarrow K$   
 ⟨proof⟩

**lemma** *implmt-is-class*:  $G \vdash C \rightsquigarrow I \implies \text{is-class } G \ C$   
 ⟨proof⟩

## widening relation

**inductive**

— widening, viz. method invocation conversion, cf. 5.3 i.e. kind of syntactic subtyping

*widen* :: *prog*  $\Rightarrow$  *ty*  $\Rightarrow$  *ty*  $\Rightarrow$  *bool* ( $\vdash \preceq -$  [71,71,71] 70)

**for** *G* :: *prog*

**where**

*refl*:  $G \vdash T \preceq T$  — identity conversion, cf. 5.1.1  
 | *subint*:  $G \vdash I \preceq I \ J \implies G \vdash \text{Iface } I \preceq \text{Iface } J$  — wid.ref.conv., cf. 5.1.4  
 | *int-obj*:  $G \vdash \text{Iface } I \preceq \text{Class } \text{Object}$   
 | *subcls*:  $G \vdash C \preceq_C D \implies G \vdash \text{Class } C \preceq \text{Class } D$   
 | *implmt*:  $G \vdash C \rightsquigarrow I \implies G \vdash \text{Class } C \preceq \text{Iface } I$   
 | *null*:  $G \vdash \text{NT} \preceq \text{RefT } R$   
 | *arr-obj*:  $G \vdash T.\boxed{\phantom{x}} \preceq \text{Class } \text{Object}$   
 | *array*:  $G \vdash \text{RefT } S \preceq \text{RefT } T \implies G \vdash \text{RefT } S.\boxed{\phantom{x}} \preceq \text{RefT } T.\boxed{\phantom{x}}$

**declare** *widen.refl* [*intro!*]

**declare** *widen.intros* [*simp*]

**lemma** *widen-PrimT*:  $G \vdash \text{PrimT } x \preceq T \implies (\exists y. T = \text{PrimT } y)$   
 ⟨proof⟩

**lemma** *widen-PrimT2*:  $G \vdash S \preceq_{\text{PrimT}} x \implies \exists y. S = \text{PrimT } y$   
 $\langle \text{proof} \rangle$

These widening lemmata hold in Bali but are too strong for ordinary Java. They would not work for real Java Integral Types, like short, long, int. These lemmata are just for documentation and are not used.

**lemma** *widen-PrimT-strong*:  $G \vdash \text{PrimT } x \preceq T \implies T = \text{PrimT } x$   
 $\langle \text{proof} \rangle$

**lemma** *widen-PrimT2-strong*:  $G \vdash S \preceq_{\text{PrimT}} x \implies S = \text{PrimT } x$   
 $\langle \text{proof} \rangle$

Specialized versions for booleans also would work for real Java

**lemma** *widen-Boolean*:  $G \vdash \text{PrimT Boolean} \preceq T \implies T = \text{PrimT Boolean}$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Boolean2*:  $G \vdash S \preceq_{\text{PrimT}} \text{Boolean} \implies S = \text{PrimT Boolean}$   
 $\langle \text{proof} \rangle$

**lemma** *widen-RefT*:  $G \vdash \text{RefT } R \preceq T \implies \exists t. T = \text{RefT } t$   
 $\langle \text{proof} \rangle$

**lemma** *widen-RefT2*:  $G \vdash S \preceq_{\text{RefT}} R \implies \exists t. S = \text{RefT } t$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Iface*:  $G \vdash \text{Iface } I \preceq T \implies T = \text{Class Object} \vee (\exists J. T = \text{Iface } J)$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Iface2*:  $G \vdash S \preceq \text{Iface } J \implies S = \text{NT} \vee (\exists I. S = \text{Iface } I) \vee (\exists D. S = \text{Class } D)$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Iface-Iface*:  $G \vdash \text{Iface } I \preceq \text{Iface } J \implies G \vdash I \preceq_I J$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Iface-Iface-eq [simp]*:  $G \vdash \text{Iface } I \preceq \text{Iface } J = G \vdash I \preceq_I J$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Class*:  $G \vdash \text{Class } C \preceq T \implies (\exists D. T = \text{Class } D) \vee (\exists I. T = \text{Iface } I)$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Class2*:  $G \vdash S \preceq \text{Class } C \implies C = \text{Object} \vee S = \text{NT} \vee (\exists D. S = \text{Class } D)$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Class-Class*:  $G \vdash \text{Class } C \preceq \text{Class } cm \implies G \vdash C \preceq_C cm$

$\langle \text{proof} \rangle$

**lemma** *widen-Class-Class-eq* [simp]:  $G \vdash \text{Class } C \preceq \text{Class } cm = G \vdash C \preceq_C cm$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Class-Iface*:  $G \vdash \text{Class } C \preceq \text{Iface } I \implies G \vdash C \rightsquigarrow I$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Class-Iface-eq* [simp]:  $G \vdash \text{Class } C \preceq \text{Iface } I = G \vdash C \rightsquigarrow I$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Array*:  $G \vdash S.\square \preceq T \implies T = \text{Class Object} \vee (\exists T'. T = T'.\square \wedge G \vdash S \preceq T')$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Array2*:  $G \vdash S \preceq T.\square \implies S = NT \vee (\exists S'. S = S'.\square \wedge G \vdash S' \preceq T)$   
 $\langle \text{proof} \rangle$

**lemma** *widen-ArrayPrimT*:  $G \vdash \text{PrimT } t.\square \preceq T \implies T = \text{Class Object} \vee T = \text{PrimT } t.\square$   
 $\langle \text{proof} \rangle$

**lemma** *widen-ArrayRefT*:  
 $G \vdash \text{RefT } t.\square \preceq T \implies T = \text{Class Object} \vee (\exists s. T = \text{RefT } s.\square \wedge G \vdash \text{RefT } t \preceq \text{RefT } s)$   
 $\langle \text{proof} \rangle$

**lemma** *widen-ArrayRefT-ArrayRefT-eq* [simp]:  
 $G \vdash \text{RefT } T.\square \preceq \text{RefT } T'.\square = G \vdash \text{RefT } T \preceq \text{RefT } T'$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Array-Array*:  $G \vdash T.\square \preceq T'.\square \implies G \vdash T \preceq T'$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Array-Class*:  $G \vdash S.\square \preceq \text{Class } C \implies C = \text{Object}$   
 $\langle \text{proof} \rangle$

**lemma** *widen-NT2*:  $G \vdash S \preceq NT \implies S = NT$   
 $\langle \text{proof} \rangle$

**lemma** *widen-Object*:  $\llbracket \text{isrtype } G \ T; \text{ws-prog } G \rrbracket \implies G \vdash \text{RefT } T \preceq \text{Class Object}$   
 $\langle \text{proof} \rangle$

**lemma** *widen-trans-lemma* [rule-format (no-asm)]:  
 $\llbracket G \vdash S \preceq U; \forall C. \text{is-class } G \ C \longrightarrow G \vdash C \preceq_C \text{Object} \rrbracket \implies \forall T. G \vdash U \preceq T \longrightarrow G \vdash S \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *ws-widen-trans*:  $\llbracket G \vdash S \preceq U; G \vdash U \preceq T; \text{ws-prog } G \rrbracket \implies G \vdash S \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *widen-antisym-lemma*  $[rule-format \ (no-asm)]$ :  $\llbracket G \vdash S \preceq T;$   
 $\forall I \ J. \ G \vdash I \preceq I \ J \wedge G \vdash J \preceq I \ I \longrightarrow I = J;$   
 $\forall C \ D. \ G \vdash C \preceq_C D \wedge G \vdash D \preceq_C C \longrightarrow C = D;$   
 $\forall I \ . \ G \vdash \text{Object} \rightsquigarrow I \longrightarrow \text{False} \rrbracket \implies G \vdash T \preceq S \longrightarrow S = T$   
 $\langle \text{proof} \rangle$

**lemmas** *subint-antisym* =  
 $\text{subint1-acyclic } [THEN \ \text{acyclic-impl-antisym-rtrancl}, \ \text{standard}]$

**lemmas** *subcls-antisym* =  
 $\text{subcls1-acyclic } [THEN \ \text{acyclic-impl-antisym-rtrancl}, \ \text{standard}]$

**lemma** *widen-antisym*:  $\llbracket G \vdash S \preceq T; G \vdash T \preceq S; \text{ws-prog } G \rrbracket \implies S = T$   
 $\langle \text{proof} \rangle$

**lemma** *widen-ObjectD*  $[dest!]$ :  $G \vdash \text{Class } \text{Object} \preceq T \implies T = \text{Class } \text{Object}$   
 $\langle \text{proof} \rangle$

**constdefs**

$\text{widens} :: \text{prog} \Rightarrow [\text{ty list}, \text{ty list}] \Rightarrow \text{bool} \ (\vdash - [\preceq] - [71, 71, 71] \ 70)$   
 $G \vdash Ts [\preceq] Ts' \equiv \text{list-all2} \ (\lambda T \ T'. \ G \vdash T \preceq T') \ Ts \ Ts'$

**lemma** *widens-Nil*  $[simp]$ :  $G \vdash [] [\preceq] []$   
 $\langle \text{proof} \rangle$

**lemma** *widens-Cons*  $[simp]$ :  $G \vdash (S \# Ss) [\preceq] (T \# Ts) = (G \vdash S \preceq T \wedge G \vdash Ss [\preceq] Ts)$   
 $\langle \text{proof} \rangle$

## narrowing relation

**inductive** — narrowing reference conversion, cf. 5.1.5

$\text{narrow} :: \text{prog} \Rightarrow \text{ty} \Rightarrow \text{ty} \Rightarrow \text{bool} \ (\vdash - \succ - [71, 71, 71] \ 70)$

**for**  $G :: \text{prog}$

**where**

$\text{subcls}: G \vdash C \preceq_C D \implies G \vdash \text{Class } D \succ \text{Class } C$   
 $\text{implmt}: \neg G \vdash C \rightsquigarrow I \implies G \vdash \text{Class } C \succ \text{Iface } I$   
 $\text{obj-arr}: G \vdash \text{Class } \text{Object} \succ T. []$   
 $\text{int-cls}: G \vdash \text{Iface } I \succ \text{Class } C$   
 $\text{subint}: \text{imethds } G \ I \ \text{hidings } \text{imethds } G \ J \ \text{entails}$   
 $(\lambda (md, mh) \ (md', mh'). \ G \vdash \text{mrt } mh \preceq \text{mrt } mh') \implies$   
 $\neg G \vdash I \preceq I \ J \implies G \vdash \text{Iface } I \succ \text{Iface } J$   
 $\text{array}: G \vdash \text{RefT } S \succ \text{RefT } T \implies G \vdash \text{RefT } S. [] \succ \text{RefT } T. []$

**lemma** *narrow-RefT*:  $G \vdash \text{RefT } R \succ T \implies \exists t. \ T = \text{RefT } t$   
 $\langle \text{proof} \rangle$

**lemma** *narrow-RefT2*:  $G \vdash S \succ \text{RefT } R \implies \exists t. \ S = \text{RefT } t$   
 $\langle \text{proof} \rangle$



**lemma** *narrow-PrimT*:  $G \vdash \text{PrimT } pt \succ T \implies \exists t. T = \text{PrimT } t$   
 $\langle \text{proof} \rangle$

**lemma** *narrow-PrimT2*:  $G \vdash S \succ \text{PrimT } pt \implies$   
 $\exists t. S = \text{PrimT } t \wedge G \vdash \text{PrimT } t \preceq \text{PrimT } pt$   
 $\langle \text{proof} \rangle$

These narrowing lemmata hold in Bali but are too strong for ordinary Java. They would not work for real Java Integral Types, like short, long, int. These lemmata are just for documentation and are not used.

**lemma** *narrow-PrimT-strong*:  $G \vdash \text{PrimT } pt \succ T \implies T = \text{PrimT } pt$   
 $\langle \text{proof} \rangle$

**lemma** *narrow-PrimT2-strong*:  $G \vdash S \succ \text{PrimT } pt \implies S = \text{PrimT } pt$   
 $\langle \text{proof} \rangle$

Specialized versions for booleans also would work for real Java

**lemma** *narrow-Boolean*:  $G \vdash \text{PrimT } \text{Boolean} \succ T \implies T = \text{PrimT } \text{Boolean}$   
 $\langle \text{proof} \rangle$

**lemma** *narrow-Boolean2*:  $G \vdash S \succ \text{PrimT } \text{Boolean} \implies S = \text{PrimT } \text{Boolean}$   
 $\langle \text{proof} \rangle$

## casting relation

**inductive** — casting conversion, cf. 5.5

*cast* :: *prog*  $\Rightarrow$  *ty*  $\Rightarrow$  *ty*  $\Rightarrow$  *bool* ( $\vdash \preceq ?$  - [71,71,71] 70)

**for** *G* :: *prog*

**where**

*widen*:  $G \vdash S \preceq T \implies G \vdash S \preceq ? T$

| *narrow*:  $G \vdash S \succ T \implies G \vdash S \preceq ? T$

**lemma** *cast-RefT*:  $G \vdash \text{RefT } R \preceq ? T \implies \exists t. T = \text{RefT } t$   
 $\langle \text{proof} \rangle$

**lemma** *cast-RefT2*:  $G \vdash S \preceq ? \text{RefT } R \implies \exists t. S = \text{RefT } t$   
 $\langle \text{proof} \rangle$

**lemma** *cast-PrimT*:  $G \vdash \text{PrimT } pt \preceq ? T \implies \exists t. T = \text{PrimT } t$   
 $\langle \text{proof} \rangle$

**lemma** *cast-PrimT2*:  $G \vdash S \preceq ? \text{PrimT } pt \implies \exists t. S = \text{PrimT } t \wedge G \vdash \text{PrimT } t \preceq \text{PrimT } pt$   
 $\langle \text{proof} \rangle$

**lemma** *cast-Boolean*:

**assumes** *bool-cast*:  $G \vdash \text{Prim}T \text{ Boolean} \preceq^? T$   
**shows**  $T = \text{Prim}T \text{ Boolean}$   
 ⟨*proof*⟩

**lemma** *cast-Boolean2*:  
**assumes** *bool-cast*:  $G \vdash S \preceq^? \text{Prim}T \text{ Boolean}$   
**shows**  $S = \text{Prim}T \text{ Boolean}$   
 ⟨*proof*⟩

**end**

## Chapter 10

# DeclConcepts

## 16 Advanced concepts on Java declarations like overriding, inheritance, dynamic method lookup

theory *DeclConcepts* imports *TypeRel* begin

access control (cf. 6.6), overriding and hiding (cf. 8.4.6.1)

**constdefs**

*is-public* :: *prog*  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*  
*is-public* *G* *qn*  $\equiv$  (case class *G* *qn* of  
     None  $\Rightarrow$  (case iface *G* *qn* of  
         None  $\Rightarrow$  False  
         | Some iface  $\Rightarrow$  access iface = Public)  
     | Some class  $\Rightarrow$  access class = Public)

## 17 accessibility of types (cf. 6.6.1)

Primitive types are always accessible, interfaces and classes are accessible in their package or if they are defined public, an array type is accessible if its element type is accessible

**consts** *accessible-in* :: *prog*  $\Rightarrow$  *ty*  $\Rightarrow$  *pname*  $\Rightarrow$  *bool*  
     (-  $\vdash$  - *accessible'-in* - [61,61,61] 60)  
     *rt-accessible-in*:: *prog*  $\Rightarrow$  *ref-ty*  $\Rightarrow$  *pname*  $\Rightarrow$  *bool*  
     (-  $\vdash$  - *accessible'-in'* - [61,61,61] 60)

**primrec**

$G \vdash (\text{PrimT } p) \text{ accessible-in pack} = \text{True}$   
*accessible-in-RefT-simp*:  
 $G \vdash (\text{RefT } r) \text{ accessible-in pack} = G \vdash r \text{ accessible-in' pack}$   
  
 $G \vdash (\text{NullT}) \text{ accessible-in' pack} = \text{True}$   
 $G \vdash (\text{IfaceT } I) \text{ accessible-in' pack} = ((\text{pid } I = \text{pack}) \vee \text{is-public } G \ I)$   
 $G \vdash (\text{ClassT } C) \text{ accessible-in' pack} = ((\text{pid } C = \text{pack}) \vee \text{is-public } G \ C)$   
 $G \vdash (\text{ArrayT } ty) \text{ accessible-in' pack} = G \vdash ty \text{ accessible-in pack}$

**declare** *accessible-in-RefT-simp* [*simp del*]

**constdefs**

*is-acc-class* :: *prog*  $\Rightarrow$  *pname*  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*  
*is-acc-class* *G* *P* *C*  $\equiv$  *is-class* *G* *C*  $\wedge$   $G \vdash (\text{Class } C) \text{ accessible-in } P$   
*is-acc-iface* :: *prog*  $\Rightarrow$  *pname*  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*  
*is-acc-iface* *G* *P* *I*  $\equiv$  *is-iface* *G* *I*  $\wedge$   $G \vdash (\text{Iface } I) \text{ accessible-in } P$   
*is-acc-type* :: *prog*  $\Rightarrow$  *pname*  $\Rightarrow$  *ty*  $\Rightarrow$  *bool*  
*is-acc-type* *G* *P* *T*  $\equiv$  *is-type* *G* *T*  $\wedge$   $G \vdash T \text{ accessible-in } P$   
*is-acc-reftype* :: *prog*  $\Rightarrow$  *pname*  $\Rightarrow$  *ref-ty*  $\Rightarrow$  *bool*  
*is-acc-reftype* *G* *P* *T*  $\equiv$  *isrtype* *G* *T*  $\wedge$   $G \vdash T \text{ accessible-in' } P$

**lemma** *is-acc-classD*:

*is-acc-class* *G* *P* *C*  $\Longrightarrow$  *is-class* *G* *C*  $\wedge$   $G \vdash (\text{Class } C) \text{ accessible-in } P$   
 <proof>

**lemma** *is-acc-class-is-class*: *is-acc-class* *G* *P* *C*  $\Longrightarrow$  *is-class* *G* *C*

<proof>

**lemma** *is-acc-ifaceD*:

*is-acc-iface* *G* *P* *I*  $\Longrightarrow$  *is-iface* *G* *I*  $\wedge$   $G \vdash (\text{Iface } I) \text{ accessible-in } P$   
 <proof>

**lemma** *is-acc-typeD*:  
*is-acc-type*  $G\ P\ T \implies is-type\ G\ T \wedge G \vdash T\ accessible-in\ P$   
 $\langle proof \rangle$

**lemma** *is-acc-reftypeD*:  
*is-acc-reftype*  $G\ P\ T \implies isrtype\ G\ T \wedge G \vdash T\ accessible-in'\ P$   
 $\langle proof \rangle$

## 18 accessibility of members

The accessibility of members is more involved as the accessibility of types. We have to distinguish several cases to model the different effects of accessibility during inheritance, overriding and ordinary member access

### Various technical conversion and selection functions

overloaded selector *accmodi* to select the access modifier out of various HOL types

**axclass** *has-accmodi* < *type*  
**consts** *accmodi*:: '*a*::*has-accmodi*  $\Rightarrow acc-modi$

**instance** *acc-modi*::*has-accmodi*  $\langle proof \rangle$

**defs** (overloaded)  
*acc-modi-accmodi-def*: *accmodi* (*a*::*acc-modi*)  $\equiv a$

**lemma** *acc-modi-accmodi-simp*[*simp*]: *accmodi* (*a*::*acc-modi*) = *a*  
 $\langle proof \rangle$

**instance** *decl-ext-type*:: (*type*) *has-accmodi*  $\langle proof \rangle$

**defs** (overloaded)  
*decl-acc-modi-def*: *accmodi* (*d*::('a::*type*) *decl-scheme*)  $\equiv access\ d$

**lemma** *decl-acc-modi-simp*[*simp*]: *accmodi* (*d*::('a::*type*) *decl-scheme*) = *access d*  
 $\langle proof \rangle$

**instance** \* :: (*type*,*has-accmodi*) *has-accmodi*  $\langle proof \rangle$

**defs** (overloaded)  
*pair-acc-modi-def*: *accmodi* *p*  $\equiv (accmodi\ (snd\ p))$

**lemma** *pair-acc-modi-simp*[*simp*]: *accmodi* (*x*,*a*) = (*accmodi a*)  
 $\langle proof \rangle$

**instance** *memberdecl* :: *has-accmodi*  $\langle proof \rangle$

**defs** (overloaded)  
*memberdecl-acc-modi-def*: *accmodi* *m*  $\equiv$  (case *m* of  
     *fdecl f*  $\Rightarrow accmodi\ f$   
     | *mdecl m*  $\Rightarrow accmodi\ m$ )

**lemma** *memberdecl-fdecl-acc-modi-simp*[simp]:  
 $\text{accmodi } (fdecl\ m) = \text{accmodi } m$   
 ⟨proof⟩

**lemma** *memberdecl-mdecl-acc-modi-simp*[simp]:  
 $\text{accmodi } (mdecl\ m) = \text{accmodi } m$   
 ⟨proof⟩

overloaded selector *declclass* to select the declaring class out of various HOL types

**axclass** *has-declclass* < type  
**consts** *declclass*:: 'a::has-declclass  $\Rightarrow$  qname

**instance** *qname-ext-type*::(type) *has-declclass* ⟨proof⟩

**defs** (overloaded)  
*qname-declclass-def*:  $\text{declclass } (q::\text{qname}) \equiv q$

**lemma** *qname-declclass-simp*[simp]:  $\text{declclass } (q::\text{qname}) = q$   
 ⟨proof⟩

**instance** \* :: (has-declclass,type) *has-declclass* ⟨proof⟩

**defs** (overloaded)  
*pair-declclass-def*:  $\text{declclass } p \equiv \text{declclass } (\text{fst } p)$

**lemma** *pair-declclass-simp*[simp]:  $\text{declclass } (c,x) = \text{declclass } c$   
 ⟨proof⟩

overloaded selector *is-static* to select the static modifier out of various HOL types

**axclass** *has-static* < type  
**consts** *is-static* :: 'a::has-static  $\Rightarrow$  bool

**instance** *decl-ext-type* :: (has-static) *has-static* ⟨proof⟩

**defs** (overloaded)  
*decl-is-static-def*:  
 $\text{is-static } (m::('a::\text{has-static})\ \text{decl-scheme}) \equiv \text{is-static } (\text{Decl.decl.more } m)$

**instance** *member-ext-type* :: (type) *has-static* ⟨proof⟩

**defs** (overloaded)  
*static-field-type-is-static-def*:  
 $\text{is-static } (m::('b::\text{type})\ \text{member-ext-type}) \equiv \text{static-sel } m$

**lemma** *member-is-static-simp*:  $\text{is-static } (m::'a\ \text{member-scheme}) = \text{static } m$   
 ⟨proof⟩

**instance** \* :: (type,has-static) *has-static* ⟨proof⟩

**defs** (overloaded)  
*pair-is-static-def*:  $\text{is-static } p \equiv \text{is-static } (\text{snd } p)$

**lemma** *pair-is-static-simp* [simp]:  $\text{is-static } (x,s) = \text{is-static } s$

*⟨proof⟩*

**lemma** *pair-is-static-simp1*: *is-static*  $p = \text{is-static } (\text{snd } p)$   
 $\langle \text{proof} \rangle$

```
instance memberdecl:: has-static <proof>
```

```

defs (overloaded)
memberdecl-is-static-def:
  is-static m  $\equiv$  (case m of
    fdecl f  $\Rightarrow$  is-static f
    | mdecl m  $\Rightarrow$  is-static m)

```

**lemma** *memberdecl-is-static-fdecl-simp*[*simp*]:  
 $is-static\ (fdecl\ f) = is-static\ f$   
*<proof>*

**lemma** *memberdecl-is-static-mdecl-simp*[simp]:  
 $is\_static\ (mdecl\ m) = is\_static\ m$   
 $\langle proof \rangle$

**lemma** *mhead-static-simp* [simp]: *is-static (mhead m) = is-static m*  
 $\langle proof \rangle$

**constdefs** — some mnemonic selectors for various pairs

$$\begin{array}{lcl} \text{declface}:: (qname \times ('a::type) \text{ decl-scheme}) \Rightarrow qname & & \\ \text{declface} \equiv fst & \text{--- get the interface component} & \end{array}$$
$$\begin{array}{ll} mbr:: (qtname \times memberdecl) \Rightarrow memberdecl & \\ mbr \equiv snd & \text{— get the memberdecl component} \end{array}$$

<i>mthd</i> ::	$('b \times 'a) \Rightarrow 'a$	
		— also used for mdecl, mhead
<i>mthd</i> $\equiv$ <i>snd</i>		— get the method component

$$\begin{array}{ll} fld :: ('b \times ('a :: type) \text{ decl-scheme}) \Rightarrow ('a :: type) \text{ decl-scheme} \\ \quad \text{--- also used for } ((vname \times qtname) \times field) \\ fld \equiv snd & \text{--- get the field component} \end{array}$$

**constdefs** — some mnemonic selectors for  $(vname \times qname)$   
 $fname:: (vname \times 'a) \Rightarrow vname$  — also used for `fdecl`  
 $fname \equiv fst$

$$\begin{aligned} \text{declclassf} &:: (vname \times qname) \Rightarrow qname \\ \text{declclassf} &\equiv \text{snd} \end{aligned}$$

**lemma** *decliface-simp[simp]: decliface (I,m) = I*  
*⟨proof⟩*

**lemma** *mbr-simp*[simp]:  $mbr\ (C,m) = m$   
 $\langle proof \rangle$

**lemma** *access-mbr-simp* [simp]:  $(accmodi\ (mbr\ m)) = accmodi\ m$   
 $\langle proof \rangle$

**lemma** *mthd-simp*[simp]:  $mthd\ (C,m) = m$   
 $\langle proof \rangle$

**lemma** *fld-simp*[simp]:  $fld\ (C,f) = f$   
 $\langle proof \rangle$

**lemma** *accmodi-simp*[simp]:  $accmodi\ (C,m) = access\ m$   
 $\langle proof \rangle$

**lemma** *access-mthd-simp* [simp]:  $(access\ (mthd\ m)) = accmodi\ m$   
 $\langle proof \rangle$

**lemma** *access-fld-simp* [simp]:  $(access\ (fld\ f)) = accmodi\ f$   
 $\langle proof \rangle$

**lemma** *static-mthd-simp*[simp]:  $static\ (mthd\ m) = is-static\ m$   
 $\langle proof \rangle$

**lemma** *mthd-is-static-simp* [simp]:  $is-static\ (mthd\ m) = is-static\ m$   
 $\langle proof \rangle$

**lemma** *static-fld-simp*[simp]:  $static\ (fld\ f) = is-static\ f$   
 $\langle proof \rangle$

**lemma** *ext-field-simp* [simp]:  $(declclass\ f,fld\ f) = f$   
 $\langle proof \rangle$

**lemma** *ext-method-simp* [simp]:  $(declclass\ m,mthd\ m) = m$   
 $\langle proof \rangle$

**lemma** *ext-mbr-simp* [simp]:  $(declclass\ m,mbr\ m) = m$   
 $\langle proof \rangle$

**lemma** *fname-simp*[simp]:  $fname\ (n,c) = n$   
 $\langle proof \rangle$

**lemma** *declclassf-simp*[simp]:  $declclassf\ (n,c) = c$   
 $\langle proof \rangle$



**constdefs** — some mnemonic selectors for  $(vname \times qname)$

$fldname :: (vname \times qname) \Rightarrow vname$

$fldname \equiv fst$

$fldclass :: (vname \times qname) \Rightarrow qname$

$fldclass \equiv snd$

**lemma** *fldname-simp*[simp]:  $fldname\ (n,c) = n$

*<proof>*

**lemma** *fldclass-simp*[simp]:  $fldclass\ (n,c) = c$

*<proof>*

**lemma** *ext-fldname-simp*[simp]:  $(fldname\ f, fldclass\ f) = f$

*<proof>*

Convert a qualified method declaration (qualified with its declaring class) to a qualified member declaration: *methdMembr*

**constdefs**

$methdMembr :: (qname \times mdecl) \Rightarrow (qname \times memberdecl)$

$methdMembr\ m \equiv (fst\ m, mdecl\ (snd\ m))$

**lemma** *methdMembr-simp*[simp]:  $methdMembr\ (c,m) = (c, mdecl\ m)$

*<proof>*

**lemma** *accmodi-methdMembr-simp*[simp]:  $accmodi\ (methdMembr\ m) = accmodi\ m$

*<proof>*

**lemma** *is-static-methdMembr-simp*[simp]:  $is-static\ (methdMembr\ m) = is-static\ m$

*<proof>*

**lemma** *declclass-methdMembr-simp*[simp]:  $declclass\ (methdMembr\ m) = declclass\ m$

*<proof>*

Convert a qualified method (qualified with its declaring class) to a qualified member declaration: *method*

**constdefs**

$method :: sig \Rightarrow (qname \times methd) \Rightarrow (qname \times memberdecl)$

$method\ sig\ m \equiv (declclass\ m, mdecl\ (sig, mthd\ m))$

**lemma** *method-simp*[simp]:  $method\ sig\ (C,m) = (C, mdecl\ (sig, m))$

*<proof>*

**lemma** *accmodi-method-simp*[simp]:  $accmodi\ (method\ sig\ m) = accmodi\ m$

*<proof>*

**lemma** *declclass-method-simp*[simp]:  $declclass\ (method\ sig\ m) = declclass\ m$

*<proof>*

**lemma** *is-static-method-simp[simp]*: *is-static* (method sig m) = *is-static* m  
 ⟨proof⟩

**lemma** *mbr-method-simp[simp]*: *mbr* (method sig m) = *mdecl* (sig, mthd m)  
 ⟨proof⟩

**lemma** *memberid-method-simp[simp]*: *memberid* (method sig m) = *mid* sig  
 ⟨proof⟩

**constdefs**

*fieldm* :: *vname*  $\Rightarrow$  (*qtname*  $\times$  *field*)  $\Rightarrow$  (*qtname*  $\times$  *memberdecl*)  
*fieldm* n f  $\equiv$  (*declclass* f, *fdecl* (n, fld f))

**lemma** *fieldm-simp[simp]*: *fieldm* n (C, f) = (C, *fdecl* (n, f))  
 ⟨proof⟩

**lemma** *accmodi-fieldm-simp[simp]*: *accmodi* (*fieldm* n f) = *accmodi* f  
 ⟨proof⟩

**lemma** *declclass-fieldm-simp[simp]*: *declclass* (*fieldm* n f) = *declclass* f  
 ⟨proof⟩

**lemma** *is-static-fieldm-simp[simp]*: *is-static* (*fieldm* n f) = *is-static* f  
 ⟨proof⟩

**lemma** *mbr-fieldm-simp[simp]*: *mbr* (*fieldm* n f) = *fdecl* (n, fld f)  
 ⟨proof⟩

**lemma** *memberid-fieldm-simp[simp]*: *memberid* (*fieldm* n f) = *fld* n  
 ⟨proof⟩

Select the signature out of a qualified method declaration: *msig*

**constdefs** *msig*:: (*qtname*  $\times$  *mdecl*)  $\Rightarrow$  *sig*  
*msig* m  $\equiv$  *fst* (*snd* m)

**lemma** *msig-simp[simp]*: *msig* (c, (s, m)) = s  
 ⟨proof⟩

Convert a qualified method (qualified with its declaring class) to a qualified method declaration:  
*qmdecl*

**constdefs** *qmdecl* :: *sig*  $\Rightarrow$  (*qtname*  $\times$  *methd*)  $\Rightarrow$  (*qtname*  $\times$  *mdecl*)  
*qmdecl* sig m  $\equiv$  (*declclass* m, (sig, mthd m))

**lemma** *qmdecl-simp[simp]*: *qmdecl* sig (C, m) = (C, (sig, m))  
 ⟨proof⟩

**lemma** *declclass-qmdecl-simp*[simp]: *declclass* (*qmdecl sig m*) = *declclass m*  
 ⟨proof⟩

**lemma** *accmodi-qmdecl-simp*[simp]: *accmodi* (*qmdecl sig m*) = *accmodi m*  
 ⟨proof⟩

**lemma** *is-static-qmdecl-simp*[simp]: *is-static* (*qmdecl sig m*) = *is-static m*  
 ⟨proof⟩

**lemma** *msig-qmdecl-simp*[simp]: *msig* (*qmdecl sig m*) = *sig*  
 ⟨proof⟩

**lemma** *mdecl-qmdecl-simp*[simp]:  
*mdecl* (*mthd* (*qmdecl sig new*)) = *mdecl* (*sig*, *mthd new*)  
 ⟨proof⟩

**lemma** *methdMembr-qmdecl-simp* [simp]:  
*methdMembr* (*qmdecl sig old*) = *method sig old*  
 ⟨proof⟩

overloaded selector *resTy* to select the result type out of various HOL types

**axclass** *has-resTy* < *type*  
**consts** *resTy*:: 'a::has-resTy  $\Rightarrow$  *ty*

**instance** *decl-ext-type* :: (*has-resTy*) *has-resTy* ⟨proof⟩

**defs** (overloaded)  
*decl-resTy-def*:  
*resTy* (*m*::('a::has-resTy) *decl-scheme*)  $\equiv$  *resTy* (*Decl.decl.more m*)

**instance** *member-ext-type* :: (*has-resTy*) *has-resTy* ⟨proof⟩

**defs** (overloaded)  
*member-ext-type-resTy-def*:  
*resTy* (*m*::('b::has-resTy) *member-ext-type*)  
 $\equiv$  *resTy* (*member.more-sel m*)

**instance** *mhead-ext-type* :: (*type*) *has-resTy* ⟨proof⟩

**defs** (overloaded)  
*mhead-ext-type-resTy-def*:  
*resTy* (*m*::('b *mhead-ext-type*))  
 $\equiv$  *resT-sel m*

**lemma** *mhead-resTy-simp*: *resTy* (*m*::'a *mhead-scheme*) = *resT m*  
 ⟨proof⟩

**lemma** *resTy-mhead* [simp]: *resTy* (*mhead m*) = *resTy m*  
 ⟨proof⟩

**instance** \* :: (*type*, *has-resTy*) *has-resTy* ⟨proof⟩

**defs (overloaded)**

*pair-resTy-def*:  $\text{resTy } p \equiv \text{resTy } (\text{snd } p)$

**lemma** *pair-resTy-simp*[simp]:  $\text{resTy } (x, m) = \text{resTy } m$   
 ⟨proof⟩

**lemma** *qmdecl-resTy-simp* [simp]:  $\text{resTy } (\text{qmdecl sig } m) = \text{resTy } m$   
 ⟨proof⟩

**lemma** *resTy-mthd* [simp]:  $\text{resTy } (\text{mthd } m) = \text{resTy } m$   
 ⟨proof⟩

**inheritable-in**

$G \vdash m$  *inheritable-in*  $P$ :  $m$  can be inherited by classes in package  $P$  if:

- the declaration class of  $m$  is accessible in  $P$  and
- the member  $m$  is declared with protected or public access or if it is declared with default (package) access, the package of the declaration class of  $m$  is also  $P$ . If the member  $m$  is declared with private access it is not accessible for inheritance at all.

**constdefs**

*inheritable-in*::

$\text{prog} \Rightarrow (\text{qname} \times \text{memberdecl}) \Rightarrow \text{pname} \Rightarrow \text{bool}$   
 $(- \vdash - \text{inheritable}'\text{-in} - [61, 61, 61] \ 60)$

$G \vdash \text{membr}$  *inheritable-in*  $\text{pack}$

$\equiv (\text{case } (\text{accmodi membr}) \text{ of}$   
    $\text{Private} \Rightarrow \text{False}$   
    $\mid \text{Package} \Rightarrow (\text{pid } (\text{declclass membr})) = \text{pack}$   
    $\mid \text{Protected} \Rightarrow \text{True}$   
    $\mid \text{Public} \Rightarrow \text{True})$

**syntax**

*Method-inheritable-in*::

$\text{prog} \Rightarrow (\text{qname} \times \text{mdecl}) \Rightarrow \text{pname} \Rightarrow \text{bool}$   
 $(- \vdash \text{Method} - \text{inheritable}'\text{-in} - [61, 61, 61] \ 60)$

**translations**

$G \vdash \text{Method } m$  *inheritable-in*  $p == G \vdash \text{methdMembr } m$  *inheritable-in*  $p$

**syntax**

*Methd-inheritable-in*::

$\text{prog} \Rightarrow \text{sig} \Rightarrow (\text{qname} \times \text{methd}) \Rightarrow \text{pname} \Rightarrow \text{bool}$   
 $(- \vdash \text{Methd} - - \text{inheritable}'\text{-in} - [61, 61, 61, 61] \ 60)$

**translations**

$G \vdash \text{Methd } s \ m$  *inheritable-in*  $p == G \vdash (\text{method } s \ m)$  *inheritable-in*  $p$

**declared-in/undeclared-in**

**constdefs** *cdeclaredmethd*::  $\text{prog} \Rightarrow \text{qname} \Rightarrow (\text{sig}, \text{methd}) \text{ table}$

*cdeclaredmethd*  $G \ C$

$\equiv (\text{case class } G \ C \text{ of}$   
    $\text{None} \Rightarrow \lambda \text{ sig. None}$

| *Some c*  $\Rightarrow$  *table-of (methods c)*  
 )

**constdefs**

*cdeclaredfield*:: *prog*  $\Rightarrow$  *qname*  $\Rightarrow$  (*vname,field*) *table*

*cdeclaredfield G C*

$\equiv$  (*case class G C of*  
     *None*  $\Rightarrow \lambda$  *sig. None*  
     | *Some c*  $\Rightarrow$  *table-of (cfields c)*  
 )

**constdefs**

*declared-in*:: *prog*  $\Rightarrow$  *memberdecl*  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*

( $\vdash$  - *declared'-in* - [61,61,61] 60)

$G \vdash m$  *declared-in C*  $\equiv$  (*case m of*  
     *fdecl (fn,f)*  $\Rightarrow$  *cdeclaredfield G C fn = Some f*  
     | *mdecl (sig,m)*  $\Rightarrow$  *cdeclaredmethd G C sig = Some m*)

**syntax**

*method-declared-in*:: *prog*  $\Rightarrow$  (*qname*  $\times$  *mdecl*)  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*

( $\vdash$  *Method* - *declared'-in* - [61,61,61] 60)

**translations**

$G \vdash \text{Method } m \text{ declared-in } C == G \vdash mdecl (mthd m) \text{ declared-in } C$

**syntax**

*methd-declared-in*:: *prog*  $\Rightarrow$  *sig*  $\Rightarrow$  (*qname*  $\times$  *methd*)  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*

( $\vdash$  *Methd* - - *declared'-in* - [61,61,61,61] 60)

**translations**

$G \vdash \text{Methd } s m \text{ declared-in } C == G \vdash mdecl (s, mthd m) \text{ declared-in } C$

**lemma** *declared-in-classD*:

$G \vdash m \text{ declared-in } C \implies \text{is-class } G C$

$\langle \text{proof} \rangle$

**constdefs**

*undeclared-in*:: *prog*  $\Rightarrow$  *memberid*  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*

( $\vdash$  - *undeclared'-in* - [61,61,61] 60)

$G \vdash m$  *undeclared-in C*  $\equiv$  (*case m of*  
     *fid fn*  $\Rightarrow$  *cdeclaredfield G C fn = None*  
     | *mid sig*  $\Rightarrow$  *cdeclaredmethd G C sig = None*)

**members****inductive**

*members*:: *prog*  $\Rightarrow$  (*qname*  $\times$  *memberdecl*)  $\Rightarrow$  *qname*  $\Rightarrow$  *bool*

( $\vdash$  - *member'-of* - [61,61,61] 60)

**for** *G*:: *prog*

**where**

*Immediate*:  $\llbracket G \vdash mbr m \text{ declared-in } C; declclass m = C \rrbracket \implies G \vdash m \text{ member-of } C$

| *Inherited*:  $\llbracket G \vdash m \text{ inheritable-in } (pid C); G \vdash memberid m \text{ undeclared-in } C;$   
      $G \vdash C \prec_{C1} S; G \vdash (Class S) \text{ accessible-in } (pid C); G \vdash m \text{ member-of } S$   
      $\rrbracket \implies G \vdash m \text{ member-of } C$

Note that in the case of an inherited member only the members of the direct superclass are concerned. If a member of a superclass of the direct superclass isn't inherited in the direct superclass (not

member of the direct superclass) than it can't be a member of the class. E.g. If a member of a class A is defined with package access it isn't member of a subclass S if S isn't in the same package as A. Any further subclasses of S will not inherit the member, regardless if they are in the same package as A or not.

#### syntax

*method-member-of*::  $prog \Rightarrow (qname \times mdecl) \Rightarrow qname \Rightarrow bool$   
 $(- \vdash Method - member'-of - [61,61,61] 60)$

#### translations

$G \vdash Method\ m\ member-of\ C \Leftrightarrow G \vdash (methdMembr\ m)\ member-of\ C$

#### syntax

*methd-member-of*::  $prog \Rightarrow sig \Rightarrow (qname \times methd) \Rightarrow qname \Rightarrow bool$   
 $(- \vdash Methd - - member'-of - [61,61,61,61] 60)$

#### translations

$G \vdash Methd\ s\ m\ member-of\ C \Leftrightarrow G \vdash (method\ s\ m)\ member-of\ C$

#### syntax

*fieldm-member-of*::  $prog \Rightarrow vname \Rightarrow (qname \times field) \Rightarrow qname \Rightarrow bool$   
 $(- \vdash Field - - member'-of - [61,61,61] 60)$

#### translations

$G \vdash Field\ n\ f\ member-of\ C \Leftrightarrow G \vdash fieldm\ n\ f\ member-of\ C$

#### constdefs

*inherits*::  $prog \Rightarrow qname \Rightarrow (qname \times memberdecl) \Rightarrow bool$   
 $(- \vdash - inherits - [61,61,61] 60)$

$G \vdash C\ inherits\ m$

$\equiv G \vdash m\ inheritable-in\ (pid\ C) \wedge G \vdash memberid\ m\ undeclared-in\ C \wedge$   
 $(\exists\ S.\ G \vdash C \prec_{C1} S \wedge G \vdash (Class\ S)\ accessible-in\ (pid\ C) \wedge G \vdash m\ member-of\ S)$

**lemma** *inherits-member*:  $G \vdash C\ inherits\ m \implies G \vdash m\ member-of\ C$   
 $\langle proof \rangle$

**constdefs** *member-in*::  $prog \Rightarrow (qname \times memberdecl) \Rightarrow qname \Rightarrow bool$   
 $(- \vdash - member'-in - [61,61,61] 60)$

$G \vdash m\ member-in\ C \equiv \exists\ provC.\ G \vdash C \preceq_C provC \wedge G \vdash m\ member-of\ provC$

A member is in a class if it is member of the class or a superclass. If a member is in a class we can select this member. This additional notion is necessary since not all members are inherited to subclasses. So such members are not member-of the subclass but member-in the subclass.

#### syntax

*method-member-in*::  $prog \Rightarrow (qname \times mdecl) \Rightarrow qname \Rightarrow bool$   
 $(- \vdash Method - member'-in - [61,61,61] 60)$

#### translations

$G \vdash Method\ m\ member-in\ C \Leftrightarrow G \vdash (methdMembr\ m)\ member-in\ C$

#### syntax

*methd-member-in*::  $prog \Rightarrow sig \Rightarrow (qname \times methd) \Rightarrow qname \Rightarrow bool$   
 $(- \vdash Methd - - member'-in - [61,61,61,61] 60)$

#### translations

$G \vdash Methd\ s\ m\ member-in\ C \Leftrightarrow G \vdash (method\ s\ m)\ member-in\ C$

**lemma** *member-inD*:  $G \vdash m \text{ member-in } C$   
 $\implies \exists \text{ provC}. G \vdash C \preceq_C \text{ provC} \wedge G \vdash m \text{ member-of } \text{provC}$   
 $\langle \text{proof} \rangle$

**lemma** *member-inI*:  $\llbracket G \vdash m \text{ member-of } \text{provC}; G \vdash C \preceq_C \text{ provC} \rrbracket \implies G \vdash m \text{ member-in } C$   
 $\langle \text{proof} \rangle$

**lemma** *member-of-to-member-in*:  $G \vdash m \text{ member-of } C \implies G \vdash m \text{ member-in } C$   
 $\langle \text{proof} \rangle$

## overriding

Unfortunately the static notion of overriding (used during the typecheck of the compiler) and the dynamic notion of overriding (used during execution in the JVM) are not exactly the same.

Static overriding (used during the typecheck of the compiler)

### inductive

*stat-overridesR* ::  $\text{prog} \Rightarrow (\text{qname} \times \text{mdecl}) \Rightarrow (\text{qname} \times \text{mdecl}) \Rightarrow \text{bool}$   
 $(- \vdash - \text{overrides}_S - [61, 61, 61] \ 60)$   
**for**  $G :: \text{prog}$   
**where**

*Direct*:  $\llbracket \neg \text{is-static new}; \text{msig new} = \text{msig old};$   
 $G \vdash \text{Method new declared-in } (\text{declclass new});$   
 $G \vdash \text{Method old declared-in } (\text{declclass old});$   
 $G \vdash \text{Method old inheritable-in pid } (\text{declclass new});$   
 $G \vdash (\text{declclass new}) \prec_{C1} \text{superNew};$   
 $G \vdash \text{Method old member-of superNew}$   
 $\rrbracket \implies G \vdash \text{new overrides}_S \text{ old}$

| *Indirect*:  $\llbracket G \vdash \text{new overrides}_S \text{ inter}; G \vdash \text{inter overrides}_S \text{ old} \rrbracket$   
 $\implies G \vdash \text{new overrides}_S \text{ old}$

Dynamic overriding (used during the typecheck of the compiler)

### inductive

*overridesR* ::  $\text{prog} \Rightarrow (\text{qname} \times \text{mdecl}) \Rightarrow (\text{qname} \times \text{mdecl}) \Rightarrow \text{bool}$   
 $(- \vdash - \text{overrides} - [61, 61, 61] \ 60)$   
**for**  $G :: \text{prog}$   
**where**

*Direct*:  $\llbracket \neg \text{is-static new}; \neg \text{is-static old}; \text{accmodi new} \neq \text{Private};$   
 $\text{msig new} = \text{msig old};$   
 $G \vdash (\text{declclass new}) \prec_C (\text{declclass old});$   
 $G \vdash \text{Method new declared-in } (\text{declclass new});$   
 $G \vdash \text{Method old declared-in } (\text{declclass old});$   
 $G \vdash \text{Method old inheritable-in pid } (\text{declclass new});$   
 $G \vdash \text{resTy new} \preceq \text{resTy old}$   
 $\rrbracket \implies G \vdash \text{new overrides old}$

| *Indirect*:  $\llbracket G \vdash \text{new overrides inter}; G \vdash \text{inter overrides old} \rrbracket$   
 $\implies G \vdash \text{new overrides old}$

## syntax

*sig-stat-overrides* ::  
 $\text{prog} \Rightarrow \text{sig} \Rightarrow (\text{qname} \times \text{methd}) \Rightarrow (\text{qname} \times \text{methd}) \Rightarrow \text{bool}$

$$(-, \vdash - \text{overrides}_S - [61, 61, 61, 61] \ 60)$$
**translations**

$$G, s \vdash \text{new overrides}_S \text{ old} \rightarrow G \vdash (\text{qmdecl } s \text{ new}) \text{ overrides}_S (\text{qmdecl } s \text{ old})$$
**syntax**

$$\text{sig-overrides}:: \text{prog} \Rightarrow \text{sig} \Rightarrow (\text{qtname} \times \text{methd}) \Rightarrow (\text{qtname} \times \text{methd}) \Rightarrow \text{bool}$$

$$(-, \vdash - \text{overrides} - [61, 61, 61, 61] \ 60)$$
**translations**

$$G, s \vdash \text{new overrides old} \rightarrow G \vdash (\text{qmdecl } s \text{ new}) \text{ overrides} (\text{qmdecl } s \text{ old})$$
**Hiding****constdefs** *hides*::
$$\text{prog} \Rightarrow (\text{qtname} \times \text{mdecl}) \Rightarrow (\text{qtname} \times \text{mdecl}) \Rightarrow \text{bool}$$

$$(\vdash - \text{hides} - [61, 61, 61] \ 60)$$

$$G \vdash \text{new hides old}$$

$$\equiv \text{is-static new} \wedge \text{msig new} = \text{msig old} \wedge$$

$$G \vdash (\text{declclass new}) \prec_C (\text{declclass old}) \wedge$$

$$G \vdash \text{Method new declared-in} (\text{declclass new}) \wedge$$

$$G \vdash \text{Method old declared-in} (\text{declclass old}) \wedge$$

$$G \vdash \text{Method old inheritable-in pid} (\text{declclass new})$$
**syntax**

$$\text{sig-hides}:: \text{prog} \Rightarrow \text{sig} \Rightarrow (\text{qtname} \times \text{mdecl}) \Rightarrow (\text{qtname} \times \text{mdecl}) \Rightarrow \text{bool}$$

$$(-, \vdash - \text{hides} - [61, 61, 61, 61] \ 60)$$
**translations**

$$G, s \vdash \text{new hides old} \rightarrow G \vdash (\text{qmdecl } s \text{ new}) \text{ hides} (\text{qmdecl } s \text{ old})$$
**lemma** *hidesI*:
$$\llbracket \text{is-static new}; \text{msig new} = \text{msig old};$$

$$G \vdash (\text{declclass new}) \prec_C (\text{declclass old});$$

$$G \vdash \text{Method new declared-in} (\text{declclass new});$$

$$G \vdash \text{Method old declared-in} (\text{declclass old});$$

$$G \vdash \text{Method old inheritable-in pid} (\text{declclass new})$$

$$\rrbracket \implies G \vdash \text{new hides old}$$

$$\langle \text{proof} \rangle$$
**lemma** *hidesD*:
$$\llbracket G \vdash \text{new hides old} \rrbracket \implies$$

$$\text{declclass new} \neq \text{Object} \wedge \text{is-static new} \wedge \text{msig new} = \text{msig old} \wedge$$

$$G \vdash (\text{declclass new}) \prec_C (\text{declclass old}) \wedge$$

$$G \vdash \text{Method new declared-in} (\text{declclass new}) \wedge$$

$$G \vdash \text{Method old declared-in} (\text{declclass old})$$

$$\langle \text{proof} \rangle$$
**lemma** *overrides-commonD*:
$$\llbracket G \vdash \text{new overrides old} \rrbracket \implies$$

$$\text{declclass new} \neq \text{Object} \wedge \neg \text{is-static new} \wedge \neg \text{is-static old} \wedge$$

$$\text{accmodi new} \neq \text{Private} \wedge$$

$$\text{msig new} = \text{msig old} \wedge$$

$$G \vdash (\text{declclass new}) \prec_C (\text{declclass old}) \wedge$$

$$G \vdash \text{Method new declared-in} (\text{declclass new}) \wedge$$

$$G \vdash \text{Method old declared-in} (\text{declclass old})$$

$$\langle \text{proof} \rangle$$



**lemma** *ws-overrides-commonD*:

$\llbracket G \vdash \text{new overrides old}; \text{ws-prog } G \rrbracket \implies$   
 $\text{declclass new} \neq \text{Object} \wedge \neg \text{is-static new} \wedge \neg \text{is-static old} \wedge$   
 $\text{accmodi new} \neq \text{Private} \wedge G \vdash \text{resTy new} \preceq \text{resTy old} \wedge$   
 $\text{msig new} = \text{msig old} \wedge$   
 $G \vdash (\text{declclass new}) \prec_C (\text{declclass old}) \wedge$   
 $G \vdash \text{Method new declared-in } (\text{declclass new}) \wedge$   
 $G \vdash \text{Method old declared-in } (\text{declclass old})$   
 $\langle \text{proof} \rangle$

**lemma** *overrides-eq-sigD*:

$\llbracket G \vdash \text{new overrides old} \rrbracket \implies \text{msig old} = \text{msig new}$   
 $\langle \text{proof} \rangle$

**lemma** *hides-eq-sigD*:

$\llbracket G \vdash \text{new hides old} \rrbracket \implies \text{msig old} = \text{msig new}$   
 $\langle \text{proof} \rangle$

## permits access

### constdefs

*permits-acc::*

$\text{prog} \Rightarrow (\text{qname} \times \text{memberdecl}) \Rightarrow \text{qname} \Rightarrow \text{qname} \Rightarrow \text{bool}$   
 $(- \vdash - \text{ in } - \text{ permits'-acc'-from } - [61, 61, 61, 61] 60)$

$G \vdash \text{membr in class permits-acc-from accclass}$   
 $\equiv (\text{case } (\text{accmodi membr}) \text{ of}$   
 $\quad \text{Private} \Rightarrow (\text{declclass membr} = \text{accclass})$   
 $\quad | \text{Package} \Rightarrow (\text{pid } (\text{declclass membr}) = \text{pid accclass})$   
 $\quad | \text{Protected} \Rightarrow (\text{pid } (\text{declclass membr}) = \text{pid accclass})$   
 $\quad \vee$   
 $\quad (G \vdash \text{accclass} \prec_C \text{declclass membr}$   
 $\quad \wedge (G \vdash \text{class} \preceq_C \text{accclass} \vee \text{is-static membr}))$   
 $\quad | \text{Public} \Rightarrow \text{True})$

The subcondition of the *Protected* case:  $G \vdash \text{accclass} \prec_C \text{declclass membr}$  could also be relaxed to:  $G \vdash \text{accclass} \preceq_C \text{declclass membr}$  since in case both classes are the same the other condition  $\text{pid } (\text{declclass membr}) = \text{pid accclass}$  holds anyway.

Like in case of overriding, the static and dynamic accessibility of members is not uniform.

- Statically the class/interface of the member must be accessible for the member to be accessible. During runtime this is not necessary. For Example, if a class is accessible and we are allowed to access a member of this class (statically) we expect that we can access this member in an arbitrary subclass (during runtime). It's not intended to restrict the access to accessible subclasses during runtime.
- Statically the member we want to access must be "member of" the class. Dynamically it must only be "member in" the class.

### inductive

*accessible-fromR :: prog  $\Rightarrow$  qname  $\Rightarrow$  (qname  $\times$  memberdecl)  $\Rightarrow$  qname  $\Rightarrow$  bool*  
**and** *accessible-from :: prog  $\Rightarrow$  (qname  $\times$  memberdecl)  $\Rightarrow$  qname  $\Rightarrow$  qname  $\Rightarrow$  bool*  
 $(- \vdash - \text{ of } - \text{ accessible'-from } - [61, 61, 61, 61] 60)$   
**and** *method-accessible-from :: prog  $\Rightarrow$  (qname  $\times$  mdecl)  $\Rightarrow$  qname  $\Rightarrow$  qname  $\Rightarrow$  bool*  
 $(- \vdash \text{Method } - \text{ of } - \text{ accessible'-from } - [61, 61, 61, 61] 60)$   
**for**  $G :: \text{prog}$  **and**  $\text{accclass} :: \text{qname}$

**where**

$G \vdash \text{membr of } cls \text{ accessible-from } accclass \equiv \text{accessible-fromR } G \text{ accclass membr } cls$

|  $G \vdash \text{Method } m \text{ of } cls \text{ accessible-from } accclass \equiv \text{accessible-fromR } G \text{ accclass (methdMembr } m) \text{ } cls$

| *Immediate*:  $\llbracket G \vdash \text{membr member-of class};$   
 $G \vdash (\text{Class } class) \text{ accessible-in (pid accclass)};$   
 $G \vdash \text{membr in class permits-acc-from accclass}$   
 $\rrbracket \Rightarrow G \vdash \text{membr of class accessible-from accclass}$

| *Overriding*:  $\llbracket G \vdash \text{membr member-of class};$   
 $G \vdash (\text{Class } class) \text{ accessible-in (pid accclass)};$   
 $\text{membr} = (C, mdecl \text{ new});$   
 $G \vdash (C, \text{new}) \text{ overrides}_S \text{ old};$   
 $G \vdash \text{class } \prec_C \text{ supr};$   
 $G \vdash \text{Method old of supr accessible-from accclass}$   
 $\rrbracket \Rightarrow G \vdash \text{membr of class accessible-from accclass}$

**syntax**

*methd-accessible-from*:

$prog \Rightarrow sig \Rightarrow (qname \times methd) \Rightarrow qname \Rightarrow qname \Rightarrow bool$   
 $(- \vdash \text{Method } - \text{ of } - \text{ accessible'-from } - [61, 61, 61, 61, 61] \ 60)$

**translations**

$G \vdash \text{Method } s \text{ m of } cls \text{ accessible-from accclass}$   
 $\Rightarrow G \vdash (\text{method } s \text{ m}) \text{ of } cls \text{ accessible-from accclass}$

**syntax**

*field-accessible-from*:

$prog \Rightarrow vname \Rightarrow (qname \times field) \Rightarrow qname \Rightarrow qname \Rightarrow bool$   
 $(- \vdash \text{Field } - \text{ of } - \text{ accessible'-from } - [61, 61, 61, 61, 61] \ 60)$

**translations**

$G \vdash \text{Field fn } f \text{ of } C \text{ accessible-from accclass}$   
 $\Rightarrow G \vdash (\text{fieldm fn } f) \text{ of } C \text{ accessible-from accclass}$

**inductive**

*dyn-accessible-fromR* ::  $prog \Rightarrow qname \Rightarrow (qname \times memberdecl) \Rightarrow qname \Rightarrow bool$   
**and** *dyn-accessible-from'* ::  $prog \Rightarrow (qname \times memberdecl) \Rightarrow qname \Rightarrow qname \Rightarrow bool$   
 $(- \vdash - \text{ in } - \text{ dyn'-accessible'-from } - [61, 61, 61, 61] \ 60)$   
**and** *method-dyn-accessible-from* ::  $prog \Rightarrow (qname \times mdecl) \Rightarrow qname \Rightarrow qname \Rightarrow bool$   
 $(- \vdash \text{Method } - \text{ in } - \text{ dyn'-accessible'-from } - [61, 61, 61, 61] \ 60)$   
**for**  $G :: prog$  **and**  $accC :: qname$

**where**

$G \vdash \text{membr in } C \text{ dyn-accessible-from accC} \equiv \text{dyn-accessible-fromR } G \text{ accC membr } C$

|  $G \vdash \text{Method } m \text{ in } C \text{ dyn-accessible-from accC} \equiv \text{dyn-accessible-fromR } G \text{ accC (methdMembr } m) \text{ } C$

| *Immediate*:  $\llbracket G \vdash \text{membr member-in class};$   
 $G \vdash \text{membr in class permits-acc-from accclass}$   
 $\rrbracket \Rightarrow G \vdash \text{membr in class dyn-accessible-from accclass}$

| *Overriding*:  $\llbracket G \vdash \text{membr member-in class};$   
 $\text{membr} = (C, mdecl \text{ new});$   
 $G \vdash (C, \text{new}) \text{ overrides}_S \text{ old};$   
 $G \vdash \text{class } \prec_C \text{ supr};$   
 $G \vdash \text{Method old in supr dyn-accessible-from accclass}$   
 $\rrbracket \Rightarrow G \vdash \text{membr in class dyn-accessible-from accclass}$

**syntax***methd-dyn-accessible-from::*

$$\text{prog} \Rightarrow \text{sig} \Rightarrow (\text{qname} \times \text{methd}) \Rightarrow \text{qname} \Rightarrow \text{qname} \Rightarrow \text{bool}$$

$$(- \vdash \text{Methd} \text{ - - in - dyn'-accessible'-from - } [61,61,61,61,61] \text{ } 60)$$
**translations** $G \vdash \text{Methd } s \text{ } m \text{ in } C \text{ dyn-accessible-from } \text{acc}C$  $\Rightarrow G \vdash (\text{method } s \text{ } m) \text{ in } C \text{ dyn-accessible-from } \text{acc}C$ **syntax***field-dyn-accessible-from::*

$$\text{prog} \Rightarrow \text{vname} \Rightarrow (\text{qname} \times \text{field}) \Rightarrow \text{qname} \Rightarrow \text{qname} \Rightarrow \text{bool}$$

$$(- \vdash \text{Field} \text{ - - in - dyn'-accessible'-from - } [61,61,61,61,61] \text{ } 60)$$
**translations** $G \vdash \text{Field } \text{fn } f \text{ in } \text{dyn}C \text{ dyn-accessible-from } \text{acc}C$  $\Rightarrow G \vdash (\text{fieldm } \text{fn } f) \text{ in } \text{dyn}C \text{ dyn-accessible-from } \text{acc}C$ **lemma** *accessible-from-commonD*:  $G \vdash m \text{ of } C \text{ accessible-from } S$  $\Rightarrow G \vdash m \text{ member-of } C \wedge G \vdash (\text{Class } C) \text{ accessible-in } (\text{pid } S)$  $\langle \text{proof} \rangle$ **lemma** *unique-declaration*: $\llbracket G \vdash m \text{ declared-in } C; G \vdash n \text{ declared-in } C; \text{memberid } m = \text{memberid } n \rrbracket$  $\Rightarrow m = n$  $\langle \text{proof} \rangle$ **lemma** *declared-not-undeclared*: $G \vdash m \text{ declared-in } C \Rightarrow \neg G \vdash \text{memberid } m \text{ undeclared-in } C$  $\langle \text{proof} \rangle$ **lemma** *undeclared-not-declared*: $G \vdash \text{memberid } m \text{ undeclared-in } C \Rightarrow \neg G \vdash m \text{ declared-in } C$  $\langle \text{proof} \rangle$ **lemma** *not-undeclared-declared*:
$$\neg G \vdash \text{memb-}id \text{ undeclared-in } C \Rightarrow (\exists m. G \vdash m \text{ declared-in } C \wedge$$

$$\text{memb-}id = \text{memberid } m)$$
 $\langle \text{proof} \rangle$ **lemma** *unique-declared-in*: $\llbracket G \vdash m \text{ declared-in } C; G \vdash n \text{ declared-in } C; \text{memberid } m = \text{memberid } n \rrbracket$  $\Rightarrow m = n$  $\langle \text{proof} \rangle$ **lemma** *unique-member-of*:**assumes**  $n$ :  $G \vdash n \text{ member-of } C$  **and** $m$ :  $G \vdash m \text{ member-of } C$  **and***eqid*:  $\text{memberid } n = \text{memberid } m$ **shows**  $n=m$  $\langle \text{proof} \rangle$

**lemma** *member-of-is-classD*:  $G \vdash m \text{ member-of } C \implies \text{is-class } G \ C$   
 $\langle \text{proof} \rangle$

**lemma** *member-of-declC*:  
 $G \vdash m \text{ member-of } C$   
 $\implies G \vdash \text{mbr } m \text{ declared-in } (\text{declclass } m)$   
 $\langle \text{proof} \rangle$

**lemma** *member-of-member-of-declC*:  
 $G \vdash m \text{ member-of } C$   
 $\implies G \vdash m \text{ member-of } (\text{declclass } m)$   
 $\langle \text{proof} \rangle$

**lemma** *member-of-class-relation*:  
 $G \vdash m \text{ member-of } C \implies G \vdash C \preceq_C \text{ declclass } m$   
 $\langle \text{proof} \rangle$

**lemma** *member-in-class-relation*:  
 $G \vdash m \text{ member-in } C \implies G \vdash C \preceq_C \text{ declclass } m$   
 $\langle \text{proof} \rangle$

**lemma** *stat-override-declclasses-relation*:  
 $\llbracket G \vdash (\text{declclass } \text{new}) \prec_{C1} \text{superNew}; G \vdash \text{Method } \text{old} \text{ member-of } \text{superNew} \rrbracket$   
 $\implies G \vdash (\text{declclass } \text{new}) \prec_C (\text{declclass } \text{old})$   
 $\langle \text{proof} \rangle$

**lemma** *stat-overrides-commonD*:  
 $\llbracket G \vdash \text{new overrides}_S \text{ old} \rrbracket \implies$   
 $\text{declclass } \text{new} \neq \text{Object} \wedge \neg \text{is-static } \text{new} \wedge \text{msig } \text{new} = \text{msig } \text{old} \wedge$   
 $G \vdash (\text{declclass } \text{new}) \prec_C (\text{declclass } \text{old}) \wedge$   
 $G \vdash \text{Method } \text{new} \text{ declared-in } (\text{declclass } \text{new}) \wedge$   
 $G \vdash \text{Method } \text{old} \text{ declared-in } (\text{declclass } \text{old})$   
 $\langle \text{proof} \rangle$

**lemma** *member-of-Package*:  
 $\llbracket G \vdash m \text{ member-of } C; \text{accomodi } m = \text{Package} \rrbracket$   
 $\implies \text{pid } (\text{declclass } m) = \text{pid } C$   
 $\langle \text{proof} \rangle$

**lemma** *member-in-declC*:  $G \vdash m \text{ member-in } C \implies G \vdash m \text{ member-in } (\text{declclass } m)$   
 $\langle \text{proof} \rangle$

**lemma** *dyn-accessible-from-commonD*:  $G \vdash m \text{ in } C \text{ dyn-accessible-from } S$   
 $\implies G \vdash m \text{ member-in } C$   
 $\langle \text{proof} \rangle$

**lemma** *no-Private-stat-override*:

$\llbracket G \vdash \text{new overrides}_S \text{ old} \rrbracket \implies \text{accmodi old} \neq \text{Private}$   
 $\langle \text{proof} \rangle$

**lemma** *no-Private-override*:  $\llbracket G \vdash \text{new overrides old} \rrbracket \implies \text{accmodi old} \neq \text{Private}$   
 $\langle \text{proof} \rangle$

**lemma** *permits-acc-inheritance*:  
 $\llbracket G \vdash m \text{ in } \text{statC} \text{ permits-acc-from } \text{accC}; G \vdash \text{dynC} \preceq_C \text{statC} \rrbracket \implies G \vdash m \text{ in } \text{dynC} \text{ permits-acc-from } \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *permits-acc-static-declC*:  
 $\llbracket G \vdash m \text{ in } C \text{ permits-acc-from } \text{accC}; G \vdash m \text{ member-in } C; \text{is-static } m \rrbracket \implies G \vdash m \text{ in } (\text{declclass } m) \text{ permits-acc-from } \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *dyn-accessible-from-static-declC*:  
**assumes**  $\text{acc-C}: G \vdash m \text{ in } C \text{ dyn-accessible-from } \text{accC}$  **and**  
 $\text{static: is-static } m$   
**shows**  $G \vdash m \text{ in } (\text{declclass } m) \text{ dyn-accessible-from } \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *field-accessible-fromD*:  
 $\llbracket G \vdash \text{membr of } C \text{ accessible-from } \text{accC}; \text{is-field } \text{membr} \rrbracket$   
 $\implies G \vdash \text{membr member-of } C \wedge$   
 $G \vdash (\text{Class } C) \text{ accessible-in } (\text{pid } \text{accC}) \wedge$   
 $G \vdash \text{membr in } C \text{ permits-acc-from } \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *field-accessible-from-permits-acc-inheritance*:  
 $\llbracket G \vdash \text{membr of } \text{statC} \text{ accessible-from } \text{accC}; \text{is-field } \text{membr}; G \vdash \text{dynC} \preceq_C \text{statC} \rrbracket$   
 $\implies G \vdash \text{membr in } \text{dynC} \text{ permits-acc-from } \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *accessible-fieldD*:  
 $\llbracket G \vdash \text{membr of } C \text{ accessible-from } \text{accC}; \text{is-field } \text{membr} \rrbracket$   
 $\implies G \vdash \text{membr member-of } C \wedge$   
 $G \vdash (\text{Class } C) \text{ accessible-in } (\text{pid } \text{accC}) \wedge$   
 $G \vdash \text{membr in } C \text{ permits-acc-from } \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *member-of-Private*:  
 $\llbracket G \vdash m \text{ member-of } C; \text{accmodi } m = \text{Private} \rrbracket \implies \text{declclass } m = C$   
 $\langle \text{proof} \rangle$

**lemma** *member-of-subclseq-declC*:

$G \vdash m \text{ member-of } C \implies G \vdash C \preceq_C \text{ declclass } m$   
 ⟨proof⟩

**lemma** *member-of-inheritance*:

**assumes**  $m$ :  $G \vdash m \text{ member-of } D$  **and**  
 $\text{subclseq-}D\text{-}C$ :  $G \vdash D \preceq_C C$  **and**  
 $\text{subclseq-}C\text{-}m$ :  $G \vdash C \preceq_C \text{ declclass } m$  **and**  
 $ws$ :  $ws\text{-prog } G$   
**shows**  $G \vdash m \text{ member-of } C$   
 ⟨proof⟩

**lemma** *member-of-subcls*:

**assumes**  $\text{old}$ :  $G \vdash \text{old member-of } C$  **and**  
 $\text{new}$ :  $G \vdash \text{new member-of } D$  **and**  
 $\text{eqid}$ :  $\text{memberid new} = \text{memberid old}$  **and**  
 $\text{subclseq-}D\text{-}C$ :  $G \vdash D \preceq_C C$  **and**  
 $\text{subcls-new-old}$ :  $G \vdash \text{declclass new} \prec_C \text{ declclass old}$  **and**  
 $ws$ :  $ws\text{-prog } G$   
**shows**  $G \vdash D \prec_C C$   
 ⟨proof⟩

**corollary** *member-of-overrides-subcls*:

$\llbracket G \vdash \text{Methd sig old member-of } C; G \vdash \text{Methd sig new member-of } D; G \vdash D \preceq_C C;$   
 $G, \text{sig} \vdash \text{new overrides old}; ws\text{-prog } G \rrbracket$   
 $\implies G \vdash D \prec_C C$   
 ⟨proof⟩

**corollary** *member-of-stat-overrides-subcls*:

$\llbracket G \vdash \text{Methd sig old member-of } C; G \vdash \text{Methd sig new member-of } D; G \vdash D \preceq_C C;$   
 $G, \text{sig} \vdash \text{new overrides}_S \text{ old}; ws\text{-prog } G \rrbracket$   
 $\implies G \vdash D \prec_C C$   
 ⟨proof⟩

**lemma** *inherited-field-access*:

**assumes**  $\text{stat-acc}$ :  $G \vdash \text{membr of statC accessible-from accC}$  **and**  
 $\text{is-field}$ :  $\text{is-field membr}$  **and**  
 $\text{subclseq}$ :  $G \vdash \text{dynC} \preceq_C \text{ statC}$   
**shows**  $G \vdash \text{membr in dynC dyn-accessible-from accC}$   
 ⟨proof⟩

**lemma** *accessible-inheritance*:

**assumes**  $\text{stat-acc}$ :  $G \vdash m \text{ of statC accessible-from accC}$  **and**  
 $\text{subclseq}$ :  $G \vdash \text{dynC} \preceq_C \text{ statC}$  **and**  
 $\text{member-dynC}$ :  $G \vdash m \text{ member-of dynC}$  **and**  
 $\text{dynC-acc}$ :  $G \vdash (\text{Class dynC}) \text{ accessible-in } (\text{pid accC})$   
**shows**  $G \vdash m \text{ of dynC accessible-from accC}$   
 ⟨proof⟩

## fields and methods

### types

$f_{\text{spec}} = v_{\text{name}} \times q_{\text{name}}$

**translations**

$$fspec \leq (type) \text{ vname} \times \text{qname}$$
**constdefs**

$$imethds:: \text{prog} \Rightarrow \text{qname} \Rightarrow (\text{sig}, \text{qname} \times \text{mhead}) \text{ tables}$$

$$imethds \ G \ I$$

$$\equiv \text{iface-rec } (G, I) \\ (\lambda I \ i \ ts. (\text{Un-tables } ts) \oplus \oplus \\ (o2s \circ \text{table-of } (\text{map } (\lambda (s, m). (s, I, m)) (imethods \ i))))$$

methods of an interface, with overriding and inheritance, cf. 9.2

**constdefs**

$$accimethds:: \text{prog} \Rightarrow \text{pname} \Rightarrow \text{qname} \Rightarrow (\text{sig}, \text{qname} \times \text{mhead}) \text{ tables}$$

$$accimethds \ G \ \text{pack } I$$

$$\equiv \text{if } G \vdash \text{Iface } I \text{ accessible-in } \text{pack} \\ \text{then } imethds \ G \ I \\ \text{else } \lambda k. \{\}$$

only returns imethds if the interface is accessible

**constdefs**

$$methd:: \text{prog} \Rightarrow \text{qname} \Rightarrow (\text{sig}, \text{qname} \times \text{methd}) \text{ table}$$

$$methd \ G \ C$$

$$\equiv \text{class-rec } (G, C) \ \text{empty} \\ (\lambda C \ c \ \text{subcls-mthds}. \\ \text{filter-tab } (\lambda \text{sig } m. G \vdash C \text{ inherits method sig } m) \\ \text{subcls-mthds} \\ ++ \\ \text{table-of } (\text{map } (\lambda (s, m). (s, C, m)) (\text{methods } c)))$$

$\text{methd } G \ C$ : methods of a class  $C$  (statically visible from  $C$ ), with inheritance and hiding cf. 8.4.6; Overriding is captured by *dynmethd*. Every new method with the same signature coalesces the method of a superclass.

**constdefs**

$$accmethd:: \text{prog} \Rightarrow \text{qname} \Rightarrow \text{qname} \Rightarrow (\text{sig}, \text{qname} \times \text{methd}) \text{ table}$$

$$accmethd \ G \ S \ C$$

$$\equiv \text{filter-tab } (\lambda \text{sig } m. G \vdash \text{method sig } m \text{ of } C \text{ accessible-from } S) \\ (\text{methd } G \ C)$$

$\text{accmethd } G \ S \ C$ : only those methods of  $\text{methd } G \ C$ , accessible from  $S$

Note the class component in the accessibility filter. The class where method  $m$  is declared (*declC*) isn't necessarily accessible from the current scope  $S$ . The method can be made accessible through inheritance, too. So we must test accessibility of method  $m$  of class  $C$  (not *declclass m*)

**constdefs**

$$\text{dynmethd}:: \text{prog} \Rightarrow \text{qname} \Rightarrow \text{qname} \Rightarrow (\text{sig}, \text{qname} \times \text{methd}) \text{ table}$$

$$\text{dynmethd } G \ \text{statC} \ \text{dynC}$$

$$\equiv \lambda \text{sig}. \\ (\text{if } G \vdash \text{dynC} \preceq_C \text{statC} \\ \text{then } (\text{case } \text{methd } G \ \text{statC} \ \text{sig} \text{ of} \\ \text{None} \Rightarrow \text{None} \\ | \text{Some } \text{statM} \\ \Rightarrow (\text{class-rec } (G, \text{dynC}) \ \text{empty} \\ (\lambda C \ c \ \text{subcls-mthds}. \\ \text{subcls-mthds} \\ ++ \\ \text{filter-tab}$$

```

      (λ - dynM. G, sig ⊢ dynM overrides statM ∨ dynM = statM)
      (methd G C) ))
    ) sig
  )
else None)

```

*dynmethd G statC dynC*: dynamic method lookup of a reference with dynamic class *dynC* and static class *statC*

Note some kind of duality between *methd* and *dynmethd* in the *class-rec* arguments. Whereas *methd* filters the subclass methods (to get only the inherited ones), *dynmethd* filters the new methods (to get only those methods which actually override the methods of the static class)

#### constdefs

```

dynimethd:: prog ⇒ qname ⇒ qname ⇒ (sig, qname × methd) table
dynimethd G I dynC
  ≡ λ sig. if imethds G I sig ≠ {}
           then methd G dynC sig
           else dynmethd G Object dynC sig

```

*dynimethd G I dynC*: dynamic method lookup of a reference with dynamic class *dynC* and static interface type *I*

When calling an interface method, we must distinguish if the method signature was defined in the interface or if it must be an Object method in the other case. If it was an interface method we search the class hierarchy starting at the dynamic class of the object up to Object to find the first matching method (*methd*). Since all interface methods have public access the method can't be coalesced due to some odd visibility effects like in case of *dynmethd*. The method will be inherited or overridden in all classes from the first class implementing the interface down to the actual dynamic class.

#### constdefs

```

dynlookup:: prog ⇒ ref-ty ⇒ qname ⇒ (sig, qname × methd) table
dynlookup G statT dynC
  ≡ (case statT of
    | NullT      ⇒ empty
    | IfaceT I    ⇒ dynimethd G I      dynC
    | ClassT statC ⇒ dynmethd G statC dynC
    | ArrayT ty   ⇒ dynmethd G Object dynC)

```

*dynlookup G statT dynC*: dynamic lookup of a method within the static reference type *statT* and the dynamic class *dynC*. In a wellformd context *statT* will not be *NullT* and in case *statT* is an array type, *dynC*=Object

#### constdefs

```

fields:: prog ⇒ qname ⇒ ((vname × qname) × field) list
fields G C
  ≡ class-rec (G, C) [] (λ C c ts. map (λ (n, t). ((n, C), t)) (cfields c) @ ts)

```

*DeclConcepts.fields G C* list of fields of a class, including all the fields of the superclasses (private, inherited and hidden ones) not only the accessible ones (an instance of a object allocates all these fields)

#### constdefs

```

accfield:: prog ⇒ qname ⇒ qname ⇒ (vname, qname × field) table
accfield G S C
  ≡ let field-tab = table-of((map (λ ((n, d), f). (n, (d, f)))) (fields G C))
    in filter-tab (λ n (declC, f). G ⊢ (declC, fdecl (n, f)) of C accessible-from S)
    field-tab

```

*accfield G C S*: fields of a class *C* which are accessible from scope of class *S* with inheritance and hiding, cf. 8.3



note the class component in the accessibility filter (see also *methd*). The class declaring field  $f$  (*declC*) isn't necessarily accessible from scope  $S$ . The field can be made visible through inheritance, too. So we must test accessibility of field  $f$  of class  $C$  (not *declclass*  $f$ )

### constdefs

$is-methd :: prog \Rightarrow qname \Rightarrow sig \Rightarrow bool$   
 $is-methd\ G \equiv \lambda C\ sig. is-class\ G\ C \wedge methd\ G\ C\ sig \neq None$

**constdefs**  $efname :: ((vname \times qname) \times field) \Rightarrow (vname \times qname)$   
 $efname \equiv fst$

**lemma**  $efname-simp[simp]: efname\ (n, f) = n$   
 $\langle proof \rangle$

## 19 imethds

**lemma**  $imethds-rec: \llbracket iface\ G\ I = Some\ i; ws-prog\ G \rrbracket \implies$   
 $imethds\ G\ I = Un-tables\ ((\lambda J. imethds\ G\ J) 'set\ (isuperIfs\ i)) \oplus \oplus$   
 $(o2s \circ table-of\ (map\ (\lambda(s, mh). (s, I, mh))\ (imethods\ i)))$   
 $\langle proof \rangle$

**lemma**  $imethds-norec:$   
 $\llbracket iface\ G\ md = Some\ i; ws-prog\ G; table-of\ (imethods\ i)\ sig = Some\ mh \rrbracket \implies$   
 $(md, mh) \in imethds\ G\ md\ sig$   
 $\langle proof \rangle$

**lemma**  $imethds-declI: \llbracket m \in imethds\ G\ I\ sig; ws-prog\ G; is-iface\ G\ I \rrbracket \implies$   
 $(\exists i. iface\ G\ (decliface\ m) = Some\ i \wedge$   
 $table-of\ (imethods\ i)\ sig = Some\ (methd\ m)) \wedge$   
 $(I, decliface\ m) \in (subint1\ G)^* \wedge m \in imethds\ G\ (decliface\ m)\ sig$   
 $\langle proof \rangle$

**lemma**  $imethds-cases\ [consumes\ 3, case-names\ NewMethod\ InheritedMethod]:$   
**assumes**  $im: im \in imethds\ G\ I\ sig$  **and**  
 $ifI: iface\ G\ I = Some\ i$  **and**  
 $ws: ws-prog\ G$  **and**  
 $hyp-new: table-of\ (map\ (\lambda(s, mh). (s, I, mh))\ (imethods\ i))\ sig$   
 $= Some\ im \implies P$  **and**  
 $hyp-inh: \bigwedge J. \llbracket J \in set\ (isuperIfs\ i); im \in imethds\ G\ J\ sig \rrbracket \implies P$   
**shows**  $P$   
 $\langle proof \rangle$

## 20 accimethd

**lemma**  $accimethds-simp\ [simp]:$   
 $G \vdash Iface\ I\ accessible-in\ pack \implies accimethds\ G\ pack\ I = imethds\ G\ I$   
 $\langle proof \rangle$

**lemma**  $accimethdsD:$   
 $im \in accimethds\ G\ pack\ I\ sig$   
 $\implies im \in imethds\ G\ I\ sig \wedge G \vdash Iface\ I\ accessible-in\ pack$

$\langle \text{proof} \rangle$

**lemma** *accimethdsI*:

$\llbracket im \in imethds\ G\ I\ sig; G \vdash I \text{face } I \text{ accessible-in } pack \rrbracket$   
 $\implies im \in accimethds\ G\ pack\ I\ sig$   
 $\langle \text{proof} \rangle$

## 21 methd

**lemma** *methd-rec*:  $\llbracket class\ G\ C = Some\ c; ws-prog\ G \rrbracket \implies$   
 $methd\ G\ C$

$= (if\ C = Object$   
 $\quad then\ empty$   
 $\quad else\ filter-tab\ (\lambda sig\ m. G \vdash C\ inherits\ method\ sig\ m)$   
 $\quad \quad \quad (methd\ G\ (super\ c)))$   
 $\quad ++\ table-of\ (map\ (\lambda(s,m). (s,C,m))\ (methods\ c))$

$\langle \text{proof} \rangle$

**lemma** *methd-norec*:

$\llbracket class\ G\ declC = Some\ c; ws-prog\ G; table-of\ (methods\ c)\ sig = Some\ m \rrbracket$   
 $\implies methd\ G\ declC\ sig = Some\ (declC,\ m)$   
 $\langle \text{proof} \rangle$

**lemma** *methd-declC*:

$\llbracket methd\ G\ C\ sig = Some\ m; ws-prog\ G; is-class\ G\ C \rrbracket \implies$   
 $(\exists d. class\ G\ (declclass\ m) = Some\ d \wedge table-of\ (methods\ d)\ sig = Some\ (methd\ m)) \wedge$   
 $G \vdash C \preceq_C (declclass\ m) \wedge methd\ G\ (declclass\ m)\ sig = Some\ m$   
 $\langle \text{proof} \rangle$

**lemma** *methd-inheritedD*:

$\llbracket class\ G\ C = Some\ c; ws-prog\ G; methd\ G\ C\ sig = Some\ m \rrbracket$   
 $\implies (declclass\ m \neq C \longrightarrow G \vdash C\ inherits\ method\ sig\ m)$   
 $\langle \text{proof} \rangle$

**lemma** *methd-diff-cls*:

$\llbracket ws-prog\ G; is-class\ G\ C; is-class\ G\ D;$   
 $methd\ G\ C\ sig = m; methd\ G\ D\ sig = n; m \neq n$   
 $\rrbracket \implies C \neq D$   
 $\langle \text{proof} \rangle$

**lemma** *method-declared-inI*:

$\llbracket table-of\ (methods\ c)\ sig = Some\ m; class\ G\ C = Some\ c \rrbracket$   
 $\implies G \vdash mdecl\ (sig,m)\ declared-in\ C$   
 $\langle \text{proof} \rangle$

**lemma** *methd-declared-in-declclass*:

$\llbracket methd\ G\ C\ sig = Some\ m; ws-prog\ G; is-class\ G\ C \rrbracket$   
 $\implies G \vdash Methd\ sig\ m\ declared-in\ (declclass\ m)$   
 $\langle \text{proof} \rangle$

**lemma** *member-methd*:

**assumes** *member-of*:  $G \vdash \text{Methd sig } m \text{ member-of } C$  **and**  
 $ws: ws\text{-prog } G$   
**shows**  $\text{methd } G \ C \ \text{sig} = \text{Some } m$   
 $\langle \text{proof} \rangle$

**lemma** *finite-methd:ws-prog*  $G \implies \text{finite } \{\text{methd } G \ C \ \text{sig} \mid \text{sig } C. \text{is-class } G \ C\}$   
 $\langle \text{proof} \rangle$

**lemma** *finite-dom-methd*:

$\llbracket ws\text{-prog } G; \text{is-class } G \ C \rrbracket \implies \text{finite } (\text{dom } (\text{methd } G \ C))$   
 $\langle \text{proof} \rangle$

## 22 accmethd

**lemma** *accmethd-SomeD*:

$\text{accmethd } G \ S \ C \ \text{sig} = \text{Some } m$   
 $\implies \text{methd } G \ C \ \text{sig} = \text{Some } m \wedge G \vdash \text{method sig } m \text{ of } C \text{ accessible-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *accmethd-SomeI*:

$\llbracket \text{methd } G \ C \ \text{sig} = \text{Some } m; G \vdash \text{method sig } m \text{ of } C \text{ accessible-from } S \rrbracket$   
 $\implies \text{accmethd } G \ S \ C \ \text{sig} = \text{Some } m$   
 $\langle \text{proof} \rangle$

**lemma** *accmethd-declC*:

$\llbracket \text{accmethd } G \ S \ C \ \text{sig} = \text{Some } m; ws\text{-prog } G; \text{is-class } G \ C \rrbracket \implies$   
 $(\exists d. \text{class } G \ (\text{declclass } m) = \text{Some } d \wedge$   
 $\text{table-of } (\text{methods } d) \ \text{sig} = \text{Some } (\text{methd } m)) \wedge$   
 $G \vdash C \preceq_C (\text{declclass } m) \wedge \text{methd } G \ (\text{declclass } m) \ \text{sig} = \text{Some } m \wedge$   
 $G \vdash \text{method sig } m \text{ of } C \text{ accessible-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *finite-dom-accmethd*:

$\llbracket ws\text{-prog } G; \text{is-class } G \ C \rrbracket \implies \text{finite } (\text{dom } (\text{accmethd } G \ S \ C))$   
 $\langle \text{proof} \rangle$

## 23 dynmethd

**lemma** *dynmethd-rec*:

$\llbracket \text{class } G \ \text{dynC} = \text{Some } c; ws\text{-prog } G \rrbracket \implies$   
 $\text{dynmethd } G \ \text{statC} \ \text{dynC} \ \text{sig}$   
 $= (\text{if } G \vdash \text{dynC} \preceq_C \text{statC}$   
 $\text{then } (\text{case methd } G \ \text{statC} \ \text{sig of}$   
 $\text{None} \Rightarrow \text{None}$   
 $\mid \text{Some statM}$   
 $\Rightarrow (\text{case methd } G \ \text{dynC} \ \text{sig of}$   
 $\text{None} \Rightarrow \text{dynmethd } G \ \text{statC} \ (\text{super } c) \ \text{sig}$   
 $\mid \text{Some dynM} \Rightarrow$   
 $(\text{if } G, \text{sig} \vdash \text{dynM overrides statM} \vee \text{dynM} = \text{statM}$   
 $\text{then } \text{Some dynM}$

else (dynmethd G statC (super c) sig)

)))

else None)

(is -  $\implies$  -  $\implies$  ?Dynmethd-def dynC sig = ?Dynmethd-rec dynC c sig)

$\langle$ proof $\rangle$

**lemma** *dynmethd-C-C*:  $\llbracket \text{is-class } G \ C; \text{ws-prog } G \rrbracket$   
 $\implies \text{dynmethd } G \ C \ C \ \text{sig} = \text{methd } G \ C \ \text{sig}$   
 $\langle$ proof $\rangle$

**lemma** *dynmethdSomeD*:  
 $\llbracket \text{dynmethd } G \ \text{statC} \ \text{dynC} \ \text{sig} = \text{Some } \text{dynM}; \text{is-class } G \ \text{dynC}; \text{ws-prog } G \rrbracket$   
 $\implies G \vdash \text{dynC} \preceq_C \text{statC} \wedge (\exists \text{statM}. \text{methd } G \ \text{statC} \ \text{sig} = \text{Some } \text{statM})$   
 $\langle$ proof $\rangle$

**lemma** *dynmethd-Some-cases* [consumes 3, case-names Static Overrides]:  
**assumes**  $\text{dynM}: \text{dynmethd } G \ \text{statC} \ \text{dynC} \ \text{sig} = \text{Some } \text{dynM}$  **and**  
 $\text{is-clc-dynC}: \text{is-class } G \ \text{dynC}$  **and**  
 $\text{ws}: \text{ws-prog } G$  **and**  
 $\text{hyp-static}: \text{methd } G \ \text{statC} \ \text{sig} = \text{Some } \text{dynM} \implies P$  **and**  
 $\text{hyp-override}: \bigwedge \text{statM}. \llbracket \text{methd } G \ \text{statC} \ \text{sig} = \text{Some } \text{statM}; \text{dynM} \neq \text{statM};$   
 $G, \text{sig} \vdash \text{dynM} \text{ overrides } \text{statM} \rrbracket \implies P$   
**shows**  $P$   
 $\langle$ proof $\rangle$

**lemma** *no-override-in-Object*:  
**assumes**  $\text{dynM}: \text{dynmethd } G \ \text{statC} \ \text{dynC} \ \text{sig} = \text{Some } \text{dynM}$  **and**  
 $\text{is-clc-dynC}: \text{is-class } G \ \text{dynC}$  **and**  
 $\text{ws}: \text{ws-prog } G$  **and**  
 $\text{statM}: \text{methd } G \ \text{statC} \ \text{sig} = \text{Some } \text{statM}$  **and**  
 $\text{neq-dynM-statM}: \text{dynM} \neq \text{statM}$   
**shows**  $\text{dynC} \neq \text{Object}$   
 $\langle$ proof $\rangle$

**lemma** *dynmethd-Some-rec-cases* [consumes 3, case-names Static Override Recursion]:  
**assumes**  $\text{dynM}: \text{dynmethd } G \ \text{statC} \ \text{dynC} \ \text{sig} = \text{Some } \text{dynM}$  **and**  
 $\text{clsDynC}: \text{class } G \ \text{dynC} = \text{Some } c$  **and**  
 $\text{ws}: \text{ws-prog } G$  **and**  
 $\text{hyp-static}: \text{methd } G \ \text{statC} \ \text{sig} = \text{Some } \text{dynM} \implies P$  **and**  
 $\text{hyp-override}: \bigwedge \text{statM}. \llbracket \text{methd } G \ \text{statC} \ \text{sig} = \text{Some } \text{statM};$   
 $\text{methd } G \ \text{dynC} \ \text{sig} = \text{Some } \text{dynM}; \text{statM} \neq \text{dynM};$   
 $G, \text{sig} \vdash \text{dynM} \text{ overrides } \text{statM} \rrbracket \implies P$  **and**  
 $\text{hyp-recursion}: \llbracket \text{dynC} \neq \text{Object};$   
 $\text{dynmethd } G \ \text{statC} \ (\text{super } c) \ \text{sig} = \text{Some } \text{dynM} \rrbracket \implies P$   
**shows**  $P$   
 $\langle$ proof $\rangle$

**lemma** *dynmethd-declC*:  
 $\llbracket \text{dynmethd } G \ \text{statC} \ \text{dynC} \ \text{sig} = \text{Some } m;$   
 $\text{is-class } G \ \text{statC}; \text{ws-prog } G$

$\llbracket \implies$   
 $(\exists d. \text{class } G \text{ (declclass } m) = \text{Some } d \wedge \text{table-of (methods } d) \text{ sig} = \text{Some (methd } m)) \wedge$   
 $G \vdash \text{dynC} \preceq_C (\text{declclass } m) \wedge \text{methd } G \text{ (declclass } m) \text{ sig} = \text{Some } m$   
 $\langle \text{proof} \rangle$

**lemma** *methd-Some-dynmethd-Some*:

**assumes**  $\text{statM}: \text{methd } G \text{ statC sig} = \text{Some statM}$  **and**  
 $\text{subclseq}: G \vdash \text{dynC} \preceq_C \text{statC}$  **and**  
 $\text{is-clc-statC}: \text{is-class } G \text{ statC}$  **and**  
 $\text{ws}: \text{ws-prog } G$   
**shows**  $\exists \text{dynM}. \text{dynmethd } G \text{ statC dynC sig} = \text{Some dynM}$   
 $(\text{is } ?P \text{ dynC})$   
 $\langle \text{proof} \rangle$

**lemma** *dynmethd-cases* [consumes 4, case-names *Static Overrides*]:

**assumes**  $\text{statM}: \text{methd } G \text{ statC sig} = \text{Some statM}$  **and**  
 $\text{subclseq}: G \vdash \text{dynC} \preceq_C \text{statC}$  **and**  
 $\text{is-clc-statC}: \text{is-class } G \text{ statC}$  **and**  
 $\text{ws}: \text{ws-prog } G$  **and**  
 $\text{hyp-static}: \text{dynmethd } G \text{ statC dynC sig} = \text{Some statM} \implies P$  **and**  
 $\text{hyp-override}: \bigwedge \text{dynM}. \llbracket \text{dynmethd } G \text{ statC dynC sig} = \text{Some dynM};$   
 $\text{dynM} \neq \text{statM};$   
 $G, \text{sig} \vdash \text{dynM overrides statM} \rrbracket \implies P$   
**shows**  $P$   
 $\langle \text{proof} \rangle$

**lemma** *ws-dynmethd*:

**assumes**  $\text{statM}: \text{methd } G \text{ statC sig} = \text{Some statM}$  **and**  
 $\text{subclseq}: G \vdash \text{dynC} \preceq_C \text{statC}$  **and**  
 $\text{is-clc-statC}: \text{is-class } G \text{ statC}$  **and**  
 $\text{ws}: \text{ws-prog } G$   
**shows**  
 $\exists \text{dynM}. \text{dynmethd } G \text{ statC dynC sig} = \text{Some dynM} \wedge$   
 $\text{is-static dynM} = \text{is-static statM} \wedge G \vdash \text{resTy dynM} \preceq_{\text{resTy}} \text{statM}$   
 $\langle \text{proof} \rangle$

## 24 dynlookup

**lemma** *dynlookup-cases* [consumes 1, case-names *NullT IfaceT ClassT ArrayT*]:

$\llbracket \text{dynlookup } G \text{ statT dynC sig} = x;$   
 $\llbracket \text{statT} = \text{NullT} \quad ; \text{empty sig} = x \rrbracket \implies P;$   
 $\bigwedge I. \llbracket \text{statT} = \text{IfaceT } I \quad ; \text{dynmethd } G \text{ } I \quad \text{dynC sig} = x \rrbracket \implies P;$   
 $\bigwedge \text{statC}. \llbracket \text{statT} = \text{ClassT statC}; \text{dynmethd } G \text{ statC dynC sig} = x \rrbracket \implies P;$   
 $\bigwedge \text{ty}. \llbracket \text{statT} = \text{ArrayT ty} \quad ; \text{dynmethd } G \text{ Object dynC sig} = x \rrbracket \implies P$   
 $\rrbracket \implies P$   
 $\langle \text{proof} \rangle$

## 25 fields

**lemma** *fields-rec*:  $\llbracket \text{class } G \text{ } C = \text{Some } c; \text{ws-prog } G \rrbracket \implies$   
 $\text{fields } G \text{ } C = \text{map } (\lambda(fn, ft). ((fn, C), ft)) (\text{cfields } c) @$   
 $(\text{if } C = \text{Object then } [] \text{ else fields } G \text{ (super } c))$   
 $\langle \text{proof} \rangle$

**lemma** *fields-norec*:

$\llbracket \text{class } G \text{ fd} = \text{Some } c; \text{ws-prog } G; \text{table-of } (c\text{fields } c) \text{ fn} = \text{Some } f \rrbracket$   
 $\implies \text{table-of } (\text{fields } G \text{ fd}) (fn, fd) = \text{Some } f$   
 $\langle \text{proof} \rangle$

**lemma** *table-of-fieldsD*:

$\text{table-of } (\text{map } (\lambda(fn, ft). ((fn, C), ft)) (c\text{fields } c)) \text{efn} = \text{Some } f$   
 $\implies (\text{declclassf efn}) = C \wedge \text{table-of } (c\text{fields } c) (fname \text{efn}) = \text{Some } f$   
 $\langle \text{proof} \rangle$

**lemma** *fields-declC*:

$\llbracket \text{table-of } (\text{fields } G \text{ C}) \text{efn} = \text{Some } f; \text{ws-prog } G; \text{is-class } G \text{ C} \rrbracket \implies$   
 $(\exists d. \text{class } G (\text{declclassf efn}) = \text{Some } d \wedge$   
 $\text{table-of } (c\text{fields } d) (fname \text{efn}) = \text{Some } f) \wedge$   
 $G \vdash C \preceq_C (\text{declclassf efn}) \wedge \text{table-of } (\text{fields } G (\text{declclassf efn})) \text{efn} = \text{Some } f$   
 $\langle \text{proof} \rangle$

**lemma** *fields-emptyI*:  $\bigwedge y. \llbracket \text{ws-prog } G; \text{class } G \text{ C} = \text{Some } c; c\text{fields } c = [];$   
 $C \neq \text{Object} \longrightarrow \text{class } G (\text{super } c) = \text{Some } y \wedge \text{fields } G (\text{super } c) = [] \rrbracket \implies$   
 $\text{fields } G \text{ C} = []$   
 $\langle \text{proof} \rangle$

**lemma** *fields-mono-lemma*:

$\llbracket x \in \text{set } (\text{fields } G \text{ C}); G \vdash D \preceq_C C; \text{ws-prog } G \rrbracket$   
 $\implies x \in \text{set } (\text{fields } G \text{ D})$   
 $\langle \text{proof} \rangle$

**lemma** *ws-unique-fields-lemma*:

$\llbracket (efn, fd) \in \text{set } (\text{fields } G (\text{super } c)); fc \in \text{set } (c\text{fields } c); \text{ws-prog } G;$   
 $fname \text{efn} = fname fc; \text{declclassf efn} = C;$   
 $\text{class } G \text{ C} = \text{Some } c; C \neq \text{Object}; \text{class } G (\text{super } c) = \text{Some } d \rrbracket \implies R$   
 $\langle \text{proof} \rangle$

**lemma** *ws-unique-fields*:  $\llbracket \text{is-class } G \text{ C}; \text{ws-prog } G;$

$\bigwedge C c. \llbracket \text{class } G \text{ C} = \text{Some } c \rrbracket \implies \text{unique } (c\text{fields } c) \rrbracket \implies$   
 $\text{unique } (\text{fields } G \text{ C})$   
 $\langle \text{proof} \rangle$

## 26 accfield

**lemma** *accfield-fields*:

$\text{accfield } G \text{ S C fn} = \text{Some } f$   
 $\implies \text{table-of } (\text{fields } G \text{ C}) (fn, \text{declclass } f) = \text{Some } (fld f)$   
 $\langle \text{proof} \rangle$

**lemma** *accfield-declC-is-class*:

$\llbracket \text{is-class } G \text{ C}; \text{accfield } G \text{ S C en} = \text{Some } (fd, f); \text{ws-prog } G \rrbracket \implies$   
 $\text{is-class } G \text{ fd}$

$\langle \text{proof} \rangle$

**lemma** *accfield-accessibleD*:

$\text{accfield } G \ S \ C \ \text{fn} = \text{Some } f \implies G \vdash \text{Field } \text{fn } f \text{ of } C \text{ accessible-from } S$   
 $\langle \text{proof} \rangle$

## 27 is methd

**lemma** *is-methdI*:

$\llbracket \text{class } G \ C = \text{Some } y; \text{methd } G \ C \ \text{sig} = \text{Some } b \rrbracket \implies \text{is-methd } G \ C \ \text{sig}$   
 $\langle \text{proof} \rangle$

**lemma** *is-methdD*:

$\text{is-methd } G \ C \ \text{sig} \implies \text{class } G \ C \neq \text{None} \wedge \text{methd } G \ C \ \text{sig} \neq \text{None}$   
 $\langle \text{proof} \rangle$

**lemma** *finite-is-methd*:

$\text{ws-prog } G \implies \text{finite } (\text{Collect } (\text{split } (\text{is-methd } G)))$   
 $\langle \text{proof} \rangle$

## calculation of the superclasses of a class

**constdefs**

$\text{superclasses}:: \text{prog} \Rightarrow \text{qtname} \Rightarrow \text{qtname set}$   
 $\text{superclasses } G \ C \equiv \text{class-rec } (G, C) \ \{\}$   
 $\quad (\lambda \ C \ c \ \text{superclss}. \ (\text{if } C = \text{Object}$   
 $\quad \quad \text{then } \{\}$   
 $\quad \quad \text{else insert } (\text{super } c) \ \text{superclss}))$

**lemma** *superclasses-rec*:  $\llbracket \text{class } G \ C = \text{Some } c; \text{ws-prog } G \rrbracket \implies$

$\text{superclasses } G \ C$   
 $= (\text{if } (C = \text{Object})$   
 $\quad \text{then } \{\}$   
 $\quad \text{else insert } (\text{super } c) \ (\text{superclasses } G \ (\text{super } c)))$   
 $\langle \text{proof} \rangle$

**lemma** *superclasses-mono*:

$\llbracket G \vdash C \prec_C D; \text{ws-prog } G; \text{class } G \ C = \text{Some } c;$   
 $\quad \wedge \ C \ c. \llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object} \rrbracket \implies \exists \ sc. \text{class } G \ (\text{super } c) = \text{Some } sc;$   
 $\quad x \in \text{superclasses } G \ D$   
 $\rrbracket \implies x \in \text{superclasses } G \ C$   
 $\langle \text{proof} \rangle$

**lemma** *subclsEval*:

$\llbracket G \vdash C \prec_C D; \text{ws-prog } G; \text{class } G \ C = \text{Some } c;$   
 $\quad \wedge \ C \ c. \llbracket \text{class } G \ C = \text{Some } c; C \neq \text{Object} \rrbracket \implies \exists \ sc. \text{class } G \ (\text{super } c) = \text{Some } sc$   
 $\rrbracket \implies D \in \text{superclasses } G \ C$   
 $\langle \text{proof} \rangle$

**end**





## Chapter 11

# WellType

## 28 Well-typedness of Java programs

**theory** *WellType* **imports** *DeclConcepts* **begin**

improvements over Java Specification 1.0:

- methods of Object can be called upon references of interface or array type

simplifications:

- the type rules include all static checks on statements and expressions, e.g. definedness of names (of parameters, locals, fields, methods)

design issues:

- unified type judgment for statements, variables, expressions, expression lists
- statements are typed like expressions with dummy type Void
- the typing rules take an extra argument that is capable of determining the dynamic type of objects. Therefore, they can be used for both checking static types and determining runtime types in transition semantics.

**types** *lenv*

= (*lname*, *ty*) *table* — local variables, including This and Result

**record** *env* =

*prg*:: *prog* — program  
*cls*:: *qtname* — current package and class name  
*lcl*:: *lenv* — local environment

**translations**

*lenv* <= (*type*) (*lname*, *ty*) *table*  
*lenv* <= (*type*) *lname*  $\Rightarrow$  *ty option*  
*env* <= (*type*) (*prg*::*prog*, *cls*::*qtname*, *lcl*::*lenv*)  
*env* <= (*type*) (*prg*::*prog*, *cls*::*qtname*, *lcl*::*lenv*, ...::'*a*)

**syntax**

*pkg* :: *env*  $\Rightarrow$  *pname* — select the current package from an environment

**translations**

*pkg e* == *pid (cls e)*

### Static overloading: maximally specific methods

**types**

*emhead* = *ref-ty*  $\times$  *mhead*

— Some mnemonic selectors for *emhead*

**constdefs**

*declrefT* :: *emhead*  $\Rightarrow$  *ref-ty*  
*declrefT*  $\equiv$  *fst*

*mhd* :: *emhead*  $\Rightarrow$  *mhead*  
*mhd*  $\equiv$  *snd*

**lemma** *declrefT-simp[simp]:declrefT (r,m) = r*

$\langle \text{proof} \rangle$

**lemma**  $\text{mhd-simp}[simp]: \text{mhd } (r, m) = m$

$\langle \text{proof} \rangle$

**lemma**  $\text{static-mhd-simp}[simp]: \text{static } (\text{mhd } m) = \text{is-static } m$

$\langle \text{proof} \rangle$

**lemma**  $\text{mhd-resTy-simp } [simp]: \text{resTy } (\text{mhd } m) = \text{resTy } m$

$\langle \text{proof} \rangle$

**lemma**  $\text{mhd-is-static-simp } [simp]: \text{is-static } (\text{mhd } m) = \text{is-static } m$

$\langle \text{proof} \rangle$

**lemma**  $\text{mhd-accmodi-simp } [simp]: \text{accmodi } (\text{mhd } m) = \text{accmodi } m$

$\langle \text{proof} \rangle$

**consts**

$\text{cmheads} \quad :: \text{prog} \Rightarrow \text{qtname} \Rightarrow \text{qtname} \Rightarrow \text{sig} \Rightarrow \text{emhead set}$

$\text{Objectmheads} \quad :: \text{prog} \Rightarrow \text{qtname} \Rightarrow \text{sig} \Rightarrow \text{emhead set}$

$\text{accObjectmheads} :: \text{prog} \Rightarrow \text{qtname} \Rightarrow \text{ref-ty} \Rightarrow \text{sig} \Rightarrow \text{emhead set}$

$\text{mheads} \quad :: \text{prog} \Rightarrow \text{qtname} \Rightarrow \text{ref-ty} \Rightarrow \text{sig} \Rightarrow \text{emhead set}$

**defs**

$\text{cmheads-def}:$

$\text{cmheads } G \ S \ C$

$\equiv \lambda \text{sig}. (\lambda (Cls, \text{mthd}). (\text{ClassT } Cls, (\text{mhead } \text{mthd}))) \text{ 'o2s } (\text{accmethd } G \ S \ C \ \text{sig})$

$\text{Objectmheads-def}:$

$\text{Objectmheads } G \ S$

$\equiv \lambda \text{sig}. (\lambda (Cls, \text{mthd}). (\text{ClassT } Cls, (\text{mhead } \text{mthd})))$

$\text{ 'o2s } (\text{filter-tab } (\lambda \text{sig } m. \text{accmodi } m \neq \text{Private}) (\text{accmethd } G \ S \ \text{Object}) \ \text{sig})$

$\text{accObjectmheads-def}:$

$\text{accObjectmheads } G \ S \ T$

$\equiv \text{if } G \vdash \text{RefT } T \text{ accessible-in } (\text{pid } S)$

$\text{ then } \text{Objectmheads } G \ S$

$\text{ else } \lambda \text{sig}. \{\}$

**primrec**

$\text{mheads } G \ S \ \text{NullT} = (\lambda \text{sig}. \{\})$

$\text{mheads } G \ S \ (\text{IfaceT } I) = (\lambda \text{sig}. (\lambda (I, h). (\text{IfaceT } I, h)))$

$\text{ 'accimethds } G \ (\text{pid } S) \ I \ \text{sig} \cup$

$\text{ accObjectmheads } G \ S \ (\text{IfaceT } I) \ \text{sig})$

$\text{mheads } G \ S \ (\text{ClassT } C) = \text{cmheads } G \ S \ C$

$\text{mheads } G \ S \ (\text{ArrayT } T) = \text{accObjectmheads } G \ S \ (\text{ArrayT } T)$

**constdefs**

— applicable methods, cf. 15.11.2.1

$\text{appl-methds} \quad :: \text{prog} \Rightarrow \text{qtname} \Rightarrow \text{ref-ty} \Rightarrow \text{sig} \Rightarrow (\text{emhead} \times \text{ty list}) \ \text{set}$

$\text{appl-methds } G \ S \ rt \equiv \lambda \text{sig}.$

$\{(mh, pTs') \mid mh \ pTs'. \ mh \in \text{mheads } G \ S \ rt \ (\text{name} = \text{name } \text{sig}, \text{parTs} = pTs') \} \wedge$   
 $G \vdash (\text{parTs } \text{sig}) [\preceq] pTs' \}$

— more specific methods, cf. 15.11.2.2

$\text{more-spec} \quad :: \text{prog} \Rightarrow \text{emhead} \times \text{ty list} \Rightarrow \text{emhead} \times \text{ty list} \Rightarrow \text{bool}$

$\text{more-spec } G \equiv \lambda (mh, pTs). \lambda (mh', pTs'). \ G \vdash pTs [\preceq] pTs'$

— maximally specific methods, cf. 15.11.2.2

$max-spec \quad :: prog \Rightarrow qname \Rightarrow ref-ty \Rightarrow sig \Rightarrow (emhead \times ty\ list) \quad set$

$max-spec\ G\ S\ rt\ sig \equiv \{m. m \in appl-methds\ G\ S\ rt\ sig \wedge$   
 $(\forall m' \in appl-methds\ G\ S\ rt\ sig. more-spec\ G\ m'\ m \longrightarrow m' = m)\}$

**lemma** *max-spec2appl-meths*:

$x \in max-spec\ G\ S\ T\ sig \implies x \in appl-methds\ G\ S\ T\ sig$

$\langle proof \rangle$

**lemma** *appl-methsD*:  $(mh, pTs') \in appl-methds\ G\ S\ T\ (\llbracket name = mn, parTs = pTs \rrbracket) \implies$   
 $mh \in mheads\ G\ S\ T\ (\llbracket name = mn, parTs = pTs \rrbracket) \wedge G \vdash pTs [\preceq] pTs'$

$\langle proof \rangle$

**lemma** *max-spec2mheads*:

$max-spec\ G\ S\ rt\ (\llbracket name = mn, parTs = pTs \rrbracket) = insert\ (mh, pTs')\ A$   
 $\implies mh \in mheads\ G\ S\ rt\ (\llbracket name = mn, parTs = pTs \rrbracket) \wedge G \vdash pTs [\preceq] pTs'$

$\langle proof \rangle$

**constdefs**

$empty-dt \quad ::\ dyn-ty$

$empty-dt \equiv \lambda a. None$

$invmode \quad ::\ ('a::type)member-scheme \Rightarrow expr \Rightarrow inv-mode$

$invmode\ m\ e \equiv if\ is-static\ m$

$\quad then\ Static$

$\quad else\ if\ e = Super\ then\ SuperM\ else\ IntVir$

**lemma** *invmode-nonstatic* [simp]:

$invmode\ (\llbracket access = a, static = False, \dots = x \rrbracket)\ (Acc\ (LVar\ e)) = IntVir$

$\langle proof \rangle$

**lemma** *invmode-Static-eq* [simp]:  $(invmode\ m\ e = Static) = is-static\ m$

$\langle proof \rangle$

**lemma** *invmode-IntVir-eq*:  $(invmode\ m\ e = IntVir) = (\neg(is-static\ m) \wedge e \neq Super)$

$\langle proof \rangle$

**lemma** *Null-staticD*:

$a' = Null \longrightarrow (is-static\ m) \implies invmode\ m\ e = IntVir \longrightarrow a' \neq Null$

$\langle proof \rangle$

## Typing for unary operations

**consts** *unop-type*  $:: unop \Rightarrow prim-ty$

**primrec**

*unop-type* *UPlus*  $= Integer$

*unop-type UMinus* = *Integer*  
*unop-type UBitNot* = *Integer*  
*unop-type UNot* = *Boolean*

**consts** *wt-unop* :: *unop*  $\Rightarrow$  *ty*  $\Rightarrow$  *bool*

**primrec**

*wt-unop UPlus* *t* = (*t* = *PrimT Integer*)  
*wt-unop UMinus* *t* = (*t* = *PrimT Integer*)  
*wt-unop UBitNot* *t* = (*t* = *PrimT Integer*)  
*wt-unop UNot* *t* = (*t* = *PrimT Boolean*)

## Typing for binary operations

**consts** *binop-type* :: *binop*  $\Rightarrow$  *prim-ty*

**primrec**

*binop-type Mul* = *Integer*  
*binop-type Div* = *Integer*  
*binop-type Mod* = *Integer*  
*binop-type Plus* = *Integer*  
*binop-type Minus* = *Integer*  
*binop-type LShift* = *Integer*  
*binop-type RShift* = *Integer*  
*binop-type RShiftU* = *Integer*  
*binop-type Less* = *Boolean*  
*binop-type Le* = *Boolean*  
*binop-type Greater* = *Boolean*  
*binop-type Ge* = *Boolean*  
*binop-type Eq* = *Boolean*  
*binop-type Neq* = *Boolean*  
*binop-type BitAnd* = *Integer*  
*binop-type And* = *Boolean*  
*binop-type BitXor* = *Integer*  
*binop-type Xor* = *Boolean*  
*binop-type BitOr* = *Integer*  
*binop-type Or* = *Boolean*  
*binop-type CondAnd* = *Boolean*  
*binop-type CondOr* = *Boolean*

**consts** *wt-binop* :: *prog*  $\Rightarrow$  *binop*  $\Rightarrow$  *ty*  $\Rightarrow$  *ty*  $\Rightarrow$  *bool*

**primrec**

*wt-binop G Mul* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Div* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Mod* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Plus* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Minus* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G LShift* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G RShift* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G RShiftU* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Less* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Le* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Greater* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Ge* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Eq* *t1 t2* = ( $G \vdash t1 \preceq t2 \vee G \vdash t2 \preceq t1$ )  
*wt-binop G Neq* *t1 t2* = ( $G \vdash t1 \preceq t2 \vee G \vdash t2 \preceq t1$ )  
*wt-binop G BitAnd* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G And* *t1 t2* = ((*t1* = *PrimT Boolean*)  $\wedge$  (*t2* = *PrimT Boolean*))  
*wt-binop G BitXor* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))  
*wt-binop G Xor* *t1 t2* = ((*t1* = *PrimT Boolean*)  $\wedge$  (*t2* = *PrimT Boolean*))  
*wt-binop G BitOr* *t1 t2* = ((*t1* = *PrimT Integer*)  $\wedge$  (*t2* = *PrimT Integer*))

$wt\text{-}binop\ G\ Or\quad t1\ t2 = ((t1 = PrimT\ Boolean) \wedge (t2 = PrimT\ Boolean))$   
 $wt\text{-}binop\ G\ CondAnd\ t1\ t2 = ((t1 = PrimT\ Boolean) \wedge (t2 = PrimT\ Boolean))$   
 $wt\text{-}binop\ G\ CondOr\ t1\ t2 = ((t1 = PrimT\ Boolean) \wedge (t2 = PrimT\ Boolean))$

## Typing for terms

**types**  $tys = ty + ty\ list$

**translations**

$tys\ \leq = (type)\ ty + ty\ list$

## inductive

$wt :: env \Rightarrow dyn\text{-}ty \Rightarrow [term, tys] \Rightarrow bool\ (-, \models :: - [51, 51, 51, 51] 50)$   
**and**  $wt\text{-}stmt :: env \Rightarrow dyn\text{-}ty \Rightarrow stmt \Rightarrow bool\ (-, \models :: \surd [51, 51, 51] 50)$   
**and**  $ty\text{-}expr :: env \Rightarrow dyn\text{-}ty \Rightarrow [expr, ty] \Rightarrow bool\ (-, \models :: - [51, 51, 51, 51] 50)$   
**and**  $ty\text{-}var :: env \Rightarrow dyn\text{-}ty \Rightarrow [var, ty] \Rightarrow bool\ (-, \models :: - [51, 51, 51, 51] 50)$   
**and**  $ty\text{-}exprs :: env \Rightarrow dyn\text{-}ty \Rightarrow [expr\ list, ty\ list] \Rightarrow bool\ (-, \models :: \dot{-} [51, 51, 51, 51] 50)$

**where**

$E, dt \models s :: \surd \equiv E, dt \models In1r\ s :: Inl\ (PrimT\ Void)$   
 $| E, dt \models e :: -T \equiv E, dt \models In1l\ e :: Inl\ T$   
 $| E, dt \models e :: T \equiv E, dt \models In2\ e :: Inl\ T$   
 $| E, dt \models e :: \dot{-}T \equiv E, dt \models In3\ e :: Inr\ T$

— well-typed statements

$| Skip: \quad E, dt \models Skip :: \surd$

$| Expr: \llbracket E, dt \models e :: -T \rrbracket \Longrightarrow$   
 $E, dt \models Expr\ e :: \surd$

— cf. 14.6

$| Lab: \quad E, dt \models c :: \surd \Longrightarrow$   
 $E, dt \models l \bullet c :: \surd$

$| Comp: \llbracket E, dt \models c1 :: \surd;$   
 $\quad \quad \quad \quad \quad \quad \quad \quad E, dt \models c2 :: \surd \rrbracket \Longrightarrow$   
 $E, dt \models c1 ;; c2 :: \surd$

— cf. 14.8

$| If: \llbracket E, dt \models e :: -PrimT\ Boolean;$   
 $\quad \quad \quad \quad \quad \quad \quad \quad E, dt \models c1 :: \surd;$   
 $\quad \quad \quad \quad \quad \quad \quad \quad E, dt \models c2 :: \surd \rrbracket \Longrightarrow$   
 $E, dt \models If(e)\ c1\ Else\ c2 :: \surd$

— cf. 14.10

$| Loop: \llbracket E, dt \models e :: -PrimT\ Boolean;$   
 $\quad \quad \quad \quad \quad \quad \quad \quad E, dt \models c :: \surd \rrbracket \Longrightarrow$   
 $E, dt \models l \bullet While(e)\ c :: \surd$

— cf. 14.13, 14.15, 14.16

$| Jmp: \quad E, dt \models Jmp\ jump :: \surd$

— cf. 14.16

$| Throw: \llbracket E, dt \models e :: -Class\ tn;$   
 $\quad \quad \quad \quad \quad \quad \quad \quad prg\ E \vdash tn \preceq_C\ SXCpt\ Throwable \rrbracket \Longrightarrow$   
 $E, dt \models Throw\ e :: \surd$

— cf. 14.18

$| Try: \llbracket E, dt \models c1 :: \surd; prg\ E \vdash tn \preceq_C\ SXCpt\ Throwable;$   
 $\quad \quad \quad \quad \quad \quad \quad \quad lcl\ E\ (VName\ vn) = None; E\ (lcl := lcl\ E\ (VName\ vn \mapsto Class\ tn)) \rrbracket, dt \models c2 :: \surd \rrbracket$

$\implies$ 

$$E, dt \models \text{Try } c1 \text{ Catch}(tn \text{ } vn) \text{ } c2 :: \checkmark$$

— cf. 14.18

$$\begin{array}{l} | \text{Fin: } \llbracket E, dt \models c1 :: \checkmark; E, dt \models c2 :: \checkmark \rrbracket \implies \\ \quad E, dt \models c1 \text{ Finally } c2 :: \checkmark \end{array}$$

$$| \text{Init: } \llbracket \text{is-class } (prg \ E) \ C \rrbracket \implies$$

$$E, dt \models \text{Init } C :: \checkmark$$

— *Init* is created on the fly during evaluation (see Eval.thy). The class isn't necessarily accessible from the points *Init* is called. Therefor we only demand *is-class* and not *is-acc-class* here.

— well-typed expressions

— cf. 15.8

$$\begin{array}{l} | \text{NewC: } \llbracket \text{is-acc-class } (prg \ E) \ (pkg \ E) \ C \rrbracket \implies \\ \quad E, dt \models \text{NewC } C :: \text{Class } C \end{array}$$

— cf. 15.9

$$\begin{array}{l} | \text{NewA: } \llbracket \text{is-acc-type } (prg \ E) \ (pkg \ E) \ T; \\ \quad E, dt \models i :: \text{PrimT Integer} \rrbracket \implies \\ \quad E, dt \models \text{New } T[i] :: \text{---} T.[] \end{array}$$

— cf. 15.15

$$\begin{array}{l} | \text{Cast: } \llbracket E, dt \models e :: \text{---} T; \text{is-acc-type } (prg \ E) \ (pkg \ E) \ T'; \\ \quad prg \ E \vdash T \preceq? T' \rrbracket \implies \\ \quad E, dt \models \text{Cast } T' \ e :: \text{---} T' \end{array}$$

— cf. 15.19.2

$$\begin{array}{l} | \text{Inst: } \llbracket E, dt \models e :: \text{RefT } T; \text{is-acc-type } (prg \ E) \ (pkg \ E) \ (\text{RefT } T'); \\ \quad prg \ E \vdash \text{RefT } T \preceq? \text{RefT } T' \rrbracket \implies \\ \quad E, dt \models e \text{ InstOf } T' :: \text{--- PrimT Boolean} \end{array}$$

— cf. 15.7.1

$$\begin{array}{l} | \text{Lit: } \llbracket \text{typeof } dt \ x = \text{Some } T \rrbracket \implies \\ \quad E, dt \models \text{Lit } x :: \text{---} T \end{array}$$

$$\begin{array}{l} | \text{UnOp: } \llbracket E, dt \models e :: \text{---} T; wt\text{-unop } unop \ Te; T = \text{PrimT } (unop\text{-type } unop) \rrbracket \\ \implies \\ \quad E, dt \models \text{UnOp } unop \ e :: \text{---} T \end{array}$$

$$\begin{array}{l} | \text{BinOp: } \llbracket E, dt \models e1 :: \text{---} T1; E, dt \models e2 :: \text{---} T2; wt\text{-binop } (prg \ E) \ binop \ T1 \ T2; \\ \quad T = \text{PrimT } (binop\text{-type } binop) \rrbracket \\ \implies \\ \quad E, dt \models \text{BinOp } binop \ e1 \ e2 :: \text{---} T \end{array}$$

— cf. 15.10.2, 15.11.1

$$\begin{array}{l} | \text{Super: } \llbracket lcl \ E \ \text{This} = \text{Some } (\text{Class } C); C \neq \text{Object}; \\ \quad class \ (prg \ E) \ C = \text{Some } c \rrbracket \implies \\ \quad E, dt \models \text{Super} :: \text{--- Class } (super \ c) \end{array}$$

— cf. 15.13.1, 15.10.1, 15.12

$$\begin{array}{l} | \text{Acc: } \llbracket E, dt \models va :: T \rrbracket \implies \\ \quad E, dt \models \text{Acc } va :: \text{---} T \end{array}$$

— cf. 15.25, 15.25.1

$$\begin{array}{l} | \text{Ass: } \llbracket E, dt \models va :: T; va \neq \text{LVar } \text{This}; \\ \quad E, dt \models v :: \text{---} T'; \\ \quad prg \ E \vdash T' \preceq T \rrbracket \implies \\ \quad E, dt \models va := v :: \text{---} T' \end{array}$$

— cf. 15.24

| *Cond*:  $\llbracket E, dt \models e0 :: - \text{PrimT Boolean};$   
 $E, dt \models e1 :: - T1; E, dt \models e2 :: - T2;$   
 $\text{prg } E \vdash T1 \preceq T2 \wedge T = T2 \vee \text{prg } E \vdash T2 \preceq T1 \wedge T = T1 \rrbracket \implies$   
 $E, dt \models e0 \text{ ? } e1 : e2 :: - T$

— cf. 15.11.1, 15.11.2, 15.11.3

| *Call*:  $\llbracket E, dt \models e :: - \text{RefT statT};$   
 $E, dt \models ps :: \doteq pTs;$   
 $\text{max-spec } (\text{prg } E) (\text{cls } E) \text{ statT } (\text{name} = mn, \text{parTs} = pTs) \rrbracket$   
 $= \{((\text{statDeclT}, m), pTs')\}$   
 $\rrbracket \implies$   
 $E, dt \models \{ \text{cls } E, \text{statT}, \text{invmode } m \ e \} e \cdot mn(\{pTs'\}ps) :: - (\text{resTy } m)$

| *Methd*:  $\llbracket \text{is-class } (\text{prg } E) \ C;$   
 $\text{methd } (\text{prg } E) \ C \ \text{sig} = \text{Some } m;$   
 $E, dt \models \text{Body } (\text{declclass } m) (\text{stmt } (\text{mbody } (\text{mthd } m))) :: - T \rrbracket \implies$   
 $E, dt \models \text{Methd } C \ \text{sig} :: - T$

— The class  $C$  is the dynamic class of the method call (cf. Eval.thy). It hasn't got to be directly accessible from the current package  $\text{pkg } E$ . Only the static class must be accessible (enshured indirectly by *Call*). Note that  $l$  is just a dummy value. It is only used in the smallstep semantics. To proof typesafety directly for the smallstep semantics we would have to assume conformance of  $l$  here!

| *Body*:  $\llbracket \text{is-class } (\text{prg } E) \ D;$   
 $E, dt \models \text{blk} :: \checkmark;$   
 $(\text{lcl } E) \ \text{Result} = \text{Some } T;$   
 $\text{is-type } (\text{prg } E) \ T \rrbracket \implies$   
 $E, dt \models \text{Body } D \ \text{blk} :: - T$

— The class  $D$  implementing the method must not directly be accessible from the current package  $\text{pkg } E$ , but can also be indirectly accessible due to inheritance (enshured in *Call*) The result type hasn't got to be accessible in Java! (If it is not accessible you can only assign it to Object). For dummy value  $l$  see rule *Methd*.

— well-typed variables

— cf. 15.13.1

| *LVar*:  $\llbracket \text{lcl } E \ \text{vn} = \text{Some } T; \text{is-acc-type } (\text{prg } E) (\text{pkg } E) \ T \rrbracket \implies$   
 $E, dt \models \text{LVar } \text{vn} :: = T$

— cf. 15.10.1

| *FVar*:  $\llbracket E, dt \models e :: - \text{Class } C;$   
 $\text{accfield } (\text{prg } E) (\text{cls } E) \ C \ \text{fn} = \text{Some } (\text{statDeclC}, f) \rrbracket \implies$   
 $E, dt \models \{ \text{cls } E, \text{statDeclC}, \text{is-static } f \} e \cdot \text{fn} :: = (\text{type } f)$

— cf. 15.12

| *AVar*:  $\llbracket E, dt \models e :: - T.[];$   
 $E, dt \models i :: - \text{PrimT Integer} \rrbracket \implies$   
 $E, dt \models e.[i] :: = T$

— well-typed expression lists

— cf. 15.11.???

| *Nil*:  $E, dt \models [] :: \doteq []$

— cf. 15.11.???

| *Cons*:  $\llbracket E, dt \models e :: - T;$   
 $E, dt \models es :: \doteq Ts \rrbracket \implies$   
 $E, dt \models e \# es :: \doteq T \# Ts$



**syntax**

$-wt \quad :: env \Rightarrow [term, tys] \Rightarrow bool \ (-|-:- [51, 51, 51] \ 50)$   
 $-wt-stmt \quad :: env \Rightarrow stmt \Rightarrow bool \ (-|-:-<> [51, 51] \ 50)$   
 $-ty-expr \quad :: env \Rightarrow [expr, ty] \Rightarrow bool \ (-|-:- [51, 51, 51] \ 50)$   
 $-ty-var \quad :: env \Rightarrow [var, ty] \Rightarrow bool \ (-|-:- [51, 51, 51] \ 50)$   
 $-ty-exprs \quad :: env \Rightarrow [expr \ list, ty \ list] \Rightarrow bool \ (-|-:-\# [51, 51, 51] \ 50)$

**syntax** (*xsymbols*)

$-wt \quad :: env \Rightarrow [term, tys] \Rightarrow bool \ (-|-:- [51, 51, 51] \ 50)$   
 $-wt-stmt \quad :: env \Rightarrow stmt \Rightarrow bool \ (-|-:-\surd [51, 51] \ 50)$   
 $-ty-expr \quad :: env \Rightarrow [expr, ty] \Rightarrow bool \ (-|-:- [51, 51, 51] \ 50)$   
 $-ty-var \quad :: env \Rightarrow [var, ty] \Rightarrow bool \ (-|-:- [51, 51, 51] \ 50)$   
 $-ty-exprs \quad :: env \Rightarrow [expr \ list, ty \ list] \Rightarrow bool \ (-|-:- [51, 51, 51] \ 50)$

**translations**

$E \vdash t :: T == E, empty-dt \models t :: T$   
 $E \vdash s :: \surd == E \vdash In1r \ s :: Inl \ (PrimT \ Void)$   
 $E \vdash e :: -T == E \vdash In1l \ e :: Inl \ T$   
 $E \vdash e :: =T == E \vdash In2 \ e :: Inl \ T$   
 $E \vdash e :: \doteq T == E \vdash In3 \ e :: Inr \ T$

**declare** *not-None-eq* [*simp del*]

**declare** *split-if* [*split del*] *split-if-asm* [*split del*]

**declare** *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]

*<ML>*

**inductive-cases** *wt-elim-cases* [*cases set*]:

$E, dt \models In2 \ (LVar \ vn) \quad :: T$   
 $E, dt \models In2 \ (\{accC, statDeclC, s\} e..fn) :: T$   
 $E, dt \models In2 \ (e.[i]) \quad :: T$   
 $E, dt \models In1l \ (NewC \ C) \quad :: T$   
 $E, dt \models In1l \ (New \ T'[i]) \quad :: T$   
 $E, dt \models In1l \ (Cast \ T' \ e) \quad :: T$   
 $E, dt \models In1l \ (e \ InstOf \ T') \quad :: T$   
 $E, dt \models In1l \ (Lit \ x) \quad :: T$   
 $E, dt \models In1l \ (UnOp \ unop \ e) \quad :: T$   
 $E, dt \models In1l \ (BinOp \ binop \ e1 \ e2) \quad :: T$   
 $E, dt \models In1l \ (Super) \quad :: T$   
 $E, dt \models In1l \ (Acc \ va) \quad :: T$   
 $E, dt \models In1l \ (Ass \ va \ v) \quad :: T$   
 $E, dt \models In1l \ (e0 \ ? \ e1 : e2) \quad :: T$   
 $E, dt \models In1l \ (\{accC, statT, mode\} e.mn(\{pT^{\wedge}p\})) :: T$   
 $E, dt \models In1l \ (Methd \ C \ sig) \quad :: T$   
 $E, dt \models In1l \ (Body \ D \ blk) \quad :: T$   
 $E, dt \models In3 \ \langle \rangle \quad :: Ts$   
 $E, dt \models In3 \ (e \# es) \quad :: Ts$   
 $E, dt \models In1r \ Skip \quad :: x$   
 $E, dt \models In1r \ (Expr \ e) \quad :: x$   
 $E, dt \models In1r \ (c1 ;; c2) \quad :: x$   
 $E, dt \models In1r \ (l \cdot c) \quad :: x$   
 $E, dt \models In1r \ (If(e) \ c1 \ Else \ c2) \quad :: x$   
 $E, dt \models In1r \ (l \cdot While(e) \ c) \quad :: x$   
 $E, dt \models In1r \ (Jmp \ jump) \quad :: x$   
 $E, dt \models In1r \ (Throw \ e) \quad :: x$   
 $E, dt \models In1r \ (Try \ c1 \ Catch(tn \ vn) \ c2) :: x$

$E, dt \models \text{In1r } (c1 \text{ Finally } c2) \quad ::x$   
 $E, dt \models \text{In1r } (\text{Init } C) \quad ::x$   
**declare** *not-None-eq* [simp]  
**declare** *split-if* [split] *split-if-asm* [split]  
**declare** *split-paired-All* [simp] *split-paired-Ex* [simp]  
 $\langle ML \rangle$

**lemma** *is-acc-class-is-accessible*:  
 $\text{is-acc-class } G \ P \ C \implies G \vdash (\text{Class } C) \text{ accessible-in } P$   
 $\langle \text{proof} \rangle$

**lemma** *is-acc-iface-is-iface*:  $\text{is-acc-iface } G \ P \ I \implies \text{is-iface } G \ I$   
 $\langle \text{proof} \rangle$

**lemma** *is-acc-iface-Iface-is-accessible*:  
 $\text{is-acc-iface } G \ P \ I \implies G \vdash (\text{Iface } I) \text{ accessible-in } P$   
 $\langle \text{proof} \rangle$

**lemma** *is-acc-type-is-type*:  $\text{is-acc-type } G \ P \ T \implies \text{is-type } G \ T$   
 $\langle \text{proof} \rangle$

**lemma** *is-acc-iface-is-accessible*:  
 $\text{is-acc-type } G \ P \ T \implies G \vdash T \text{ accessible-in } P$   
 $\langle \text{proof} \rangle$

**lemma** *wt-Methd-is-methd*:  
 $E \vdash \text{In1l } (\text{Methd } C \ \text{sig}) :: T \implies \text{is-methd } (\text{prg } E) \ C \ \text{sig}$   
 $\langle \text{proof} \rangle$

Special versions of some typing rules, better suited to pattern match the conclusion (no selectors in the conclusion)

**lemma** *wt-Call*:  
 $\llbracket E, dt \models e :: - \text{RefT } \text{statT}; E, dt \models ps :: \dot{=} pTs;$   
 $\text{max-spec } (\text{prg } E) \ (\text{cls } E) \ \text{statT} \ (\text{name} = mn, \text{parTs} = pTs) \rrbracket$   
 $= \{((\text{statDeclC}, m), pTs')\}; rT = (\text{resTy } m); \text{accC} = \text{cls } E;$   
 $\text{mode} = \text{invmode } m \ e \rrbracket \implies E, dt \models \{\text{accC}, \text{statT}, \text{mode}\} e \cdot mn(\{pTs'\}ps) :: - rT$   
 $\langle \text{proof} \rangle$

**lemma** *invocationTypeExpr-noClassD*:  
 $\llbracket E \vdash e :: - \text{RefT } \text{statT} \rrbracket$   
 $\implies (\forall \text{ statC}. \text{statT} \neq \text{ClassT } \text{statC}) \longrightarrow \text{invmode } m \ e \neq \text{SuperM}$   
 $\langle \text{proof} \rangle$

**lemma** *wt-Super*:  
 $\llbracket \text{lcl } E \ \text{This} = \text{Some } (\text{Class } C); C \neq \text{Object}; \text{class } (\text{prg } E) \ C = \text{Some } c; D = \text{super } c \rrbracket$   
 $\implies E, dt \models \text{Super} :: - \text{Class } D$   
 $\langle \text{proof} \rangle$

**lemma** *wt-FVar*:

$\llbracket E, dt \models e :: - \text{Class } C; \text{accfield } (\text{prg } E) (\text{cls } E) C \text{ fn} = \text{Some } (\text{statDeclC}, f);$   
 $\text{sf} = \text{is-static } f; fT = (\text{type } f); \text{accC} = \text{cls } E \rrbracket$   
 $\implies E, dt \models \{ \text{accC}, \text{statDeclC}, \text{sf} \} e.. \text{fn} :: fT$   
 $\langle \text{proof} \rangle$

**lemma** *wt-init* [iff]:  $E, dt \models \text{Init } C :: \surd = \text{is-class } (\text{prg } E) C$   
 $\langle \text{proof} \rangle$

**declare** *wt.Skip* [iff]

**lemma** *wt-StatRef*:  
 $\text{is-acc-type } (\text{prg } E) (\text{pkg } E) (\text{RefT } rt) \implies E \vdash \text{StatRef } rt :: - \text{RefT } rt$   
 $\langle \text{proof} \rangle$

**lemma** *wt-Inj-elim*:  
 $\bigwedge E. E, dt \models t :: U \implies \text{case } t \text{ of}$   
 $\quad \text{In1 } ec \Rightarrow (\text{case } ec \text{ of}$   
 $\quad \quad \text{Inl } e \Rightarrow \exists T. U = \text{Inl } T$   
 $\quad \quad | \text{Inr } s \Rightarrow U = \text{Inl } (\text{PrimT } \text{Void}))$   
 $\quad | \text{In2 } e \Rightarrow (\exists T. U = \text{Inl } T)$   
 $\quad | \text{In3 } e \Rightarrow (\exists T. U = \text{Inr } T)$   
 $\langle \text{proof} \rangle$

**lemma** *wt-expr-eq*:  $E, dt \models \text{In1l } t :: U = (\exists T. U = \text{Inl } T \wedge E, dt \models t :: - T)$   
 $\langle \text{proof} \rangle$

**lemma** *wt-var-eq*:  $E, dt \models \text{In2 } t :: U = (\exists T. U = \text{Inl } T \wedge E, dt \models t :: T)$   
 $\langle \text{proof} \rangle$

**lemma** *wt-exprs-eq*:  $E, dt \models \text{In3 } t :: U = (\exists Ts. U = \text{Inr } Ts \wedge E, dt \models t :: Ts)$   
 $\langle \text{proof} \rangle$

**lemma** *wt-stmt-eq*:  $E, dt \models \text{In1r } t :: U = (U = \text{Inl } (\text{PrimT } \text{Void}) \wedge E, dt \models t :: \surd)$   
 $\langle \text{proof} \rangle$

$\langle \text{ML} \rangle$

**lemma** *wt-elim-BinOp*:  
 $\llbracket E, dt \models \text{In1l } (\text{BinOp } \text{binop } e1 e2) :: T;$   
 $\bigwedge T1 T2 T3.$   
 $\llbracket E, dt \models e1 :: - T1; E, dt \models e2 :: - T2; \text{wt-binop } (\text{prg } E) \text{binop } T1 T2;$   
 $E, dt \models (\text{if } b \text{ then } \text{In1l } e2 \text{ else } \text{In1r } \text{Skip}) :: T3;$   
 $T = \text{Inl } (\text{PrimT } (\text{binop-type } \text{binop})) \rrbracket$   
 $\implies P \rrbracket$   
 $\implies P$   
 $\langle \text{proof} \rangle$

**lemma** *Inj-eq-lemma* [simp]:  
 $(\forall T. (\exists T'. T = \text{Inj } T' \wedge P T') \longrightarrow Q T) = (\forall T'. P T' \longrightarrow Q (\text{Inj } T'))$

$\langle proof \rangle$

**lemma** *single-valued-tys-lemma* [rule-format (no-asm)]:  
 $\forall S\ T. G \vdash S \preceq T \longrightarrow G \vdash T \preceq S \longrightarrow S = T \Longrightarrow E, dt \models t :: T \Longrightarrow$   
 $G = \text{prg } E \longrightarrow (\forall T'. E, dt \models t :: T' \longrightarrow T = T')$   
 $\langle proof \rangle$

**lemma** *single-valued-tys*:  
 $\text{ws-prog } (\text{prg } E) \Longrightarrow \text{single-valued } \{(t, T). E, dt \models t :: T\}$   
 $\langle proof \rangle$

**lemma** *typeof-empty-is-type* [rule-format (no-asm)]:  
 $\text{typeof } (\lambda a. \text{None})\ v = \text{Some } T \longrightarrow \text{is-type } G\ T$   
 $\langle proof \rangle$

**lemma** *typeof-is-type* [rule-format (no-asm)]:  
 $(\forall a. v \neq \text{Addr } a) \longrightarrow (\exists T. \text{typeof } dt\ v = \text{Some } T \wedge \text{is-type } G\ T)$   
 $\langle proof \rangle$

**end**

## Chapter 12

# DefiniteAssignment

## 29 Definite Assignment

**theory** *DefiniteAssignment* **imports** *WellType* **begin**

Definite Assignment Analysis (cf. 16)

The definite assignment analysis approximates the sets of local variables that will be assigned at a certain point of evaluation, and ensures that we will only read variables which previously were assigned. It should conform to the following idea: If the evaluation of a term completes normally (no abrupton (exception, break, continue, return) appeared) , the set of local variables calculated by the analysis is a subset of the variables that were actually assigned during evaluation.

To get more precise information about the sets of assigned variables the analysis includes the following optimisations:

- Inside of a while loop we also take care of the variables assigned before break statements, since the break causes the while loop to continue normally.
- For conditional statements we take care of constant conditions to statically determine the path of evaluation.
- Inside a distinct path of a conditional statements we know to which boolean value the condition has evaluated to, and so can retrieve more information about the variables assigned during evaluation of the boolean condition.

Since in our model of Java the return values of methods are stored in a local variable we also ensure that every path of (normal) evaluation will assign the result variable, or in the sense of real Java every path ends up in and return instruction.

Not covered yet:

- analysis of definite unassigned
- special treatment of final fields

### Correct nesting of jump statements

For definite assignment it becomes crucial, that jumps (break, continue, return) are nested correctly i.e. a continue jump is nested in a matching while statement, a break jump is nested in a proper label statement, a class initialiser does not terminate abruptly with a return. With this we can for example ensure that evaluation of an expression will never end up with a jump, since no breaks, continues or returns are allowed in an expression.

**consts** *jumpNestingOkS* :: *jump set*  $\Rightarrow$  *stmt*  $\Rightarrow$  *bool*

**primrec**

*jumpNestingOkS jmps (Skip)* = *True*

*jumpNestingOkS jmps (Expr e)* = *True*

*jumpNestingOkS jmps (j• s)* = *jumpNestingOkS* ( $\{j\} \cup$  *jmps*) *s*

*jumpNestingOkS jmps (c1;;c2)* = (*jumpNestingOkS jmps c1*  $\wedge$   
*jumpNestingOkS jmps c2*)

*jumpNestingOkS jmps (If(e) c1 Else c2)* = (*jumpNestingOkS jmps c1*  $\wedge$   
*jumpNestingOkS jmps c2*)

*jumpNestingOkS jmps (l• While(e) c)* = *jumpNestingOkS* ( $\{Cont\ l\} \cup$  *jmps*) *c*

— The label of the while loop only handles continue jumps. Breaks are only handled by *Lab*

*jumpNestingOkS jmps (Jmp j)* = ( $j \in$  *jmps*)

*jumpNestingOkS jmps (Throw e)* = *True*

*jumpNestingOkS jmps (Try c1 Catch(*C vn*) c2)* = (*jumpNestingOkS jmps c1*  $\wedge$   
*jumpNestingOkS jmps c2*)

*jumpNestingOkS jmps (c1 Finally c2)* = (*jumpNestingOkS jmps c1*  $\wedge$

*jumpNestingOkS jmps c2)*

*jumpNestingOkS jmps (Init C) = True*  
 — wellformedness of the program must enshure that for all initializers *jumpNestingOkS* holds  
 — Dummy analysis for intermediate smallstep term *FinA*  
*jumpNestingOkS jmps (FinA a c) = False*

**constdefs** *jumpNestingOk* :: *jump set*  $\Rightarrow$  *term*  $\Rightarrow$  *bool*  
*jumpNestingOk jmps t*  $\equiv$  (case *t* of  
     *In1 se*  $\Rightarrow$  (case *se* of  
         *Inl e*  $\Rightarrow$  *True*  
         | *Inr s*  $\Rightarrow$  *jumpNestingOkS jmps s*)  
     | *In2 v*  $\Rightarrow$  *True*  
     | *In3 es*  $\Rightarrow$  *True*)

**lemma** *jumpNestingOk-expr-simp* [*simp*]: *jumpNestingOk jmps (In1l e) = True*  
 <proof>

**lemma** *jumpNestingOk-expr-simp1* [*simp*]: *jumpNestingOk jmps <e::expr> = True*  
 <proof>

**lemma** *jumpNestingOk-stmt-simp* [*simp*]:  
*jumpNestingOk jmps (In1r s) = jumpNestingOkS jmps s*  
 <proof>

**lemma** *jumpNestingOk-stmt-simp1* [*simp*]:  
*jumpNestingOk jmps <s::stmt> = jumpNestingOkS jmps s*  
 <proof>

**lemma** *jumpNestingOk-var-simp* [*simp*]: *jumpNestingOk jmps (In2 v) = True*  
 <proof>

**lemma** *jumpNestingOk-var-simp1* [*simp*]: *jumpNestingOk jmps <v::var> = True*  
 <proof>

**lemma** *jumpNestingOk-expr-list-simp* [*simp*]: *jumpNestingOk jmps (In3 es) = True*  
 <proof>

**lemma** *jumpNestingOk-expr-list-simp1* [*simp*]:  
*jumpNestingOk jmps <es::expr list> = True*  
 <proof>

## Calculation of assigned variables for boolean expressions

### 30 Very restricted calculation fallback calculation

**consts** *the-LVar-name*:: *var*  $\Rightarrow$  *lname*  
**primrec**  
*the-LVar-name* (*LVar n*) = *n*

**consts** *assignsE* :: *expr*  $\Rightarrow$  *lname set*

$assignsV :: var \Rightarrow lname\ set$   
 $assignsEs :: expr\ list \Rightarrow lname\ set$

**primrec**

$assignsE\ (NewC\ c) = \{\}$   
 $assignsE\ (NewA\ t\ e) = assignsE\ e$   
 $assignsE\ (Cast\ t\ e) = assignsE\ e$   
 $assignsE\ (e\ InstOf\ r) = assignsE\ e$   
 $assignsE\ (Lit\ val) = \{\}$   
 $assignsE\ (UnOp\ unop\ e) = assignsE\ e$   
 $assignsE\ (BinOp\ binop\ e1\ e2) = (if\ binop=CondAnd\ \vee\ binop=CondOr$   
 $\quad then\ (assignsE\ e1)$   
 $\quad else\ (assignsE\ e1) \cup (assignsE\ e2))$   
 $assignsE\ (Super) = \{\}$   
 $assignsE\ (Acc\ v) = assignsV\ v$   
 $assignsE\ (v:=e)$   
 $\quad = (assignsV\ v) \cup (assignsE\ e) \cup$   
 $\quad (if\ \exists\ n.\ v=(LVar\ n)\ then\ \{the-LVar-name\ v\}$   
 $\quad \quad else\ \{\})$

$assignsE\ (b? e1 : e2) = (assignsE\ b) \cup ((assignsE\ e1) \cap (assignsE\ e2))$   
 $assignsE\ (\{accC, statT, mode\}objRef.mn(\{pTs\}args))$   
 $\quad = (assignsE\ objRef) \cup (assignsEs\ args)$

— Only dummy analysis for intermediate expressions *Method*, *Body*, *InsInitE* and *Callee*

$assignsE\ (Method\ C\ sig) = \{\}$   
 $assignsE\ (Body\ C\ s) = \{\}$   
 $assignsE\ (InsInitE\ s\ e) = \{\}$   
 $assignsE\ (Callee\ l\ e) = \{\}$

$assignsV\ (LVar\ n) = \{\}$   
 $assignsV\ (\{accC, statDeclC, stat\}objRef..fn) = assignsE\ objRef$   
 $assignsV\ (e1.[e2]) = assignsE\ e1 \cup assignsE\ e2$

$assignsEs\ [] = \{\}$   
 $assignsEs\ (e\#es) = assignsE\ e \cup assignsEs\ es$

**constdefs**  $assigns :: term \Rightarrow lname\ set$

$assigns\ t \equiv (case\ t\ of$   
 $\quad In1\ se \Rightarrow (case\ se\ of$   
 $\quad \quad Inl\ e \Rightarrow assignsE\ e$   
 $\quad \quad | Inr\ s \Rightarrow \{\})$   
 $\quad | In2\ v \Rightarrow assignsV\ v$   
 $\quad | In3\ es \Rightarrow assignsEs\ es)$

**lemma** *assigns-expr-simp* [simp]:  $assigns\ (In1l\ e) = assignsE\ e$   
 $\langle proof \rangle$

**lemma** *assigns-expr-simp1* [simp]:  $assigns\ (\langle e \rangle) = assignsE\ e$   
 $\langle proof \rangle$

**lemma** *assigns-stmt-simp* [simp]:  $assigns\ (In1r\ s) = \{\}$   
 $\langle proof \rangle$

**lemma** *assigns-stmt-simp1* [simp]:  $assigns\ (\langle s::stmt \rangle) = \{\}$   
 $\langle proof \rangle$



**lemma** *assigns-var-simp* [simp]: *assigns (In2 v) = assignsV v*  
 ⟨proof⟩

**lemma** *assigns-var-simp1* [simp]: *assigns (⟨v⟩) = assignsV v*  
 ⟨proof⟩

**lemma** *assigns-expr-list-simp* [simp]: *assigns (In3 es) = assignsEs es*  
 ⟨proof⟩

**lemma** *assigns-expr-list-simp1* [simp]: *assigns (⟨es⟩) = assignsEs es*  
 ⟨proof⟩

### 31 Analysis of constant expressions

**consts** *constVal* :: *expr*  $\Rightarrow$  *val option*

**primrec**

*constVal* (*NewC* *c*) = *None*

*constVal* (*NewA* *t* *e*) = *None*

*constVal* (*Cast* *t* *e*) = *None*

*constVal* (*Inst* *e* *r*) = *None*

*constVal* (*Lit* *val*) = *Some val*

*constVal* (*UnOp* *unop* *e*) = (case (*constVal* *e*) of  
     *None*  $\Rightarrow$  *None*  
     | *Some v*  $\Rightarrow$  *Some (eval-unop unop v)*)

*constVal* (*BinOp* *binop* *e1* *e2*) = (case (*constVal* *e1*) of  
     *None*  $\Rightarrow$  *None*  
     | *Some v1*  $\Rightarrow$  (case (*constVal* *e2*) of  
         *None*  $\Rightarrow$  *None*  
         | *Some v2*  $\Rightarrow$  *Some (eval-binop binop v1 v2)*)))

*constVal* (*Super*) = *None*

*constVal* (*Acc* *v*) = *None*

*constVal* (*Ass* *v* *e*) = *None*

*constVal* (*Cond* *b* *e1* *e2*) = (case (*constVal* *b*) of  
     *None*  $\Rightarrow$  *None*  
     | *Some bv*  $\Rightarrow$  (case *the-Bool* *bv* of  
         *True*  $\Rightarrow$  (case (*constVal* *e2*) of  
             *None*  $\Rightarrow$  *None*  
             | *Some v*  $\Rightarrow$  *constVal* *e1*)  
         | *False*  $\Rightarrow$  (case (*constVal* *e1*) of  
             *None*  $\Rightarrow$  *None*  
             | *Some v*  $\Rightarrow$  *constVal* *e2*))))

— Note that *constVal* (*Cond* *b* *e1* *e2*) is stricter as it could be. It requires that all tree expressions are constant even if we can decide which branch to choose, provided the constant value of *b*

*constVal* (*Call* *accC* *statT* *mode* *objRef* *mn* *pTs* *args*) = *None*

*constVal* (*Methd* *C* *sig*) = *None*

*constVal* (*Body* *C* *s*) = *None*

*constVal* (*InsInitE* *s* *e*) = *None*

*constVal* (*Callee* *l* *e*) = *None*

**lemma** *constVal-Some-induct* [consumes 1, case-names *Lit* *UnOp* *BinOp* *CondL* *CondR*]:

**assumes** *const*: *constVal* *e* = *Some v* **and**

*hyp-Lit*:  $\bigwedge v. P (Lit v)$  **and**

*hyp-UnOp*:  $\bigwedge unop e'. P e' \Longrightarrow P (UnOp unop e')$  **and**

*hyp-BinOp*:  $\bigwedge binop e1 e2. \llbracket P e1; P e2 \rrbracket \Longrightarrow P (BinOp binop e1 e2)$  **and**

$$\text{hyp-CondL: } \bigwedge b \text{ bv } e1 \ e2. \llbracket \text{constVal } b = \text{Some } bv; \text{the-Bool } bv; P \ b; P \ e1 \rrbracket$$

$$\implies P \ (b? \ e1 : e2) \text{ and}$$

$$\text{hyp-CondR: } \bigwedge b \text{ bv } e1 \ e2. \llbracket \text{constVal } b = \text{Some } bv; \neg \text{the-Bool } bv; P \ b; P \ e2 \rrbracket$$

$$\implies P \ (b? \ e1 : e2)$$

shows  $P \ e$   
 ⟨proof⟩

**lemma** *assignsE-const-simp*:  $\text{constVal } e = \text{Some } v \implies \text{assignsE } e = \{\}$   
 ⟨proof⟩

## 32 Main analysis for boolean expressions

Assigned local variables after evaluating the expression if it evaluates to a specific boolean value. If the expression cannot evaluate to a *Boolean* value UNIV is returned. If we expect true/false the opposite constant false/true will also lead to UNIV.

**consts** *assigns-if*::  $\text{bool} \Rightarrow \text{expr} \Rightarrow \text{lname set}$

**primrec**

$$\begin{aligned} \text{assigns-if } b \ (\text{NewC } c) &= \text{UNIV} \text{ — can never evaluate to Boolean} \\ \text{assigns-if } b \ (\text{NewA } t \ e) &= \text{UNIV} \text{ — can never evaluate to Boolean} \\ \text{assigns-if } b \ (\text{Cast } t \ e) &= \text{assigns-if } b \ e \\ \text{assigns-if } b \ (\text{Inst } e \ r) &= \text{assignsE } e \text{ — Inst has type Boolean but } e \text{ is a reference type} \\ \text{assigns-if } b \ (\text{Lit } val) &= (\text{if } val = \text{Bool } b \text{ then } \{\} \text{ else UNIV}) \\ \text{assigns-if } b \ (\text{UnOp } unop \ e) &= (\text{case constVal } (\text{UnOp } unop \ e) \text{ of} \\ &\quad \text{None} \Rightarrow (\text{if } unop = \text{UNot} \\ &\quad \text{then assigns-if } (\neg b) \ e \\ &\quad \text{else UNIV}) \\ &\quad | \text{Some } v \Rightarrow (\text{if } v = \text{Bool } b \\ &\quad \text{then } \{\} \\ &\quad \text{else UNIV})) \end{aligned}$$

$$\begin{aligned} \text{assigns-if } b \ (\text{BinOp } binop \ e1 \ e2) &= (\text{case constVal } (\text{BinOp } binop \ e1 \ e2) \text{ of} \\ &\quad \text{None} \Rightarrow (\text{if } binop = \text{CondAnd} \text{ then} \\ &\quad \quad (\text{case } b \text{ of} \\ &\quad \quad \quad \text{True} \Rightarrow \text{assigns-if True } e1 \cup \text{assigns-if True } e2 \\ &\quad \quad \quad | \text{False} \Rightarrow \text{assigns-if False } e1 \cap \\ &\quad \quad \quad \quad (\text{assigns-if True } e1 \cup \text{assigns-if False } e2)) \\ &\quad \text{else} \\ &\quad (\text{if } binop = \text{CondOr} \text{ then} \\ &\quad \quad (\text{case } b \text{ of} \\ &\quad \quad \quad \text{True} \Rightarrow \text{assigns-if True } e1 \cap \\ &\quad \quad \quad \quad (\text{assigns-if False } e1 \cup \text{assigns-if True } e2) \\ &\quad \quad \quad | \text{False} \Rightarrow \text{assigns-if False } e1 \cup \text{assigns-if False } e2) \\ &\quad \quad \text{else assignsE } e1 \cup \text{assignsE } e2)) \\ &\quad | \text{Some } v \Rightarrow (\text{if } v = \text{Bool } b \text{ then } \{\} \text{ else UNIV})) \end{aligned}$$

$$\begin{aligned} \text{assigns-if } b \ (\text{Super}) &= \text{UNIV} \text{ — can never evaluate to Boolean} \\ \text{assigns-if } b \ (\text{Acc } v) &= (\text{assignsV } v) \\ \text{assigns-if } b \ (v := e) &= (\text{assignsE } (\text{Ass } v \ e)) \\ \text{assigns-if } b \ (c? \ e1 : e2) &= (\text{assignsE } c) \cup \\ &\quad (\text{case } (\text{constVal } c) \text{ of} \\ &\quad \quad \text{None} \Rightarrow (\text{assigns-if } b \ e1) \cap \\ &\quad \quad \quad (\text{assigns-if } b \ e2) \\ &\quad \quad | \text{Some } bv \Rightarrow (\text{case the-Bool } bv \text{ of} \\ &\quad \quad \quad \text{True} \Rightarrow \text{assigns-if } b \ e1 \\ &\quad \quad \quad | \text{False} \Rightarrow \text{assigns-if } b \ e2)) \end{aligned}$$

$$\begin{aligned} \text{assigns-if } b \ (\{\text{accC}, \text{statT}, \text{mode}\} \text{objRef} \cdot \text{mn}(\{pTs\} \text{args})) &= \text{assignsE } (\{\text{accC}, \text{statT}, \text{mode}\} \text{objRef} \cdot \text{mn}(\{pTs\} \text{args})) \end{aligned}$$

— Only dummy analysis for intermediate expressions *Method*, *Body*, *InsInitE* and *Callee*

*assigns-if* *b* (*Method* *C* *sig*) = {}  
*assigns-if* *b* (*Body* *C* *s*) = {}  
*assigns-if* *b* (*InsInitE* *s* *e*) = {}  
*assigns-if* *b* (*Callee* *l* *e*) = {}

**lemma** *assigns-if-const-b-simp*:

**assumes** *boolConst*: *constVal* *e* = *Some* (*Bool* *b*) (**is** ?*Const* *b* *e*)  
**shows** *assigns-if* *b* *e* = {} (**is** ?*Ass* *b* *e*)  
 <proof>

**lemma** *assigns-if-const-not-b-simp*:

**assumes** *boolConst*: *constVal* *e* = *Some* (*Bool* *b*) (**is** ?*Const* *b* *e*)  
**shows** *assigns-if* ( $\neg$ *b*) *e* = *UNIV* (**is** ?*Ass* *b* *e*)  
 <proof>

### 33 Lifting set operations to range of tables (map to a set)

**constdefs**

*union-ts*:: ('*a*, '*b*) *tables*  $\Rightarrow$  ('*a*, '*b*) *tables*  $\Rightarrow$  ('*a*, '*b*) *tables*  
 ( $- \Rightarrow \cup$  - [67,67] 65)  
 $A \Rightarrow \cup B \equiv \lambda k. A\ k \cup B\ k$

**constdefs**

*intersect-ts*:: ('*a*, '*b*) *tables*  $\Rightarrow$  ('*a*, '*b*) *tables*  $\Rightarrow$  ('*a*, '*b*) *tables*  
 ( $- \Rightarrow \cap$  - [72,72] 71)  
 $A \Rightarrow \cap B \equiv \lambda k. A\ k \cap B\ k$

**constdefs**

*all-union-ts*:: ('*a*, '*b*) *tables*  $\Rightarrow$  '*b* *set*  $\Rightarrow$  ('*a*, '*b*) *tables*  
 (**infixl**  $\Rightarrow \cup_{\forall}$  40)  
 $A \Rightarrow \cup_{\forall} B \equiv \lambda k. A\ k \cup B$

### Binary union of tables

**lemma** *union-ts-iff* [*simp*]:  $(c \in (A \Rightarrow \cup B)\ k) = (c \in A\ k \vee c \in B\ k)$   
 <proof>

**lemma** *union-tsI1* [*elim?*]:  $c \in A\ k \Longrightarrow c \in (A \Rightarrow \cup B)\ k$   
 <proof>

**lemma** *union-tsI2* [*elim?*]:  $c \in B\ k \Longrightarrow c \in (A \Rightarrow \cup B)\ k$   
 <proof>

**lemma** *union-tsCI* [*intro!*]:  $(c \notin B\ k \Longrightarrow c \in A\ k) \Longrightarrow c \in (A \Rightarrow \cup B)\ k$   
 <proof>

**lemma** *union-tsE* [*elim!*]:

$\llbracket c \in (A \Rightarrow \cup B)\ k; (c \in A\ k \Longrightarrow P); (c \in B\ k \Longrightarrow P) \rrbracket \Longrightarrow P$   
 <proof>

### Binary intersection of tables

**lemma** *intersect-ts-iff* [simp]:  $c \in (A \Rightarrow \cap B) \ k = (c \in A \ k \wedge c \in B \ k)$   
 ⟨proof⟩

**lemma** *intersect-tsI* [intro!]:  $\llbracket c \in A \ k; c \in B \ k \rrbracket \Longrightarrow c \in (A \Rightarrow \cap B) \ k$   
 ⟨proof⟩

**lemma** *intersect-tsD1*:  $c \in (A \Rightarrow \cap B) \ k \Longrightarrow c \in A \ k$   
 ⟨proof⟩

**lemma** *intersect-tsD2*:  $c \in (A \Rightarrow \cap B) \ k \Longrightarrow c \in B \ k$   
 ⟨proof⟩

**lemma** *intersect-tsE* [elim!]:  
 $\llbracket c \in (A \Rightarrow \cap B) \ k; \llbracket c \in A \ k; c \in B \ k \rrbracket \Longrightarrow P \rrbracket \Longrightarrow P$   
 ⟨proof⟩

### All-Union of tables and set

**lemma** *all-union-ts-iff* [simp]:  $(c \in (A \Rightarrow \cup B) \ k) = (c \in A \ k \vee c \in B)$   
 ⟨proof⟩

**lemma** *all-union-tsI1* [elim?]:  $c \in A \ k \Longrightarrow c \in (A \Rightarrow \cup B) \ k$   
 ⟨proof⟩

**lemma** *all-union-tsI2* [elim?]:  $c \in B \Longrightarrow c \in (A \Rightarrow \cup B) \ k$   
 ⟨proof⟩

**lemma** *all-union-tsCI* [intro!]:  $(c \notin B \Longrightarrow c \in A \ k) \Longrightarrow c \in (A \Rightarrow \cup B) \ k$   
 ⟨proof⟩

**lemma** *all-union-tsE* [elim!]:  
 $\llbracket c \in (A \Rightarrow \cup B) \ k; (c \in A \ k \Longrightarrow P); (c \in B \Longrightarrow P) \rrbracket \Longrightarrow P$   
 ⟨proof⟩

### The rules of definite assignment

**types** *breakass* = (*label*, *lname*) *tables*

— Mapping from a break label, to the set of variables that will be assigned if the evaluation terminates with this break

**record** *assigned* =

*nrm* :: *lname set* — Definetly assigned variables for normal completion

*brk* :: *breakass* — Definetly assigned variables for abrupt completion with a break

**constdefs** *rmlab* :: '*a*  $\Rightarrow$  ('*a*, '*b*) *tables*  $\Rightarrow$  ('*a*, '*b*) *tables*

*rmlab* *k A*  $\equiv \lambda x. \text{if } x=k \text{ then } UNIV \text{ else } A \ x$

**constdefs** *range-inter-ts* :: ('*a*, '*b*) *tables*  $\Rightarrow$  '*b set* ( $\Rightarrow \bigcap$  - 80)

$$\Rightarrow \bigcap A \equiv \{x \mid x. \forall k. x \in A\ k\}$$

In  $E \vdash B \gg t \gg A$ ,  $B$  denotes the "assigned" variables before evaluating term  $t$ , whereas  $A$  denotes the "assigned" variables after evaluating term  $t$ . The environment  $E$  is only needed for the conditional - ? - : -. The definite assignment rules refer to the typing rules here to distinguish boolean and other expressions.

### inductive

$da :: env \Rightarrow lname\ set \Rightarrow term \Rightarrow assigned \Rightarrow bool$  (-|-|-|-|- [65,65,65,65] 71)

### where

$Skip: Env \vdash B \gg \langle Skip \rangle \gg (\text{norm}=B, \text{brk}=\lambda l. UNIV)$

|  $Expr: Env \vdash B \gg \langle e \rangle \gg A$

$\Rightarrow$

$Env \vdash B \gg \langle Expr\ e \rangle \gg A$

|  $Lab: \llbracket Env \vdash B \gg \langle c \rangle \gg C; \text{norm}\ A = \text{norm}\ C \cap (\text{brk}\ C)\ l; \text{brk}\ A = \text{rmlab}\ l\ (\text{brk}\ C) \rrbracket$

$\Rightarrow$

$Env \vdash B \gg \langle Break\ l.\ c \rangle \gg A$

|  $Comp: \llbracket Env \vdash B \gg \langle c1 \rangle \gg C1; Env \vdash \text{norm}\ C1 \gg \langle c2 \rangle \gg C2; \text{norm}\ A = \text{norm}\ C2; \text{brk}\ A = (\text{brk}\ C1) \Rightarrow \cap (\text{brk}\ C2) \rrbracket$

$\Rightarrow$

$Env \vdash B \gg \langle c1;; c2 \rangle \gg A$

|  $If: \llbracket Env \vdash B \gg \langle e \rangle \gg E;$

$Env \vdash (B \cup \text{assigns-if}\ True\ e) \gg \langle c1 \rangle \gg C1;$

$Env \vdash (B \cup \text{assigns-if}\ False\ e) \gg \langle c2 \rangle \gg C2;$

$\text{norm}\ A = \text{norm}\ C1 \cap \text{norm}\ C2;$

$\text{brk}\ A = \text{brk}\ C1 \Rightarrow \cap \text{brk}\ C2 \rrbracket$

$\Rightarrow$

$Env \vdash B \gg \langle If(e)\ c1\ Else\ c2 \rangle \gg A$

— Note that  $E$  is not further used, because we take the specialized sets that also consider if the expression evaluates to true or false. Inside of  $e$  there is no **break** or **finally**, so the break map of  $E$  will be the trivial one. So  $Env \vdash B \gg \langle e \rangle \gg E$  is just used to ensure the definite assignment in expression  $e$ . Notice the implicit analysis of a constant boolean expression  $e$  in this rule. For example, if  $e$  is constantly *True* then *assigns-if False e* = *UNIV* and therefor  $\text{norm}\ C2 = UNIV$ . So finally  $\text{norm}\ A = \text{norm}\ C1$ . For the break maps this trick workd too, because the trival break map will map all labels to *UNIV*. In the example, if no break occurs in  $c2$  the break maps will trivially map to *UNIV* and if a break occurs it will map to *UNIV* too, because *assigns-if False e* = *UNIV*. So in the intersection of the break maps the path  $c2$  will have no contribution.

|  $Loop: \llbracket Env \vdash B \gg \langle e \rangle \gg E;$

$Env \vdash (B \cup \text{assigns-if}\ True\ e) \gg \langle c \rangle \gg C;$

$\text{norm}\ A = \text{norm}\ C \cap (B \cup \text{assigns-if}\ False\ e);$

$\text{brk}\ A = \text{brk}\ C \rrbracket$

$\Rightarrow$

$Env \vdash B \gg \langle l.\ While(e)\ c \rangle \gg A$

— The *Loop* rule resembles some of the ideas of the *If* rule. For the  $\text{norm}\ A$  the set  $B \cup \text{assigns-if False e}$  will be *UNIV* if the condition is constantly true. To normally exit the while loop, we must consider the body  $c$  to be completed normally ( $\text{norm}\ C$ ) or with a break. But in this model, the label  $l$  of the loop only handles continue labels, not break labels. The break label will be handled by an enclosing *Lab* statement. So we don't have to handle the breaks specially.

|  $Jmp: \llbracket \text{jump}=\text{Ret} \longrightarrow \text{Result} \in B;$

$\text{norm}\ A = UNIV;$

$\text{brk}\ A = (\text{case jump of}$

$\text{Break}\ l \Rightarrow \lambda k. \text{if } k=l \text{ then } B \text{ else } UNIV$

|  $\text{Cont}\ l \Rightarrow \lambda k. UNIV$

|  $\text{Ret} \Rightarrow \lambda k. UNIV) \rrbracket$

$$\Rightarrow$$

$$Env \vdash B \gg \langle \text{Jump } jump \rangle \gg A$$

— In case of a break to label  $l$  the corresponding break set is all variables assigned before the break. The assigned variables for normal completion of the *Jump* is *UNIV*, because the statement will never complete normally. For continue and return the break map is the trivial one. In case of a return we ensure that the result value is assigned.

$$\begin{aligned} | \text{Throw: } & \llbracket Env \vdash B \gg \langle e \rangle \gg E; nrm\ A = UNIV; brk\ A = (\lambda\ l.\ UNIV) \rrbracket \\ & \Rightarrow Env \vdash B \gg \langle \text{Throw } e \rangle \gg A \end{aligned}$$

$$\begin{aligned} | \text{Try: } & \llbracket Env \vdash B \gg \langle c1 \rangle \gg C1; \\ & Env(\llbracket lcl := lcl\ Env(VName\ vn \mapsto Class\ C) \rrbracket) \vdash (B \cup \{VName\ vn\}) \gg \langle c2 \rangle \gg C2; \\ & nrm\ A = nrm\ C1 \cap nrm\ C2; \\ & brk\ A = brk\ C1 \Rightarrow \cap\ brk\ C2 \rrbracket \\ & \Rightarrow Env \vdash B \gg \langle \text{Try } c1\ \text{Catch}(C\ vn)\ c2 \rangle \gg A \end{aligned}$$

$$\begin{aligned} | \text{Fin: } & \llbracket Env \vdash B \gg \langle c1 \rangle \gg C1; \\ & Env \vdash B \gg \langle c2 \rangle \gg C2; \\ & nrm\ A = nrm\ C1 \cup nrm\ C2; \\ & brk\ A = ((brk\ C1) \Rightarrow \cup_{\vee}\ (nrm\ C2)) \Rightarrow \cap\ (brk\ C2) \rrbracket \\ & \Rightarrow \\ & Env \vdash B \gg \langle c1\ \text{Finally } c2 \rangle \gg A \end{aligned}$$

— The set of assigned variables before execution  $c2$  are the same as before execution  $c1$ , because  $c1$  could throw an exception and so we can't guarantee that any variable will be assigned in  $c1$ . The *Finally* statement completes normally if both  $c1$  and  $c2$  complete normally. If  $c1$  completes abruptly with a break, then  $c2$  also will be executed and may terminate normally or with a break. The overall break map then is the intersection of the maps of both paths. If  $c2$  terminates normally we have to extend all break sets in  $brk\ C1$  with  $nrm\ C2$  ( $\Rightarrow \cup_{\vee}$ ). If  $c2$  exits with a break this break will appear in the overall result state. We don't know if  $c1$  completed normally or abruptly (maybe with an exception not only a break) so  $c1$  has no contribution to the break map following this path.

— Evaluation of expressions and the break sets of definite assignment: Thinking of a Java expression we assume that we can never have a break statement inside of an expression. So for all expressions the break sets could be set to the trivial one:  $\lambda l.\ UNIV$ . But we can't trivially prove, that evaluating an expression will never result in a break, although Java expressions already syntactically don't allow nested statements in them. The reason are the nested class initialization statements which are inserted by the evaluation rules. So to prove the absence of a break we need to ensure, that the initialization statements will never end up in a break. In a wellformed initialization statement, of course, where breaks are nested correctly inside of *Lab* or *Loop* statements evaluation of the whole initialization statement will never result in a break, because this break will be handled inside of the statement. But for simplicity we haven't added the analysis of the correct nesting of breaks in the typing judgments right now. So we have decided to adjust the rules of definite assignment to fit to these circumstances. If an initialization is involved during evaluation of the expression (evaluation rules *FVar*, *NewC* and *NewA*

$$| \text{Init: } Env \vdash B \gg \langle \text{Init } C \rangle \gg (nrm=B, brk=\lambda\ l.\ UNIV)$$

— Wellformedness of a program will ensure, that every static initialiser is definitely assigned and the jumps are nested correctly. The case here for *Init* is just for convenience, to get a proper precondition for the induction hypothesis in various proofs, so that we don't have to expand the initialisation on every point where it is triggered by the evaluation rules.

$$| \text{NewC: } Env \vdash B \gg \langle \text{NewC } C \rangle \gg (nrm=B, brk=\lambda\ l.\ UNIV)$$

$$\begin{aligned} | \text{NewA: } & Env \vdash B \gg \langle e \rangle \gg A \\ & \Rightarrow \\ & Env \vdash B \gg \langle \text{New } T[e] \rangle \gg A \end{aligned}$$

$$\begin{aligned} | \text{Cast: } & Env \vdash B \gg \langle e \rangle \gg A \\ & \Rightarrow \\ & Env \vdash B \gg \langle \text{Cast } T\ e \rangle \gg A \end{aligned}$$

- | *Inst*:  $Env \vdash B \gg \langle e \rangle \gg A$   
 $\implies$   
 $Env \vdash B \gg \langle e \text{ InstOf } T \rangle \gg A$
  - | *Lit*:  $Env \vdash B \gg \langle Lit \ v \rangle \gg (\llbracket nrm=B, brk=\lambda l. UNIV \rrbracket)$
  - | *UnOp*:  $Env \vdash B \gg \langle e \rangle \gg A$   
 $\implies$   
 $Env \vdash B \gg \langle UnOp \ unop \ e \rangle \gg A$
  - | *CondAnd*:  $\llbracket Env \vdash B \gg \langle e1 \rangle \gg E1; Env \vdash (B \cup assigns\text{-}if \ True \ e1) \gg \langle e2 \rangle \gg E2;$   
 $nrm \ A = B \cup (assigns\text{-}if \ True \ (BinOp \ CondAnd \ e1 \ e2) \cap$   
 $assigns\text{-}if \ False \ (BinOp \ CondAnd \ e1 \ e2));$   
 $brk \ A = (\lambda l. UNIV) \rrbracket$   
 $\implies$   
 $Env \vdash B \gg \langle BinOp \ CondAnd \ e1 \ e2 \rangle \gg A$
  - | *CondOr*:  $\llbracket Env \vdash B \gg \langle e1 \rangle \gg E1; Env \vdash (B \cup assigns\text{-}if \ False \ e1) \gg \langle e2 \rangle \gg E2;$   
 $nrm \ A = B \cup (assigns\text{-}if \ True \ (BinOp \ CondOr \ e1 \ e2) \cap$   
 $assigns\text{-}if \ False \ (BinOp \ CondOr \ e1 \ e2));$   
 $brk \ A = (\lambda l. UNIV) \rrbracket$   
 $\implies$   
 $Env \vdash B \gg \langle BinOp \ CondOr \ e1 \ e2 \rangle \gg A$
  - | *BinOp*:  $\llbracket Env \vdash B \gg \langle e1 \rangle \gg E1; Env \vdash nrm \ E1 \gg \langle e2 \rangle \gg A;$   
 $binop \neq CondAnd; binop \neq CondOr \rrbracket$   
 $\implies$   
 $Env \vdash B \gg \langle BinOp \ binop \ e1 \ e2 \rangle \gg A$
  - | *Super*:  $This \in B$   
 $\implies$   
 $Env \vdash B \gg \langle Super \rangle \gg (\llbracket nrm=B, brk=\lambda l. UNIV \rrbracket)$
  - | *AccLVar*:  $\llbracket vn \in B;$   
 $nrm \ A = B; brk \ A = (\lambda k. UNIV) \rrbracket$   
 $\implies$   
 $Env \vdash B \gg \langle Acc \ (LVar \ vn) \rangle \gg A$
- To properly access a local variable we have to test the definite assignment here. The variable must occur in the set  $B$
- | *Acc*:  $\llbracket \forall \ vn. v \neq LVar \ vn;$   
 $Env \vdash B \gg \langle v \rangle \gg A \rrbracket$   
 $\implies$   
 $Env \vdash B \gg \langle Acc \ v \rangle \gg A$
  - | *AssLVar*:  $\llbracket Env \vdash B \gg \langle e \rangle \gg E; nrm \ A = nrm \ E \cup \{vn\}; brk \ A = brk \ E \rrbracket$   
 $\implies$   
 $Env \vdash B \gg \langle (LVar \ vn) := e \rangle \gg A$
  - | *Ass*:  $\llbracket \forall \ vn. v \neq LVar \ vn; Env \vdash B \gg \langle v \rangle \gg V; Env \vdash nrm \ V \gg \langle e \rangle \gg A \rrbracket$   
 $\implies$   
 $Env \vdash B \gg \langle v := e \rangle \gg A$
  - | *CondBool*:  $\llbracket Env \vdash (c \ ? \ e1 : e2) :: \neg (PrimT \ Boolean);$   
 $Env \vdash B \gg \langle c \rangle \gg C;$   
 $Env \vdash (B \cup assigns\text{-}if \ True \ c) \gg \langle e1 \rangle \gg E1;$   
 $Env \vdash (B \cup assigns\text{-}if \ False \ c) \gg \langle e2 \rangle \gg E2;$   
 $nrm \ A = B \cup (assigns\text{-}if \ True \ (c \ ? \ e1 : e2) \cap$   
 $assigns\text{-}if \ False \ (c \ ? \ e1 : e2)); \rrbracket$

$$\begin{aligned}
& \text{brk } A = (\lambda l. \text{UNIV}) \\
& \implies \\
& \text{Env} \vdash B \gg \langle c ? e1 : e2 \rangle \gg A
\end{aligned}$$

$$\begin{aligned}
| \text{Cond: } & \llbracket \neg \text{Env} \vdash (c ? e1 : e2) :: \neg (\text{PrimT Boolean}); \\
& \text{Env} \vdash B \gg \langle c \rangle \gg C; \\
& \text{Env} \vdash (B \cup \text{assigns-if True } c) \gg \langle e1 \rangle \gg E1; \\
& \text{Env} \vdash (B \cup \text{assigns-if False } c) \gg \langle e2 \rangle \gg E2; \\
& \text{nrm } A = \text{nrm } E1 \cap \text{nrm } E2; \text{brk } A = (\lambda l. \text{UNIV}) \\
& \implies \\
& \text{Env} \vdash B \gg \langle c ? e1 : e2 \rangle \gg A
\end{aligned}$$

$$\begin{aligned}
| \text{Call: } & \llbracket \text{Env} \vdash B \gg \langle e \rangle \gg E; \text{Env} \vdash \text{nrm } E \gg \langle \text{args} \rangle \gg A \rrbracket \\
& \implies \\
& \text{Env} \vdash B \gg \langle \{ \text{accC}, \text{statT}, \text{mode} \} e \cdot \text{mn}(\{ pTs \} \text{args}) \rangle \gg A
\end{aligned}$$

— The interplay of *Call*, *Method* and *Body*: Why rules for *Method* and *Body* at all? Note that a Java source program will not include bare *Method* or *Body* terms. These terms are just introduced during evaluation. So definite assignment of *Call* does not consider *Method* or *Body* at all. So for definite assignment alone we could omit the rules for *Method* and *Body*. But since evaluation of the method invocation is split up into three rules we must ensure that we have enough information about the call even in the *Body* term to make sure that we can proof type safety. Also we must be able transport this information from *Call* to *Method* and then further to *Body* during evaluation to establish the definite assignment of *Method* during evaluation of *Call*, and of *Body* during evaluation of *Method*. This is necessary since definite assignment will be a precondition for each induction hypothesis coming out of the evaluation rules, and therefor we have to establish the definite assignment of the sub-evaluation during the type-safety proof. Note that well-typedness is also a precondition for type-safety and so we can omit some assertion that are already ensured by well-typedness.

$$\begin{aligned}
| \text{Method: } & \llbracket \text{method } (\text{prg } \text{Env}) \text{ } D \text{ sig} = \text{Some } m; \\
& \text{Env} \vdash B \gg \langle \text{Body } (\text{declclass } m) (\text{stmt } (\text{mbody } (\text{mthd } m))) \rangle \gg A \\
& \rrbracket \\
& \implies \\
& \text{Env} \vdash B \gg \langle \text{Method } D \text{ sig} \rangle \gg A
\end{aligned}$$

$$\begin{aligned}
| \text{Body: } & \llbracket \text{Env} \vdash B \gg \langle c \rangle \gg C; \text{jumpNestingOkS } \{ \text{Ret} \} c; \text{Result} \in \text{nrm } C; \\
& \text{nrm } A = B; \text{brk } A = (\lambda l. \text{UNIV}) \rrbracket \\
& \implies \\
& \text{Env} \vdash B \gg \langle \text{Body } D c \rangle \gg A
\end{aligned}$$

— Note that  $A$  is not correlated to  $C$ . If the body statement returns abruptly with return, evaluation of *Body* will absorb this return and complete normally. So we cannot trivially get the assigned variables of the body statement since it has not completed normally or with a break. If the body completes normally we guarantee that the result variable is set with this rule. But if the body completes abruptly with a return we can't guarantee that the result variable is set here, since definite assignment only talks about normal completion and breaks. So for a return the *Jump* rule ensures that the result variable is set and then this information must be carried over to the *Body* rule by the conformance predicate of the state.

$$| \text{LVar: } \text{Env} \vdash B \gg \langle \text{LVar } vn \rangle \gg (\text{nrm} = B, \text{brk} = \lambda l. \text{UNIV})$$

$$\begin{aligned}
| \text{FVar: } & \text{Env} \vdash B \gg \langle e \rangle \gg A \\
& \implies \\
& \text{Env} \vdash B \gg \langle \{ \text{accC}, \text{statDeclC}, \text{stat} \} e \cdot \text{fn} \rangle \gg A
\end{aligned}$$

$$\begin{aligned}
| \text{AVar: } & \llbracket \text{Env} \vdash B \gg \langle e1 \rangle \gg E1; \text{Env} \vdash \text{nrm } E1 \gg \langle e2 \rangle \gg A \rrbracket \\
& \implies \\
& \text{Env} \vdash B \gg \langle e1.[e2] \rangle \gg A
\end{aligned}$$

$$| \text{Nil: } \text{Env} \vdash B \gg \langle [] :: \text{expr list} \rangle \gg (\text{nrm} = B, \text{brk} = \lambda l. \text{UNIV})$$

$$\begin{aligned}
| \text{Cons: } & \llbracket \text{Env} \vdash B \gg \langle e :: \text{expr} \rangle \gg E; \text{Env} \vdash \text{nrm } E \gg \langle es \rangle \gg A \rrbracket \\
& \implies \\
& \text{Env} \vdash B \gg \langle e \# es \rangle \gg A
\end{aligned}$$



**declare** *inj-term-sym-simps* [*simp*]  
**declare** *assigns-if.simps* [*simp del*]  
**declare** *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]  
 $\langle ML \rangle$

**inductive-cases** *da-elim-cases* [*cases set*]:

$Env \vdash B \gg \langle Skip \rangle \gg A$   
 $Env \vdash B \gg In1r\ Skip \gg A$   
 $Env \vdash B \gg \langle Expr\ e \rangle \gg A$   
 $Env \vdash B \gg In1r\ (Expr\ e) \gg A$   
 $Env \vdash B \gg \langle l \cdot c \rangle \gg A$   
 $Env \vdash B \gg In1r\ (l \cdot c) \gg A$   
 $Env \vdash B \gg \langle c1;; c2 \rangle \gg A$   
 $Env \vdash B \gg In1r\ (c1;; c2) \gg A$   
 $Env \vdash B \gg \langle If(e)\ c1\ Else\ c2 \rangle \gg A$   
 $Env \vdash B \gg In1r\ (If(e)\ c1\ Else\ c2) \gg A$   
 $Env \vdash B \gg \langle l \cdot While(e)\ c \rangle \gg A$   
 $Env \vdash B \gg In1r\ (l \cdot While(e)\ c) \gg A$   
 $Env \vdash B \gg \langle Jmp\ jump \rangle \gg A$   
 $Env \vdash B \gg In1r\ (Jmp\ jump) \gg A$   
 $Env \vdash B \gg \langle Throw\ e \rangle \gg A$   
 $Env \vdash B \gg In1r\ (Throw\ e) \gg A$   
 $Env \vdash B \gg \langle Try\ c1\ Catch(C\ vn)\ c2 \rangle \gg A$   
 $Env \vdash B \gg In1r\ (Try\ c1\ Catch(C\ vn)\ c2) \gg A$   
 $Env \vdash B \gg \langle c1\ Finally\ c2 \rangle \gg A$   
 $Env \vdash B \gg In1r\ (c1\ Finally\ c2) \gg A$   
 $Env \vdash B \gg \langle Init\ C \rangle \gg A$   
 $Env \vdash B \gg In1r\ (Init\ C) \gg A$   
 $Env \vdash B \gg \langle NewC\ C \rangle \gg A$   
 $Env \vdash B \gg In1l\ (NewC\ C) \gg A$   
 $Env \vdash B \gg \langle New\ T[e] \rangle \gg A$   
 $Env \vdash B \gg In1l\ (New\ T[e]) \gg A$   
 $Env \vdash B \gg \langle Cast\ T\ e \rangle \gg A$   
 $Env \vdash B \gg In1l\ (Cast\ T\ e) \gg A$   
 $Env \vdash B \gg \langle e\ InstOf\ T \rangle \gg A$   
 $Env \vdash B \gg In1l\ (e\ InstOf\ T) \gg A$   
 $Env \vdash B \gg \langle Lit\ v \rangle \gg A$   
 $Env \vdash B \gg In1l\ (Lit\ v) \gg A$   
 $Env \vdash B \gg \langle UnOp\ unop\ e \rangle \gg A$   
 $Env \vdash B \gg In1l\ (UnOp\ unop\ e) \gg A$   
 $Env \vdash B \gg \langle BinOp\ binop\ e1\ e2 \rangle \gg A$   
 $Env \vdash B \gg In1l\ (BinOp\ binop\ e1\ e2) \gg A$   
 $Env \vdash B \gg \langle Super \rangle \gg A$   
 $Env \vdash B \gg In1l\ (Super) \gg A$   
 $Env \vdash B \gg \langle Acc\ v \rangle \gg A$   
 $Env \vdash B \gg In1l\ (Acc\ v) \gg A$   
 $Env \vdash B \gg \langle v := e \rangle \gg A$   
 $Env \vdash B \gg In1l\ (v := e) \gg A$   
 $Env \vdash B \gg \langle c\ ?\ e1 : e2 \rangle \gg A$   
 $Env \vdash B \gg In1l\ (c\ ?\ e1 : e2) \gg A$   
 $Env \vdash B \gg \langle \{accC, statT, mode\} e \cdot mn(\{pTs\} args) \rangle \gg A$   
 $Env \vdash B \gg In1l\ (\{accC, statT, mode\} e \cdot mn(\{pTs\} args)) \gg A$   
 $Env \vdash B \gg \langle Methd\ C\ sig \rangle \gg A$   
 $Env \vdash B \gg In1l\ (Methd\ C\ sig) \gg A$   
 $Env \vdash B \gg \langle Body\ D\ c \rangle \gg A$   
 $Env \vdash B \gg In1l\ (Body\ D\ c) \gg A$   
 $Env \vdash B \gg \langle LVar\ vn \rangle \gg A$

$Env \vdash B \gg In2 (LVar\ vn) \gg A$   
 $Env \vdash B \gg \langle \{accC, statDeclC, stat\} e..fn \rangle \gg A$   
 $Env \vdash B \gg In2 (\{accC, statDeclC, stat\} e..fn) \gg A$   
 $Env \vdash B \gg \langle e1.[e2] \rangle \gg A$   
 $Env \vdash B \gg In2 (e1.[e2]) \gg A$   
 $Env \vdash B \gg \langle [] :: expr\ list \rangle \gg A$   
 $Env \vdash B \gg In3 ([] :: expr\ list) \gg A$   
 $Env \vdash B \gg \langle e \# es \rangle \gg A$   
 $Env \vdash B \gg In3 (e \# es) \gg A$   
**declare** *inj-term-sym-simps* [*simp del*]  
**declare** *assigns-if.simps* [*simp*]  
**declare** *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]  
 $\langle ML \rangle$

**lemma** *da-Skip*:  $A = \langle nrm=B, brk=\lambda\ l.\ UNIV \rangle \implies Env \vdash B \gg \langle Skip \rangle \gg A$   
 $\langle proof \rangle$

**lemma** *da-NewC*:  $A = \langle nrm=B, brk=\lambda\ l.\ UNIV \rangle \implies Env \vdash B \gg \langle NewC\ C \rangle \gg A$   
 $\langle proof \rangle$

**lemma** *da-Lit*:  $A = \langle nrm=B, brk=\lambda\ l.\ UNIV \rangle \implies Env \vdash B \gg \langle Lit\ v \rangle \gg A$   
 $\langle proof \rangle$

**lemma** *da-Super*:  $\llbracket This \in B; A = \langle nrm=B, brk=\lambda\ l.\ UNIV \rangle \rrbracket \implies Env \vdash B \gg \langle Super \rangle \gg A$   
 $\langle proof \rangle$

**lemma** *da-Init*:  $A = \langle nrm=B, brk=\lambda\ l.\ UNIV \rangle \implies Env \vdash B \gg \langle Init\ C \rangle \gg A$   
 $\langle proof \rangle$

**lemma** *assignsE-subseteq-assigns-ifs*:  
**assumes** *boolEx*:  $E \vdash e :: \neg PrimT\ Boolean\ (is\ ?Boolean\ e)$   
**shows**  $assignsE\ e \subseteq assigns\text{-}if\ True\ e \cap assigns\text{-}if\ False\ e\ (is\ ?Incl\ e)$   
 $\langle proof \rangle$

**lemma** *rmlab-same-label* [*simp*]:  $(rmlab\ l\ A)\ l = UNIV$   
 $\langle proof \rangle$

**lemma** *rmlab-same-label1* [*simp*]:  $l=l' \implies (rmlab\ l\ A)\ l' = UNIV$   
 $\langle proof \rangle$

**lemma** *rmlab-other-label* [*simp*]:  $l \neq l' \implies (rmlab\ l\ A)\ l' = A\ l'$   
 $\langle proof \rangle$

**lemma** *range-inter-ts-subseteq* [intro]:  $\forall k. A \ k \subseteq B \ k \implies \Rightarrow \bigcap A \subseteq \Rightarrow \bigcap B$   
 ⟨proof⟩

**lemma** *range-inter-ts-subseteq'*:  
 $\llbracket \forall k. A \ k \subseteq B \ k; x \in \Rightarrow \bigcap A \rrbracket \implies x \in \Rightarrow \bigcap B$   
 ⟨proof⟩

**lemma** *da-monotone*:  
**assumes** *da*:  $Env \vdash B \gg t \gg A$  **and**  
 $B \subseteq B'$  **and**  
 $da'$ :  $Env \vdash B' \gg t \gg A'$   
**shows**  $(nrm \ A \subseteq nrm \ A') \wedge (\forall l. (brk \ A \ l \subseteq brk \ A' \ l))$   
 ⟨proof⟩

**lemma** *da-weaken*:  
**assumes** *da*:  $Env \vdash B \gg t \gg A$  **and**  $B \subseteq B'$   
**shows**  $\exists A'. Env \vdash B' \gg t \gg A'$   
 ⟨proof⟩

**corollary** *da-weakenE* [consumes 2]:  
**assumes**  $da$ :  $Env \vdash B \gg t \gg A$  **and**  
 $B': B \subseteq B'$  **and**  
 $ex\text{-}mono$ :  $\bigwedge A'. \llbracket Env \vdash B' \gg t \gg A'; nrm \ A \subseteq nrm \ A';$   
 $\bigwedge l. brk \ A \ l \subseteq brk \ A' \ l \rrbracket \implies P$   
**shows**  $P$   
 ⟨proof⟩

**end**



## Chapter 13

# WellForm

### 34 Well-formedness of Java programs

**theory** *WellForm* **imports** *DefiniteAssignment* **begin**

For static checks on expressions and statements, see *WellType.thy*  
improvements over Java Specification 1.0 (cf. 8.4.6.3, 8.4.6.4, 9.4.1):

- a method implementing or overwriting another method may have a result type that widens to the result type of the other method (instead of identical type)
- if a method hides another method (both methods have to be static!) there are no restrictions to the result type since the methods have to be static and there is no dynamic binding of static methods
- if an interface inherits more than one method with the same signature, the methods need not have identical return types

simplifications:

- Object and standard exceptions are assumed to be declared like normal classes

#### well-formed field declarations

well-formed field declaration (common part for classes and interfaces), cf. 8.3 and (9.3)

**constdefs**

$$\begin{aligned} wf\_fdecl &:: prog \Rightarrow pname \Rightarrow fdecl \Rightarrow bool \\ wf\_fdecl \ G \ P &\equiv \lambda(fn,f). \ is\_acc\_type \ G \ P \ (type \ f) \end{aligned}$$

**lemma** *wf-fdecl-def2*:  $\bigwedge fd. \ wf\_fdecl \ G \ P \ fd = is\_acc\_type \ G \ P \ (type \ (snd \ fd))$   
*<proof>*

#### well-formed method declarations

A method head is wellformed if:

- the signature and the method head agree in the number of parameters
- all types of the parameters are visible
- the result type is visible
- the parameter names are unique

**constdefs**

$$\begin{aligned} wf\_mhead &:: prog \Rightarrow pname \Rightarrow sig \Rightarrow mhead \Rightarrow bool \\ wf\_mhead \ G \ P &\equiv \lambda \ sig \ mh. \ length \ (parTs \ sig) = length \ (pars \ mh) \wedge \\ &\quad (\ \forall \ T \in set \ (parTs \ sig). \ is\_acc\_type \ G \ P \ T) \wedge \\ &\quad is\_acc\_type \ G \ P \ (resTy \ mh) \wedge \\ &\quad distinct \ (pars \ mh) \end{aligned}$$

A method declaration is wellformed if:

- the method head is wellformed
- the names of the local variables are unique
- the types of the local variables must be accessible

- the local variables don't shadow the parameters
- the class of the method is defined
- the body statement is welltyped with respect to the modified environment of local names, where the local variables, the parameters the special result variable (Res) and This are associated with their types.

**constdefs** *callee-lcl* :: *qname*  $\Rightarrow$  *sig*  $\Rightarrow$  *methd*  $\Rightarrow$  *lenv*  
*callee-lcl* *C* *sig* *m*  
 $\equiv \lambda k. (case\ k\ of$   
 $\quad EName\ e$   
 $\quad \Rightarrow (case\ e\ of$   
 $\quad \quad VName\ v$   
 $\quad \quad \Rightarrow (table-of\ (lcls\ (mbody\ m))((pars\ m)[\mapsto](parTs\ sig)))\ v$   
 $\quad \quad | Res \Rightarrow Some\ (resTy\ m))$   
 $\quad | This \Rightarrow if\ is-static\ m\ then\ None\ else\ Some\ (Class\ C))$

**constdefs** *parameters* :: *methd*  $\Rightarrow$  *lname* *set*  
*parameters* *m*  $\equiv set\ (map\ (EName\ o\ VName)\ (pars\ m))$   
 $\cup (if\ (static\ m)\ then\ \{\}\ else\ \{This\})$

**constdefs**  
*wf-mdecl* :: *prog*  $\Rightarrow$  *qname*  $\Rightarrow$  *mdecl*  $\Rightarrow$  *bool*  
*wf-mdecl* *G* *C*  $\equiv$   
 $\lambda(sig, m).$   
 $\quad wf-mhead\ G\ (pid\ C)\ sig\ (mhead\ m) \wedge$   
 $\quad unique\ (lcls\ (mbody\ m)) \wedge$   
 $\quad (\forall (vn, T) \in set\ (lcls\ (mbody\ m)).\ is-acc-type\ G\ (pid\ C)\ T) \wedge$   
 $\quad (\forall pn \in set\ (pars\ m).\ table-of\ (lcls\ (mbody\ m))\ pn = None) \wedge$   
 $\quad jumpNestingOkS\ \{Ret\}\ (stmt\ (mbody\ m)) \wedge$   
 $\quad is-class\ G\ C \wedge$   
 $\quad (\llbracket prg = G, cls = C, lcl = callee-lcl\ C\ sig\ m \rrbracket \vdash (stmt\ (mbody\ m))) :: \checkmark \wedge$   
 $\quad (\exists\ A.\ \llbracket prg = G, cls = C, lcl = callee-lcl\ C\ sig\ m \rrbracket$   
 $\quad \quad \vdash\ parameters\ m\ \gg (stmt\ (mbody\ m)) \gg A$   
 $\quad \quad \wedge\ Result \in nrm\ A)$

**lemma** *callee-lcl-VName-simp* [simp]:  
*callee-lcl* *C* *sig* *m* (*EName* (*VName* *v*))  
 $= (table-of\ (lcls\ (mbody\ m))((pars\ m)[\mapsto](parTs\ sig)))\ v$   
 $\langle proof \rangle$

**lemma** *callee-lcl-Res-simp* [simp]:  
*callee-lcl* *C* *sig* *m* (*EName* *Res*) = *Some* (*resTy* *m*)  
 $\langle proof \rangle$

**lemma** *callee-lcl-This-simp* [simp]:  
*callee-lcl* *C* *sig* *m* (*This*) = (*if* *is-static* *m* *then* *None* *else* *Some* (*Class* *C*))  
 $\langle proof \rangle$

**lemma** *callee-lcl-This-static-simp*:  
*is-static* *m*  $\implies$  *callee-lcl* *C* *sig* *m* (*This*) = *None*  
 $\langle proof \rangle$

**lemma** *callee-lcl-This-not-static-simp*:

$\neg \text{is-static } m \implies \text{callee-lcl } C \text{ sig } m \text{ (This) = Some (Class } C)$

$\langle \text{proof} \rangle$

**lemma** *wf-mheadI*:

$\llbracket \text{length (parTs sig) = length (pars m)}; \forall T \in \text{set (parTs sig)}. \text{is-acc-type } G \text{ P } T;$   
 $\text{is-acc-type } G \text{ P (resTy } m); \text{distinct (pars m)} \rrbracket \implies$   
 $\text{wf-mhead } G \text{ P sig } m$

$\langle \text{proof} \rangle$

**lemma** *wf-mdeclI*:  $\llbracket$

$\text{wf-mhead } G \text{ (pid } C) \text{ sig (mhead } m); \text{unique (lcls (mbody } m));$   
 $(\forall pn \in \text{set (pars } m). \text{table-of (lcls (mbody } m)) pn = None);$   
 $\forall (vn, T) \in \text{set (lcls (mbody } m)). \text{is-acc-type } G \text{ (pid } C) \text{ } T;$   
 $\text{jumpNestingOkS \{Ret\} (stmt (mbody } m));$   
 $\text{is-class } G \text{ } C;$   
 $(\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = \text{callee-lcl } C \text{ sig } m \rrbracket \vdash \text{stmt (mbody } m) :: \sqrt{ };$   
 $(\exists A. (\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = \text{callee-lcl } C \text{ sig } m \rrbracket \vdash \text{parameters } m \gg \langle \text{stmt (mbody } m) \rangle \gg A$   
 $\wedge \text{Result} \in \text{nrm } A)$

$\rrbracket \implies$

$\text{wf-mdecl } G \text{ } C \text{ (sig, } m)$

$\langle \text{proof} \rangle$

**lemma** *wf-mdeclE [consumes 1]*:

$\llbracket \text{wf-mdecl } G \text{ } C \text{ (sig, } m);$   
 $\llbracket \text{wf-mhead } G \text{ (pid } C) \text{ sig (mhead } m); \text{unique (lcls (mbody } m));$   
 $\forall pn \in \text{set (pars } m). \text{table-of (lcls (mbody } m)) pn = None;$   
 $\forall (vn, T) \in \text{set (lcls (mbody } m)). \text{is-acc-type } G \text{ (pid } C) \text{ } T;$   
 $\text{jumpNestingOkS \{Ret\} (stmt (mbody } m));$   
 $\text{is-class } G \text{ } C;$   
 $(\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = \text{callee-lcl } C \text{ sig } m \rrbracket \vdash \text{stmt (mbody } m) :: \sqrt{ };$   
 $(\exists A. (\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = \text{callee-lcl } C \text{ sig } m \rrbracket \vdash \text{parameters } m \gg \langle \text{stmt (mbody } m) \rangle \gg A$   
 $\wedge \text{Result} \in \text{nrm } A)$

$\rrbracket \implies P$

$\rrbracket \implies P$

$\langle \text{proof} \rangle$

**lemma** *wf-mdeclD1*:

$\text{wf-mdecl } G \text{ } C \text{ (sig, } m) \implies$   
 $\text{wf-mhead } G \text{ (pid } C) \text{ sig (mhead } m) \wedge \text{unique (lcls (mbody } m)) \wedge$   
 $(\forall pn \in \text{set (pars } m). \text{table-of (lcls (mbody } m)) pn = None) \wedge$   
 $(\forall (vn, T) \in \text{set (lcls (mbody } m)). \text{is-acc-type } G \text{ (pid } C) \text{ } T)$

$\langle \text{proof} \rangle$

**lemma** *wf-mdecl-bodyD*:

$\text{wf-mdecl } G \text{ } C \text{ (sig, } m) \implies$   
 $(\exists T. (\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = \text{callee-lcl } C \text{ sig } m \rrbracket \vdash \text{Body } C \text{ (stmt (mbody } m)) :: -T \wedge$   
 $G \vdash T \preceq (\text{resTy } m))$

$\langle \text{proof} \rangle$



**lemma** *rT-is-acc-type*:

*wf-mhead G P sig m*  $\implies$  *is-acc-type G P (resTy m)*  
 $\langle \text{proof} \rangle$

### well-formed interface declarations

A interface declaration is wellformed if:

- the interface hierarchy is wellstructured
- there is no class with the same name
- the method heads are wellformed and not static and have Public access
- the methods are uniquely named
- all superinterfaces are accessible
- the result type of a method overriding a method of Object widens to the result type of the overridden method. Shadowing static methods is forbidden.
- the result type of a method overriding a set of methods defined in the superinterfaces widens to each of the corresponding result types

### constdefs

*wf-idecl* :: *prog*  $\Rightarrow$  *idecl*  $\Rightarrow$  *bool*  
*wf-idecl G*  $\equiv$   
 $\lambda(I, i).$   
 $ws-idecl\ G\ I\ (isuperIfs\ i) \wedge$   
 $\neg is-class\ G\ I \wedge$   
 $(\forall (sig, mh) \in set\ (imethods\ i). wf-mhead\ G\ (pid\ I)\ sig\ mh \wedge$   
 $\neg is-static\ mh \wedge$   
 $accmodi\ mh = Public) \wedge$   
 $unique\ (imethods\ i) \wedge$   
 $(\forall J \in set\ (isuperIfs\ i). is-acc-iface\ G\ (pid\ I)\ J) \wedge$   
 $(table-of\ (imethods\ i)$   
 $hiding\ (methd\ G\ Object)$   
 $under\ (\lambda new\ old. accmodi\ old \neq Private)$   
 $entails\ (\lambda new\ old. G \vdash resTy\ new \preceq resTy\ old \wedge$   
 $is-static\ new = is-static\ old)) \wedge$   
 $(o2s \circ table-of\ (imethods\ i)$   
 $hidings\ Un-tables((\lambda J. (imethds\ G\ J)) 'set\ (isuperIfs\ i))$   
 $entails\ (\lambda new\ old. G \vdash resTy\ new \preceq resTy\ old))$

**lemma** *wf-idecl-mhead*:  $\llbracket wf-idecl\ G\ (I, i); (sig, mh) \in set\ (imethods\ i) \rrbracket \implies$

*wf-mhead G (pid I) sig mh*  $\wedge \neg is-static\ mh \wedge accmodi\ mh = Public$   
 $\langle \text{proof} \rangle$

**lemma** *wf-idecl-hidings*:

*wf-idecl G (I, i)*  $\implies$   
 $(\lambda s. o2s\ (table-of\ (imethods\ i)\ s))$   
 $hidings\ Un-tables\ ((\lambda J. imethds\ G\ J)\ 'set\ (isuperIfs\ i))$   
 $entails\ \lambda new\ old. G \vdash resTy\ new \preceq resTy\ old$   
 $\langle \text{proof} \rangle$

**lemma** *wf-idecl-hiding*:

*wf-idecl*  $G (I, i) \implies$   
*table-of* (*imethods*  $i$ )  
           *hiding* (*methd*  $G$  *Object*)  
           *under*  $(\lambda \text{ new old. } \text{accmod } i \text{ old} \neq \text{Private})$   
           *entails*  $(\lambda \text{ new old. } G \vdash_{\text{resTy}} \text{new} \leq_{\text{resTy}} \text{old} \wedge$   
   *is-static*  $\text{new} = \text{is-static old})$ )  
 $\langle \text{proof} \rangle$

**lemma** *wf-idecl-supD*:

$\llbracket \text{wf-idecl } G (I, i); J \in \text{set } (\text{isuperIfs } i) \rrbracket$   
 $\implies \text{is-acc-iface } G (\text{pid } I) J \wedge (J, I) \notin (\text{subint1 } G)^+ +$   
 $\langle \text{proof} \rangle$

## well-formed class declarations

A class declaration is wellformed if:

- there is no interface with the same name
- all superinterfaces are accessible and for all methods implementing an interface method the result type widens to the result type of the interface method, the method is not static and offers at least as much access (this actually means that the method has Public access, since all interface methods have public access)
- all field declarations are wellformed and the field names are unique
- all method declarations are wellformed and the method names are unique
- the initialization statement is welltyped
- the classhierarchy is wellstructured
- Unless the class is *Object*:
  - the superclass is accessible
  - for all methods overriding another method (of a superclass) the result type widens to the result type of the overridden method, the access modifier of the new method provides at least as much access as the overwritten one.
  - for all methods hiding a method (of a superclass) the hidden method must be static and offer at least as much access rights. Remark: In contrast to the Java Language Specification we don't restrict the result types of the method (as in case of overriding), because there seems to be no reason, since there is no dynamic binding of static methods. (cf. 8.4.6.3 vs. 15.12.1). Stricly speaking the restrictions on the access rights aren't necessary to, since the static type and the access rights together determine which method is to be called statically. But if a class gains more then one static method with the same signature due to inheritance, it is confusing when the method selection depends on the access rights only: e.g. Class C declares static public method *foo()*. Class D is subclass of C and declares static method *foo()* with default package access. *D.foo()* ? if this call is in the same package as D then *foo* of class D is called, otherwise *foo* of class C.

**constdefs** *entails*::  $(\text{'a, 'b}) \text{ table} \Rightarrow (\text{'b} \Rightarrow \text{bool}) \Rightarrow \text{bool}$   
    $(- \text{ entails } - 20)$

$t \text{ entails } P \equiv \forall k. \forall x \in t \ k: P \ x$

**lemma** *entailsD*:

$\llbracket t \text{ entails } P; t \text{ k} = \text{Some } x \rrbracket \implies P \ x$   
 $\langle \text{proof} \rangle$

**lemma** *empty-entails[simp]*: *empty entails P*

$\langle \text{proof} \rangle$

**constdefs**

*wf-cdecl* :: *prog*  $\Rightarrow$  *cdecl*  $\Rightarrow$  *bool*

*wf-cdecl* *G*  $\equiv$

$\lambda(C, c).$   
 $\neg \text{is-iface } G \ C \wedge$   
 $(\forall I \in \text{set } (\text{superIfs } c). \text{is-acc-iface } G \ (\text{pid } C) \ I \wedge$   
 $(\forall s. \forall im \in \text{imethds } G \ I \ s.$   
 $(\exists cm \in \text{methd } G \ C \ s: G \vdash \text{resTy } cm \preceq \text{resTy } im \wedge$   
 $\neg \text{is-static } cm \wedge$   
 $\text{accmodi } im \leq \text{accmodi } cm))) \wedge$   
 $(\forall f \in \text{set } (\text{cfields } c). \text{wf-fdecl } G \ (\text{pid } C) \ f) \wedge \text{unique } (\text{cfields } c) \wedge$   
 $(\forall m \in \text{set } (\text{methods } c). \text{wf-mdecl } G \ C \ m) \wedge \text{unique } (\text{methods } c) \wedge$   
 $\text{jumpNestingOkS } \{\} \ (\text{init } c) \wedge$   
 $(\exists A. (\text{prg} = G, \text{cls} = C, \text{lcl} = \text{empty}) \vdash \{\} \gg \langle \text{init } c \rangle \gg A) \wedge$   
 $(\text{prg} = G, \text{cls} = C, \text{lcl} = \text{empty}) \vdash (\text{init } c) :: \checkmark \wedge \text{ws-cdecl } G \ C \ (\text{super } c) \wedge$   
 $(C \neq \text{Object} \longrightarrow$   
 $(\text{is-acc-class } G \ (\text{pid } C) \ (\text{super } c) \wedge$   
 $(\text{table-of } (\text{map } (\lambda (s, m). (s, C, m)) (\text{methods } c))$   
 $\text{entails } (\lambda \text{ new}. \forall \text{ old sig.}$   
 $(G, \text{sig} \vdash \text{new overrides}_S \text{ old}$   
 $\longrightarrow (G \vdash \text{resTy } \text{new} \preceq \text{resTy } \text{old} \wedge$   
 $\text{accmodi } \text{old} \leq \text{accmodi } \text{new} \wedge$   
 $\neg \text{is-static } \text{old})) \wedge$   
 $(G, \text{sig} \vdash \text{new hides old}$   
 $\longrightarrow (\text{accmodi } \text{old} \leq \text{accmodi } \text{new} \wedge$   
 $\text{is-static } \text{old}))))$   
 $)$

**lemma** *wf-cdeclE* [*consumes 1*]:

$\llbracket \text{wf-cdecl } G \ (C, c);$   
 $\llbracket \neg \text{is-iface } G \ C;$   
 $(\forall I \in \text{set } (\text{superIfs } c). \text{is-acc-iface } G \ (\text{pid } C) \ I \wedge$   
 $(\forall s. \forall im \in \text{imethds } G \ I \ s.$   
 $(\exists cm \in \text{methd } G \ C \ s: G \vdash \text{resTy } cm \preceq \text{resTy } im \wedge$   
 $\neg \text{is-static } cm \wedge$   
 $\text{accmodi } im \leq \text{accmodi } cm)))$ ;  
 $\forall f \in \text{set } (\text{cfields } c). \text{wf-fdecl } G \ (\text{pid } C) \ f; \text{unique } (\text{cfields } c);$   
 $\forall m \in \text{set } (\text{methods } c). \text{wf-mdecl } G \ C \ m; \text{unique } (\text{methods } c);$   
 $\text{jumpNestingOkS } \{\} \ (\text{init } c);$   
 $\exists A. (\text{prg} = G, \text{cls} = C, \text{lcl} = \text{empty}) \vdash \{\} \gg \langle \text{init } c \rangle \gg A;$   
 $(\text{prg} = G, \text{cls} = C, \text{lcl} = \text{empty}) \vdash (\text{init } c) :: \checkmark;$   
 $\text{ws-cdecl } G \ C \ (\text{super } c);$   
 $(C \neq \text{Object} \longrightarrow$   
 $(\text{is-acc-class } G \ (\text{pid } C) \ (\text{super } c) \wedge$   
 $(\text{table-of } (\text{map } (\lambda (s, m). (s, C, m)) (\text{methods } c))$   
 $\text{entails } (\lambda \text{ new}. \forall \text{ old sig.}$   
 $(G, \text{sig} \vdash \text{new overrides}_S \text{ old}$   
 $\longrightarrow (G \vdash \text{resTy } \text{new} \preceq \text{resTy } \text{old} \wedge$

$$\begin{aligned}
& \text{accmodi old} \leq \text{accmodi new} \wedge \\
& \neg \text{is-static old})) \wedge \\
& (G, \text{sig} \vdash \text{new hides old} \\
& \longrightarrow (\text{accmodi old} \leq \text{accmodi new} \wedge \\
& \text{is-static old}))) \\
& )) \Rightarrow P \\
& \Rightarrow P \\
& \langle \text{proof} \rangle
\end{aligned}$$

**lemma** *wf-cdecl-unique*:

$\text{wf-cdecl } G \ (C, c) \Rightarrow \text{unique } (\text{cfields } c) \wedge \text{unique } (\text{methods } c)$   
 $\langle \text{proof} \rangle$

**lemma** *wf-cdecl-fdecl*:

$\llbracket \text{wf-cdecl } G \ (C, c); f \in \text{set } (\text{cfields } c) \rrbracket \Rightarrow \text{wf-fdecl } G \ (\text{pid } C) \ f$   
 $\langle \text{proof} \rangle$

**lemma** *wf-cdecl-mdecl*:

$\llbracket \text{wf-cdecl } G \ (C, c); m \in \text{set } (\text{methods } c) \rrbracket \Rightarrow \text{wf-mdecl } G \ C \ m$   
 $\langle \text{proof} \rangle$

**lemma** *wf-cdecl-impD*:

$\llbracket \text{wf-cdecl } G \ (C, c); I \in \text{set } (\text{superIfs } c) \rrbracket$   
 $\Rightarrow \text{is-acc-iface } G \ (\text{pid } C) \ I \wedge$   
 $(\forall s. \forall im \in \text{imethds } G \ I \ s.$   
 $(\exists cm \in \text{methd } G \ C \ s: G \vdash \text{resTy } cm \preceq \text{resTy } im \wedge \neg \text{is-static } cm \wedge$   
 $\text{accmodi } im \leq \text{accmodi } cm))$   
 $\langle \text{proof} \rangle$

**lemma** *wf-cdecl-supD*:

$\llbracket \text{wf-cdecl } G \ (C, c); C \neq \text{Object} \rrbracket \Rightarrow$   
 $\text{is-acc-class } G \ (\text{pid } C) \ (\text{super } c) \wedge (\text{super } c, C) \notin (\text{subcls1 } G)^\wedge + \wedge$   
 $(\text{table-of } (\text{map } (\lambda (s, m). (s, C, m)) (\text{methods } c))$   
 $\text{entails } (\lambda \text{ new. } \forall \text{ old sig.}$   
 $(G, \text{sig} \vdash \text{new overrides}_S \text{ old}$   
 $\longrightarrow (G \vdash \text{resTy } \text{new} \preceq \text{resTy } \text{old} \wedge$   
 $\text{accmodi } \text{old} \leq \text{accmodi } \text{new} \wedge$   
 $\neg \text{is-static } \text{old})) \wedge$   
 $(G, \text{sig} \vdash \text{new hides old}$   
 $\longrightarrow (\text{accmodi } \text{old} \leq \text{accmodi } \text{new} \wedge$   
 $\text{is-static } \text{old}))))$   
 $\langle \text{proof} \rangle$

**lemma** *wf-cdecl-overrides-SomeD*:

$\llbracket \text{wf-cdecl } G \ (C, c); C \neq \text{Object}; \text{table-of } (\text{methods } c) \text{ sig} = \text{Some newM};$   
 $G, \text{sig} \vdash (C, \text{newM}) \text{ overrides}_S \text{ old}$   
 $\rrbracket \Rightarrow G \vdash \text{resTy } \text{newM} \preceq \text{resTy } \text{old} \wedge$   
 $\text{accmodi } \text{old} \leq \text{accmodi } \text{newM} \wedge$   
 $\neg \text{is-static } \text{old}$   
 $\langle \text{proof} \rangle$

**lemma** *wf-cdecl-hides-SomeD*:

$\llbracket wf\text{-}cdecl\ G\ (C, c); C \neq \text{Object}; \text{table-of}\ (methods\ c)\ sig = \text{Some}\ newM;$   
 $G, sig \vdash (C, newM)\ \text{hides}\ old$   
 $\rrbracket \implies accmodi\ old \leq access\ newM \wedge$   
 $is\text{-}static\ old$   
 $\langle proof \rangle$

**lemma** *wf-cdecl-wt-init*:

$wf\text{-}cdecl\ G\ (C, c) \implies (\llbracket prg = G, cls = C, lcl = empty \rrbracket) \vdash init\ c :: \checkmark$   
 $\langle proof \rangle$

## well-formed programs

A program declaration is wellformed if:

- the class `ObjectC` of `Object` is defined
- every method of `Object` has an access modifier distinct from `Package`. This is necessary since every interface automatically inherits from `Object`. We must know, that every time a `Object` method is "overridden" by an interface method this is also overridden by the class implementing the the interface (see *implement-dynmethd* and *class-mheadsD*)
- all standard Exceptions are defined
- all defined interfaces are wellformed
- all defined classes are wellformed

**constdefs**

$wf\text{-}prog :: prog \Rightarrow bool$   
 $wf\text{-}prog\ G \equiv let\ is = ifaces\ G; cs = classes\ G\ in$   
 $ObjectC \in set\ cs \wedge$   
 $(\forall\ m \in set\ Object\text{-}mdecls.\ accmodi\ m \neq Package) \wedge$   
 $(\forall\ xn.\ SXcptC\ xn \in set\ cs) \wedge$   
 $(\forall\ i \in set\ is.\ wf\text{-}idecl\ G\ i) \wedge unique\ is \wedge$   
 $(\forall\ c \in set\ cs.\ wf\text{-}cdecl\ G\ c) \wedge unique\ cs$

**lemma** *wf-prog-idecl*:  $\llbracket iface\ G\ I = \text{Some}\ i; wf\text{-}prog\ G \rrbracket \implies wf\text{-}idecl\ G\ (I, i)$

$\langle proof \rangle$

**lemma** *wf-prog-cdecl*:  $\llbracket class\ G\ C = \text{Some}\ c; wf\text{-}prog\ G \rrbracket \implies wf\text{-}cdecl\ G\ (C, c)$

$\langle proof \rangle$

**lemma** *wf-prog-Object-mdecls*:

$wf\text{-}prog\ G \implies (\forall\ m \in set\ Object\text{-}mdecls.\ accmodi\ m \neq Package)$

$\langle proof \rangle$

**lemma** *wf-prog-acc-superD*:

$\llbracket wf\text{-}prog\ G; class\ G\ C = \text{Some}\ c; C \neq \text{Object} \rrbracket$   
 $\implies is\text{-}acc\text{-}class\ G\ (pid\ C)\ (super\ c)$

$\langle proof \rangle$

**lemma** *wf-ws-prog* [*elim!*, *simp*]:  $wf\text{-}prog\ G \implies ws\text{-}prog\ G$

$\langle \text{proof} \rangle$

**lemma** *class-Object* [simp]:

$\text{wf-prog } G \implies$

$\text{class } G \text{ Object} = \text{Some } (\llbracket \text{access}=\text{Public}, \text{cfields}=\llbracket, \text{methods}=\text{Object-mdecls},$   
 $\text{init}=\text{Skip}, \text{super}=\text{arbitrary}, \text{superIfs}=\llbracket \rrbracket$

$\langle \text{proof} \rangle$

**lemma** *methd-Object*[simp]:  $\text{wf-prog } G \implies \text{methd } G \text{ Object} =$

$\text{table-of } (\text{map } (\lambda(s,m). (s, \text{Object}, m)) \text{ Object-mdecls})$

$\langle \text{proof} \rangle$

**lemma** *wf-prog-Object-methd*:

$\llbracket \text{wf-prog } G; \text{methd } G \text{ Object sig} = \text{Some } m \rrbracket \implies \text{accmodi } m \neq \text{Package}$

$\langle \text{proof} \rangle$

**lemma** *wf-prog-Object-is-public*[intro]:

$\text{wf-prog } G \implies \text{is-public } G \text{ Object}$

$\langle \text{proof} \rangle$

**lemma** *class-SXcpt* [simp]:

$\text{wf-prog } G \implies$

$\text{class } G (\text{SXcpt } xn) = \text{Some } (\llbracket \text{access}=\text{Public}, \text{cfields}=\llbracket, \text{methods}=\text{SXcpt-mdecls},$   
 $\text{init}=\text{Skip},$   
 $\text{super}=\text{if } xn = \text{Throwable} \text{ then } \text{Object}$   
 $\text{else } \text{SXcpt } \text{Throwable},$   
 $\text{superIfs}=\llbracket \rrbracket$

$\langle \text{proof} \rangle$

**lemma** *wf-ObjectC* [simp]:

$\text{wf-cdecl } G \text{ ObjectC} = (\neg \text{is-iface } G \text{ Object} \wedge \text{Ball } (\text{set } \text{Object-mdecls})$   
 $(\text{wf-mdecl } G \text{ Object}) \wedge \text{unique } \text{Object-mdecls})$

$\langle \text{proof} \rangle$

**lemma** *Object-is-class* [simp, elim!]:  $\text{wf-prog } G \implies \text{is-class } G \text{ Object}$

$\langle \text{proof} \rangle$

**lemma** *Object-is-acc-class* [simp, elim!]:  $\text{wf-prog } G \implies \text{is-acc-class } G \text{ S Object}$

$\langle \text{proof} \rangle$

**lemma** *SXcpt-is-class* [simp, elim!]:  $\text{wf-prog } G \implies \text{is-class } G (\text{SXcpt } xn)$

$\langle \text{proof} \rangle$

**lemma** *SXcpt-is-acc-class* [simp, elim!]:

$\text{wf-prog } G \implies \text{is-acc-class } G \text{ S } (\text{SXcpt } xn)$

$\langle \text{proof} \rangle$

**lemma** *fields-Object* [simp]:  $\text{wf-prog } G \implies \text{DeclConcepts.fields } G \text{ Object} = \llbracket$

$\langle \text{proof} \rangle$

**lemma** *accfield-Object* [simp]:  
 $\text{wf-prog } G \implies \text{accfield } G \text{ S Object} = \text{empty}$   
 $\langle \text{proof} \rangle$

**lemma** *fields-Throwable* [simp]:  
 $\text{wf-prog } G \implies \text{DeclConcepts.fields } G \text{ (SXcpt Throwable)} = []$   
 $\langle \text{proof} \rangle$

**lemma** *fields-SXcpt* [simp]:  $\text{wf-prog } G \implies \text{DeclConcepts.fields } G \text{ (SXcpt } xn) = []$   
 $\langle \text{proof} \rangle$

**lemmas** *widen-trans* = *ws-widen-trans* [OF - - *wf-ws-prog*, *elim*]

**lemma** *widen-trans2* [elim]:  $\llbracket G \vdash U \preceq T; G \vdash S \preceq U; \text{wf-prog } G \rrbracket \implies G \vdash S \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *Xcpt-subcls-Throwable* [simp]:  
 $\text{wf-prog } G \implies G \vdash \text{SXcpt } xn \preceq_C \text{ SXcpt Throwable}$   
 $\langle \text{proof} \rangle$

**lemma** *unique-fields*:  
 $\llbracket \text{is-class } G \text{ C}; \text{wf-prog } G \rrbracket \implies \text{unique } (\text{DeclConcepts.fields } G \text{ C})$   
 $\langle \text{proof} \rangle$

**lemma** *fields-mono*:  
 $\llbracket \text{table-of } (\text{DeclConcepts.fields } G \text{ C}) \text{ fn} = \text{Some } f; G \vdash D \preceq_C C; \text{is-class } G \text{ D}; \text{wf-prog } G \rrbracket$   
 $\implies \text{table-of } (\text{DeclConcepts.fields } G \text{ D}) \text{ fn} = \text{Some } f$   
 $\langle \text{proof} \rangle$

**lemma** *fields-is-type* [elim]:  
 $\llbracket \text{table-of } (\text{DeclConcepts.fields } G \text{ C}) \text{ m} = \text{Some } f; \text{wf-prog } G; \text{is-class } G \text{ C} \rrbracket \implies$   
 $\text{is-type } G \text{ (type } f)$   
 $\langle \text{proof} \rangle$

**lemma** *imethds-wf-mhead* [rule-format (no-asm)]:  
 $\llbracket m \in \text{imethds } G \text{ I sig}; \text{wf-prog } G; \text{is-iface } G \text{ I} \rrbracket \implies$   
 $\text{wf-mhead } G \text{ (pid (decliface m)) sig (mthd m)} \wedge$   
 $\neg \text{is-static } m \wedge \text{accmodi } m = \text{Public}$   
 $\langle \text{proof} \rangle$

**lemma** *methd-wf-mdecl*:  
 $\llbracket \text{methd } G \text{ C sig} = \text{Some } m; \text{wf-prog } G; \text{class } G \text{ C} = \text{Some } y \rrbracket \implies$   
 $G \vdash C \preceq_C (\text{declclass } m) \wedge \text{is-class } G \text{ (declclass } m) \wedge$   
 $\text{wf-mdecl } G \text{ (declclass } m) \text{ (sig, (mthd } m))$   
 $\langle \text{proof} \rangle$

**lemma** *methd-rT-is-type*:  
 $\llbracket wf\text{-prog } G; methd\ G\ C\ sig = Some\ m; \\ class\ G\ C = Some\ y \rrbracket \\ \implies is\text{-type}\ G\ (resTy\ m) \\ \langle proof \rangle$

**lemma** *accmethd-rT-is-type*:  
 $\llbracket wf\text{-prog } G; accmethd\ G\ S\ C\ sig = Some\ m; \\ class\ G\ C = Some\ y \rrbracket \\ \implies is\text{-type}\ G\ (resTy\ m) \\ \langle proof \rangle$

**lemma** *methd-Object-SomeD*:  
 $\llbracket wf\text{-prog } G; methd\ G\ Object\ sig = Some\ m \rrbracket \\ \implies declclass\ m = Object \\ \langle proof \rangle$

**lemma** *wf-imethdsD*:  
 $\llbracket im \in imethds\ G\ I\ sig; wf\text{-prog } G; is\text{-iface}\ G\ I \rrbracket \\ \implies \neg is\text{-static}\ im \wedge accmodi\ im = Public \\ \langle proof \rangle$

**lemma** *wf-prog-hidesD*:  
**assumes** *hides*:  $G \vdash new\ overrides\ old$  **and** *wf*:  $wf\text{-prog } G$   
**shows**  
 $accmodi\ old \leq accmodi\ new \wedge \\ is\text{-static}\ old \\ \langle proof \rangle$

Compare this lemma about static overriding  $G \vdash new\ overrides_S\ old$  with the definition of dynamic overriding  $G \vdash new\ overrides\ old$ . Conforming result types and restrictions on the access modifiers of the old and the new method are not part of the predicate for static overriding. But they are enshured in a wellfromed program. Dynamic overriding has no restrictions on the access modifiers but enforces confrom result types as precondition. But with some efford we can guarantee the access modifier restriction for dynamic overriding, too. See lemma *wf-prog-dyn-override-prop*.

**lemma** *wf-prog-stat-overridesD*:  
**assumes** *stat-override*:  $G \vdash new\ overrides_S\ old$  **and** *wf*:  $wf\text{-prog } G$   
**shows**  
 $G \vdash resTy\ new \preceq resTy\ old \wedge \\ accmodi\ old \leq accmodi\ new \wedge \\ \neg is\text{-static}\ old \\ \langle proof \rangle$

**lemma** *static-to-dynamic-overriding*:  
**assumes** *stat-override*:  $G \vdash new\ overrides_S\ old$  **and** *wf* :  $wf\text{-prog } G$   
**shows**  $G \vdash new\ overrides\ old \\ \langle proof \rangle$



**lemma** *non-Package-instance-method-inheritance:*

**assumes** *old-inheritable*:  $G \vdash \text{Method old inheritable-in (pid C)}$  **and**  
*accmodi-old*:  $\text{accmodi old} \neq \text{Package}$  **and**  
*instance-method*:  $\neg \text{is-static old}$  **and**  
*subcls*:  $G \vdash C \prec_C \text{declclass old}$  **and**  
*old-declared*:  $G \vdash \text{Method old declared-in (declclass old)}$  **and**  
*wf*:  $\text{wf-prog } G$   
**shows**  $G \vdash \text{Method old member-of } C \vee$   
 $(\exists \text{ new. } G \vdash \text{new overrides}_S \text{ old} \wedge G \vdash \text{Method new member-of } C)$   
 $\langle \text{proof} \rangle$

**lemma** *non-Package-instance-method-inheritance-cases* [consumes 6, case-names *Inheritance Overriding*]:

**assumes** *old-inheritable*:  $G \vdash \text{Method old inheritable-in (pid C)}$  **and**  
*accmodi-old*:  $\text{accmodi old} \neq \text{Package}$  **and**  
*instance-method*:  $\neg \text{is-static old}$  **and**  
*subcls*:  $G \vdash C \prec_C \text{declclass old}$  **and**  
*old-declared*:  $G \vdash \text{Method old declared-in (declclass old)}$  **and**  
*wf*:  $\text{wf-prog } G$  **and**  
*inheritance*:  $G \vdash \text{Method old member-of } C \implies P$  **and**  
*overriding*:  $\bigwedge \text{ new. } \llbracket G \vdash \text{new overrides}_S \text{ old}; G \vdash \text{Method new member-of } C \rrbracket \implies P$   
**shows**  $P$   
 $\langle \text{proof} \rangle$

**lemma** *dynamic-to-static-overriding:*

**assumes** *dyn-override*:  $G \vdash \text{new overrides old}$  **and**  
*accmodi-old*:  $\text{accmodi old} \neq \text{Package}$  **and**  
*wf*:  $\text{wf-prog } G$   
**shows**  $G \vdash \text{new overrides}_S \text{ old}$   
 $\langle \text{proof} \rangle$

**lemma** *wf-prog-dyn-override-prop:*

**assumes** *dyn-override*:  $G \vdash \text{new overrides old}$  **and**  
*wf*:  $\text{wf-prog } G$   
**shows**  $\text{accmodi old} \leq \text{accmodi new}$   
 $\langle \text{proof} \rangle$

**lemma** *overrides-Package-old:*

**assumes** *dyn-override*:  $G \vdash \text{new overrides old}$  **and**  
*accmodi-new*:  $\text{accmodi new} = \text{Package}$  **and**  
*wf*:  $\text{wf-prog } G$   
**shows**  $\text{accmodi old} = \text{Package}$   
 $\langle \text{proof} \rangle$

**lemma** *dyn-override-Package:*

**assumes** *dyn-override*:  $G \vdash \text{new overrides old}$  **and**  
*accmodi-old*:  $\text{accmodi old} = \text{Package}$  **and**  
*accmodi-new*:  $\text{accmodi new} = \text{Package}$  **and**  
*wf*:  $\text{wf-prog } G$   
**shows**  $\text{pid (declclass old)} = \text{pid (declclass new)}$   
 $\langle \text{proof} \rangle$

**lemma** *dyn-override-Package-escape*:

**assumes** *dyn-override*:  $G \vdash \text{new overrides old}$  **and**  
*accmodi-old*:  $\text{accmodi old} = \text{Package}$  **and**  
*outside-pack*:  $\text{pid}(\text{declclass old}) \neq \text{pid}(\text{declclass new})$  **and**  
*wf*: *wf-prog*  $G$   
**shows**  $\exists \text{ inter. } G \vdash \text{new overrides inter} \wedge G \vdash \text{inter overrides old} \wedge$   
 $\text{pid}(\text{declclass old}) = \text{pid}(\text{declclass inter}) \wedge$   
 $\text{Protected} \leq \text{accmodi inter}$

$\langle \text{proof} \rangle$

**lemma** *declclass-widen*[*rule-format*]:

*wf-prog*  $G$   
 $\longrightarrow (\forall c \ m. \text{class } G \ C = \text{Some } c \longrightarrow \text{methd } G \ C \ \text{sig} = \text{Some } m$   
 $\longrightarrow G \vdash C \preceq_C \text{declclass } m) \text{ (is ?P } G \ C)$

$\langle \text{proof} \rangle$

**lemma** *declclass-methd-Object*:

$\llbracket \text{wf-prog } G; \text{methd } G \ \text{Object} \ \text{sig} = \text{Some } m \rrbracket \Longrightarrow \text{declclass } m = \text{Object}$   
 $\langle \text{proof} \rangle$

**lemma** *methd-declaredD*:

$\llbracket \text{wf-prog } G; \text{is-class } G \ C; \text{methd } G \ C \ \text{sig} = \text{Some } m \rrbracket$   
 $\Longrightarrow G \vdash (\text{mdecl}(\text{sig}, \text{methd } m)) \text{ declared-in } (\text{declclass } m)$   
 $\langle \text{proof} \rangle$

**lemma** *methd-rec-Some-cases* [*consumes 4*, *case-names NewMethod InheritedMethod*]:

**assumes** *methd-C*:  $\text{methd } G \ C \ \text{sig} = \text{Some } m$  **and**  
*ws*: *ws-prog*  $G$  **and**  
*clsC*:  $\text{class } G \ C = \text{Some } c$  **and**  
*neq-C-Obj*:  $C \neq \text{Object}$   
**shows**  
 $\llbracket \text{table-of}(\text{map}(\lambda(s, m). (s, C, m))(\text{methods } c)) \ \text{sig} = \text{Some } m \rrbracket \Longrightarrow P;$   
 $\llbracket G \vdash C \text{ inherits } (\text{method } \text{sig } m); \text{methd } G \ (\text{super } c) \ \text{sig} = \text{Some } m \rrbracket \Longrightarrow P$   
 $\rrbracket \Longrightarrow P$   
 $\langle \text{proof} \rangle$

**lemma** *methd-member-of*:

**assumes** *wf*: *wf-prog*  $G$   
**shows**  
 $\llbracket \text{is-class } G \ C; \text{methd } G \ C \ \text{sig} = \text{Some } m \rrbracket \Longrightarrow G \vdash \text{Methd } \text{sig } m \text{ member-of } C$   
 $(\text{is ?Class } C \Longrightarrow \text{?Method } C \Longrightarrow \text{?MemberOf } C)$   
 $\langle \text{proof} \rangle$

**lemma** *current-methd*:

$\llbracket \text{table-of}(\text{methods } c) \ \text{sig} = \text{Some } \text{new};$   
 $\text{ws-prog } G; \text{class } G \ C = \text{Some } c; C \neq \text{Object};$   
 $\text{methd } G \ (\text{super } c) \ \text{sig} = \text{Some } \text{old} \rrbracket$   
 $\Longrightarrow \text{methd } G \ C \ \text{sig} = \text{Some } (C, \text{new})$   
 $\langle \text{proof} \rangle$

**lemma** *wf-prog-staticD*:

**assumes** *wf*: wf-prog *G* **and**  
*clsC*: class *G* *C* = Some *c* **and**  
*neq-C-Obj*: *C*  $\neq$  Object **and**  
*old*: methd *G* (super *c*) sig = Some *old* **and**  
*accmodi-old*: Protected  $\leq$  accmodi *old* **and**  
*new*: table-of (methods *c*) sig = Some *new*  
**shows** is-static *new* = is-static *old*  
 ⟨proof⟩

**lemma** *inheritable-instance-methd*:

**assumes** *subclseq-C-D*:  $G \vdash C \preceq_C D$  **and**  
*is-cls-D*: is-class *G* *D* **and**  
*wf*: wf-prog *G* **and**  
*old*: methd *G* *D* sig = Some *old* **and**  
*accmodi-old*: Protected  $\leq$  accmodi *old* **and**  
*not-static-old*:  $\neg$  is-static *old*  
**shows**  
 $\exists$  *new*. methd *G* *C* sig = Some *new*  $\wedge$   
 (*new* = *old*  $\vee$  *G*,sig $\vdash$ *new* overrides<sub>*S*</sub> *old*)  
 (is ( $\exists$  *new*. (?Constraint *C* *new* *old*)))  
 ⟨proof⟩

**lemma** *inheritable-instance-methd-cases* [consumes 6  
 , case-names *Inheritance* *Overriding*]:

**assumes** *subclseq-C-D*:  $G \vdash C \preceq_C D$  **and**  
*is-cls-D*: is-class *G* *D* **and**  
*wf*: wf-prog *G* **and**  
*old*: methd *G* *D* sig = Some *old* **and**  
*accmodi-old*: Protected  $\leq$  accmodi *old* **and**  
*not-static-old*:  $\neg$  is-static *old* **and**  
*inheritance*: methd *G* *C* sig = Some *old*  $\implies$  *P* **and**  
*overriding*:  $\bigwedge$  *new*.  $\llbracket$ methd *G* *C* sig = Some *new*;  
*G*,sig $\vdash$ *new* overrides<sub>*S*</sub> *old* $\rrbracket \implies$  *P*  
**shows** *P*  
 ⟨proof⟩

**lemma** *inheritable-instance-methd-props*:

**assumes** *subclseq-C-D*:  $G \vdash C \preceq_C D$  **and**  
*is-cls-D*: is-class *G* *D* **and**  
*wf*: wf-prog *G* **and**  
*old*: methd *G* *D* sig = Some *old* **and**  
*accmodi-old*: Protected  $\leq$  accmodi *old* **and**  
*not-static-old*:  $\neg$  is-static *old*  
**shows**  
 $\exists$  *new*. methd *G* *C* sig = Some *new*  $\wedge$   
 $\neg$  is-static *new*  $\wedge$  *G* $\vdash$ resTy *new*  $\preceq$ resTy *old*  $\wedge$  accmodi *old*  $\leq$  accmodi *new*  
 (is ( $\exists$  *new*. (?Constraint *C* *new* *old*)))  
 ⟨proof⟩

**lemma** *bexI'*:  $x \in A \implies P\ x \implies \exists x \in A. P\ x$  ⟨proof⟩

**lemma** *ballE'*:  $\forall x \in A. P\ x \implies (x \notin A \implies Q) \implies (P\ x \implies Q) \implies Q$  ⟨proof⟩

**lemma** *subint-widen-imethds*:

$$\begin{aligned} & \llbracket G \vdash I \preceq I \ J; \text{wf-prog } G; \text{is-iface } G \ J; jm \in \text{imethds } G \ J \ \text{sig} \rrbracket \implies \\ & \quad \exists im \in \text{imethds } G \ I \ \text{sig}. \text{is-static } im = \text{is-static } jm \wedge \\ & \quad \quad \text{accmodi } im = \text{accmodi } jm \wedge \\ & \quad \quad G \vdash \text{resTy } im \preceq \text{resTy } jm \end{aligned}$$

$\langle \text{proof} \rangle$

**lemma** *implmt1-methd*:

$$\begin{aligned} & \bigwedge \text{sig}. \llbracket G \vdash C \rightsquigarrow I; \text{wf-prog } G; im \in \text{imethds } G \ I \ \text{sig} \rrbracket \implies \\ & \quad \exists cm \in \text{methd } G \ C \ \text{sig}: \neg \text{is-static } cm \wedge \neg \text{is-static } im \wedge \\ & \quad \quad G \vdash \text{resTy } cm \preceq \text{resTy } im \wedge \\ & \quad \quad \text{accmodi } im = \text{Public} \wedge \text{accmodi } cm = \text{Public} \end{aligned}$$

$\langle \text{proof} \rangle$

**lemma** *implmt-methd* [rule-format (no-asm)]:

$$\begin{aligned} & \llbracket \text{wf-prog } G; G \vdash C \rightsquigarrow I \rrbracket \implies \text{is-iface } G \ I \longrightarrow \\ & \quad (\forall im \in \text{imethds } G \ I \ \text{sig}. \\ & \quad \quad \exists cm \in \text{methd } G \ C \ \text{sig}: \neg \text{is-static } cm \wedge \neg \text{is-static } im \wedge \\ & \quad \quad \quad G \vdash \text{resTy } cm \preceq \text{resTy } im \wedge \\ & \quad \quad \quad \text{accmodi } im = \text{Public} \wedge \text{accmodi } cm = \text{Public}) \end{aligned}$$

$\langle \text{proof} \rangle$

**lemma** *mheadsD* [rule-format (no-asm)]:

$$\begin{aligned} & emh \in \text{mheads } G \ S \ t \ \text{sig} \longrightarrow \text{wf-prog } G \longrightarrow \\ & \quad (\exists C \ D \ m. t = \text{ClassT } C \wedge \text{declrefT } emh = \text{ClassT } D \wedge \\ & \quad \quad \text{accmethd } G \ S \ C \ \text{sig} = \text{Some } m \wedge \\ & \quad \quad (\text{declclass } m = D) \wedge \text{mhead } (mthd \ m) = (mhd \ emh)) \vee \\ & \quad (\exists I. t = \text{IfaceT } I \wedge ((\exists im. im \in \text{accimethds } G \ (pid \ S) \ I \ \text{sig} \wedge \\ & \quad \quad mthd \ im = mhd \ emh) \vee \\ & \quad \quad (\exists m. G \vdash \text{Iface } I \ \text{accessible-in } (pid \ S) \wedge \text{accmethd } G \ S \ \text{Object } \text{sig} = \text{Some } m \wedge \\ & \quad \quad \text{accmodi } m \neq \text{Private} \wedge \\ & \quad \quad \text{declrefT } emh = \text{ClassT } \text{Object} \wedge \text{mhead } (mthd \ m) = mhd \ emh))) \vee \\ & \quad (\exists T \ m. t = \text{ArrayT } T \wedge G \vdash \text{Array } T \ \text{accessible-in } (pid \ S) \wedge \\ & \quad \quad \text{accmethd } G \ S \ \text{Object } \text{sig} = \text{Some } m \wedge \text{accmodi } m \neq \text{Private} \wedge \\ & \quad \quad \text{declrefT } emh = \text{ClassT } \text{Object} \wedge \text{mhead } (mthd \ m) = mhd \ emh) \end{aligned}$$

$\langle \text{proof} \rangle$

**lemma** *mheads-cases* [consumes 2, case-names Class-methd

*Iface-methd Iface-Object-methd Array-Object-methd*]:

$$\begin{aligned} & \llbracket emh \in \text{mheads } G \ S \ t \ \text{sig}; \text{wf-prog } G; \\ & \quad \bigwedge C \ D \ m. \llbracket t = \text{ClassT } C; \text{declrefT } emh = \text{ClassT } D; \text{accmethd } G \ S \ C \ \text{sig} = \text{Some } m; \\ & \quad \quad (\text{declclass } m = D); \text{mhead } (mthd \ m) = (mhd \ emh) \rrbracket \implies P \ emh; \\ & \quad \bigwedge I \ im. \llbracket t = \text{IfaceT } I; im \in \text{accimethds } G \ (pid \ S) \ I \ \text{sig}; mthd \ im = mhd \ emh \rrbracket \\ & \quad \implies P \ emh; \\ & \quad \bigwedge I \ m. \llbracket t = \text{IfaceT } I; G \vdash \text{Iface } I \ \text{accessible-in } (pid \ S); \\ & \quad \quad \text{accmethd } G \ S \ \text{Object } \text{sig} = \text{Some } m; \text{accmodi } m \neq \text{Private}; \end{aligned}$$

$$\begin{aligned}
& \text{declrefT } emh = \text{ClassT Object}; \text{mhead (methd m) = mhd emh} \implies P \text{ emh}; \\
\wedge \quad & T \text{ m. } \llbracket t = \text{ArrayT T}; G \vdash \text{Array T accessible-in (pid S)}; \\
& \text{accmethd G S Object sig = Some m; accmodi m} \neq \text{Private}; \\
& \text{declrefT } emh = \text{ClassT Object}; \text{mhead (methd m) = mhd emh} \implies P \text{ emh} \\
& \implies P \text{ emh} \\
& \langle \text{proof} \rangle
\end{aligned}$$

**lemma** *declclassD[rule-format]*:  
 $\llbracket \text{wf-prog G}; \text{class G C} = \text{Some c}; \text{methd G C sig} = \text{Some m};$   
 $\text{class G (declclass m) = Some d} \rrbracket$   
 $\implies \text{table-of (methods d) sig} = \text{Some (methd m)}$   
 $\langle \text{proof} \rangle$

**lemma** *dynmethd-Object*:  
**assumes** *statM*:  $\text{methd G Object sig} = \text{Some statM}$  **and**  
 $\text{private: accmodi statM} = \text{Private}$  **and**  
 $\text{is-clc-C: is-class G C}$  **and**  
 $\text{wf: wf-prog G}$   
**shows**  $\text{dynmethd G Object C sig} = \text{Some statM}$   
 $\langle \text{proof} \rangle$

**lemma** *wf-imethds-hiding-objmethdsD*:  
**assumes**  $\text{old: methd G Object sig} = \text{Some old}$  **and**  
 $\text{is-if-I: is-iface G I}$  **and**  
 $\text{wf: wf-prog G}$  **and**  
 $\text{not-private: accmodi old} \neq \text{Private}$  **and**  
 $\text{new: new} \in \text{imethds G I sig}$   
**shows**  $G \vdash \text{resTy new} \preceq \text{resTy old} \wedge \text{is-static new} = \text{is-static old (is ?P new)}$   
 $\langle \text{proof} \rangle$

Which dynamic classes are valid to look up a member of a distinct static type? We have to distinct class members (named static members in Java) from instance members. Class members are global to all Objects of a class, instance members are local to a single Object instance. If a member is equipped with the static modifier it is a class member, else it is an instance member. The following table gives an overview of the current framework. We assume to have a reference with static type *statT* and a dynamic class *dynC*. Between both of these types the widening relation holds  $G \vdash \text{Class dynC} \preceq \text{statT}$ . Unfortunately this ordinary widening relation isn't enough to describe the valid lookup classes, since we must cope the special cases of arrays and interfaces, too. If we statically expect an array or interface we may lookup a field or a method in Object which isn't covered in the widening relation.

statT	field	instance	method	static	(class)	method	_____
_____	NullT	/	/	/	Iface	/	dynC Object Class dynC dynC dynC Array / Object Object

In most cases we can lookup the member in the dynamic class. But as an interface can't declare new static methods, nor an array can define new methods at all, we have to lookup methods in the base class Object.

The limitation to classes in the field column is artificial and comes out of the typing rule for the field access (see rule *FVar* in the welltyping relation *wt* in theory WellType). It stems out of the fact, that Object indeed has no non private fields. So interfaces and arrays can actually have no fields at all and a field access would be senseless. (In Java interfaces are allowed to declare new fields but in current Bali not!). So there is no principal reason why we should not allow Objects to declare non

private fields. Then we would get the following column:

statT field ————— NullT / Iface Object Class dynC Array Object

**consts** *valid-lookup-cls*:: *prog*  $\Rightarrow$  *ref-ty*  $\Rightarrow$  *qtname*  $\Rightarrow$  *bool*  $\Rightarrow$  *bool*  
 $(-, - \vdash - \text{ valid'-lookup'-cls'-for } - [61, 61, 61, 61] \ 60)$

**primrec**

$G, \text{NullT} \vdash \text{dynC valid-lookup-cls-for static-membr} = \text{False}$

$G, \text{IfaceT } I \vdash \text{dynC valid-lookup-cls-for static-membr}$   
 $= (\text{if static-membr}$

$\text{then dynC=Object}$

$\text{else } G \vdash \text{Class dynC} \preceq \text{Iface } I)$

$G, \text{ClassT } C \vdash \text{dynC valid-lookup-cls-for static-membr} = G \vdash \text{Class dynC} \preceq \text{Class } C$

$G, \text{ArrayT } T \vdash \text{dynC valid-lookup-cls-for static-membr} = (\text{dynC=Object})$

**lemma** *valid-lookup-cls-is-class*:

**assumes** *dynC*:  $G, \text{statT} \vdash \text{dynC valid-lookup-cls-for static-membr}$  **and**  
 $\text{ty-statT}: \text{isrtype } G \ \text{statT}$  **and**

$\text{wf}: \text{wf-prog } G$

**shows** *is-class*  $G \ \text{dynC}$

$\langle \text{proof} \rangle$

**declare** *split-paired-All* [simp del] *split-paired-Ex* [simp del]

$\langle \text{ML} \rangle$

**lemma** *dynamic-mheadsD*:

$\llbracket \text{emh} \in \text{mheads } G \ S \ \text{statT} \ \text{sig};$

$G, \text{statT} \vdash \text{dynC valid-lookup-cls-for (is-static emh)};$

$\text{isrtype } G \ \text{statT}; \text{wf-prog } G$

$\rrbracket \implies \exists m \in \text{dynlookup } G \ \text{statT} \ \text{dynC} \ \text{sig}:$

$\text{is-static } m = \text{is-static } \text{emh} \wedge G \vdash \text{resTy } m \preceq \text{resTy } \text{emh}$

$\langle \text{proof} \rangle$

**declare** *split-paired-All* [simp] *split-paired-Ex* [simp]

$\langle \text{ML} \rangle$

**lemma** *methd-declclass*:

$\llbracket \text{class } G \ C = \text{Some } c; \text{wf-prog } G; \text{methd } G \ C \ \text{sig} = \text{Some } m \rrbracket$

$\implies \text{methd } G \ (\text{declclass } m) \ \text{sig} = \text{Some } m$

$\langle \text{proof} \rangle$

**lemma** *dynmethd-declclass*:

$\llbracket \text{dynmethd } G \ \text{statC} \ \text{dynC} \ \text{sig} = \text{Some } m;$

$\text{wf-prog } G; \text{is-class } G \ \text{statC}$

$\rrbracket \implies \text{methd } G \ (\text{declclass } m) \ \text{sig} = \text{Some } m$

$\langle \text{proof} \rangle$

**lemma** *dynlookup-declC*:

$\llbracket \text{dynlookup } G \ \text{statT} \ \text{dynC} \ \text{sig} = \text{Some } m; \text{wf-prog } G;$

$\text{is-class } G \ \text{dynC}; \text{isrtype } G \ \text{statT}$

$\rrbracket \implies G \vdash \text{dynC} \preceq_C (\text{declclass } m) \wedge \text{is-class } G \ (\text{declclass } m)$

$\langle \text{proof} \rangle$

**lemma** *dynlookup-Array-declclassD* [simp]:  
 $\llbracket \text{dynlookup } G \text{ (ArrayT } T) \text{ Object sig} = \text{Some } dm; \text{wf-prog } G \rrbracket$   
 $\implies \text{declclass } dm = \text{Object}$   
 <proof>

**declare** *split-paired-All* [simp del] *split-paired-Ex* [simp del]  
 <ML>

**lemma** *wt-is-type*:  $E, dt \models v :: T \implies \text{wf-prog } (\text{prg } E) \longrightarrow$   
 $dt = \text{empty-dt} \longrightarrow (\text{case } T \text{ of}$   
 $\quad \text{Inl } T \Rightarrow \text{is-type } (\text{prg } E) \text{ } T$   
 $\quad | \text{Inr } Ts \Rightarrow \text{Ball } (\text{set } Ts) (\text{is-type } (\text{prg } E)))$   
 <proof>  
**declare** *split-paired-All* [simp] *split-paired-Ex* [simp]  
 <ML>

**lemma** *ty-expr-is-type*:  
 $\llbracket E \vdash e :: -T; \text{wf-prog } (\text{prg } E) \rrbracket \implies \text{is-type } (\text{prg } E) \text{ } T$   
 <proof>

**lemma** *ty-var-is-type*:  
 $\llbracket E \vdash v :: T; \text{wf-prog } (\text{prg } E) \rrbracket \implies \text{is-type } (\text{prg } E) \text{ } T$   
 <proof>

**lemma** *ty-exprs-is-type*:  
 $\llbracket E \vdash es :: Ts; \text{wf-prog } (\text{prg } E) \rrbracket \implies \text{Ball } (\text{set } Ts) (\text{is-type } (\text{prg } E))$   
 <proof>

**lemma** *static-mheadsD*:  
 $\llbracket emh \in \text{mheads } G \text{ } S \text{ } t \text{ } sig; \text{wf-prog } G; E \vdash e :: -\text{RefT } t; \text{prg } E = G ;$   
 $\quad \text{invmode } (mhd \text{ } emh) \text{ } e \neq \text{IntVir}$   
 $\rrbracket \implies \exists m. ( (\exists C. t = \text{ClassT } C \wedge \text{accmethd } G \text{ } S \text{ } C \text{ } sig = \text{Some } m)$   
 $\quad \vee (\forall C. t \neq \text{ClassT } C \wedge \text{accmethd } G \text{ } S \text{ } \text{Object } sig = \text{Some } m )) \wedge$   
 $\quad \text{declrefT } emh = \text{ClassT } (\text{declclass } m) \wedge \text{mhead } (mthd \text{ } m) = (mhd \text{ } emh)$   
 <proof>

**lemma** *wt-MethdI*:  
 $\llbracket \text{methd } G \text{ } C \text{ } sig = \text{Some } m; \text{wf-prog } G;$   
 $\quad \text{class } G \text{ } C = \text{Some } c \rrbracket \implies$   
 $\exists T. (\text{prg} = G, \text{cls} = (\text{declclass } m),$   
 $\quad \text{lcl} = \text{callee-lcl } (\text{declclass } m) \text{ } sig \text{ } (mthd \text{ } m)) \Vdash \text{Methd } C \text{ } sig :: -T \wedge G \vdash T \preceq_{\text{resTy}} m$   
 <proof>

### 35 accessibility concerns

**lemma** *mheads-type-accessible*:  
 $\llbracket emh \in \text{mheads } G \text{ } S \text{ } T \text{ } sig; \text{wf-prog } G \rrbracket$   
 $\implies G \vdash \text{RefT } T \text{ accessible-in } (\text{pid } S)$   
 <proof>

**lemma** *static-to-dynamic-accessible-from-aux:*

$\llbracket G \vdash m \text{ of } C \text{ accessible-from } accC; wf\text{-prog } G \rrbracket$   
 $\implies G \vdash m \text{ in } C \text{ dyn-accessible-from } accC$   
 <proof>

**lemma** *static-to-dynamic-accessible-from:*

**assumes** *stat-acc:*  $G \vdash m \text{ of } statC \text{ accessible-from } accC$  **and**  
*subclseq:*  $G \vdash dynC \preceq_C statC$  **and**  
*wf:*  $wf\text{-prog } G$   
**shows**  $G \vdash m \text{ in } dynC \text{ dyn-accessible-from } accC$   
 <proof>

**lemma** *static-to-dynamic-accessible-from-static:*

**assumes** *stat-acc:*  $G \vdash m \text{ of } statC \text{ accessible-from } accC$  **and**  
*static:* *is-static*  $m$  **and**  
*wf:*  $wf\text{-prog } G$   
**shows**  $G \vdash m \text{ in } (declclass\ m) \text{ dyn-accessible-from } accC$   
 <proof>

**lemma** *dynmethd-member-in:*

**assumes**  $m: dynmethd\ G\ statC\ dynC\ sig = Some\ m$  **and**  
*iscls-statC:* *is-class*  $G\ statC$  **and**  
*wf:*  $wf\text{-prog } G$   
**shows**  $G \vdash Methd\ sig\ m \text{ member-in } dynC$   
 <proof>

**lemma** *dynmethd-access-prop:*

**assumes** *statM:*  $methd\ G\ statC\ sig = Some\ statM$  **and**  
*stat-acc:*  $G \vdash Methd\ sig\ statM \text{ of } statC \text{ accessible-from } accC$  **and**  
*dynM:*  $dynmethd\ G\ statC\ dynC\ sig = Some\ dynM$  **and**  
*wf:*  $wf\text{-prog } G$   
**shows**  $G \vdash Methd\ sig\ dynM \text{ in } dynC \text{ dyn-accessible-from } accC$   
 <proof>

**lemma** *implmt-methd-access:*

**fixes**  $accC::qname$   
**assumes** *iface-methd:*  $imethds\ G\ I\ sig \neq \{\}$  **and**  
*implements:*  $G \vdash dynC \rightsquigarrow I$  **and**  
*isif-I:* *is-iface*  $G\ I$  **and**  
*wf:*  $wf\text{-prog } G$   
**shows**  $\exists\ dynM. methd\ G\ dynC\ sig = Some\ dynM \wedge$   
 $G \vdash Methd\ sig\ dynM \text{ in } dynC \text{ dyn-accessible-from } accC$   
 <proof>

**corollary** *implmt-dynimethd-access:*

**fixes**  $accC::qname$   
**assumes** *iface-methd:*  $imethds\ G\ I\ sig \neq \{\}$  **and**  
*implements:*  $G \vdash dynC \rightsquigarrow I$  **and**  
*isif-I:* *is-iface*  $G\ I$  **and**  
*wf:*  $wf\text{-prog } G$   
**shows**  $\exists\ dynM. dynimethd\ G\ I\ dynC\ sig = Some\ dynM \wedge$   
 $G \vdash Methd\ sig\ dynM \text{ in } dynC \text{ dyn-accessible-from } accC$   
 <proof>



**lemma** *dynlookup-access-prop*:

**assumes** *emh*:  $emh \in mheads\ G\ accC\ statT\ sig$  **and**  
 $dynM$ :  $dynlookup\ G\ statT\ dynC\ sig = Some\ dynM$  **and**  
 $dynC$ -prop:  $G, statT \vdash dynC\ valid\ lookup\ cls\ for\ is\ static\ emh$  **and**  
 $isT$ -statT:  $isrtype\ G\ statT$  **and**  
 $wf$ :  $wf\ prog\ G$

**shows**  $G \vdash Methd\ sig\ dynM\ in\ dynC\ dyn\ accessible\ from\ accC$   
 $\langle proof \rangle$

**lemma** *dynlookup-access*:

**assumes** *emh*:  $emh \in mheads\ G\ accC\ statT\ sig$  **and**  
 $dynC$ -prop:  $G, statT \vdash dynC\ valid\ lookup\ cls\ for\ (is\ static\ emh)$  **and**  
 $isT$ -statT:  $isrtype\ G\ statT$  **and**  
 $wf$ :  $wf\ prog\ G$

**shows**  $\exists\ dynM. dynlookup\ G\ statT\ dynC\ sig = Some\ dynM \wedge$   
 $G \vdash Methd\ sig\ dynM\ in\ dynC\ dyn\ accessible\ from\ accC$   
 $\langle proof \rangle$

**lemma** *stat-overrides-Package-old*:

**assumes**  $stat\ override$ :  $G \vdash new\ overrides\ old$  **and**  
 $accmodi$ -new:  $accmodi\ new = Package$  **and**  
 $wf$ :  $wf\ prog\ G$

**shows**  $accmodi\ old = Package$   
 $\langle proof \rangle$

## Properties of dynamic accessibility

**lemma** *dyn-accessible-Private*:

**assumes**  $dyn\ acc$ :  $G \vdash m\ in\ C\ dyn\ accessible\ from\ accC$  **and**  
 $priv$ :  $accmodi\ m = Private$

**shows**  $accC = declclass\ m$   
 $\langle proof \rangle$

*dyn-accessible-Package* only works with the *wf-prog* assumption. Without it. it is easy to leaf the Package!

**lemma** *dyn-accessible-Package*:

$\llbracket G \vdash m\ in\ C\ dyn\ accessible\ from\ accC; accmodi\ m = Package;$   
 $wf\ prog\ G \rrbracket$

$\implies pid\ accC = pid\ (declclass\ m)$   
 $\langle proof \rangle$

For fields we don't need the wellformedness of the program, since there is no overriding

**lemma** *dyn-accessible-field-Package*:

**assumes**  $dyn\ acc$ :  $G \vdash f\ in\ C\ dyn\ accessible\ from\ accC$  **and**  
 $pack$ :  $accmodi\ f = Package$  **and**  
 $field$ :  $is\ field\ f$

**shows**  $pid\ accC = pid\ (declclass\ f)$   
 $\langle proof \rangle$

*dyn-accessible-instance-field-Protected* only works for fields since methods can break the package bounds due to overriding

**lemma** *dyn-accessible-instance-field-Protected*:

**assumes**  $dyn\ acc$ :  $G \vdash f\ in\ C\ dyn\ accessible\ from\ accC$  **and**  
 $prot$ :  $accmodi\ f = Protected$  **and**  
 $field$ :  $is\ field\ f$  **and**

*instance-field*:  $\neg \text{is-static } f$  **and**  
*outside*:  $\text{pid } (\text{declclass } f) \neq \text{pid } \text{accC}$   
**shows**  $G \vdash C \preceq_C \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *dyn-accessible-static-field-Protected*:  
**assumes** *dyn-acc*:  $G \vdash f \text{ in } C \text{ dyn-accessible-from } \text{accC}$  **and**  
*prot*:  $\text{accmodi } f = \text{Protected}$  **and**  
*field*: *is-field*  $f$  **and**  
*static-field*: *is-static*  $f$  **and**  
*outside*:  $\text{pid } (\text{declclass } f) \neq \text{pid } \text{accC}$   
**shows**  $G \vdash \text{accC} \preceq_C \text{declclass } f \wedge G \vdash C \preceq_C \text{declclass } f$   
 $\langle \text{proof} \rangle$

**end**

## Chapter 14

# State

### 36 State for evaluation of Java expressions and statements

**theory** *State* **imports** *DeclConcepts* **begin**

design issues:

- all kinds of objects (class instances, arrays, and class objects) are handled via a general object abstraction
- the heap and the map for class objects are combined into a single table (*recall* (*loc*, *obj*) *table*  $\times$  (*qtname*, *obj*) *table*  $\sim =$  (*loc* + *qtname*, *obj*) *table*)

#### objects

**datatype** *obj-tag* = — tag for generic object

*CInst qtname* — class instance

| *Arr ty int* — array with component type and length

— — CStat *qtname* the tag is irrelevant for a class object, i.e. the static fields of a class, since its type is given already by the reference to it (see below)

**types** *vn* = *fspec* + *int* — variable name

**record** *obj* =

*tag* :: *obj-tag*

— generalized object

*values* :: (*vn*, *val*) *table*

#### translations

*fspec* <= (*type*) *vname*  $\times$  *qtname*

*vn* <= (*type*) *fspec* + *int*

*obj* <= (*type*) ( $\llbracket \text{tag}::\text{obj-tag}, \text{values}::\text{vn} \Rightarrow \text{val option} \rrbracket$ )

*obj* <= (*type*) ( $\llbracket \text{tag}::\text{obj-tag}, \text{values}::\text{vn} \Rightarrow \text{val option}, \dots::'a \rrbracket$ )

#### constdefs

*the-Arr* :: *obj option*  $\Rightarrow$  *ty*  $\times$  *int*  $\times$  (*vn*, *val*) *table*

*the-Arr obj*  $\equiv$  *SOME* (*T,k,t*). *obj* = *Some* ( $\llbracket \text{tag}=\text{Arr } T \text{ } k, \text{values}=t \rrbracket$ )

**lemma** *the-Arr-Arr* [*simp*]: *the-Arr* (*Some* ( $\llbracket \text{tag}=\text{Arr } T \text{ } k, \text{values}=cs \rrbracket$ )) = (*T,k,cs*)

$\langle \text{proof} \rangle$

**lemma** *the-Arr-Arr1* [*simp,intro,dest*]:

$\llbracket \text{tag } \text{obj} = \text{Arr } T \text{ } k \rrbracket \Longrightarrow \text{the-Arr } (\text{Some } \text{obj}) = (T,k,\text{values } \text{obj})$

$\langle \text{proof} \rangle$

#### constdefs

*upd-obj* :: *vn*  $\Rightarrow$  *val*  $\Rightarrow$  *obj*  $\Rightarrow$  *obj*

*upd-obj n v*  $\equiv$   $\lambda \text{ obj} . \text{obj } (\llbracket \text{values}:= (\text{values } \text{obj}) (n \mapsto v) \rrbracket)$

**lemma** *upd-obj-def2* [*simp*]:

*upd-obj n v obj* = *obj* ( $\llbracket \text{values}:= (\text{values } \text{obj}) (n \mapsto v) \rrbracket$ )

$\langle \text{proof} \rangle$

#### constdefs

*obj-ty* :: *obj*  $\Rightarrow$  *ty*

*obj-ty obj*  $\equiv$  *case tag obj of*

$$\begin{array}{l} CInst\ C \Rightarrow Class\ C \\ | Arr\ T\ k \Rightarrow T.[] \end{array}$$

**lemma** *obj-ty-eq* [intro!]: *obj-ty* ( $\langle tag=oi, values=x \rangle$ ) = *obj-ty* ( $\langle tag=oi, values=y \rangle$ )  
 ⟨proof⟩

**lemma** *obj-ty-eq1* [intro!,dest]:  
 $tag\ obj = tag\ obj' \implies obj\text{-}ty\ obj = obj\text{-}ty\ obj'$   
 ⟨proof⟩

**lemma** *obj-ty-cong* [simp]:  
 $obj\text{-}ty\ (obj\ \langle values:=vs \rangle) = obj\text{-}ty\ obj$   
 ⟨proof⟩

**lemma** *obj-ty-CInst* [simp]:  
 $obj\text{-}ty\ \langle tag=CInst\ C, values=vs \rangle = Class\ C$   
 ⟨proof⟩

**lemma** *obj-ty-CInst1* [simp,intro!,dest]:  
 $\llbracket tag\ obj = CInst\ C \rrbracket \implies obj\text{-}ty\ obj = Class\ C$   
 ⟨proof⟩

**lemma** *obj-ty-Arr* [simp]:  
 $obj\text{-}ty\ \langle tag=Arr\ T\ i, values=vs \rangle = T.[]$   
 ⟨proof⟩

**lemma** *obj-ty-Arr1* [simp,intro!,dest]:  
 $\llbracket tag\ obj = Arr\ T\ i \rrbracket \implies obj\text{-}ty\ obj = T.[]$   
 ⟨proof⟩

**lemma** *obj-ty-widenD*:  
 $G \vdash obj\text{-}ty\ obj \preceq RefT\ t \implies (\exists C. tag\ obj = CInst\ C) \vee (\exists T\ k. tag\ obj = Arr\ T\ k)$   
 ⟨proof⟩

**constdefs**

$$\begin{array}{l} obj\text{-}class :: obj \Rightarrow qname \\ obj\text{-}class\ obj \equiv case\ tag\ obj\ of \\ \quad CInst\ C \Rightarrow C \\ \quad | Arr\ T\ k \Rightarrow Object \end{array}$$

**lemma** *obj-class-CInst* [simp]: *obj-class* ( $\langle tag=CInst\ C, values=vs \rangle$ ) = *C*  
 ⟨proof⟩

**lemma** *obj-class-CInst1* [simp,intro!,dest]:  
 $tag\ obj = CInst\ C \implies obj\text{-}class\ obj = C$   
 ⟨proof⟩

**lemma** *obj-class-Arr* [simp]: *obj-class* ( $\langle \text{tag} = \text{Arr } T \ k, \text{values} = \text{vs} \rangle$ ) = *Object*  
 $\langle \text{proof} \rangle$

**lemma** *obj-class-Arr1* [simp,intro!,dest]:  
 $\text{tag } \text{obj} = \text{Arr } T \ k \implies \text{obj-class } \text{obj} = \text{Object}$   
 $\langle \text{proof} \rangle$

**lemma** *obj-ty-obj-class*:  $G \vdash \text{obj-ty } \text{obj} \preceq \text{Class } \text{statC} = G \vdash \text{obj-class } \text{obj} \preceq_C \text{statC}$   
 $\langle \text{proof} \rangle$

## object references

**types** *oref* = *loc* + *qname* — generalized object reference

### syntax

*Heap* :: *loc*  $\Rightarrow$  *oref*  
*Stat* :: *qname*  $\Rightarrow$  *oref*

### translations

*Heap*  $\Rightarrow$  *Inl*  
*Stat*  $\Rightarrow$  *Inr*  
*oref*  $\leq$  (*type*) *loc* + *qname*

### constdefs

*fields-table*::  
 $\text{prog} \Rightarrow \text{qname} \Rightarrow (\text{fspec} \Rightarrow \text{field} \Rightarrow \text{bool}) \Rightarrow (\text{fspec}, \text{ty}) \text{ table}$   
*fields-table* *G C P*  
 $\equiv \text{option-map } \text{type} \circ \text{table-of } (\text{filter } (\text{split } P) (\text{DeclConcepts.fields } G \ C))$

**lemma** *fields-table-SomeI*:  
 $\llbracket \text{table-of } (\text{DeclConcepts.fields } G \ C) \ n = \text{Some } f; P \ n \ f \rrbracket$   
 $\implies \text{fields-table } G \ C \ P \ n = \text{Some } (\text{type } f)$   
 $\langle \text{proof} \rangle$

**lemma** *fields-table-SomeD'*: *fields-table* *G C P fn* = *Some T*  $\implies$   
 $\exists f. (fn, f) \in \text{set}(\text{DeclConcepts.fields } G \ C) \wedge \text{type } f = T$   
 $\langle \text{proof} \rangle$

**lemma** *fields-table-SomeD*:  
 $\llbracket \text{fields-table } G \ C \ P \ fn = \text{Some } T; \text{unique } (\text{DeclConcepts.fields } G \ C) \rrbracket \implies$   
 $\exists f. \text{table-of } (\text{DeclConcepts.fields } G \ C) \ fn = \text{Some } f \wedge \text{type } f = T$   
 $\langle \text{proof} \rangle$

### constdefs

*in-bounds* :: *int*  $\Rightarrow$  *int*  $\Rightarrow$  *bool*  $((-/ \text{in'-bounds } -) [50, 51] \ 50)$   
 $i \text{ in-bounds } k \equiv 0 \leq i \wedge i < k$

*arr-comps* :: '*a*  $\Rightarrow$  *int*  $\Rightarrow$  *int*  $\Rightarrow$  '*a* option  
 $\text{arr-comps } T \ k \equiv \lambda i. \text{if } i \text{ in-bounds } k \text{ then } \text{Some } T \text{ else } \text{None}$

*var-tys* :: *prog*  $\Rightarrow$  *obj-tag*  $\Rightarrow$  *oref*  $\Rightarrow$  (*vn*, *ty*) *table*  
*var-tys* *G oi r*

$$\begin{aligned}
&\equiv \text{case } r \text{ of} \\
&\quad \text{Heap } a \Rightarrow (\text{case } oi \text{ of} \\
&\quad\quad CInst \ C \Rightarrow \text{fields-table } G \ C \ (\lambda n \ f. \neg \text{static } f) \ (+) \ \text{empty} \\
&\quad\quad | \text{Arr } T \ k \Rightarrow \text{empty } (+) \ \text{arr-comps } T \ k) \\
&\quad | \text{Stat } C \Rightarrow \text{fields-table } G \ C \ (\lambda fn \ f. \text{declclassf } fn = C \wedge \text{static } f) \\
&\quad\quad (+) \ \text{empty}
\end{aligned}$$

**lemma** *var-tys-Some-eq*:

*var-tys*  $G \ oi \ r \ n = \text{Some } T$

$$\begin{aligned}
&= (\text{case } r \text{ of} \\
&\quad \text{Inl } a \Rightarrow (\text{case } oi \text{ of} \\
&\quad\quad CInst \ C \Rightarrow (\exists nt. n = \text{Inl } nt \wedge \text{fields-table } G \ C \ (\lambda n \ f. \\
&\quad\quad\quad \neg \text{static } f) \ nt = \text{Some } T)) \\
&\quad\quad | \text{Arr } t \ k \Rightarrow (\exists i. n = \text{Inr } i \wedge i \text{ in-bounds } k \wedge t = T)) \\
&\quad | \text{Inr } C \Rightarrow (\exists nt. n = \text{Inl } nt \wedge \\
&\quad\quad \text{fields-table } G \ C \ (\lambda fn \ f. \text{declclassf } fn = C \wedge \text{static } f) \ nt \\
&\quad\quad = \text{Some } T))
\end{aligned}$$

$\langle \text{proof} \rangle$

## stores

**types** *globs* — global variables: heap and static variables

$$\begin{aligned}
&= (\text{oref} \ , \ \text{obj}) \ \text{table} \\
&\text{heap} \\
&= (\text{loc} \ , \ \text{obj}) \ \text{table}
\end{aligned}$$

## translations

$$\begin{aligned}
&\text{globs} \leq (\text{type}) \ (\text{oref} \ , \ \text{obj}) \ \text{table} \\
&\text{heap} \leq (\text{type}) \ (\text{loc} \ , \ \text{obj}) \ \text{table}
\end{aligned}$$

**datatype** *st* =

*st* *globs* *locals*

## 37 access

### constdefs

$$\begin{aligned}
&\text{globs} :: st \Rightarrow \text{globs} \\
&\text{globs} \equiv \text{st-case } (\lambda g \ l. \ g)
\end{aligned}$$

$$\begin{aligned}
&\text{locals} :: st \Rightarrow \text{locals} \\
&\text{locals} \equiv \text{st-case } (\lambda g \ l. \ l)
\end{aligned}$$

$$\begin{aligned}
&\text{heap} :: st \Rightarrow \text{heap} \\
&\text{heap } s \equiv \text{globs } s \circ \text{Heap}
\end{aligned}$$

**lemma** *globs-def2 [simp]*:  $\text{globs } (st \ g \ l) = g$

$\langle \text{proof} \rangle$

**lemma** *locals-def2 [simp]*:  $\text{locals } (st \ g \ l) = l$

$\langle \text{proof} \rangle$

**lemma** *heap-def2 [simp]: heap s a = globs s (Heap a)*  
 ⟨proof⟩

#### **syntax**

*val-this*     :: *st*  $\Rightarrow$  *val*  
*lookup-obj*   :: *st*  $\Rightarrow$  *val*  $\Rightarrow$  *obj*

#### **translations**

*val-this s*        == *the (locals s This)*  
*lookup-obj s a'* == *the (heap s (the-Addr a'))*

### **38 memory allocation**

#### **constdefs**

*new-Addr*     :: *heap*  $\Rightarrow$  *loc option*  
*new-Addr h*   ≡ *if* ( $\forall a. h a \neq \text{None}$ ) *then None else Some (SOME a. h a = None)*

**lemma** *new-AddrD: new-Addr h = Some a  $\implies$  h a = None*  
 ⟨proof⟩

**lemma** *new-AddrD2: new-Addr h = Some a  $\implies \forall b. h b \neq \text{None} \longrightarrow b \neq a$*   
 ⟨proof⟩

**lemma** *new-Addr-SomeI: h a = None  $\implies \exists b. \text{new-Addr h} = \text{Some b} \wedge h b = \text{None}$*   
 ⟨proof⟩

### **39 initialization**

#### **syntax**

*init-vals*     :: (*a, ty*) *table*  $\Rightarrow$  (*a, val*) *table*

#### **translations**

*init-vals vs*    == *option-map default-val*  $\circ$  *vs*

**lemma** *init-arr-comps-base [simp]: init-vals (arr-comps T 0) = empty*  
 ⟨proof⟩

**lemma** *init-arr-comps-step [simp]:*  
 $0 < j \implies \text{init-vals (arr-comps T } j \text{ )} =$   
 $\text{init-vals (arr-comps T (j - 1))(j - 1} \mapsto \text{default-val T)}$   
 ⟨proof⟩

### **40 update**

#### **constdefs**

*gupd*        :: *oref*  $\Rightarrow$  *obj*  $\Rightarrow$  *st*  $\Rightarrow$  *st*        (*gupd'(- $\mapsto$ -')[10,10]1000*)  
*gupd r obj* ≡ *st-case* ( $\lambda g l. \text{st } (g(r \mapsto \text{obj})) l$ )

*lupd*        :: *lname*  $\Rightarrow$  *val*  $\Rightarrow$  *st*  $\Rightarrow$  *st*        (*lupd'(- $\mapsto$ -')[10,10]1000*)  
*lupd vn v*   ≡ *st-case* ( $\lambda g l. \text{st } g (l(vn \mapsto v))$ )

*upd-gobj*    :: *oref*  $\Rightarrow$  *vn*  $\Rightarrow$  *val*  $\Rightarrow$  *st*  $\Rightarrow$  *st*



$upd-gobj\ r\ n\ v \equiv st-case\ (\lambda g\ l.\ st\ (chg-map\ (upd-obj\ n\ v)\ r\ g)\ l)$

$set-locals\ ::\ locals \Rightarrow st \Rightarrow st$   
 $set-locals\ l \equiv st-case\ (\lambda g\ l'.\ st\ g\ l)$

$init-obj\ ::\ prog \Rightarrow obj-tag \Rightarrow oref \Rightarrow st \Rightarrow st$   
 $init-obj\ G\ oi\ r \equiv gupd(r \mapsto \langle tag=oi, values=init-vals\ (var-tys\ G\ oi\ r) \rangle)$

### **syntax**

$init-class-obj\ ::\ prog \Rightarrow qtname \Rightarrow st \Rightarrow st$

### **translations**

$init-class-obj\ G\ C == init-obj\ G\ arbitrary\ (Inr\ C)$

**lemma**  $gupd-def2\ [simp]:\ gupd(r \mapsto obj)\ (st\ g\ l) = st\ (g(r \mapsto obj))\ l$   
 $\langle proof \rangle$

**lemma**  $lupd-def2\ [simp]:\ lupd(vn \mapsto v)\ (st\ g\ l) = st\ g\ (l(vn \mapsto v))$   
 $\langle proof \rangle$

**lemma**  $globs-gupd\ [simp]:\ globs\ (gupd(r \mapsto obj)\ s) = globs\ s(r \mapsto obj)$   
 $\langle proof \rangle$

**lemma**  $globs-lupd\ [simp]:\ globs\ (lupd(vn \mapsto v)\ s) = globs\ s$   
 $\langle proof \rangle$

**lemma**  $locals-gupd\ [simp]:\ locals\ (gupd(r \mapsto obj)\ s) = locals\ s$   
 $\langle proof \rangle$

**lemma**  $locals-lupd\ [simp]:\ locals\ (lupd(vn \mapsto v)\ s) = locals\ s(vn \mapsto v)$   
 $\langle proof \rangle$

**lemma**  $globs-upd-gobj-new\ [rule-format\ (no-asm),\ simp]:$   
 $globs\ s\ r = None \longrightarrow globs\ (upd-gobj\ r\ n\ v\ s) = globs\ s$   
 $\langle proof \rangle$

**lemma**  $globs-upd-gobj-upd\ [rule-format\ (no-asm),\ simp]:$   
 $globs\ s\ r = Some\ obj \longrightarrow globs\ (upd-gobj\ r\ n\ v\ s) = globs\ s(r \mapsto upd-obj\ n\ v\ obj)$   
 $\langle proof \rangle$

**lemma**  $locals-upd-gobj\ [simp]:\ locals\ (upd-gobj\ r\ n\ v\ s) = locals\ s$   
 $\langle proof \rangle$

**lemma**  $globs-init-obj\ [simp]:\ globs\ (init-obj\ G\ oi\ r\ s)\ t =$   
 $(if\ t=r\ then\ Some\ \langle tag=oi, values=init-vals\ (var-tys\ G\ oi\ r) \rangle\ else\ globs\ s\ t)$   
 $\langle proof \rangle$

**lemma** *locals-init-obj* [simp]:  $\text{locals } (\text{init-obj } G \text{ oi } r \ s) = \text{locals } s$   
 $\langle \text{proof} \rangle$

**lemma** *surjective-st* [simp]:  $\text{st } (\text{globs } s) (\text{locals } s) = s$   
 $\langle \text{proof} \rangle$

**lemma** *surjective-st-init-obj*:  
 $\text{st } (\text{globs } (\text{init-obj } G \text{ oi } r \ s)) (\text{locals } s) = \text{init-obj } G \text{ oi } r \ s$   
 $\langle \text{proof} \rangle$

**lemma** *heap-heap-upd* [simp]:  
 $\text{heap } (\text{st } (g(\text{Inl } a \mapsto \text{obj}))) \ l = \text{heap } (\text{st } g \ l)(a \mapsto \text{obj})$   
 $\langle \text{proof} \rangle$

**lemma** *heap-stat-upd* [simp]:  $\text{heap } (\text{st } (g(\text{Inr } C \mapsto \text{obj}))) \ l = \text{heap } (\text{st } g \ l)$   
 $\langle \text{proof} \rangle$

**lemma** *heap-local-upd* [simp]:  $\text{heap } (\text{st } g \ (l(vn \mapsto v))) = \text{heap } (\text{st } g \ l)$   
 $\langle \text{proof} \rangle$

**lemma** *heap-gupd-Heap* [simp]:  $\text{heap } (\text{gupd}(\text{Heap } a \mapsto \text{obj}) \ s) = \text{heap } s(a \mapsto \text{obj})$   
 $\langle \text{proof} \rangle$

**lemma** *heap-gupd-Stat* [simp]:  $\text{heap } (\text{gupd}(\text{Stat } C \mapsto \text{obj}) \ s) = \text{heap } s$   
 $\langle \text{proof} \rangle$

**lemma** *heap-lupd* [simp]:  $\text{heap } (\text{lupd}(vn \mapsto v) \ s) = \text{heap } s$   
 $\langle \text{proof} \rangle$

**lemma** *heap-upd-gobj-Stat* [simp]:  $\text{heap } (\text{upd-gobj } (\text{Stat } C) \ n \ v \ s) = \text{heap } s$   
 $\langle \text{proof} \rangle$

**lemma** *set-locals-def2* [simp]:  $\text{set-locals } l \ (\text{st } g \ l') = \text{st } g \ l$   
 $\langle \text{proof} \rangle$

**lemma** *set-locals-id* [simp]:  $\text{set-locals } (\text{locals } s) \ s = s$   
 $\langle \text{proof} \rangle$

**lemma** *set-set-locals* [simp]:  $\text{set-locals } l \ (\text{set-locals } l' \ s) = \text{set-locals } l \ s$   
 $\langle \text{proof} \rangle$

**lemma** *locals-set-locals* [simp]:  $\text{locals } (\text{set-locals } l \ s) = l$   
 $\langle \text{proof} \rangle$

**lemma** *globs-set-locals* [simp]:  $\text{globs } (\text{set-locals } l \ s) = \text{globs } s$   
 $\langle \text{proof} \rangle$

**lemma** *heap-set-locals* [simp]:  $\text{heap } (\text{set-locals } l \ s) = \text{heap } s$

$\langle proof \rangle$

## abrupt completion

### consts

$the-Xcpt :: abrupt \Rightarrow xcpt$   
 $the-Jump :: abrupt \Rightarrow jump$   
 $the-Loc :: xcpt \Rightarrow loc$   
 $the-Std :: xcpt \Rightarrow xname$

**primrec**  $the-Xcpt (Xcpt x) = x$   
**primrec**  $the-Jump (Jump j) = j$   
**primrec**  $the-Loc (Loc a) = a$   
**primrec**  $the-Std (Std x) = x$

### constdefs

$abrupt-if :: bool \Rightarrow abopt \Rightarrow abopt \Rightarrow abopt$   
 $abrupt-if\ c\ x'\ x \equiv if\ c \wedge (x = None)\ then\ x'\ else\ x$

**lemma**  $abrupt-if-True-None [simp]: abrupt-if\ True\ x\ None = x$   
 $\langle proof \rangle$

**lemma**  $abrupt-if-True-not-None [simp]: x \neq None \implies abrupt-if\ True\ x\ y \neq None$   
 $\langle proof \rangle$

**lemma**  $abrupt-if-False [simp]: abrupt-if\ False\ x\ y = y$   
 $\langle proof \rangle$

**lemma**  $abrupt-if-Some [simp]: abrupt-if\ c\ x\ (Some\ y) = Some\ y$   
 $\langle proof \rangle$

**lemma**  $abrupt-if-not-None [simp]: y \neq None \implies abrupt-if\ c\ x\ y = y$   
 $\langle proof \rangle$

**lemma**  $split-abrupt-if:$   
 $P\ (abrupt-if\ c\ x'\ x) =$   
 $((c \wedge x = None \longrightarrow P\ x') \wedge (\neg (c \wedge x = None) \longrightarrow P\ x))$   
 $\langle proof \rangle$

### syntax

$raise-if :: bool \Rightarrow xname \Rightarrow abopt \Rightarrow abopt$   
 $np :: val \Rightarrow abopt \Rightarrow abopt$   
 $check-neg :: val \Rightarrow abopt \Rightarrow abopt$   
 $error-if :: bool \Rightarrow error \Rightarrow abopt \Rightarrow abopt$

### translations

$\text{raise-if } c \text{ } xn == \text{abrupt-if } c \text{ } (\text{Some } (Xcpt \text{ } (Std \text{ } xn)))$   
 $\text{np } v == \text{raise-if } (v = \text{Null}) \text{ } \text{NullPointer}$   
 $\text{check-neg } i' == \text{raise-if } (\text{the-Intg } i' < 0) \text{ } \text{NegArrSize}$   
 $\text{error-if } c \text{ } e == \text{abrupt-if } c \text{ } (\text{Some } (\text{Error } e))$

**lemma** *raise-if-None* [simp]:  $(\text{raise-if } c \text{ } x \text{ } y = \text{None}) = (\neg c \wedge y = \text{None})$   
 $\langle \text{proof} \rangle$   
**declare** *raise-if-None* [THEN iffD1, dest!]

**lemma** *if-raise-if-None* [simp]:  
 $((\text{if } b \text{ then } y \text{ else } \text{raise-if } c \text{ } x \text{ } y) = \text{None}) = ((c \longrightarrow b) \wedge y = \text{None})$   
 $\langle \text{proof} \rangle$

**lemma** *raise-if-SomeD* [dest!]:  
 $\text{raise-if } c \text{ } x \text{ } y = \text{Some } z \implies c \wedge z = (Xcpt \text{ } (Std \text{ } x)) \wedge y = \text{None} \vee (y = \text{Some } z)$   
 $\langle \text{proof} \rangle$

**lemma** *error-if-None* [simp]:  $(\text{error-if } c \text{ } e \text{ } y = \text{None}) = (\neg c \wedge y = \text{None})$   
 $\langle \text{proof} \rangle$   
**declare** *error-if-None* [THEN iffD1, dest!]

**lemma** *if-error-if-None* [simp]:  
 $((\text{if } b \text{ then } y \text{ else } \text{error-if } c \text{ } e \text{ } y) = \text{None}) = ((c \longrightarrow b) \wedge y = \text{None})$   
 $\langle \text{proof} \rangle$

**lemma** *error-if-SomeD* [dest!]:  
 $\text{error-if } c \text{ } e \text{ } y = \text{Some } z \implies c \wedge z = (\text{Error } e) \wedge y = \text{None} \vee (y = \text{Some } z)$   
 $\langle \text{proof} \rangle$

**constdefs**  
 $\text{absorb} :: \text{jump} \Rightarrow \text{abopt} \Rightarrow \text{abopt}$   
 $\text{absorb } j \text{ } a \equiv \text{if } a = \text{Some } (\text{Jump } j) \text{ then } \text{None} \text{ else } a$

**lemma** *absorb-SomeD* [dest!]:  $\text{absorb } j \text{ } a = \text{Some } x \implies a = \text{Some } x$   
 $\langle \text{proof} \rangle$

**lemma** *absorb-same* [simp]:  $\text{absorb } j \text{ } (\text{Some } (\text{Jump } j)) = \text{None}$   
 $\langle \text{proof} \rangle$

**lemma** *absorb-other* [simp]:  $a \neq \text{Some } (\text{Jump } j) \implies \text{absorb } j \text{ } a = a$   
 $\langle \text{proof} \rangle$

**lemma** *absorb-Some-NoneD*:  $\text{absorb } j \text{ } (\text{Some } \text{abr}) = \text{None} \implies \text{abr} = \text{Jump } j$   
 $\langle \text{proof} \rangle$

**lemma** *absorb-Some-JumpD*:  $\text{absorb } j \text{ } s = \text{Some } (\text{Jump } j') \implies j' \neq j$   
 $\langle \text{proof} \rangle$

**full program state****types**

$state = abopt \times st$  — state including abrupton information

**syntax**

$Norm :: st \Rightarrow state$   
 $abrupt :: state \Rightarrow abopt$   
 $store :: state \Rightarrow st$

**translations**

$Norm\ s == (None, s)$   
 $abrupt ==> fst$   
 $store ==> snd$   
 $abopt <= (type)\ State.abrupt\ option$   
 $abopt <= (type)\ abrupt\ option$   
 $state <= (type)\ abopt \times State.st$   
 $state <= (type)\ abopt \times st$

**lemma** *single-stateE*:  $\forall Z. Z = (s::state) \implies False$   
 $\langle proof \rangle$

**lemma** *state-not-single*:  $All\ (op = (x::state)) \implies R$   
 $\langle proof \rangle$

**constdefs**

$normal :: state \Rightarrow bool$   
 $normal \equiv \lambda s. abrupt\ s = None$

**lemma** *normal-def2* [*simp*]:  $normal\ s = (abrupt\ s = None)$   
 $\langle proof \rangle$

**constdefs**

$heap-free :: nat \Rightarrow state \Rightarrow bool$   
 $heap-free\ n \equiv \lambda s. atleast-free\ (heap\ (store\ s))\ n$

**lemma** *heap-free-def2* [*simp*]:  $heap-free\ n\ s = atleast-free\ (heap\ (store\ s))\ n$   
 $\langle proof \rangle$

**41 update****constdefs**

$abupd :: (abopt \Rightarrow abopt) \Rightarrow state \Rightarrow state$   
 $abupd\ f \equiv prod-fun\ f\ id$

$supd :: (st \Rightarrow st) \Rightarrow state \Rightarrow state$   
 $supd \equiv prod-fun\ id$

**lemma** *abupd-def2* [*simp*]:  $abupd\ f\ (x, s) = (f\ x, s)$

$\langle \text{proof} \rangle$

**lemma** *abupd-abrupt-if-False* [simp]:  $\bigwedge s. \text{abupd} (\text{abrupt-if False } xo) s = s$   
 $\langle \text{proof} \rangle$

**lemma** *supd-def2* [simp]:  $\text{supd } f (x, s) = (x, f s)$   
 $\langle \text{proof} \rangle$

**lemma** *supd-lupd* [simp]:  
 $\bigwedge s. \text{supd} (\text{lupd } vn v) s = (\text{abrupt } s, \text{lupd } vn v (\text{store } s))$   
 $\langle \text{proof} \rangle$

**lemma** *supd-gupd* [simp]:  
 $\bigwedge s. \text{supd} (\text{gupd } r \text{ obj}) s = (\text{abrupt } s, \text{gupd } r \text{ obj} (\text{store } s))$   
 $\langle \text{proof} \rangle$

**lemma** *supd-init-obj* [simp]:  
 $\text{supd} (\text{init-obj } G \text{ oi } r) s = (\text{abrupt } s, \text{init-obj } G \text{ oi } r (\text{store } s))$   
 $\langle \text{proof} \rangle$

**lemma** *abupd-store-invariant* [simp]:  $\text{store} (\text{abupd } f s) = \text{store } s$   
 $\langle \text{proof} \rangle$

**lemma** *supd-abrupt-invariant* [simp]:  $\text{abrupt} (\text{supd } f s) = \text{abrupt } s$   
 $\langle \text{proof} \rangle$

## syntax

*set-lvars* :: *locals*  $\Rightarrow$  *state*  $\Rightarrow$  *state*  
*restore-lvars* :: *state*  $\Rightarrow$  *state*  $\Rightarrow$  *state*

## translations

*set-lvars* *l* == *supd* (*set-locals* *l*)  
*restore-lvars* *s'* *s* == *set-lvars* (*locals* (*store* *s'*)) *s*

**lemma** *set-set-lvars* [simp]:  $\bigwedge s. \text{set-lvars } l (\text{set-lvars } l' s) = \text{set-lvars } l s$   
 $\langle \text{proof} \rangle$

**lemma** *set-lvars-id* [simp]:  $\bigwedge s. \text{set-lvars} (\text{locals} (\text{store } s)) s = s$   
 $\langle \text{proof} \rangle$

## initialisation test

### constdefs

*inited* :: *qtname*  $\Rightarrow$  *globs*  $\Rightarrow$  *bool*  
*inited* *C* *g*  $\equiv g (\text{Stat } C) \neq \text{None}$

$initd \quad :: \text{qname} \Rightarrow \text{state} \Rightarrow \text{bool}$   
 $initd \ C \equiv initd \ C \circ \text{globs} \circ \text{store}$

**lemma** *not-initd-empty* [simp]:  $\neg initd \ C \ \text{empty}$   
 <proof>

**lemma** *initd-gupdate* [simp]:  $initd \ C \ (g(r \mapsto obj)) = (initd \ C \ g \ \vee \ r = \text{Stat } C)$   
 <proof>

**lemma** *initd-init-class-obj* [intro!]:  $initd \ C \ (\text{globs} \ (\text{init-class-obj } G \ C \ s))$   
 <proof>

**lemma** *not-initdD*:  $\neg initd \ C \ g \implies g \ (\text{Stat } C) = \text{None}$   
 <proof>

**lemma** *initdD*:  $initd \ C \ g \implies \exists \text{ obj. } g \ (\text{Stat } C) = \text{Some obj}$   
 <proof>

**lemma** *initd-def2* [simp]:  $initd \ C \ s = initd \ C \ (\text{globs} \ (\text{store } s))$   
 <proof>

*error-free*

**constdefs** *error-free*::  $\text{state} \Rightarrow \text{bool}$   
 $\text{error-free } s \equiv \neg (\exists \text{ err. abrupt } s = \text{Some } (\text{Error err}))$

**lemma** *error-free-Norm* [simp,intro]:  $\text{error-free } (\text{Norm } s)$   
 <proof>

**lemma** *error-free-normal* [simp,intro]:  $\text{normal } s \implies \text{error-free } s$   
 <proof>

**lemma** *error-free-Xcpt* [simp]:  $\text{error-free } (\text{Some } (\text{Xcpt } x), s)$   
 <proof>

**lemma** *error-free-Jump* [simp,intro]:  $\text{error-free } (\text{Some } (\text{Jump } j), s)$   
 <proof>

**lemma** *error-free-Error* [simp]:  $\text{error-free } (\text{Some } (\text{Error } e), s) = \text{False}$   
 <proof>

**lemma** *error-free-Some* [simp,intro]:  
 $\neg (\exists \text{ err. } x = \text{Error err}) \implies \text{error-free } ((\text{Some } x), s)$   
 <proof>

**lemma** *error-free-abupd-absorb* [simp,intro]:

$error\text{-}free\ s \implies error\text{-}free\ (abupd\ (absorb\ j)\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-absorb* [simp,intro]:  
 $error\text{-}free\ (a,s) \implies error\text{-}free\ (absorb\ j\ a,\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-abrupt-if* [simp,intro]:  
 $\llbracket error\text{-}free\ s; \neg (\exists\ err.\ x=Error\ err) \rrbracket$   
 $\implies error\text{-}free\ (abupd\ (abrupt\text{-}if\ p\ (Some\ x))\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-abrupt-if1* [simp,intro]:  
 $\llbracket error\text{-}free\ (a,s); \neg (\exists\ err.\ x=Error\ err) \rrbracket$   
 $\implies error\text{-}free\ (abrupt\text{-}if\ p\ (Some\ x)\ a,\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-abrupt-if-Xcpt* [simp,intro]:  
 $error\text{-}free\ s$   
 $\implies error\text{-}free\ (abupd\ (abrupt\text{-}if\ p\ (Some\ (Xcpt\ x)))\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-abrupt-if-Xcpt1* [simp,intro]:  
 $error\text{-}free\ (a,s)$   
 $\implies error\text{-}free\ (abrupt\text{-}if\ p\ (Some\ (Xcpt\ x))\ a,\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-abrupt-if-Jump* [simp,intro]:  
 $error\text{-}free\ s$   
 $\implies error\text{-}free\ (abupd\ (abrupt\text{-}if\ p\ (Some\ (Jump\ j)))\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-abrupt-if-Jump1* [simp,intro]:  
 $error\text{-}free\ (a,s)$   
 $\implies error\text{-}free\ (abrupt\text{-}if\ p\ (Some\ (Jump\ j))\ a,\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-raise-if* [simp,intro]:  
 $error\text{-}free\ s \implies error\text{-}free\ (abupd\ (raise\text{-}if\ p\ x)\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-raise-if1* [simp,intro]:  
 $error\text{-}free\ (a,s) \implies error\text{-}free\ ((raise\text{-}if\ p\ x\ a),\ s)$   
 $\langle proof \rangle$

**lemma** *error-free-supd* [simp,intro]:  
 $error\text{-}free\ s \implies error\text{-}free\ (supd\ f\ s)$   
 $\langle proof \rangle$



**lemma** *error-free-supd1* [*simp,intro*]:  
 $\text{error-free } (a,s) \implies \text{error-free } (a,f\ s)$   
 ⟨*proof*⟩

**lemma** *error-free-set-lvars* [*simp,intro*]:  
 $\text{error-free } s \implies \text{error-free } ((\text{set-lvars } l)\ s)$   
 ⟨*proof*⟩

**lemma** *error-free-set-locals* [*simp,intro*]:  
 $\text{error-free } (x, s) \implies \text{error-free } (x, \text{set-locals } l\ s')$   
 ⟨*proof*⟩

**end**



## Chapter 15

### Eval

## 42 Operational evaluation (big-step) semantics of Java expressions and statements

theory *Eval* imports *State DeclConcepts* begin

improvements over Java Specification 1.0:

- dynamic method lookup does not need to consider the return type (cf.15.11.4.4)
- throw raises a NullPointerException if a null reference is given, and each throw of a standard exception yield a fresh exception object (was not specified)
- if there is not enough memory even to allocate an OutOfMemory exception, evaluation/execution fails, i.e. simply stops (was not specified)
- array assignment checks lhs (and may throw exceptions) before evaluating rhs
- fixed exact positions of class initializations (immediate at first active use)

design issues:

- evaluation vs. (single-step) transition semantics evaluation semantics chosen, because:
  - ++ less verbose and therefore easier to read (and to handle in proofs)
  - + more abstract
  - + intermediate values (appearing in recursive rules) need not be stored explicitly, e.g. no call body construct or stack of invocation frames containing local variables and return addresses for method calls needed
  - + convenient rule induction for subject reduction theorem
    - no interleaving (for parallelism) can be described
    - stating a property of infinite executions requires the meta-level argument that this property holds for any finite prefixes of it (e.g. stopped using a counter that is decremented to zero and then throwing an exception)
- unified evaluation for variables, expressions, expression lists, statements
- the value entry in statement rules is redundant
- the value entry in rules is irrelevant in case of exceptions, but its full inclusion helps to make the rule structure independent of exception occurrence.
- as irrelevant value entries are ignored, it does not matter if they are unique. For simplicity, (fixed) arbitrary values are preferred over "free" values.
- the rule format is such that the start state may contain an exception.
  - ++ facilitates exception handling
  - + symmetry
- the rules are defined carefully in order to be applicable even in not type-correct situations (yielding undefined values), e.g. *the-Addr (Val (Bool b)) = arbitrary*.
  - ++ fewer rules
    - less readable because of auxiliary functions like *the-Addr*

Alternative: "defensive" evaluation throwing some InternalError exception in case of (impossible, for correct programs) type mismatches

- there is exactly one rule per syntactic construct
  - + no redundancy in case distinctions
- `halloc` fails iff there is no free heap address. When there is only one free heap address left, it returns an `OutOfMemory` exception. In this way it is guaranteed that when an `OutOfMemory` exception is thrown for the first time, there is a free location on the heap to allocate it.
- the allocation of objects that represent standard exceptions is deferred until execution of any enclosing catch clause, which is transparent to the program.
  - requires an auxiliary execution relation
  - ++ avoids copies of allocation code and awkward case distinctions (whether there is enough memory to allocate the exception) in evaluation rules
- unfortunately `new-Addr` is not directly executable because of Hilbert operator.

simplifications:

- local variables are initialized with default values (no definite assignment)
- garbage collection not considered, therefore also no finalizers
- stack overflow and memory overflow during class initialization not modelled
- exceptions in initializations not replaced by `ExceptionInInitializerError`

**types**  $vvar = val \times (val \Rightarrow state \Rightarrow state)$   
 $vals = (val, vvar, val\ list)\ sum3$

**translations**

$vvar \leq (type)\ val \times (val \Rightarrow state \Rightarrow state)$   
 $vals \leq (type)(val, vvar, val\ list)\ sum3$

To avoid redundancy and to reduce the number of rules, there is only one evaluation rule for each syntactic term. This is also true for variables (e.g. see the rules below for *LVar*, *FVar* and *AVar*). So evaluation of a variable must capture both possible further uses: read (rule *Acc*) or write (rule *Ass*) to the variable. Therefore a variable evaluates to a special value *vvar*, which is a pair, consisting of the current value (for later read access) and an update function (for later write access). Because during assignment to an array variable an exception may occur if the types don't match, the update function is very generic: it transforms the full state. This generic update function causes some technical trouble during some proofs (e.g. type safety, correctness of definite assignment). There we need to prove some additional invariant on this update function to prove the assignment correct, since the update function could potentially alter the whole state in an arbitrary manner. This invariant must be carried around through the whole induction. So for future approaches it may be better not to take such a generic update function, but only to store the address and the kind of variable (array (+ element type), local variable or field) for later assignment.

**syntax** (*xsymbols*)  
 $dummy-res :: vals\ (\Diamond)$

**translations**

$\Diamond == In1\ Unit$

**syntax**

$val-inj-vals :: expr \Rightarrow term\ ([\_ ]_e\ 1000)$   
 $var-inj-vals :: var \Rightarrow term\ ([\_ ]_v\ 1000)$   
 $lst-inj-vals :: expr\ list \Rightarrow term\ ([\_ ]_l\ 1000)$

**translations**

$$\begin{aligned} [e]_e &\rightarrow In1\ e \\ [v]_v &\rightarrow In2\ v \\ [es]_l &\rightarrow In3\ es \end{aligned}$$
**constdefs**

$$\begin{aligned} arbitrary3 &:: ('al + 'ar, 'b, 'c)\ sum3 \Rightarrow vals \\ arbitrary3 &\equiv sum3\text{-}case\ (In1 \circ sum\text{-}case\ (\lambda x. arbitrary))\ (\lambda x. Unit)) \\ &\quad (\lambda x. In2\ arbitrary)\ (\lambda x. In3\ arbitrary) \end{aligned}$$

**lemma** [simp]:  $arbitrary3\ (In1\ x) = In1\ arbitrary$   
 $\langle proof \rangle$

**lemma** [simp]:  $arbitrary3\ (In1r\ x) = \Diamond$   
 $\langle proof \rangle$

**lemma** [simp]:  $arbitrary3\ (In2\ x) = In2\ arbitrary$   
 $\langle proof \rangle$

**lemma** [simp]:  $arbitrary3\ (In3\ x) = In3\ arbitrary$   
 $\langle proof \rangle$

**exception throwing and catching****constdefs**

$$\begin{aligned} throw &:: val \Rightarrow abopt \Rightarrow abopt \\ throw\ a'\ x &\equiv abrupt\text{-}if\ True\ (Some\ (Xcpt\ (Loc\ (the\text{-}Addr\ a'))))\ (np\ a'\ x) \end{aligned}$$
**lemma** *throw-def2*:
$$throw\ a'\ x = abrupt\text{-}if\ True\ (Some\ (Xcpt\ (Loc\ (the\text{-}Addr\ a'))))\ (np\ a'\ x)$$
 $\langle proof \rangle$ 
**constdefs**

$$\begin{aligned} fits &:: prog \Rightarrow st \Rightarrow val \Rightarrow ty \Rightarrow bool\ (\neg, \vdash\text{-}fits\text{-}[61,61,61,61]60) \\ G, s \vdash a' fits\ T &\equiv (\exists rt. T = RefT\ rt) \longrightarrow a' = Null \vee G \vdash obj\text{-}ty(lookup\text{-}obj\ s\ a') \preceq T \end{aligned}$$

**lemma** *fits-Null* [simp]:  $G, s \vdash Null\ fits\ T$   
 $\langle proof \rangle$

**lemma** *fits-Addr-RefT* [simp]:
$$G, s \vdash Addr\ a\ fits\ RefT\ t = G \vdash obj\text{-}ty\ (the\ (heap\ s\ a)) \preceq RefT\ t$$
 $\langle proof \rangle$ 

**lemma** *fitsD*:  $\bigwedge X. G, s \vdash a' fits\ T \implies (\exists pt. T = PrimT\ pt) \vee$   
 $(\exists t. T = RefT\ t) \wedge a' = Null \vee$   
 $(\exists t. T = RefT\ t) \wedge a' \neq Null \wedge G \vdash obj\text{-}ty\ (lookup\text{-}obj\ s\ a') \preceq T$   
 $\langle proof \rangle$

**constdefs**

$$\begin{aligned} catch &:: prog \Rightarrow state \Rightarrow qname \Rightarrow bool\ (\neg, \vdash\text{-}catch\text{-}[61,61,61]60) \\ G, s \vdash catch\ C &\equiv \exists xc. abrupt\ s = Some\ (Xcpt\ xc) \wedge \end{aligned}$$

$G, store\ s \vdash Addr\ (the\text{-}Loc\ xc)\ fits\ Class\ C$

**lemma** *catch-Norm* [simp]:  $\neg G, Norm\ s \vdash catch\ tn$   
 $\langle proof \rangle$

**lemma** *catch-XcptLoc* [simp]:  
 $G, (Some\ (Xcpt\ (Loc\ a)), s) \vdash catch\ C = G, s \vdash Addr\ a\ fits\ Class\ C$   
 $\langle proof \rangle$

**lemma** *catch-Jump* [simp]:  $\neg G, (Some\ (Jump\ j), s) \vdash catch\ tn$   
 $\langle proof \rangle$

**lemma** *catch-Error* [simp]:  $\neg G, (Some\ (Error\ e), s) \vdash catch\ tn$   
 $\langle proof \rangle$

### constdefs

$new\text{-}xcpt\text{-}var :: vname \Rightarrow state \Rightarrow state$   
 $new\text{-}xcpt\text{-}var\ vn \equiv$   
 $\lambda(x, s). Norm\ (lupd\ (VName\ vn \mapsto Addr\ (the\text{-}Loc\ (the\text{-}Xcpt\ (the\ x))))\ s)$

**lemma** *new-xcpt-var-def2* [simp]:  
 $new\text{-}xcpt\text{-}var\ vn\ (x, s) =$   
 $Norm\ (lupd\ (VName\ vn \mapsto Addr\ (the\text{-}Loc\ (the\text{-}Xcpt\ (the\ x))))\ s)$   
 $\langle proof \rangle$

### misc

#### constdefs

$assign :: ('a \Rightarrow state \Rightarrow state) \Rightarrow 'a \Rightarrow state \Rightarrow state$   
 $assign\ f\ v \equiv \lambda(x, s). let\ (x', s') = (if\ x = None\ then\ f\ v\ else\ id)\ (x, s)$   
 $in\ (x', if\ x' = None\ then\ s'\ else\ s)$

**lemma** *assign-Norm-Norm* [simp]:  
 $f\ v\ (Norm\ s) = Norm\ s' \implies assign\ f\ v\ (Norm\ s) = Norm\ s'$   
 $\langle proof \rangle$

**lemma** *assign-Norm-Some* [simp]:  
 $\llbracket abrupt\ (f\ v\ (Norm\ s)) = Some\ y \rrbracket$   
 $\implies assign\ f\ v\ (Norm\ s) = (Some\ y, s)$   
 $\langle proof \rangle$

**lemma** *assign-Some* [simp]:  
 $assign\ f\ v\ (Some\ x, s) = (Some\ x, s)$   
 $\langle proof \rangle$

**lemma** *assign-Some1* [simp]:  $\neg \text{normal } s \implies \text{assign } f \ v \ s = s$   
 ⟨proof⟩

**lemma** *assign-supd* [simp]:  
 $\text{assign } (\lambda v. \text{supd } (f \ v)) \ v \ (x, s)$   
 $= (x, \text{if } x = \text{None} \text{ then } f \ v \ s \text{ else } s)$   
 ⟨proof⟩

**lemma** *assign-raise-if* [simp]:  
 $\text{assign } (\lambda v \ (x, s). ((\text{raise-if } (b \ s \ v) \ \text{xcpt}) \ x, f \ v \ s)) \ v \ (x, s) =$   
 $(\text{raise-if } (b \ s \ v) \ \text{xcpt} \ x, \text{if } x = \text{None} \wedge \neg b \ s \ v \text{ then } f \ v \ s \text{ else } s)$   
 ⟨proof⟩

### constdefs

*init-comp-ty* :: *ty*  $\Rightarrow$  *stmt*  
*init-comp-ty* *T*  $\equiv$  *if* ( $\exists C. T = \text{Class } C$ ) *then* *Init* (*the-Class* *T*) *else* *Skip*

**lemma** *init-comp-ty-PrimT* [simp]: *init-comp-ty* (*PrimT* *pt*) = *Skip*  
 ⟨proof⟩

### constdefs

*invocation-class* :: *inv-mode*  $\Rightarrow$  *st*  $\Rightarrow$  *val*  $\Rightarrow$  *ref-ty*  $\Rightarrow$  *qname*  
*invocation-class* *m* *s* *a'* *statT*  
 $\equiv$  (*case* *m* *of*  
   *Static*  $\Rightarrow$  *if* ( $\exists \text{statC}. \text{statT} = \text{ClassT } \text{statC}$ )  
     *then* *the-Class* (*RefT* *statT*)  
     *else* *Object*  
   | *SuperM*  $\Rightarrow$  *the-Class* (*RefT* *statT*)  
   | *IntVir*  $\Rightarrow$  *obj-class* (*lookup-obj* *s* *a'*)

*invocation-declclass::prog*  $\Rightarrow$  *inv-mode*  $\Rightarrow$  *st*  $\Rightarrow$  *val*  $\Rightarrow$  *ref-ty*  $\Rightarrow$  *sig*  $\Rightarrow$  *qname*  
*invocation-declclass* *G* *m* *s* *a'* *statT* *sig*  
 $\equiv$  *declclass* (*the* (*dynlookup* *G* *statT*)  
   (*invocation-class* *m* *s* *a'* *statT*)  
   *sig*)

**lemma** *invocation-class-IntVir* [simp]:  
*invocation-class* *IntVir* *s* *a'* *statT* = *obj-class* (*lookup-obj* *s* *a'*)  
 ⟨proof⟩

**lemma** *dynclass-SuperM* [simp]:  
*invocation-class* *SuperM* *s* *a'* *statT* = *the-Class* (*RefT* *statT*)  
 ⟨proof⟩

**lemma** *invocation-class-Static* [simp]:  
*invocation-class* *Static* *s* *a'* *statT* = (*if* ( $\exists \text{statC}. \text{statT} = \text{ClassT } \text{statC}$ )  
   *then* *the-Class* (*RefT* *statT*)



*else Object)*

*<proof>*

### constdefs

*init-lvars* :: *prog*  $\Rightarrow$  *qname*  $\Rightarrow$  *sig*  $\Rightarrow$  *inv-mode*  $\Rightarrow$  *val*  $\Rightarrow$  *val list*  $\Rightarrow$   
                   *state*  $\Rightarrow$  *state*  
*init-lvars* *G C sig mode a' pvs*  
 $\equiv \lambda (x,s).$   
     *let* *m* = *mthd* (*the* (*methd* *G C sig*));  
     *l* =  $\lambda k.$   
         (*case* *k* of  
           *ENam* *e*  
              $\Rightarrow$  (*case* *e* of  
               *VNam* *v*  $\Rightarrow$  (*empty* ((*pars* *m*) $\mapsto$ *pvs*)) *v*  
               | *Res*  $\Rightarrow$  *None*)  
           | *This*  
              $\Rightarrow$  (*if* *mode*=*Static* *then* *None* *else* *Some a'*))  
     *in* *set-lvars* *l* (*if* *mode* = *Static* *then* *x* *else* *np a' x,s*)

**lemma** *init-lvars-def2*: — better suited for simplification

*init-lvars* *G C sig mode a' pvs* (*x,s*) =  
   *set-lvars*  
   ( $\lambda k.$   
     (*case* *k* of  
       *ENam* *e*  
          $\Rightarrow$  (*case* *e* of  
           *VNam* *v*  
            $\Rightarrow$  (*empty* ((*pars* (*mthd* (*the* (*methd* *G C sig*)))) $\mapsto$ *pvs*)) *v*  
           | *Res*  $\Rightarrow$  *None*)  
       | *This*  
          $\Rightarrow$  (*if* *mode*=*Static* *then* *None* *else* *Some a'*))  
     (*if* *mode* = *Static* *then* *x* *else* *np a' x,s*)  
*<proof>*

### constdefs

*body* :: *prog*  $\Rightarrow$  *qname*  $\Rightarrow$  *sig*  $\Rightarrow$  *expr*  
*body* *G C sig*  $\equiv$  *let* *m* = *the* (*methd* *G C sig*)  
                   *in* *Body* (*declclass* *m*) (*stmt* (*mbody* (*mthd* *m*)))

**lemma** *body-def2*: — better suited for simplification

*body* *G C sig* = *Body* (*declclass* (*the* (*methd* *G C sig*)))  
                   (*stmt* (*mbody* (*mthd* (*the* (*methd* *G C sig*)))))  
*<proof>*

### variables

#### constdefs

*lvar* :: *lname*  $\Rightarrow$  *st*  $\Rightarrow$  *vvar*  
*lvar* *vn s*  $\equiv$  (*the* (*locals* *s vn*),  $\lambda v. \text{supd} (\text{lupd}(vn \mapsto v))$ )  
  
*fvar* :: *qname*  $\Rightarrow$  *bool*  $\Rightarrow$  *vname*  $\Rightarrow$  *val*  $\Rightarrow$  *state*  $\Rightarrow$  *vvar*  $\times$  *state*  
*fvar* *C stat fn a' s*  
 $\equiv$  *let* (*oref,xf*) = *if* *stat* *then* (*Stat* *C,id*)  
                   *else* (*Heap* (*the-Addr* *a'*),*np a'*);

$$\begin{aligned}
& n = \text{Inl } (fn, C); \\
& f = (\lambda v. \text{supd } (\text{upd-gobj } \text{oref } n \ v)) \\
& \text{in } ((\text{the } (\text{values } (\text{the } (\text{globs } (\text{store } s) \ \text{oref}))) \ n), f), \text{abupd } xf \ s) \\
\\
& \text{avar} :: \text{prog} \Rightarrow \text{val} \Rightarrow \text{val} \Rightarrow \text{state} \Rightarrow \text{vvar} \times \text{state} \\
& \text{avar } G \ i' \ a' \ s = \\
& \quad \equiv \text{let } \text{oref} = \text{Heap } (\text{the-Addr } a'); \\
& \quad \quad i = \text{the-Intg } i'; \\
& \quad \quad n = \text{Inr } i; \\
& \quad \quad (T, k, cs) = \text{the-Arr } (\text{globs } (\text{store } s) \ \text{oref}); \\
& \quad \quad f = (\lambda v \ (x, s). \ (\text{raise-if } (\neg G, s \vdash v \ \text{fits } T) \\
& \quad \quad \quad \text{ArrStore } x \\
& \quad \quad \quad , \text{upd-gobj } \text{oref } n \ v \ s)) \\
& \text{in } ((\text{the } (cs \ n), f) \\
& \quad , \text{abupd } (\text{raise-if } (\neg i \ \text{in-bounds } k) \ \text{IndOutBound} \circ np \ a') \ s)
\end{aligned}$$

**lemma** *fvar-def2*: — better suited for simplification

$$\begin{aligned}
& \text{fvar } C \ \text{stat} \ fn \ a' \ s = \\
& \quad ((\text{the} \\
& \quad \quad (\text{values} \\
& \quad \quad \quad (\text{the } (\text{globs } (\text{store } s) \ (\text{if } \text{stat} \ \text{then } \text{Stat } C \ \text{else } \text{Heap } (\text{the-Addr } a')))) \\
& \quad \quad \quad (\text{Inl } (fn, C))) \\
& \quad \quad , (\lambda v. \text{supd } (\text{upd-gobj } (\text{if } \text{stat} \ \text{then } \text{Stat } C \ \text{else } \text{Heap } (\text{the-Addr } a')) \\
& \quad \quad \quad \quad (\text{Inl } (fn, C)) \\
& \quad \quad \quad \quad v))) \\
& \quad \quad , \text{abupd } (\text{if } \text{stat} \ \text{then } \text{id} \ \text{else } np \ a') \ s)
\end{aligned}$$

$\langle \text{proof} \rangle$

**lemma** *avar-def2*: — better suited for simplification

$$\begin{aligned}
& \text{avar } G \ i' \ a' \ s = \\
& \quad ((\text{the } ((\text{snd}(\text{snd}(\text{the-Arr } (\text{globs } (\text{store } s) \ (\text{Heap } (\text{the-Addr } a')))))) \\
& \quad \quad (\text{Inr } (\text{the-Intg } i')) \\
& \quad \quad , (\lambda v \ (x, s'). \ (\text{raise-if } (\neg G, s \vdash v \ \text{fits } (\text{fst}(\text{the-Arr } (\text{globs } (\text{store } s) \\
& \quad \quad \quad \quad (\text{Heap } (\text{the-Addr } a')))))) \\
& \quad \quad \quad \text{ArrStore } x \\
& \quad \quad \quad , \text{upd-gobj } (\text{Heap } (\text{the-Addr } a')) \\
& \quad \quad \quad \quad (\text{Inr } (\text{the-Intg } i')) \ v \ s')) \\
& \quad \quad , \text{abupd } (\text{raise-if } (\neg(\text{the-Intg } i') \ \text{in-bounds } (\text{fst}(\text{snd}(\text{the-Arr } (\text{globs } (\text{store } s) \\
& \quad \quad \quad \quad (\text{Heap } (\text{the-Addr } a')))))) \ \text{IndOutBound} \circ np \ a') \\
& \quad \quad s)
\end{aligned}$$

$\langle \text{proof} \rangle$

**constdefs**

*check-field-access*:

$\text{prog} \Rightarrow \text{qname} \Rightarrow \text{qname} \Rightarrow \text{vname} \Rightarrow \text{bool} \Rightarrow \text{val} \Rightarrow \text{state} \Rightarrow \text{state}$

*check-field-access*  $G \ \text{accC} \ \text{statDeclC} \ fn \ \text{stat} \ a' \ s$

$$\begin{aligned}
& \equiv \text{let } \text{oref} = \text{if } \text{stat} \ \text{then } \text{Stat } \text{statDeclC} \\
& \quad \quad \text{else } \text{Heap } (\text{the-Addr } a'); \\
& \quad \text{dynC} = \text{case } \text{oref} \ \text{of} \\
& \quad \quad \quad \text{Heap } a \Rightarrow \text{obj-class } (\text{the } (\text{globs } (\text{store } s) \ \text{oref})) \\
& \quad \quad \quad | \ \text{Stat } C \Rightarrow C; \\
& \quad f = (\text{the } (\text{table-of } (\text{DeclConcepts.fields } G \ \text{dynC}) \ (fn, \text{statDeclC}))) \\
& \text{in } \text{abupd} \\
& \quad (\text{error-if } (\neg G \vdash \text{Field } fn \ (\text{statDeclC}, f) \ \text{in } \text{dynC} \ \text{dyn-accessible-from } \text{accC}) \\
& \quad \quad \text{AccessViolation})
\end{aligned}$$

$s$

**constdefs**

*check-method-access* ::

$prog \Rightarrow qname \Rightarrow ref\text{-}ty \Rightarrow inv\text{-}mode \Rightarrow sig \Rightarrow val \Rightarrow state \Rightarrow state$   
*check-method-access*  $G\ accC\ statT\ mode\ sig\ a'\ s$   
 $\equiv let\ invC = invocation\text{-}class\ mode\ (store\ s)\ a'\ statT;$   
 $\quad dynM = the\ (dynlookup\ G\ statT\ invC\ sig)$   
 $in\ abrupt$   
 $(error\text{-}if\ (\neg\ G \vdash Methd\ sig\ dynM\ in\ invC\ dyn\text{-}accessible\text{-}from\ accC)$   
 $\quad AccessViolation)$   
 $s$

**evaluation judgments****inductive**

*halloc* ::  $[prog, state, obj\text{-}tag, loc, state] \Rightarrow bool\ (\vdash - \text{-}halloc \text{-}\succ \rightarrow -[61, 61, 61, 61, 61]60)$  **for**  $G::prog$   
**where** — allocating objects on the heap, cf. 12.5

*Abrupt*:

$G \vdash (Some\ x, s) \text{-}halloc\ oi \succ arbitrary \rightarrow (Some\ x, s)$

| *New*:  $\llbracket new\text{-}Addr\ (heap\ s) = Some\ a;$   
 $(x, oi') = (if\ atleast\text{-}free\ (heap\ s)\ (Suc\ (Suc\ 0))\ then\ (None, oi)$   
 $\quad else\ (Some\ (Xcpt\ (Loc\ a)), CInst\ (SXcpt\ OutOfMemory)) \rrbracket$   
 $\implies$   
 $G \vdash Norm\ s \text{-}halloc\ oi \succ a \rightarrow (x, init\text{-}obj\ G\ oi'\ (Heap\ a)\ s)$

**inductive** *sxalloc* ::  $[prog, state, state] \Rightarrow bool\ (\vdash - \text{-}sxalloc \rightarrow -[61, 61, 61]60)$  **for**  $G::prog$   
**where** — allocating exception objects for standard exceptions (other than OutOfMemory)

*Norm*:  $G \vdash Norm \quad s \text{-}sxalloc \rightarrow Norm \quad s$

| *Jmp*:  $G \vdash (Some\ (Jump\ j), s) \text{-}sxalloc \rightarrow (Some\ (Jump\ j), s)$

| *Error*:  $G \vdash (Some\ (Error\ e), s) \text{-}sxalloc \rightarrow (Some\ (Error\ e), s)$

| *XcptL*:  $G \vdash (Some\ (Xcpt\ (Loc\ a)), s) \text{-}sxalloc \rightarrow (Some\ (Xcpt\ (Loc\ a)), s)$

| *SXcpt*:  $\llbracket G \vdash Norm\ s0 \text{-}halloc\ (CInst\ (SXcpt\ xn)) \succ a \rightarrow (x, s1) \rrbracket \implies$   
 $G \vdash (Some\ (Xcpt\ (Std\ xn)), s0) \text{-}sxalloc \rightarrow (Some\ (Xcpt\ (Loc\ a)), s1)$

**inductive**

*eval* ::  $[prog, state, term, vals, state] \Rightarrow bool\ (\vdash - \text{-}\succ \rightarrow '(-, -)' [61, 61, 80, 0, 0]60)$   
**and** *exec* ::  $[prog, state, stmt, state] \Rightarrow bool\ (\vdash - \text{-}\rightarrow - [61, 61, 65, 61]60)$   
**and** *eval'* ::  $[prog, state, var, vvar, state] \Rightarrow bool\ (\vdash - \text{-}\rightarrow - [61, 61, 90, 61, 61]60)$   
**and** *eval''* ::  $[prog, state, expr, val, state] \Rightarrow bool\ (\vdash - \text{-}\rightarrow - [61, 61, 80, 61, 61]60)$   
**and** *evals* ::  $[prog, state, expr\ list, val\ list, state] \Rightarrow bool\ (\vdash - \text{-}\rightarrow - [61, 61, 61, 61, 61]60)$   
**for**  $G::prog$   
**where**

$G \vdash s \text{-}c \rightarrow s' \equiv G \vdash s \text{-}In1r\ c \succ \rightarrow (\Diamond, s')$   
|  $G \vdash s \text{-}e \text{-}\succ v \rightarrow s' \equiv G \vdash s \text{-}In1l\ e \succ \rightarrow (In1\ v, s')$   
|  $G \vdash s \text{-}e \text{-}\succ vf \rightarrow s' \equiv G \vdash s \text{-}In2\ e \succ \rightarrow (In2\ vf, s')$   
|  $G \vdash s \text{-}e \text{-}\dot{\succ} v \rightarrow s' \equiv G \vdash s \text{-}In3\ e \succ \rightarrow (In3\ v, s')$

— propagation of abrupt completion

— cf. 14.1, 15.5

| *Abrupt*:

$$G \vdash (\text{Some } xc, s) -t \succ \rightarrow (\text{arbitrary3 } t, (\text{Some } xc, s))$$

— execution of statements

— cf. 14.5

| *Skip*:

$$G \vdash \text{Norm } s -\text{Skip} \rightarrow \text{Norm } s$$

— cf. 14.7

| *Expr*:  $\llbracket G \vdash \text{Norm } s0 -e \succ v \rightarrow s1 \rrbracket \implies$

$$G \vdash \text{Norm } s0 -\text{Expr } e \rightarrow s1$$

| *Lab*:  $\llbracket G \vdash \text{Norm } s0 -c \rightarrow s1 \rrbracket \implies$

$$G \vdash \text{Norm } s0 -l \cdot c \rightarrow \text{abupd } (\text{absorb } l) s1$$

— cf. 14.2

| *Comp*:  $\llbracket G \vdash \text{Norm } s0 -c1 \rightarrow s1;$

$$G \vdash s1 -c2 \rightarrow s2 \rrbracket \implies$$

$$G \vdash \text{Norm } s0 -c1;; c2 \rightarrow s2$$

— cf. 14.8.2

| *If*:  $\llbracket G \vdash \text{Norm } s0 -e \succ b \rightarrow s1;$

$$G \vdash s1 -(\text{if the-Bool } b \text{ then } c1 \text{ else } c2) \rightarrow s2 \rrbracket \implies$$

$$G \vdash \text{Norm } s0 -\text{If}(e) c1 \text{ Else } c2 \rightarrow s2$$

— cf. 14.10, 14.10.1

— A continue jump from the while body  $c$  is handled by this rule. If a continue jump with the proper label was invoked inside  $c$  this label (Cont  $l$ ) is deleted out of the abrupt component of the state before the iterative evaluation of the while statement. A break jump is handled by the Lab Statement *Lab*  $l$  (*while*...).

| *Loop*:  $\llbracket G \vdash \text{Norm } s0 -e \succ b \rightarrow s1;$

*if the-Bool*  $b$

*then*  $(G \vdash s1 -c \rightarrow s2 \wedge$

$$G \vdash (\text{abupd } (\text{absorb } (\text{Cont } l)) s2) -l \cdot \text{While}(e) c \rightarrow s3)$$

*else*  $s3 = s1 \rrbracket \implies$

$$G \vdash \text{Norm } s0 -l \cdot \text{While}(e) c \rightarrow s3$$

| *Jmp*:  $G \vdash \text{Norm } s -\text{Jmp } j \rightarrow (\text{Some } (\text{Jump } j), s)$

— cf. 14.16

| *Throw*:  $\llbracket G \vdash \text{Norm } s0 -e \succ a' \rightarrow s1 \rrbracket \implies$

$$G \vdash \text{Norm } s0 -\text{Throw } e \rightarrow \text{abupd } (\text{throw } a') s1$$

— cf. 14.18.1

| *Try*:  $\llbracket G \vdash \text{Norm } s0 -c1 \rightarrow s1; G \vdash s1 -\text{xalloc} \rightarrow s2;$

*if*  $G, s2 \vdash \text{catch } C \text{ then } G \vdash \text{new-xcpt-var } vn s2 -c2 \rightarrow s3 \text{ else } s3 = s2 \rrbracket \implies$

$$G \vdash \text{Norm } s0 -\text{Try } c1 \text{ Catch}(C vn) c2 \rightarrow s3$$

— cf. 14.18.2

| *Fin*:  $\llbracket G \vdash \text{Norm } s0 -c1 \rightarrow (x1, s1);$

$$G \vdash \text{Norm } s1 -c2 \rightarrow s2;$$

$s3 = (\text{if } (\exists \text{ err. } x1 = \text{Some } (\text{Error } \text{err})))$

*then*  $(x1, s1)$

*else*  $\text{abupd } (\text{abrupt-if } (x1 \neq \text{None}) x1) s2) \rrbracket$

$\implies$

$$G \vdash \text{Norm } s0 -c1 \text{ Finally } c2 \rightarrow s3$$

— cf. 12.4.2, 8.5

| *Init*:  $\llbracket \text{the } (\text{class } G C) = c; \rrbracket$

$$\begin{aligned}
& \text{if } \text{inited } C \text{ (globs } s0) \text{ then } s3 = \text{Norm } s0 \\
& \text{else } (G \vdash \text{Norm } (\text{init-class-obj } G \ C \ s0) \\
& \quad \neg(\text{if } C = \text{Object then Skip else Init } (\text{super } c)) \rightarrow s1 \wedge \\
& \quad G \vdash \text{set-lvars empty } s1 \neg \text{init } c \rightarrow s2 \wedge s3 = \text{restore-lvars } s1 \ s2) \parallel \\
& \implies \\
& \quad G \vdash \text{Norm } s0 \neg \text{Init } C \rightarrow s3
\end{aligned}$$

— This class initialisation rule is a little bit inaccurate. Look at the exact sequence: (1) The current class object (the static fields) are initialised (*init-class-obj*), (2) the superclasses are initialised, (3) the static initialiser of the current class is invoked. More precisely we should expect another ordering, namely 2 1 3. But we can't just naively toggle 1 and 2. By calling *init-class-obj* before initialising the superclasses, we also implicitly record that we have started to initialise the current class (by setting an value for the class object). This becomes crucial for the completeness proof of the axiomatic semantics *AxCompl.thy*. Static initialisation requires an induction on the number of classes not yet initialised (or to be more precise, classes where the initialisation has not yet begun). So we could first assign a dummy value to the class before superclass initialisation and afterwards set the correct values. But as long as we don't take memory overflow into account when allocating class objects, we can leave things as they are for convenience.

— evaluation of expressions

— cf. 15.8.1, 12.4.1

$$\begin{aligned}
& | \text{NewC: } \parallel G \vdash \text{Norm } s0 \neg \text{Init } C \rightarrow s1; \\
& \quad G \vdash \quad s1 \neg \text{halloc } (C \text{Inst } C) \succ a \rightarrow s2 \parallel \implies \\
& \quad G \vdash \text{Norm } s0 \neg \text{NewC } C \neg \text{Addr } a \rightarrow s2
\end{aligned}$$

— cf. 15.9.1, 12.4.1

$$\begin{aligned}
& | \text{NewA: } \parallel G \vdash \text{Norm } s0 \neg \text{init-comp-ty } T \rightarrow s1; G \vdash s1 \neg e \neg i' \rightarrow s2; \\
& \quad G \vdash \text{abupd } (\text{check-neg } i') \ s2 \neg \text{halloc } (\text{Arr } T \ (\text{the-Intg } i')) \succ a \rightarrow s3 \parallel \implies \\
& \quad G \vdash \text{Norm } s0 \neg \text{New } T[e] \neg \text{Addr } a \rightarrow s3
\end{aligned}$$

— cf. 15.15

$$\begin{aligned}
& | \text{Cast: } \parallel G \vdash \text{Norm } s0 \neg e \neg v \rightarrow s1; \\
& \quad s2 = \text{abupd } (\text{raise-if } (\neg G, \text{store } s1 \vdash v \text{ fits } T) \ \text{ClassCast}) \ s1 \parallel \implies \\
& \quad G \vdash \text{Norm } s0 \neg \text{Cast } T \ e \neg v \rightarrow s2
\end{aligned}$$

— cf. 15.19.2

$$\begin{aligned}
& | \text{Inst: } \parallel G \vdash \text{Norm } s0 \neg e \neg v \rightarrow s1; \\
& \quad b = (v \neq \text{Null} \wedge G, \text{store } s1 \vdash v \text{ fits } \text{RefT } T) \parallel \implies \\
& \quad G \vdash \text{Norm } s0 \neg e \text{InstOf } T \neg \text{Bool } b \rightarrow s1
\end{aligned}$$

— cf. 15.7.1

$$| \text{Lit: } G \vdash \text{Norm } s \neg \text{Lit } v \neg v \rightarrow \text{Norm } s$$

$$\begin{aligned}
& | \text{UnOp: } \parallel G \vdash \text{Norm } s0 \neg e \neg v \rightarrow s1 \parallel \\
& \implies G \vdash \text{Norm } s0 \neg \text{UnOp } \text{unop } e \neg (\text{eval-unop } \text{unop } v) \rightarrow s1
\end{aligned}$$

$$\begin{aligned}
& | \text{BinOp: } \parallel G \vdash \text{Norm } s0 \neg e1 \neg v1 \rightarrow s1; \\
& \quad G \vdash s1 \neg (\text{if need-second-arg binop } v1 \text{ then } (\text{In1l } e2) \text{ else } (\text{In1r Skip})) \\
& \quad \quad \neg \rightarrow (\text{In1 } v2, \ s2) \\
& \parallel \\
& \implies G \vdash \text{Norm } s0 \neg \text{BinOp } \text{binop } e1 \ e2 \neg (\text{eval-binop } \text{binop } v1 \ v2) \rightarrow s2
\end{aligned}$$

— cf. 15.10.2

$$| \text{Super: } G \vdash \text{Norm } s \neg \text{Super} \neg \text{val-this } s \rightarrow \text{Norm } s$$

— cf. 15.2

$$\begin{aligned}
& | \text{Acc: } \parallel G \vdash \text{Norm } s0 \neg va = \succ (v, f) \rightarrow s1 \parallel \implies \\
& \quad G \vdash \text{Norm } s0 \neg \text{Acc } va \neg v \rightarrow s1
\end{aligned}$$

— cf. 15.25.1

$$| \text{Ass: } \parallel G \vdash \text{Norm } s0 \neg va = \succ (w, f) \rightarrow s1;$$

$$G \vdash \quad s1 \text{ --} e \text{--} \succ v \rightarrow s2 \parallel \implies \\ G \vdash \text{Norm } s0 \text{ --} va := e \text{--} \succ v \rightarrow \text{assign } f \ v \ s2$$

— cf. 15.24

$$\mid \text{Cond: } \llbracket G \vdash \text{Norm } s0 \text{ --} e0 \text{--} \succ b \rightarrow s1; \\ G \vdash \quad s1 \text{ --} (\text{if the-Bool } b \text{ then } e1 \text{ else } e2) \text{--} \succ v \rightarrow s2 \parallel \implies \\ G \vdash \text{Norm } s0 \text{ --} e0 \text{ ? } e1 : e2 \text{--} \succ v \rightarrow s2$$

— The interplay of *Call*, *Method* and *Body*: Method invocation is split up into these three rules:

*Call* Calculates the target address and evaluates the arguments of the method, and then performs dynamic or static lookup of the method, corresponding to the call mode. Then the *Method* rule is evaluated on the calculated declaration class of the method invocation.

*Method* A syntactic bridge for the folded method body. It is used by the axiomatic semantics to add the proper hypothesis for recursive calls of the method.

*Body* An extra syntactic entity for the unfolded method body was introduced to properly trigger class initialisation. Without class initialisation we could just evaluate the body statement.

— cf. 15.11.4.1, 15.11.4.2, 15.11.4.4, 15.11.4.5

$$\mid \text{Call:} \\ \llbracket G \vdash \text{Norm } s0 \text{ --} e \text{--} \succ a' \rightarrow s1; G \vdash s1 \text{ --} args \dot{=} \succ vs \rightarrow s2; \\ D = \text{invocation-declclass } G \text{ mode } (\text{store } s2) \ a' \ \text{statT } (\llbracket name = mn, parTs = pTs \rrbracket); \\ s3 = \text{init-lvars } G \ D \ (\llbracket name = mn, parTs = pTs \rrbracket) \ \text{mode } a' \ vs \ s2; \\ s3' = \text{check-method-access } G \ \text{accC} \ \text{statT} \ \text{mode } (\llbracket name = mn, parTs = pTs \rrbracket) \ a' \ s3; \\ G \vdash s3' \text{ --} \text{Method } D \ (\llbracket name = mn, parTs = pTs \rrbracket) \text{--} \succ v \rightarrow s4 \parallel \\ \implies \\ G \vdash \text{Norm } s0 \text{ --} \{accC, statT, mode\} e \cdot mn(\{pTs\} args) \text{--} \succ v \rightarrow (\text{restore-lvars } s2 \ s4)$$

— The accessibility check is after *init-lvars*, to keep it simple. *init-lvars* already tests for the absence of a null-pointer reference in case of an instance method invocation.

$$\mid \text{Method:} \quad \llbracket G \vdash \text{Norm } s0 \text{ --} \text{body } G \ D \ \text{sig} \text{--} \succ v \rightarrow s1 \parallel \implies \\ G \vdash \text{Norm } s0 \text{ --} \text{Method } D \ \text{sig} \text{--} \succ v \rightarrow s1$$

$$\mid \text{Body: } \llbracket G \vdash \text{Norm } s0 \text{ --} \text{Init } D \rightarrow s1; G \vdash s1 \text{ --} c \rightarrow s2; \\ s3 = (\text{if } (\exists l. \text{abrupt } s2 = \text{Some } (\text{Jump } (\text{Break } l))) \vee \\ \text{abrupt } s2 = \text{Some } (\text{Jump } (\text{Cont } l))) \\ \text{then } \text{abupd } (\lambda x. \text{Some } (\text{Error CrossMethodJump})) \ s2 \\ \text{else } s2 \parallel \implies \\ G \vdash \text{Norm } s0 \text{ --} \text{Body } D \ c \text{--} \succ \text{the } (\text{locals } (\text{store } s2) \ \text{Result}) \\ \rightarrow \text{abupd } (\text{absorb Ret}) \ s3$$

— cf. 14.15, 12.4.1

— We filter out a break/continue in *s2*, so that we can proof definite assignment correct, without the need of conformance of the state. By this the different parts of the typesafety proof can be disentangled a little.

— evaluation of variables

— cf. 15.13.1, 15.7.2

$$\mid \text{LVar: } G \vdash \text{Norm } s \text{ --} \text{LVar } vn \dot{=} \succ \text{lvar } vn \ s \rightarrow \text{Norm } s$$

— cf. 15.10.1, 12.4.1

$$\mid \text{FVar: } \llbracket G \vdash \text{Norm } s0 \text{ --} \text{Init } \text{statDeclC} \rightarrow s1; G \vdash s1 \text{ --} e \text{--} \succ a \rightarrow s2; \\ (v, s2') = \text{fvar } \text{statDeclC} \ \text{stat fn } a \ s2; \\ s3 = \text{check-field-access } G \ \text{accC} \ \text{statDeclC} \ \text{fn } \text{stat } a \ s2' \parallel \implies \\ G \vdash \text{Norm } s0 \text{ --} \{accC, \text{statDeclC}, \text{stat}\} e \cdot \text{fn} \dot{=} \succ v \rightarrow s3$$

— The accessibility check is after *fvar*, to keep it simple. *fvar* already tests for the absence of a null-pointer reference in case of an instance field

$$\begin{array}{l} G \vdash (\text{Some } xc, s) -t \quad \quad \quad \gamma \rightarrow (v, s') \\ G \vdash \text{Norm } s -\text{In1r Skip} \quad \quad \quad \gamma \rightarrow (x, s') \end{array}$$

$G \vdash \text{Norm } s - \text{In1r } (\text{Jmp } j)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1r } (l \bullet c)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In3 } ([\ ])$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In3 } (e \# es)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{Lit } w)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{UnOp } \text{unop } e)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{BinOp } \text{binop } e1 \ e2)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In2 } (\text{LVar } vn)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{Cast } T \ e)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (e \ \text{InstOf } T)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{Super})$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{Acc } va)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1r } (\text{Expr } e)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1r } (c1;; c2)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{Methd } C \ \text{sig})$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{Body } D \ c)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1l } (e0 \ ? \ e1 : e2)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1r } (\text{If}(e) \ c1 \ \text{Else } c2)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1r } (l \bullet \text{While}(e) \ c)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1r } (c1 \ \text{Finally } c2)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1r } (\text{Throw } e)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{NewC } C)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{New } T[e])$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\text{Ass } va \ e)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1r } (\text{Try } c1 \ \text{Catch}(tn \ vn) \ c2)$	$\succ \rightarrow (x, s')$
$G \vdash \text{Norm } s - \text{In2 } (\{\text{accC}, \text{statDeclC}, \text{stat}\} e..fn)$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In2 } (e1.[e2])$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1l } (\{\text{accC}, \text{statT}, \text{mode}\} e.mn(\{pT\}p))$	$\succ \rightarrow (v, s')$
$G \vdash \text{Norm } s - \text{In1r } (\text{Init } C)$	$\succ \rightarrow (x, s')$

**declare** *not-None-eq* [*simp*]

**declare** *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]

$\langle ML \rangle$

**declare** *split-if* [*split*] *split-if-asm* [*split*]  
*option.split* [*split*] *option.split-asm* [*split*]

**lemma** *eval-Inj-elim*:

$G \vdash s - t \succ \rightarrow (w, s')$   
 $\implies \text{case } t \text{ of}$   
 $\quad \text{In1 } ec \Rightarrow (\text{case } ec \text{ of}$   
 $\quad \quad \text{Inl } e \Rightarrow (\exists v. w = \text{In1 } v)$   
 $\quad \quad | \text{Inr } c \Rightarrow w = \Diamond)$   
 $\quad | \text{In2 } e \Rightarrow (\exists v. w = \text{In2 } v)$   
 $\quad | \text{In3 } e \Rightarrow (\exists v. w = \text{In3 } v)$   
 $\langle \text{proof} \rangle$

The following simplification procedures set up the proper injections of terms and their corresponding values in the evaluation relation: E.g. an expression (injection *In1l* into terms) always evaluates to ordinary values (injection *In1* into generalised values *vals*).

**lemma** *eval-expr-eq*:  $G \vdash s - \text{In1l } t \succ \rightarrow (w, s') = (\exists v. w = \text{In1 } v \wedge G \vdash s - t = \succ v \rightarrow s')$   
 $\langle \text{proof} \rangle$

**lemma** *eval-var-eq*:  $G \vdash s - \text{In2 } t \succ \rightarrow (w, s') = (\exists vf. w = \text{In2 } vf \wedge G \vdash s - t = \succ vf \rightarrow s')$   
 $\langle \text{proof} \rangle$

**lemma** *eval-exprs-eq*:  $G \vdash s - \text{In3 } t \succ \rightarrow (w, s') = (\exists vs. w = \text{In3 } vs \wedge G \vdash s - t = \succ vs \rightarrow s')$



$\langle proof \rangle$

**lemma** *eval-stmt-eq*:  $G \vdash s - \text{In1r } t \rightarrow (w, s') = (w = \Diamond \wedge G \vdash s - t \rightarrow s')$   
 $\langle proof \rangle$

$\langle ML \rangle$

**declare** *halloc.Abrupt* [intro!] *eval.Abrupt* [intro!] *AbruptIs* [intro!]

*Callee*, *InsInitE*, *InsInitV*, *FinA* are only used in smallstep semantics, not in the bigstep semantics.  
 So their is no valid evaluation of these terms

**lemma** *eval-Callee*:  $G \vdash \text{Norm } s - \text{Callee } l \ e - \succ v \rightarrow s' = \text{False}$   
 $\langle proof \rangle$

**lemma** *eval-InsInitE*:  $G \vdash \text{Norm } s - \text{InsInitE } c \ e - \succ v \rightarrow s' = \text{False}$   
 $\langle proof \rangle$

**lemma** *eval-InsInitV*:  $G \vdash \text{Norm } s - \text{InsInitV } c \ w - \succ v \rightarrow s' = \text{False}$   
 $\langle proof \rangle$

**lemma** *eval-FinA*:  $G \vdash \text{Norm } s - \text{FinA } a \ c \rightarrow s' = \text{False}$   
 $\langle proof \rangle$

**lemma** *eval-no-abrupt-lemma*:  
 $\bigwedge s \ s'. G \vdash s - t \rightarrow (w, s') \implies \text{normal } s' \longrightarrow \text{normal } s$   
 $\langle proof \rangle$

**lemma** *eval-no-abrupt*:  
 $G \vdash (x, s) - t \rightarrow (w, \text{Norm } s') =$   
 $(x = \text{None} \wedge G \vdash \text{Norm } s - t \rightarrow (w, \text{Norm } s'))$   
 $\langle proof \rangle$

$\langle ML \rangle$

**lemma** *eval-abrupt-lemma*:  
 $G \vdash s - t \rightarrow (v, s') \implies \text{abrupt } s = \text{Some } xc \longrightarrow s' = s \wedge v = \text{arbitrary3 } t$   
 $\langle proof \rangle$

**lemma** *eval-abrupt*:  
 $G \vdash (\text{Some } xc, s) - t \rightarrow (w, s') =$   
 $(s' = (\text{Some } xc, s) \wedge w = \text{arbitrary3 } t \wedge$   
 $G \vdash (\text{Some } xc, s) - t \rightarrow (\text{arbitrary3 } t, (\text{Some } xc, s)))$   
 $\langle proof \rangle$

$\langle ML \rangle$

**lemma** *LitI*:  $G \vdash s - \text{Lit } v - \succ (\text{if normal } s \text{ then } v \text{ else arbitrary}) \rightarrow s$   
 $\langle proof \rangle$

**lemma** *SkipI* [intro!]:  $G \vdash s \text{ --Skip} \rightarrow s$   
 ⟨proof⟩

**lemma** *ExprI*:  $G \vdash s \text{ --}e\text{--}\rightarrow v \rightarrow s' \implies G \vdash s \text{ --Expr } e \rightarrow s'$   
 ⟨proof⟩

**lemma** *CompI*:  $\llbracket G \vdash s \text{ --}c1 \rightarrow s1; G \vdash s1 \text{ --}c2 \rightarrow s2 \rrbracket \implies G \vdash s \text{ --}c1;; c2 \rightarrow s2$   
 ⟨proof⟩

**lemma** *CondI*:  
 $\bigwedge s1. \llbracket G \vdash s \text{ --}e\text{--}\rightarrow v \rightarrow s1; G \vdash s1 \text{ --(if the-Bool } b \text{ then } e1 \text{ else } e2)\text{--}\rightarrow v \rightarrow s2 \rrbracket \implies$   
 $G \vdash s \text{ --}e \text{ ? } e1 : e2\text{--}\rightarrow v \text{ --(if normal } s1 \text{ then } v \text{ else arbitrary)} \rightarrow s2$   
 ⟨proof⟩

**lemma** *IfI*:  $\llbracket G \vdash s \text{ --}e\text{--}\rightarrow v \rightarrow s1; G \vdash s1 \text{ --(if the-Bool } v \text{ then } c1 \text{ else } c2)\text{--}\rightarrow s2 \rrbracket$   
 $\implies G \vdash s \text{ --If}(e) \text{ } c1 \text{ Else } c2 \rightarrow s2$   
 ⟨proof⟩

**lemma** *MethdI*:  $G \vdash s \text{ --body } G \text{ } C \text{ sig--}\rightarrow v \rightarrow s'$   
 $\implies G \vdash s \text{ --Methd } C \text{ sig--}\rightarrow v \rightarrow s'$   
 ⟨proof⟩

**lemma** *eval-Call*:  
 $\llbracket G \vdash \text{Norm } s0 \text{ --}e\text{--}\rightarrow a' \rightarrow s1; G \vdash s1 \text{ --ps--}\rightarrow pvs \rightarrow s2;$   
 $D = \text{invocation-declclass } G \text{ mode (store } s2) \text{ } a' \text{ statT } (\llbracket \text{name=mn,parTs=pTs} \rrbracket);$   
 $s3 = \text{init-lvars } G \text{ } D \text{ } (\llbracket \text{name=mn,parTs=pTs} \rrbracket) \text{ mode } a' \text{ pvs } s2;$   
 $s3' = \text{check-method-access } G \text{ accC statT mode } (\llbracket \text{name=mn,parTs=pTs} \rrbracket) \text{ } a' \text{ } s3;$   
 $G \vdash s3' \text{ --Methd } D \text{ } (\llbracket \text{name=mn,parTs=pTs} \rrbracket) \text{ --}\rightarrow v \rightarrow s4;$   
 $s4' = \text{restore-lvars } s2 \text{ } s4 \rrbracket \implies$   
 $G \vdash \text{Norm } s0 \text{ --}\{accC, statT, mode\}e.mn(\{pTs\}ps)\text{--}\rightarrow v \rightarrow s4'$   
 ⟨proof⟩

**lemma** *eval-Init*:  
 $\llbracket \text{if initd } C \text{ (globs } s0) \text{ then } s3 = \text{Norm } s0$   
 $\text{else } G \vdash \text{Norm (init-class-obj } G \text{ } C \text{ } s0)$   
 $\text{--(if } C = \text{Object then Skip else Init (super (the (class } G \text{ } C)))}\text{--}\rightarrow s1 \wedge$   
 $G \vdash \text{set-lvars empty } s1 \text{ --(init (the (class } G \text{ } C)))}\text{--}\rightarrow s2 \wedge$   
 $s3 = \text{restore-lvars } s1 \text{ } s2 \rrbracket \implies$   
 $G \vdash \text{Norm } s0 \text{ --Init } C \rightarrow s3$   
 ⟨proof⟩

**lemma** *init-done*:  $\text{initd } C \text{ } s \implies G \vdash s \text{ --Init } C \rightarrow s$   
 ⟨proof⟩

**lemma** *eval-StatRef*:  
 $G \vdash s \text{ --StatRef } rt\text{--}\rightarrow v \text{ --(if abrupt } s = \text{None then Null else arbitrary)} \rightarrow s$   
 ⟨proof⟩

**lemma** *SkipD* [*dest!*]:  $G \vdash s -\text{Skip} \rightarrow s' \implies s' = s$   
 ⟨*proof*⟩

**lemma** *Skip-eq* [*simp*]:  $G \vdash s -\text{Skip} \rightarrow s' = (s = s')$   
 ⟨*proof*⟩

**lemma** *init-retains-locals* [*rule-format* (*no-asm*)]:  $G \vdash s -t \succ \rightarrow (w, s') \implies$   
 $(\forall C. t = \text{In1r} (\text{Init } C) \longrightarrow \text{locals} (\text{store } s) = \text{locals} (\text{store } s'))$   
 ⟨*proof*⟩

**lemma** *halloc-xcpt* [*dest!*]:  
 $\bigwedge s'. G \vdash (\text{Some } xc, s) -\text{halloc } oi \succ a \rightarrow s' \implies s' = (\text{Some } xc, s)$   
 ⟨*proof*⟩

**lemma** *eval-Methd*:  
 $G \vdash s -\text{In1l}(\text{body } G \ C \ sig) \succ \rightarrow (w, s')$   
 $\implies G \vdash s -\text{In1l}(\text{Methd } C \ sig) \succ \rightarrow (w, s')$   
 ⟨*proof*⟩

**lemma** *eval-Body*:  $\llbracket G \vdash \text{Norm } s0 -\text{Init } D \rightarrow s1; G \vdash s1 -c \rightarrow s2;$   
 $\text{res} = \text{the} (\text{locals} (\text{store } s2) \ \text{Result});$   
 $s3 = (\text{if } (\exists l. \text{abrupt } s2 = \text{Some } (\text{Jump } (\text{Break } l))) \vee$   
 $\text{abrupt } s2 = \text{Some } (\text{Jump } (\text{Cont } l)))$   
 $\text{then } \text{abupd } (\lambda x. \text{Some } (\text{Error CrossMethodJump})) \ s2$   
 $\text{else } s2);$   
 $s4 = \text{abupd } (\text{absorb Ret}) \ s3 \rrbracket \implies$   
 $G \vdash \text{Norm } s0 -\text{Body } D \ c \succ \text{res} \rightarrow s4$   
 ⟨*proof*⟩

**lemma** *eval-binop-arg2-indep*:  
 $\neg \text{need-second-arg binop } v1 \implies \text{eval-binop binop } v1 \ x = \text{eval-binop binop } v1 \ y$   
 ⟨*proof*⟩

**lemma** *eval-BinOp-arg2-indepI*:  
**assumes** *eval-e1*:  $G \vdash \text{Norm } s0 -e1 \succ v1 \rightarrow s1$  **and**  
*no-need*:  $\neg \text{need-second-arg binop } v1$   
**shows**  $G \vdash \text{Norm } s0 -\text{BinOp binop } e1 \ e2 \succ (\text{eval-binop binop } v1 \ v2) \rightarrow s1$   
**(is ?EvalBinOp v2)**  
 ⟨*proof*⟩

## single valued

**lemma** *unique-halloc* [*rule-format* (*no-asm*)]:  
 $G \vdash s -\text{halloc } oi \succ a \rightarrow s' \implies G \vdash s -\text{halloc } oi \succ a' \rightarrow s'' \longrightarrow a' = a \wedge s'' = s'$   
 ⟨*proof*⟩

**lemma** *single-valued-halloc*:

*single-valued*  $\{((s, oi), (a, s')). G \vdash s -halloc\ oi \succ a \rightarrow s'\}$   
 $\langle proof \rangle$

**lemma** *unique-sxalloc* [rule-format (no-asm)]:

$G \vdash s -sxalloc \rightarrow s' \implies G \vdash s -sxalloc \rightarrow s'' \longrightarrow s'' = s'$   
 $\langle proof \rangle$

**lemma** *single-valued-sxalloc*: *single-valued*  $\{(s, s'). G \vdash s -sxalloc \rightarrow s'\}$

$\langle proof \rangle$

**lemma** *split-pairD*:  $(x, y) = p \implies x = fst\ p \ \& \ y = snd\ p$

$\langle proof \rangle$

**lemma** *unique-eval* [rule-format (no-asm)]:

$G \vdash s -t \succ \rightarrow (w, s') \implies (\forall w' s''. G \vdash s -t \succ \rightarrow (w', s'') \longrightarrow w' = w \wedge s'' = s')$   
 $\langle proof \rangle$

**lemma** *single-valued-eval*:

*single-valued*  $\{((s, t), (v, s')). G \vdash s -t \succ \rightarrow (v, s')\}$   
 $\langle proof \rangle$

**end**

## Chapter 16

### Example

### 43 Example Bali program

**theory** *Example* **imports** *Eval WellForm* **begin**

The following example Bali program includes:

- class and interface declarations with inheritance, hiding of fields, overriding of methods (with refined result type), array type,
- method call (with dynamic binding), parameter access, return expressions,
- expression statements, sequential composition, literal values, local assignment, local access, field assignment, type cast,
- exception generation and propagation, try and catch statement, throw statement
- instance creation and (default) static initialization

```
package java_lang

public interface HasFoo {
  public Base foo(Base z);
}

public class Base implements HasFoo {
  static boolean arr[] = new boolean[2];
  public HasFoo vee;
  public Base foo(Base z) {
    return z;
  }
}

public class Ext extends Base {
  public int vee;
  public Ext foo(Base z) {
    ((Ext)z).vee = 1;
    return null;
  }
}

public class Main {
  public static void main(String args[]) throws Throwable {
    Base e = new Ext();
    try {e.foo(null); }
    catch(NullPointerException z) {
      while(Ext.arr[2]) ;
    }
  }
}
```

**declare** *widen.null* [*intro*]

**lemma** *wf-fdecl-def2*:  $\bigwedge fd. wf-fdecl\ G\ P\ fd = is-acc-type\ G\ P\ (type\ (snd\ fd))$   
*<proof>*

**declare** *wf-fdecl-def2* [*iff*]

**type and expression names**

**datatype**  $tnam' = HasFoo' \mid Base' \mid Ext' \mid Main'$   
**datatype**  $vnam' = arr' \mid vee' \mid z' \mid e'$   
**datatype**  $label' = lab1'$

**consts**

$tnam' :: tnam' \Rightarrow tnam$   
 $vnam' :: vnam' \Rightarrow vname$   
 $label' :: label' \Rightarrow label$

**axioms**

$inj-tnam' [simp]: (tnam' x = tnam' y) = (x = y)$   
 $inj-vnam' [simp]: (vnam' x = vnam' y) = (x = y)$   
 $inj-label' [simp]: (label' x = label' y) = (x = y)$

$surj-tnam': \exists m. n = tnam' m$   
 $surj-vnam': \exists m. n = vnam' m$   
 $surj-label': \exists m. n = label' m$

**abbreviation**

$HasFoo :: qname$  **where**  
 $HasFoo == (\text{pid}=\text{java-lang}, \text{tid}=TName (tnam' HasFoo'))$

**abbreviation**

$Base :: qname$  **where**  
 $Base == (\text{pid}=\text{java-lang}, \text{tid}=TName (tnam' Base'))$

**abbreviation**

$Ext :: qname$  **where**  
 $Ext == (\text{pid}=\text{java-lang}, \text{tid}=TName (tnam' Ext'))$

**abbreviation**

$Main :: qname$  **where**  
 $Main == (\text{pid}=\text{java-lang}, \text{tid}=TName (tnam' Main'))$

**abbreviation**

$arr :: vname$  **where**  
 $arr == (vnam' arr')$

**abbreviation**

$vee :: vname$  **where**  
 $vee == (vnam' vee')$

**abbreviation**

$z :: vname$  **where**  
 $z == (vnam' z')$

**abbreviation**

$e :: vname$  **where**  
 $e == (vnam' e')$

**abbreviation**

$lab1 :: label$  **where**  
 $lab1 == label' lab1'$

**lemma** *neq-Base-Object* [simp]: *Base* ≠ *Object*  
 ⟨proof⟩

**lemma** *neq-Ext-Object* [simp]: *Ext* ≠ *Object*  
 ⟨proof⟩

**lemma** *neq-Main-Object* [simp]: *Main* ≠ *Object*  
 ⟨proof⟩

**lemma** *neq-Base-SXcpt* [simp]: *Base* ≠ *SXcpt* *xn*  
 ⟨proof⟩

**lemma** *neq-Ext-SXcpt* [simp]: *Ext* ≠ *SXcpt* *xn*  
 ⟨proof⟩

**lemma** *neq-Main-SXcpt* [simp]: *Main* ≠ *SXcpt* *xn*  
 ⟨proof⟩

## classes and interfaces

### defs

*Object-mdecls-def*: *Object-mdecls* ≡ []  
*SXcpt-mdecls-def*: *SXcpt-mdecls* ≡ []

### consts

*foo* :: *mname*

### constdefs

*foo-sig* :: *sig*  
*foo-sig* ≡ (⟦name=foo,parTs=[Class Base]⟧)

*foo-mhead* :: *mhead*  
*foo-mhead* ≡ (⟦access=Public,static=False,pars=[z],resT=Class Base⟧)

### constdefs

*Base-foo* :: *mdecl*  
*Base-foo* ≡ (foo-sig, (⟦access=Public,static=False,pars=[z],resT=Class Base,  
 mbody=⟦lcls=[],stmt=Return (!!z)⟧⟧))

### constdefs

*Ext-foo* :: *mdecl*  
*Ext-foo* ≡ (foo-sig,  
 (⟦access=Public,static=False,pars=[z],resT=Class Ext,  
 mbody=⟦lcls=[],  
 stmt=Expr({Ext,Ext,False} Cast (Class Ext) (!!z)..vee :=  
 Lit (Intg 1)) ;;  
 Return (Lit Null)⟧)  
 ⟧)



**constdefs**

*arr-viewed-from* :: *qtname*  $\Rightarrow$  *qtname*  $\Rightarrow$  *var*  
*arr-viewed-from* *accC C*  $\equiv$  {*accC*,*Base*,*True*}*StatRef* (*ClassT C*)..*arr*

*BaseCl* :: *class*  
*BaseCl*  $\equiv$  (*access*=*Public*,  
     *cfields*=[(*arr*, (*access*=*Public*,*static*=*True*,*type*=*PrimT Boolean*)),  
               (*vee*, (*access*=*Public*,*static*=*False*,*type*=*Iface HasFoo*))],  
     *methods*=[*Base-foo*],  
     *init*=*Expr*(*arr-viewed-from Base Base*  
               :=*New* (*PrimT Boolean*)[*Lit* (*Intg 2*)]),  
     *super*=*Object*,  
     *superIfs*=[*HasFoo*])

*ExtCl* :: *class*  
*ExtCl*  $\equiv$  (*access*=*Public*,  
     *cfields*=[(*vee*, (*access*=*Public*,*static*=*False*,*type*=*PrimT Integer*))],  
     *methods*=[*Ext-foo*],  
     *init*=*Skip*,  
     *super*=*Base*,  
     *superIfs*=[])

*MainCl* :: *class*  
*MainCl*  $\equiv$  (*access*=*Public*,  
     *cfields*=[],  
     *methods*=[],  
     *init*=*Skip*,  
     *super*=*Object*,  
     *superIfs*=[])

**constdefs**

*HasFooInt* :: *iface*  
*HasFooInt*  $\equiv$  (*access*=*Public*,*imethods*=[(*foo-sig*, *foo-mhead*)],*isuperIfs*=[])

*Ifaces* :: *idecl list*  
*Ifaces*  $\equiv$  [(*HasFoo*,*HasFooInt*)]

*Classes* :: *cdecl list*  
*Classes*  $\equiv$  [(*Base*,*BaseCl*),(*Ext*,*ExtCl*),(*Main*,*MainCl*)]@*standard-classes*

**lemmas** *table-classes-defs* =  
     *Classes-def standard-classes-def ObjectC-def SXcptC-def*

**lemma** *table-ifaces* [*simp*]: *table-of Ifaces* = *empty*(*HasFoo* $\mapsto$ *HasFooInt*)  
 <*proof*>

**lemma** *table-classes-Object* [*simp*]:  
     *table-of Classes Object* = *Some* (*access*=*Public*,*cfields*=[],  
                                     *methods*=*Object-mdecls*,  
                                     *init*=*Skip*,*super*=*arbitrary*,*superIfs*=[])  
 <*proof*>

**lemma** *table-classes-SXcpt* [*simp*]:

*table-of Classes (SXcpt xn)*  
 = *Some* ( $\langle \text{access} = \text{Public}, \text{cfields} = [], \text{methods} = \text{SXcpt-mdecls},$   
 $\text{init} = \text{Skip},$   
 $\text{super} = \text{if } xn = \text{Throwable} \text{ then } \text{Object} \text{ else } \text{SXcpt Throwable},$   
 $\text{superIfs} = [] \rangle$ )  
 $\langle \text{proof} \rangle$

**lemma** *table-classes-HasFoo* [simp]: *table-of Classes HasFoo* = *None*  
 $\langle \text{proof} \rangle$

**lemma** *table-classes-Base* [simp]: *table-of Classes Base* = *Some BaseCl*  
 $\langle \text{proof} \rangle$

**lemma** *table-classes-Ext* [simp]: *table-of Classes Ext* = *Some ExtCl*  
 $\langle \text{proof} \rangle$

**lemma** *table-classes-Main* [simp]: *table-of Classes Main* = *Some MainCl*  
 $\langle \text{proof} \rangle$

## program

### abbreviation

*tprg* :: *prog* **where**  
*tprg* == ( $\langle \text{ifaces} = \text{Ifaces}, \text{classes} = \text{Classes} \rangle$ )

### constdefs

*test* :: (*ty*)*list*  $\Rightarrow$  *stmt*  
*test pTs*  $\equiv e ::= \text{NewC Ext};;$   
 $\text{Try Expr}(\{ \text{Main}, \text{ClassT Base}, \text{Int Vir} \} !! e \cdot \text{foo}(\{ pTs \} [\text{Lit Null}]))$   
 $\text{Catch}((\text{SXcpt NullPointerException}) z)$   
 $(\text{lab1} \cdot \text{While}(\text{Acc}$   
 $\quad (\text{Acc } (\text{arr-viewed-from Main Ext}).[\text{Lit } (\text{Intg } 2)])) \text{ Skip})$

## well-structuredness

**lemma** *not-Object-subcls-any* [elim!]:  $(\text{Object}, C) \in (\text{subcls1 tprg})^+ \Longrightarrow R$   
 $\langle \text{proof} \rangle$

**lemma** *not-Throwable-subcls-SXcpt* [elim!]:  
 $(\text{SXcpt Throwable}, \text{SXcpt xn}) \in (\text{subcls1 tprg})^+ \Longrightarrow R$   
 $\langle \text{proof} \rangle$

**lemma** *not-SXcpt-n-subcls-SXcpt-n* [elim!]:  
 $(\text{SXcpt xn}, \text{SXcpt xn}) \in (\text{subcls1 tprg})^+ \Longrightarrow R$   
 $\langle \text{proof} \rangle$

**lemma** *not-Base-subcls-Ext* [elim!]:  $(\text{Base}, \text{Ext}) \in (\text{subcls1 tprg})^+ \Longrightarrow R$   
 $\langle \text{proof} \rangle$

**lemma** *not-TName-n-subcls-TName-n* [rule-format (no-asm), elim!]:  
 $(\langle \text{pid} = \text{java-lang}, \text{tid} = \text{TName tn} \rangle, \langle \text{pid} = \text{java-lang}, \text{tid} = \text{TName tn} \rangle)$

$\in (\text{subcls1 } \text{tprg})^+ \longrightarrow R$   
 $\langle \text{proof} \rangle$

**lemma** *ws-idecl-HasFoo*: *ws-idecl tprg HasFoo* []  
 $\langle \text{proof} \rangle$

**lemma** *ws-cdecl-Object*: *ws-cdecl tprg Object any*  
 $\langle \text{proof} \rangle$

**lemma** *ws-cdecl-Throwable*: *ws-cdecl tprg (SXcpt Throwable) Object*  
 $\langle \text{proof} \rangle$

**lemma** *ws-cdecl-SXcpt*: *ws-cdecl tprg (SXcpt xn) (SXcpt Throwable)*  
 $\langle \text{proof} \rangle$

**lemma** *ws-cdecl-Base*: *ws-cdecl tprg Base Object*  
 $\langle \text{proof} \rangle$

**lemma** *ws-cdecl-Ext*: *ws-cdecl tprg Ext Base*  
 $\langle \text{proof} \rangle$

**lemma** *ws-cdecl-Main*: *ws-cdecl tprg Main Object*  
 $\langle \text{proof} \rangle$

**lemmas** *ws-cdecls* = *ws-cdecl-SXcpt ws-cdecl-Object ws-cdecl-Throwable*  
*ws-cdecl-Base ws-cdecl-Ext ws-cdecl-Main*

**declare** *not-Object-subcls-any* [rule del]  
*not-Throwable-subcls-SXcpt* [rule del]  
*not-SXcpt-n-subcls-SXcpt-n* [rule del]  
*not-Base-subcls-Ext* [rule del] *not-TName-n-subcls-TName-n* [rule del]

**lemma** *ws-idecl-all*:  
 $G = \text{tprg} \implies (\forall (I, i) \in \text{set Ifaces. } \text{ws-idecl } G \ I \ (\text{isuperIfs } i))$   
 $\langle \text{proof} \rangle$

**lemma** *ws-cdecl-all*:  $G = \text{tprg} \implies (\forall (C, c) \in \text{set Classes. } \text{ws-cdecl } G \ C \ (\text{super } c))$   
 $\langle \text{proof} \rangle$

**lemma** *ws-tprg*: *ws-prog tprg*  
 $\langle \text{proof} \rangle$

**misc program properties (independent of well-structuredness)**

**lemma** *single-iface* [simp]: *is-iface tprg I = (I = HasFoo)*  
 $\langle \text{proof} \rangle$

**lemma** *empty-subint1* [simp]: *subint1 tprg* = {}  
 ⟨proof⟩

**lemma** *unique-ifaces*: *unique Ifaces*  
 ⟨proof⟩

**lemma** *unique-classes*: *unique Classes*  
 ⟨proof⟩

**lemma** *SXcpt-subcls-Throwable* [simp]: *tprg* ⊢ *SXcpt xn* ≤<sub>C</sub> *SXcpt Throwable*  
 ⟨proof⟩

**lemma** *Ext-subclseq-Base* [simp]: *tprg* ⊢ *Ext* ≤<sub>C</sub> *Base*  
 ⟨proof⟩

**lemma** *Ext-subcls-Base* [simp]: *tprg* ⊢ *Ext* <<sub>C</sub> *Base*  
 ⟨proof⟩

### fields and method lookup

**lemma** *fields-tprg-Object* [simp]: *DeclConcepts.fields tprg Object* = []  
 ⟨proof⟩

**lemma** *fields-tprg-Throwable* [simp]:  
*DeclConcepts.fields tprg (SXcpt Throwable)* = []  
 ⟨proof⟩

**lemma** *fields-tprg-SXcpt* [simp]: *DeclConcepts.fields tprg (SXcpt xn)* = []  
 ⟨proof⟩

**lemmas** *fields-rec'* = *fields-rec* [OF - *ws-tprg*]

**lemma** *fields-Base* [simp]:  
*DeclConcepts.fields tprg Base*  
 = [((*arr*, *Base*), (⊥*access*=*Public*, *static*=*True*, *type*=*PrimT Boolean*.⊥)),  
 ((*vee*, *Base*), (⊥*access*=*Public*, *static*=*False*, *type*=*Iface HasFoo* ⊥))]  
 ⟨proof⟩

**lemma** *fields-Ext* [simp]:  
*DeclConcepts.fields tprg Ext*  
 = [((*vee*, *Ext*), (⊥*access*=*Public*, *static*=*False*, *type*= *PrimT Integer*⊥))]  
 @ *DeclConcepts.fields tprg Base*  
 ⟨proof⟩

**lemmas** *imethds-rec'* = *imethds-rec* [OF - *ws-tprg*]

**lemmas** *methd-rec'* = *methd-rec* [OF - *ws-tprg*]

**lemma** *imethds-HasFoo* [simp]:  
*imethds tprg HasFoo* = *o2s* ∘ *empty*(*foo-sig* ↦ (*HasFoo*, *foo-mhead*))

$\langle \text{proof} \rangle$

**lemma** *methd-tprg-Object* [simp]: *methd tprg Object = empty*  
 $\langle \text{proof} \rangle$

**lemma** *methd-Base* [simp]:  
*methd tprg Base = table-of  $[(\lambda(s,m). (s, \text{Base}, m)) \text{Base-foo}]$*   
 $\langle \text{proof} \rangle$

**lemma** *memberid-Base-foo-simp* [simp]:  
*memberid (mdecl Base-foo) = mid foo-sig*  
 $\langle \text{proof} \rangle$

**lemma** *memberid-Ext-foo-simp* [simp]:  
*memberid (mdecl Ext-foo) = mid foo-sig*  
 $\langle \text{proof} \rangle$

**lemma** *Base-declares-foo*:  
*tprg  $\vdash$  mdecl Base-foo declared-in Base*  
 $\langle \text{proof} \rangle$

**lemma** *foo-sig-not-undeclared-in-Base*:  
 $\neg \text{tprg} \vdash \text{mid foo-sig undeclared-in Base}$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-declares-foo*:  
*tprg  $\vdash$  mdecl Ext-foo declared-in Ext*  
 $\langle \text{proof} \rangle$

**lemma** *foo-sig-not-undeclared-in-Ext*:  
 $\neg \text{tprg} \vdash \text{mid foo-sig undeclared-in Ext}$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-not-inherited-in-Ext*:  
 $\neg \text{tprg} \vdash \text{Ext inherits (Base, mdecl Base-foo)}$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-method-inheritance*:  
*filter-tab  $(\lambda \text{sig } m. \text{tprg} \vdash \text{Ext inherits method sig } m)$*   
*(empty(fst  $((\lambda(s, m). (s, \text{Base}, m)) \text{Base-foo}) \mapsto$*   
*snd  $((\lambda(s, m). (s, \text{Base}, m)) \text{Base-foo}))$*   
*= empty*  
 $\langle \text{proof} \rangle$

**lemma** *methd-Ext* [simp]: *methd tprg Ext =*  
*table-of  $[(\lambda(s,m). (s, \text{Ext}, m)) \text{Ext-foo}]$*   
 $\langle \text{proof} \rangle$

**accessibility****lemma** *classesDefined*:
$$\llbracket \text{class } \text{tprg } C = \text{Some } c; C \neq \text{Object} \rrbracket \implies \exists \text{ sc. class } \text{tprg } (\text{super } c) = \text{Some } \text{sc}$$

*<proof>*

**lemma** *superclassesBase* [simp]: *superclasses* *tprg* *Base* = { *Object* }*<proof>***lemma** *superclassesExt* [simp]: *superclasses* *tprg* *Ext* = { *Base*, *Object* }*<proof>***lemma** *superclassesMain* [simp]: *superclasses* *tprg* *Main* = { *Object* }*<proof>***lemma** *HasFoo-accessible* [simp]: *tprg* ⊢ ( *Iface* *HasFoo* ) *accessible-in* *P**<proof>***lemma** *HasFoo-is-acc-iface* [simp]: *is-acc-iface* *tprg* *P* *HasFoo**<proof>***lemma** *HasFoo-is-acc-type* [simp]: *is-acc-type* *tprg* *P* ( *Iface* *HasFoo* )*<proof>***lemma** *Base-accessible* [simp]: *tprg* ⊢ ( *Class* *Base* ) *accessible-in* *P**<proof>***lemma** *Base-is-acc-class* [simp]: *is-acc-class* *tprg* *P* *Base**<proof>***lemma** *Base-is-acc-type* [simp]: *is-acc-type* *tprg* *P* ( *Class* *Base* )*<proof>***lemma** *Ext-accessible* [simp]: *tprg* ⊢ ( *Class* *Ext* ) *accessible-in* *P**<proof>***lemma** *Ext-is-acc-class* [simp]: *is-acc-class* *tprg* *P* *Ext**<proof>***lemma** *Ext-is-acc-type* [simp]: *is-acc-type* *tprg* *P* ( *Class* *Ext* )*<proof>***lemma** *accmethd-tprg-Object* [simp]: *accmethd* *tprg* *S* *Object* = *empty**<proof>*

**lemma** *snd-special-simp*:  $\text{snd } ((\lambda(s, m). (s, a, m)) x) = (a, \text{snd } x)$   
 $\langle \text{proof} \rangle$

**lemma** *fst-special-simp*:  $\text{fst } ((\lambda(s, m). (s, a, m)) x) = \text{fst } x$   
 $\langle \text{proof} \rangle$

**lemma** *foo-sig-undeclared-in-Object*:  
 $\text{tpg} \vdash \text{mid } \text{foo-sig undeclared-in Object}$   
 $\langle \text{proof} \rangle$

**lemma** *unique-sig-Base-foo*:  
 $\text{tpg} \vdash \text{mdecl } (\text{sig}, \text{snd Base-foo}) \text{ declared-in Base} \implies \text{sig} = \text{foo-sig}$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-no-override*:  
 $\text{tpg}, \text{sig} \vdash (\text{Base}, (\text{snd Base-foo})) \text{ overrides old} \implies P$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-no-stat-override*:  
 $\text{tpg}, \text{sig} \vdash (\text{Base}, (\text{snd Base-foo})) \text{ overrides}_S \text{ old} \implies P$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-no-hide*:  
 $\text{tpg}, \text{sig} \vdash (\text{Base}, (\text{snd Base-foo})) \text{ hides old} \implies P$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-no-hide*:  
 $\text{tpg}, \text{sig} \vdash (\text{Ext}, (\text{snd Ext-foo})) \text{ hides old} \implies P$   
 $\langle \text{proof} \rangle$

**lemma** *unique-sig-Ext-foo*:  
 $\text{tpg} \vdash \text{mdecl } (\text{sig}, \text{snd Ext-foo}) \text{ declared-in Ext} \implies \text{sig} = \text{foo-sig}$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-override*:  
 $\text{tpg}, \text{sig} \vdash (\text{Ext}, (\text{snd Ext-foo})) \text{ overrides old}$   
 $\implies \text{old} = (\text{Base}, (\text{snd Base-foo}))$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-stat-override*:  
 $\text{tpg}, \text{sig} \vdash (\text{Ext}, (\text{snd Ext-foo})) \text{ overrides}_S \text{ old}$   
 $\implies \text{old} = (\text{Base}, (\text{snd Base-foo}))$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-member-of-Base*:  
 $\text{tpg} \vdash (\text{Base}, \text{mdecl Base-foo}) \text{ member-of Base}$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-member-in-Base*:  
 $\text{tprg} \vdash (\text{Base}, \text{mdecl } \text{Base-foo}) \text{ member-in } \text{Base}$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-member-of-Ext*:  
 $\text{tprg} \vdash (\text{Ext}, \text{mdecl } \text{Ext-foo}) \text{ member-of } \text{Ext}$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-member-in-Ext*:  
 $\text{tprg} \vdash (\text{Ext}, \text{mdecl } \text{Ext-foo}) \text{ member-in } \text{Ext}$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-permits-acc*:  
 $\text{tprg} \vdash (\text{Base}, \text{mdecl } \text{Base-foo}) \text{ in } \text{Base} \text{ permits-acc-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-accessible [simp]*:  
 $\text{tprg} \vdash (\text{Base}, \text{mdecl } \text{Base-foo}) \text{ of } \text{Base} \text{ accessible-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *Base-foo-dyn-accessible [simp]*:  
 $\text{tprg} \vdash (\text{Base}, \text{mdecl } \text{Base-foo}) \text{ in } \text{Base} \text{ dyn-accessible-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *accmethd-Base [simp]*:  
 $\text{accmethd } \text{tprg } S \text{ Base} = \text{methd } \text{tprg } \text{Base}$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-permits-acc*:  
 $\text{tprg} \vdash (\text{Ext}, \text{mdecl } \text{Ext-foo}) \text{ in } \text{Ext} \text{ permits-acc-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-accessible [simp]*:  
 $\text{tprg} \vdash (\text{Ext}, \text{mdecl } \text{Ext-foo}) \text{ of } \text{Ext} \text{ accessible-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-dyn-accessible [simp]*:  
 $\text{tprg} \vdash (\text{Ext}, \text{mdecl } \text{Ext-foo}) \text{ in } \text{Ext} \text{ dyn-accessible-from } S$   
 $\langle \text{proof} \rangle$

**lemma** *Ext-foo-overrides-Base-foo*:  
 $\text{tprg} \vdash (\text{Ext}, \text{Ext-foo}) \text{ overrides } (\text{Base}, \text{Base-foo})$   
 $\langle \text{proof} \rangle$

**lemma** *accmethd-Ext [simp]*:  
 $\text{accmethd } \text{tprg } S \text{ Ext} = \text{methd } \text{tprg } \text{Ext}$



$\langle \text{proof} \rangle$

**lemma** *cls-Ext*: *class tprg Ext = Some ExtCl*

$\langle \text{proof} \rangle$

**lemma** *dynmethd-Ext-foo*:

*dynmethd tprg Base Ext* ( $\text{!name} = \text{foo}, \text{parTs} = [\text{Class Base}]$ )  
 $= \text{Some } (\text{Ext}, \text{snd Ext-foo})$

$\langle \text{proof} \rangle$

**lemma** *Base-fields-accessible[simp]*:

*accfield tprg S Base*  
 $= \text{table-of}((\text{map } (\lambda((n,d),f).(n,(d,f)))) (\text{DeclConcepts.fields tprg Base}))$   
 $\langle \text{proof} \rangle$

**lemma** *arr-member-of-Base*:

*tprg*  $\vdash$  (*Base*, *fdecl* (*arr*,  
 $\text{!access} = \text{Public}, \text{static} = \text{True}, \text{type} = \text{PrimT Boolean.[]}$ )))  
*member-of Base*  
 $\langle \text{proof} \rangle$

**lemma** *arr-member-in-Base*:

*tprg*  $\vdash$  (*Base*, *fdecl* (*arr*,  
 $\text{!access} = \text{Public}, \text{static} = \text{True}, \text{type} = \text{PrimT Boolean.[]}$ )))  
*member-in Base*  
 $\langle \text{proof} \rangle$

**lemma** *arr-member-of-Ext*:

*tprg*  $\vdash$  (*Base*, *fdecl* (*arr*,  
 $\text{!access} = \text{Public}, \text{static} = \text{True}, \text{type} = \text{PrimT Boolean.[]}$ )))  
*member-of Ext*  
 $\langle \text{proof} \rangle$

**lemma** *arr-member-in-Ext*:

*tprg*  $\vdash$  (*Base*, *fdecl* (*arr*,  
 $\text{!access} = \text{Public}, \text{static} = \text{True}, \text{type} = \text{PrimT Boolean.[]}$ )))  
*member-in Ext*  
 $\langle \text{proof} \rangle$

**lemma** *Ext-fields-accessible[simp]*:

*accfield tprg S Ext*  
 $= \text{table-of}((\text{map } (\lambda((n,d),f).(n,(d,f)))) (\text{DeclConcepts.fields tprg Ext}))$   
 $\langle \text{proof} \rangle$

**lemma** *arr-Base-dyn-accessible [simp]*:

*tprg*  $\vdash$  (*Base*, *fdecl* (*arr*, ( $\text{!access} = \text{Public}, \text{static} = \text{True}, \text{type} = \text{PrimT Boolean.[]}$ )))  
*in Base dyn-accessible-from S*  
 $\langle \text{proof} \rangle$

**lemma** *arr-Ext-dyn-accessible* [simp]:  
*tprg*-(*Base*, *fdecl* (*arr*, ( $\lambda$  *access*=*Public*, *static*=*True* , *type*=*PrimT Boolean*. $\lambda$ ()))  
*in Ext dyn-accessible-from S*  
 <proof>

**lemma** *array-of-PrimT-acc* [simp]:  
*is-acc-type tprg java-lang (PrimT t. $\lambda$ )*  
 <proof>

**lemma** *PrimT-acc* [simp]:  
*is-acc-type tprg java-lang (PrimT t)*  
 <proof>

**lemma** *Object-acc* [simp]:  
*is-acc-class tprg java-lang Object*  
 <proof>

### well-formedness

**lemma** *wf-HasFoo*: *wf-idecl tprg (HasFoo, HasFooInt)*  
 <proof>

**declare** *member-is-static-simp* [simp]  
**declare** *wt.Skip* [rule del] *wt.Init* [rule del]  
 <ML>  
**lemmas** *wtIs* = *wt-Call wt-Super wt-FVar wt-StatRef wt-intros*  
**lemmas** *daIs* = *assigned.select-convs da-Skip da-NewC da-Lit da-Super da.intros*  
  
**lemmas** *Base-foo-defs* = *Base-foo-def foo-sig-def foo-mhead-def*  
**lemmas** *Ext-foo-defs* = *Ext-foo-def foo-sig-def*

**lemma** *wf-Base-foo*: *wf-mdecl tprg Base Base-foo*  
 <proof>

**lemma** *wf-Ext-foo*: *wf-mdecl tprg Ext Ext-foo*  
 <proof>

**declare** *mhead-resTy-simp* [simp add]  
**declare** *member-is-static-simp* [simp add]

**lemma** *wf-BaseC*: *wf-cdecl tprg (Base, BaseCl)*  
 <proof>

**lemma** *wf-ExtC*: *wf-cdecl tprg (Ext, ExtCl)*  
 <proof>

**lemma** *wf-MainC*: *wf-cdecl tprg (Main,MainCl)*  
*<proof>*

**lemma** *wf-idecl-all*: *p=tprg  $\implies$  Ball (set Ifaces) (wf-idecl p)*  
*<proof>*

**lemma** *wf-cdecl-all-standard-classes*:  
*Ball (set standard-classes) (wf-cdecl tprg)*  
*<proof>*

**lemma** *wf-cdecl-all*: *p=tprg  $\implies$  Ball (set Classes) (wf-cdecl p)*  
*<proof>*

**theorem** *wf-tprg*: *wf-prog tprg*  
*<proof>*

### max spec

**lemma** *appl-methds-Base-foo*:  
*appl-methds tprg S (ClassT Base) ( $\downarrow$ name=foo, parTs=[NT]) =*  
*{((ClassT Base, ( $\downarrow$ access=Public,static=False,pars=[z],resT=Class Base))*  
*,[Class Base])}*  
*<proof>*

**lemma** *max-spec-Base-foo*: *max-spec tprg S (ClassT Base) ( $\downarrow$ name=foo,parTs=[NT]) =*  
*{((ClassT Base, ( $\downarrow$ access=Public,static=False,pars=[z],resT=Class Base))*  
*, [Class Base])}*  
*<proof>*

### well-typedness

**lemma** *wt-test*: *( $\downarrow$ prg=tprg,cls=Main,lcl=empty(VName  $e \mapsto$  Class Base))  $\vdash$  test ?pTs:: $\checkmark$*   
*<proof>*

### definite assignment

**lemma** *da-test*: *( $\downarrow$ prg=tprg,cls=Main,lcl=empty(VName  $e \mapsto$  Class Base))*  
 *$\vdash \{ \} \gg \langle \text{test ?pTs} \rangle \gg (\downarrow \text{nrn} = \{ \text{VName } e \}, \text{brk} = \lambda l. \text{UNIV})$*   
*<proof>*

### execution

**lemma** *alloc-one*:  *$\bigwedge a \text{ obj. } \llbracket \text{the (new-Addr } h) = a; \text{atleast-free } h \text{ (Suc } n) \rrbracket \implies$*   
*new-Addr } h = Some a  $\wedge$  atleast-free (h( $a \mapsto$  obj)) n*  
*<proof>*

**declare** *fvar-def2* [simp] *avar-def2* [simp] *init-lvars-def2* [simp]  
**declare** *init-obj-def* [simp] *var-tys-def* [simp] *fields-table-def* [simp]  
**declare** *BaseCl-def* [simp] *ExtCl-def* [simp] *Ext-foo-def* [simp]  
*Base-foo-defs* [simp]

*<ML>*

**lemmas** *eval-Is = eval-Init eval-StatRef AbruptIs eval-intros*

### consts

*a* :: *loc*  
*b* :: *loc*  
*c* :: *loc*

**abbreviation** *one* == *Suc 0*  
**abbreviation** *two* == *Suc one*  
**abbreviation** *tree* == *Suc two*  
**abbreviation** *four* == *Suc tree*

### syntax

*obj-a* :: *obj*  
*obj-b* :: *obj*  
*obj-c* :: *obj*  
*arr-N* :: (*vn*, *val*) *table*  
*arr-a* :: (*vn*, *val*) *table*  
*globs1* :: *globs*  
*globs2* :: *globs*  
*globs3* :: *globs*  
*globs8* :: *globs*  
*locs3* :: *locals*  
*locs4* :: *locals*  
*locs8* :: *locals*  
*s0* :: *state*  
*s0'* :: *state*  
*s9'* :: *state*  
*s1* :: *state*  
*s1'* :: *state*  
*s2* :: *state*  
*s2'* :: *state*  
*s3* :: *state*  
*s3'* :: *state*  
*s4* :: *state*  
*s4'* :: *state*  
*s6'* :: *state*  
*s7'* :: *state*  
*s8* :: *state*  
*s8'* :: *state*

### translations

*obj-a* <= ( $\text{tag} = \text{Arr } (\text{PrimT Boolean}) \ (\text{CONST two})$   
 $\text{, values} = \text{CONST empty}(\text{Inr } 0 \mapsto \text{Bool False})(\text{Inr } (\text{CONST one}) \mapsto \text{Bool False})$ )  
*obj-b* <= ( $\text{tag} = \text{CInst } (\text{CONST Ext})$   
 $\text{, values} = (\text{CONST empty}(\text{Inl } (\text{CONST vee}, \text{CONST Base}) \mapsto \text{Null } \quad)$   
 $\quad (\text{Inl } (\text{CONST vee}, \text{CONST Ext } ) \mapsto \text{Intg } 0))$ )  
*obj-c* == ( $\text{tag} = \text{CInst } (\text{SXcpt NullPointer}), \text{values} = \text{CONST empty}$ )  
*arr-N* ==  $\text{CONST empty}(\text{Inl } (\text{CONST arr}, \text{CONST Base}) \mapsto \text{Null})$   
*arr-a* ==  $\text{CONST empty}(\text{Inl } (\text{CONST arr}, \text{CONST Base}) \mapsto \text{Addr } a)$   
*globs1* ==  $\text{CONST empty}(\text{Inr } (\text{CONST Ext}) \mapsto (\text{tag} = \text{arbitrary}, \text{values} = \text{CONST empty}))$   
 $\quad (\text{Inr } (\text{CONST Base}) \mapsto (\text{tag} = \text{arbitrary}, \text{values} = \text{arr-N}))$   
 $\quad (\text{Inr } \text{Object} \mapsto (\text{tag} = \text{arbitrary}, \text{values} = \text{CONST empty}))$   
*globs2* ==  $\text{CONST empty}(\text{Inr } (\text{CONST Ext}) \mapsto (\text{tag} = \text{arbitrary}, \text{values} = \text{CONST empty}))$   
 $\quad (\text{Inr } \text{Object} \mapsto (\text{tag} = \text{arbitrary}, \text{values} = \text{CONST empty}))$   
 $\quad (\text{Inl } a \mapsto \text{obj-a})$   
 $\quad (\text{Inr } (\text{CONST Base}) \mapsto (\text{tag} = \text{arbitrary}, \text{values} = \text{arr-a}))$   
*globs3* ==  $\text{globs2}(\text{Inl } b \mapsto \text{obj-b})$   
*globs8* ==  $\text{globs3}(\text{Inl } c \mapsto \text{obj-c})$   
*locs3* ==  $\text{CONST empty}(\text{VName } (\text{CONST } e) \mapsto \text{Addr } b)$   
*locs4* ==  $\text{CONST empty}(\text{VName } (\text{CONST } z) \mapsto \text{Null})(\text{Inr } () \mapsto \text{Addr } b)$   
*locs8* ==  $\text{locs3}(\text{VName } (\text{CONST } z) \mapsto \text{Addr } c)$

```

s0 ==      st (CONST empty) (CONST empty)
s0' == Norm s0
s1 ==      st globs1 (CONST empty)
s1' == Norm s1
s2 ==      st globs2 (CONST empty)
s2' == Norm s2
s3 ==      st globs3 locs3
s3' == Norm s3
s4 ==      st globs3 locs4
s4' == Norm s4
s6' == (Some (Xcpt (Std NullPointer)), s4)
s7' == (Some (Xcpt (Std NullPointer)), s3)
s8 ==      st globs8 locs8
s8' == Norm s8
s9' == (Some (Xcpt (Std IndOutBound)), s8)

```

**declare** *Pair-eq* [*simp del*]

**lemma** *exec-test*:

```

[[the (new-Addr (heap s1)) = a;
  the (new-Addr (heap ?s2)) = b;
  the (new-Addr (heap ?s3)) = c]] ==>
  atleast-free (heap s0) four ==>
  tprg-s0'-test [Class Base] -> ?s9'

```

*<proof>*

**declare** *Pair-eq* [*simp*]

**end**



## Chapter 17

## Conform

#### 44 Conformance notions for the type soundness proof for Java

**theory** *Conform* **imports** *State* **begin**

design issues:

- lconf allows for (arbitrary) inaccessible values
- "conforms" does not directly imply that the dynamic types of all objects on the heap are indeed existing classes. Yet this can be inferred for all referenced objs.

**types**  $env' = prog \times (lname, ty) \text{ table}$

**extension of global store**

**constdefs**

$$\begin{aligned} gext &:: st \Rightarrow st \Rightarrow bool & (-\leq|- & [71,71] \quad 70) \\ s\leq|s' &\equiv \forall r. \forall obj \in globs \ s \ r: \exists obj' \in globs \ s' \ r: tag \ obj' = tag \ obj \end{aligned}$$

For the the proof of type soundness we will need the property that during execution, objects are not lost and moreover retain the values of their tags. So the object store grows conservatively. Note that if we considered garbage collection, we would have to restrict this property to accessible objects.

**lemma** *gext-objD*:

$$\begin{aligned} &\llbracket s\leq|s'; globs \ s \ r = Some \ obj \rrbracket \\ &\implies \exists obj'. globs \ s' \ r = Some \ obj' \wedge tag \ obj' = tag \ obj \\ &\langle proof \rangle \end{aligned}$$

**lemma** *rev-gext-objD*:

$$\begin{aligned} &\llbracket globs \ s \ r = Some \ obj; s\leq|s' \rrbracket \\ &\implies \exists obj'. globs \ s' \ r = Some \ obj' \wedge tag \ obj' = tag \ obj \\ &\langle proof \rangle \end{aligned}$$

**lemma** *init-class-obj-inited*:

$$\begin{aligned} &init-class-obj \ G \ C \ s1\leq|s2 \implies init-ed \ C \ (globs \ s2) \\ &\langle proof \rangle \end{aligned}$$

**lemma** *gext-refl* [*intro!*, *simp*]:  $s\leq|s$

$\langle proof \rangle$

**lemma** *gext-gupd* [*simp*, *elim!*]:  $\bigwedge s. globs \ s \ r = None \implies s\leq|gupd(r \mapsto x)s$

$\langle proof \rangle$

**lemma** *gext-new* [*simp*, *elim!*]:  $\bigwedge s. globs \ s \ r = None \implies s\leq|init-obj \ G \ oi \ r \ s$

$\langle proof \rangle$

**lemma** *gext-trans* [*elim*]:  $\bigwedge X. \llbracket s\leq|s'; s'\leq|s'' \rrbracket \implies s\leq|s''$

$\langle proof \rangle$

**lemma** *gext-upd-gobj* [*intro!*]:  $s\leq|upd-gobj \ r \ n \ v \ s$

$\langle proof \rangle$



**lemma** *gext-cong1* [simp]:  $\text{set-locals } l \ s1 \leq |s2 = s1 \leq |s2$   
 $\langle \text{proof} \rangle$

**lemma** *gext-cong2* [simp]:  $s1 \leq | \text{set-locals } l \ s2 = s1 \leq |s2$   
 $\langle \text{proof} \rangle$

**lemma** *gext-lupd1* [simp]:  $\text{lupd}(vn \mapsto v) s1 \leq |s2 = s1 \leq |s2$   
 $\langle \text{proof} \rangle$

**lemma** *gext-lupd2* [simp]:  $s1 \leq | \text{lupd}(vn \mapsto v) s2 = s1 \leq |s2$   
 $\langle \text{proof} \rangle$

**lemma** *inited-gext*:  $\llbracket \text{inited } C \ (\text{globs } s); s \leq |s \rrbracket \implies \text{inited } C \ (\text{globs } s')$   
 $\langle \text{proof} \rangle$

## value conformance

### constdefs

$\text{conf} :: \text{prog} \Rightarrow \text{st} \Rightarrow \text{val} \Rightarrow \text{ty} \Rightarrow \text{bool} \quad (\_, \vdash :: \preceq - \quad [71, 71, 71, 71] \ 70)$   
 $G, s \vdash v :: \preceq T \equiv \exists T' \in \text{typeof} \ (\lambda a. \text{option-map obj-ty} \ (\text{heap } s \ a)) \ v : G \vdash T' \preceq T$

**lemma** *conf-cong* [simp]:  $G, \text{set-locals } l \ s \vdash v :: \preceq T = G, s \vdash v :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *conf-lupd* [simp]:  $G, \text{lupd}(vn \mapsto va) s \vdash v :: \preceq T = G, s \vdash v :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *conf-PrimT* [simp]:  $\forall dt. \text{typeof } dt \ v = \text{Some} \ (\text{PrimT } t) \implies G, s \vdash v :: \preceq \text{PrimT } t$   
 $\langle \text{proof} \rangle$

**lemma** *conf-Boolean*:  $G, s \vdash v :: \preceq \text{PrimT Boolean} \implies \exists b. v = \text{Bool } b$   
 $\langle \text{proof} \rangle$

**lemma** *conf-litval* [rule-format (no-asm)]:  
 $\text{typeof} \ (\lambda a. \text{None}) \ v = \text{Some } T \longrightarrow G, s \vdash v :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *conf-Null* [simp]:  $G, s \vdash \text{Null} :: \preceq T = G \vdash NT \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *conf-Addr*:  
 $G, s \vdash \text{Addr } a :: \preceq T = (\exists \text{obj}. \text{heap } s \ a = \text{Some obj} \wedge G \vdash \text{obj-ty obj} \preceq T)$   
 $\langle \text{proof} \rangle$

**lemma** *conf-AddrI*:  $\llbracket \text{heap } s \ a = \text{Some } \text{obj}; G \vdash \text{obj-ty } \text{obj} \preceq T \rrbracket \implies G, s \vdash \text{Addr } a :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *defval-conf* [*rule-format* (*no-asm*), *elim*]:  
 $\text{is-type } G \ T \longrightarrow G, s \vdash \text{default-val } T :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *conf-widen* [*rule-format* (*no-asm*), *elim*]:  
 $G \vdash T \preceq T' \implies G, s \vdash x :: \preceq T \longrightarrow \text{ws-prog } G \longrightarrow G, s \vdash x :: \preceq T'$   
 $\langle \text{proof} \rangle$

**lemma** *conf-gext* [*rule-format* (*no-asm*), *elim*]:  
 $G, s \vdash v :: \preceq T \longrightarrow s \leq |s' \longrightarrow G, s \upharpoonright v :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *conf-list-widen* [*rule-format* (*no-asm*)]:  
 $\text{ws-prog } G \implies$   
 $\forall Ts \ Ts'. \text{list-all2 } (\text{conf } G \ s) \ \text{vs } Ts$   
 $\longrightarrow G \vdash Ts[\preceq] \ Ts' \longrightarrow \text{list-all2 } (\text{conf } G \ s) \ \text{vs } Ts'$   
 $\langle \text{proof} \rangle$

**lemma** *conf-RefTD* [*rule-format* (*no-asm*)]:  
 $G, s \vdash a' :: \preceq \text{RefT } T$   
 $\longrightarrow a' = \text{Null} \vee (\exists a \ \text{obj } T'. a' = \text{Addr } a \wedge \text{heap } s \ a = \text{Some } \text{obj} \wedge$   
 $\text{obj-ty } \text{obj} = T' \wedge G \vdash T' \preceq \text{RefT } T)$   
 $\langle \text{proof} \rangle$

## value list conformance

### constdefs

$\text{lconf} :: \text{prog} \Rightarrow \text{st} \Rightarrow ('a, \text{val}) \text{ table} \Rightarrow ('a, \text{ty}) \text{ table} \Rightarrow \text{bool}$   
 $(-, \vdash -[\preceq]) - [71, 71, 71, 71] \ 70)$   
 $G, s \vdash \text{vs}[\preceq] \ Ts \equiv \forall n. \forall T \in Ts \ n: \exists v \in \text{vs } n: G, s \vdash v :: \preceq T$

**lemma** *lconfD*:  $\llbracket G, s \vdash \text{vs}[\preceq] \ Ts; Ts \ n = \text{Some } T \rrbracket \implies G, s \vdash (\text{the } (\text{vs } n)) :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-cong* [*simp*]:  $\bigwedge s. G, \text{set-locals } x \ s \vdash l[\preceq] \ L = G, s \vdash l[\preceq] \ L$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-lupd* [*simp*]:  $G, \text{lupd}(vn \mapsto v) \ s \vdash l[\preceq] \ L = G, s \vdash l[\preceq] \ L$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-new*:  $\llbracket L \ vn = \text{None}; G, s \vdash l[\preceq] \ L \rrbracket \implies G, s \vdash l(vn \mapsto v)[\preceq] \ L$

$\langle \text{proof} \rangle$

**lemma** *lconf-upd*:  $\llbracket G, s \vdash l[::\preceq] L; G, s \vdash v :: \preceq T; L \text{ vn} = \text{Some } T \rrbracket \implies$   
 $G, s \vdash l(vn \mapsto v)[::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-ext*:  $\llbracket G, s \vdash l[::\preceq] L; G, s \vdash v :: \preceq T \rrbracket \implies G, s \vdash l(vn \mapsto v)[::\preceq] L(vn \mapsto T)$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-map-sum* [simp]:  
 $G, s \vdash l1 (+) l2[::\preceq] L1 (+) L2 = (G, s \vdash l1[::\preceq] L1 \wedge G, s \vdash l2[::\preceq] L2)$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-ext-list* [rule-format (no-asm)]:  
 $\bigwedge X. \llbracket G, s \vdash l[::\preceq] L \rrbracket \implies$   
 $\forall vs \ Ts. \text{distinct } vns \longrightarrow \text{length } Ts = \text{length } vns$   
 $\longrightarrow \text{list-all2 } (\text{conf } G \ s) \ vs \ Ts \longrightarrow G, s \vdash l(vns \mapsto vs)[::\preceq] L(vns \mapsto Ts)$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-deallocL*:  $\llbracket G, s \vdash l[::\preceq] L(vn \mapsto T); L \text{ vn} = \text{None} \rrbracket \implies G, s \vdash l[::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-gext* [elim]:  $\llbracket G, s \vdash l[::\preceq] L; s \leq |s| \rrbracket \implies G, s \vdash l[::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-empty* [simp, intro!]:  $G, s \vdash vs[::\preceq] \text{empty}$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-init-vals* [intro!]:  
 $\forall n. \forall T \in fs \ n : \text{is-type } G \ T \implies G, s \vdash \text{init-vals } fs[::\preceq] fs$   
 $\langle \text{proof} \rangle$

## weak value list conformance

Only if the value is defined it has to conform to its type. This is the contribution of the definite assignment analysis to the notion of conformance. The definite assignment analysis ensures that the program only attempts to access local variables that actually have a defined value in the state. So conformance must only ensure that the defined values are of the right type, and not also that the value is defined.

## constdefs

$wlconf :: \text{prog} \Rightarrow st \Rightarrow ('a, \text{val}) \text{ table} \Rightarrow ('a, \text{ty}) \text{ table} \Rightarrow \text{bool}$   
 $(\neg, \vdash, \vdash[::\preceq]) - [71, 71, 71, 71] \ 70)$   
 $G, s \vdash vs[\sim::\preceq] Ts \equiv \forall n. \forall T \in Ts \ n : \forall v \in vs \ n : G, s \vdash v :: \preceq T$

**lemma** *wlconfD*:  $\llbracket G, s \vdash vs[\sim::\preceq] Ts; Ts \ n = \text{Some } T; vs \ n = \text{Some } v \rrbracket \implies G, s \vdash v :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-cong* [simp]:  $\bigwedge s. G, \text{set-locals } x \vdash l[\sim::\preceq] L = G, s \vdash l[\sim::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-lupd* [simp]:  $G, \text{lupd}(vn \mapsto v) \vdash l[\sim::\preceq] L = G, s \vdash l[\sim::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-upd*:  $\llbracket G, s \vdash l[\sim::\preceq] L; G, s \vdash v::\preceq T; L \text{ } vn = \text{Some } T \rrbracket \implies$   
 $G, s \vdash l(vn \mapsto v)[\sim::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-ext*:  $\llbracket G, s \vdash l[\sim::\preceq] L; G, s \vdash v::\preceq T \rrbracket \implies G, s \vdash l(vn \mapsto v)[\sim::\preceq] L(vn \mapsto T)$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-map-sum* [simp]:  
 $G, s \vdash l1 (+) l2[\sim::\preceq] L1 (+) L2 = (G, s \vdash l1[\sim::\preceq] L1 \wedge G, s \vdash l2[\sim::\preceq] L2)$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-ext-list* [rule-format (no-asm)]:  
 $\bigwedge X. \llbracket G, s \vdash l[\sim::\preceq] L \rrbracket \implies$   
 $\forall vs \ Ts. \text{distinct } vns \longrightarrow \text{length } Ts = \text{length } vns$   
 $\longrightarrow \text{list-all2 } (\text{conf } G \ s) \ vs \ Ts \longrightarrow G, s \vdash l(vns[\mapsto] vs)[\sim::\preceq] L(vns[\mapsto] Ts)$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-deallocL*:  $\llbracket G, s \vdash l[\sim::\preceq] L(vn \mapsto T); L \text{ } vn = \text{None} \rrbracket \implies G, s \vdash l[\sim::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-geat* [elim]:  $\llbracket G, s \vdash l[\sim::\preceq] L; s \leq s' \rrbracket \implies G, s' \vdash l[\sim::\preceq] L$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-empty* [simp, intro!]:  $G, s \vdash vs[\sim::\preceq] \text{empty}$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-empty-vals*:  $G, s \vdash \text{empty}[\sim::\preceq] ts$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-init-vals* [intro!]:  
 $\forall n. \forall T \in fs \ n:\text{is-type } G \ T \implies G, s \vdash \text{init-vals } fs[\sim::\preceq] fs$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-wlconf*:  
 $G, s \vdash l[\sim::\preceq] L \implies G, s \vdash l[\sim::\preceq] L$   
 $\langle \text{proof} \rangle$

**object conformance****constdefs**

$$\begin{aligned}
oconf &:: prog \Rightarrow st \Rightarrow obj \Rightarrow oref \Rightarrow bool \quad (-, \vdash :: \preceq \sqrt{-} \quad [71, 71, 71, 71] \quad 70) \\
G, st \vdash obj &:: \preceq \sqrt{r} \equiv G, st \vdash values \quad obj[:: \preceq] var-tys \quad G \quad (tag \quad obj) \quad r \wedge \\
&\quad (case \quad r \quad of \\
&\quad \quad Heap \quad a \Rightarrow is-type \quad G \quad (obj-ty \quad obj) \\
&\quad \quad | \quad Stat \quad C \Rightarrow True)
\end{aligned}$$

**lemma** *oconf-is-type*:  $G, st \vdash obj :: \preceq \sqrt{Heap \quad a} \implies is-type \quad G \quad (obj-ty \quad obj)$   
 $\langle proof \rangle$

**lemma** *oconf-lconf*:  $G, st \vdash obj :: \preceq \sqrt{r} \implies G, st \vdash values \quad obj[:: \preceq] var-tys \quad G \quad (tag \quad obj) \quad r$   
 $\langle proof \rangle$

**lemma** *oconf-cong [simp]*:  $G, set-locals \quad l \quad st \vdash obj :: \preceq \sqrt{r} = G, st \vdash obj :: \preceq \sqrt{r}$   
 $\langle proof \rangle$

**lemma** *oconf-init-obj-lemma*:

$$\begin{aligned}
&\llbracket \bigwedge C \quad c. \quad class \quad G \quad C = Some \quad c \implies unique \quad (DeclConcepts.fields \quad G \quad C); \\
&\quad \bigwedge C \quad c \quad fld. \quad \llbracket class \quad G \quad C = Some \quad c; \\
&\quad \quad table-of \quad (DeclConcepts.fields \quad G \quad C) \quad f = Some \quad fld \rrbracket \\
&\quad \implies is-type \quad G \quad (type \quad fld); \\
&\quad (case \quad r \quad of \\
&\quad \quad Heap \quad a \Rightarrow is-type \quad G \quad (obj-ty \quad obj) \\
&\quad \quad | \quad Stat \quad C \Rightarrow is-class \quad G \quad C) \\
&\rrbracket \implies G, st \vdash obj \quad (\llbracket values := init-vals \quad (var-tys \quad G \quad (tag \quad obj) \quad r) \rrbracket) :: \preceq \sqrt{r} \\
&\langle proof \rangle
\end{aligned}$$
**state conformance****constdefs**

$$\begin{aligned}
conforms &:: state \Rightarrow env' \Rightarrow bool \quad ( \quad - :: \preceq - \quad [71, 71] \quad 70) \\
xs :: \preceq E &\equiv let \quad (G, L) = E; \quad s = snd \quad xs; \quad l = locals \quad s \quad in \\
&\quad (\forall r. \quad \forall obj \in globs \quad s \quad r: \quad G, st \vdash obj \quad :: \preceq \sqrt{r}) \wedge \\
&\quad \quad G, st \vdash l \quad [\sim :: \preceq] L \wedge \\
&\quad (\forall a. \quad fst \quad xs = Some(Xcpt \quad (Loc \quad a)) \longrightarrow G, st \vdash Addr \quad a :: \preceq \quad Class \quad (SXcpt \quad Throwable)) \wedge \\
&\quad (fst \quad xs = Some(Jump \quad Ret) \longrightarrow l \quad Result \neq None)
\end{aligned}$$
**conforms**

**lemma** *conforms-globsD*:

$$\llbracket (x, s) :: \preceq (G, L); \quad globs \quad s \quad r = Some \quad obj \rrbracket \implies G, st \vdash obj :: \preceq \sqrt{r} \\
\langle proof \rangle$$

**lemma** *conforms-localD*:  $(x, s) :: \preceq (G, L) \implies G, st \vdash locals \quad s [\sim :: \preceq] L$   
 $\langle proof \rangle$

**lemma** *conforms-XcptLocD*:  $\llbracket (x, s) :: \preceq (G, L); \quad x = Some \quad (Xcpt \quad (Loc \quad a)) \rrbracket \implies$   
 $G, st \vdash Addr \quad a :: \preceq \quad Class \quad (SXcpt \quad Throwable)$   
 $\langle proof \rangle$

**lemma** *conforms-RetD*:  $\llbracket (x, s) :: \preceq (G, L); x = \text{Some } (\text{Jump Ret}) \rrbracket \implies$   
 $(\text{locals } s) \text{ Result} \neq \text{None}$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-RefTD*:  
 $\llbracket G, s \vdash a' :: \preceq \text{RefT } t; a' \neq \text{Null}; (x, s) :: \preceq (G, L) \rrbracket \implies$   
 $\exists a \text{ obj. } a' = \text{Addr } a \wedge \text{globs } s (\text{Inl } a) = \text{Some obj} \wedge$   
 $G \vdash \text{obj-ty obj} \preceq \text{RefT } t \wedge \text{is-type } G (\text{obj-ty obj})$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-Jump* [iff]:  
 $j = \text{Ret} \longrightarrow \text{locals } s \text{ Result} \neq \text{None}$   
 $\implies ((\text{Some } (\text{Jump } j), s) :: \preceq (G, L)) = (\text{Norm } s :: \preceq (G, L))$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-StdXcpt* [iff]:  
 $((\text{Some } (\text{Xcpt } (\text{Std } xn)), s) :: \preceq (G, L)) = (\text{Norm } s :: \preceq (G, L))$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-Err* [iff]:  
 $((\text{Some } (\text{Error } e), s) :: \preceq (G, L)) = (\text{Norm } s :: \preceq (G, L))$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-raise-if* [iff]:  
 $((\text{raise-if } c \text{ } xn \text{ } x, s) :: \preceq (G, L)) = ((x, s) :: \preceq (G, L))$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-error-if* [iff]:  
 $((\text{error-if } c \text{ } err \text{ } x, s) :: \preceq (G, L)) = ((x, s) :: \preceq (G, L))$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-NormI*:  $(x, s) :: \preceq (G, L) \implies \text{Norm } s :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-absorb* [rule-format]:  
 $(a, b) :: \preceq (G, L) \longrightarrow (\text{absorb } j \text{ } a, b) :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *conformsI*:  $\llbracket \forall r. \forall \text{obj} \in \text{globs } s \text{ } r: G, s \vdash \text{obj} :: \preceq \sqrt{r};$   
 $G, s \vdash \text{locals } s [\sim :: \preceq] L;$   
 $\forall a. x = \text{Some } (\text{Xcpt } (\text{Loc } a)) \longrightarrow G, s \vdash \text{Addr } a :: \preceq \text{Class } (\text{SXcpt Throwable});$   
 $x = \text{Some } (\text{Jump Ret}) \longrightarrow \text{locals } s \text{ Result} \neq \text{None} \rrbracket \implies$   
 $(x, s) :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-xconf*:  $\llbracket (x, s) :: \preceq (G, L);$   
 $\forall a. x' = \text{Some } (\text{Xcpt } (\text{Loc } a)) \longrightarrow G, s \vdash \text{Addr } a :: \preceq \text{Class } (\text{SXcpt Throwable});$   
 $\rrbracket$

$x' = \text{Some } (\text{Jump Ret}) \longrightarrow \text{locals } s \text{ Result} \neq \text{None}] \implies$   
 $\langle (x', s) :: \preceq (G, L) \rangle$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-lupd*:

$\llbracket (x, s) :: \preceq (G, L); L \text{ vn} = \text{Some } T; G, s \vdash v :: \preceq T \rrbracket \implies (x, \text{lupd}(vn \mapsto v)s) :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemmas** *conforms-allocL-aux* = *conforms-localD* [THEN *wlconf-ext*]

**lemma** *conforms-allocL*:

$\llbracket (x, s) :: \preceq (G, L); G, s \vdash v :: \preceq T \rrbracket \implies (x, \text{lupd}(vn \mapsto v)s) :: \preceq (G, L(vn \mapsto T))$   
 $\langle \text{proof} \rangle$

**lemmas** *conforms-deallocL-aux* = *conforms-localD* [THEN *wlconf-deallocL*]

**lemma** *conforms-deallocL*:  $\bigwedge s. \llbracket s :: \preceq (G, L(vn \mapsto T)); L \text{ vn} = \text{None} \rrbracket \implies s :: \preceq (G, L)$

$\langle \text{proof} \rangle$

**lemma** *conforms-gext*:  $\llbracket (x, s) :: \preceq (G, L); s \leq | s' \rrbracket;$

$\forall r. \forall \text{obj} \in \text{globs } s' \text{ } r: G, s \vdash \text{obj} :: \preceq \sqrt{r};$

$\text{locals } s' = \text{locals } s \rrbracket \implies (x, s') :: \preceq (G, L)$

$\langle \text{proof} \rangle$

**lemma** *conforms-xgext*:

$\llbracket (x, s) :: \preceq (G, L); (x', s') :: \preceq (G, L); s' \leq | s; \text{dom } (\text{locals } s') \subseteq \text{dom } (\text{locals } s) \rrbracket$

$\implies (x', s) :: \preceq (G, L)$

$\langle \text{proof} \rangle$

**lemma** *conforms-gupd*:  $\bigwedge \text{obj}. \llbracket (x, s) :: \preceq (G, L); G, s \vdash \text{obj} :: \preceq \sqrt{r}; s \leq | \text{gupd}(r \mapsto \text{obj})s \rrbracket$

$\implies (x, \text{gupd}(r \mapsto \text{obj})s) :: \preceq (G, L)$

$\langle \text{proof} \rangle$

**lemma** *conforms-upd-gobj*:  $\llbracket (x, s) :: \preceq (G, L); \text{globs } s \text{ } r = \text{Some } \text{obj};$

$\text{var-tys } G \text{ (tag obj) } r \text{ n} = \text{Some } T; G, s \vdash v :: \preceq T \rrbracket \implies (x, \text{upd-gobj } r \text{ n } v \text{ } s) :: \preceq (G, L)$

$\langle \text{proof} \rangle$

**lemma** *conforms-set-locals*:

$\llbracket (x, s) :: \preceq (G, L); G, s \vdash l [\sim :: \preceq] L; x = \text{Some } (\text{Jump Ret}) \longrightarrow l \text{ Result} \neq \text{None} \rrbracket$

$\implies (x, \text{set-locals } l \text{ } s) :: \preceq (G, L)$

$\langle \text{proof} \rangle$

**lemma** *conforms-locals*:

$\llbracket (a, b) :: \preceq (G, L); L \text{ } x = \text{Some } T; \text{locals } b \text{ } x \neq \text{None} \rrbracket$

$\implies G, b \vdash \text{the } (\text{locals } b \text{ } x) :: \preceq T$

$\langle \text{proof} \rangle$

**lemma** *conforms-return:*

$\bigwedge s'. \llbracket (x, s) :: \preceq (G, L); (x', s') :: \preceq (G, L'); s \leq |s'; x' \neq \text{Some } (\text{Jump Ret}) \rrbracket \implies$   
 $(x', \text{set-locals } (\text{locals } s) s') :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**end**



## Chapter 18

# DefiniteAssignmentCorrect

## 45 Correctness of Definite Assignment

**theory** *DefiniteAssignmentCorrect* **imports** *WellForm Eval begin*

**declare**  $[[simproc\ del:\ wt\text{-}expr\ wt\text{-}var\ wt\text{-}exprs\ wt\text{-}stmt]]$

**lemma** *sxalloc-no-jump*:

**assumes** *sxalloc*:  $G \vdash s0 \text{ --sxalloc--> } s1$  **and**  
*no-jmp*:  $abrupt\ s0 \neq Some\ (Jump\ j)$   
**shows**  $abrupt\ s1 \neq Some\ (Jump\ j)$

$\langle proof \rangle$

**lemma** *sxalloc-no-jump'*:

**assumes** *sxalloc*:  $G \vdash s0 \text{ --sxalloc--> } s1$  **and**  
*jump*:  $abrupt\ s1 = Some\ (Jump\ j)$   
**shows**  $abrupt\ s0 = Some\ (Jump\ j)$

$\langle proof \rangle$

**lemma** *halloc-no-jump*:

**assumes** *halloc*:  $G \vdash s0 \text{ --halloc } oi \succ a \rightarrow s1$  **and**  
*no-jmp*:  $abrupt\ s0 \neq Some\ (Jump\ j)$   
**shows**  $abrupt\ s1 \neq Some\ (Jump\ j)$

$\langle proof \rangle$

**lemma** *halloc-no-jump'*:

**assumes** *halloc*:  $G \vdash s0 \text{ --halloc } oi \succ a \rightarrow s1$  **and**  
*jump*:  $abrupt\ s1 = Some\ (Jump\ j)$   
**shows**  $abrupt\ s0 = Some\ (Jump\ j)$

$\langle proof \rangle$

**lemma** *Body-no-jump*:

**assumes** *eval*:  $G \vdash s0 \text{ --Body } D\ c \text{ --> } v \rightarrow s1$  **and**  
*jump*:  $abrupt\ s0 \neq Some\ (Jump\ j)$   
**shows**  $abrupt\ s1 \neq Some\ (Jump\ j)$

$\langle proof \rangle$

**lemma** *Methd-no-jump*:

**assumes** *eval*:  $G \vdash s0 \text{ --Methd } D\ sig \text{ --> } v \rightarrow s1$  **and**  
*jump*:  $abrupt\ s0 \neq Some\ (Jump\ j)$   
**shows**  $abrupt\ s1 \neq Some\ (Jump\ j)$

$\langle proof \rangle$

**lemma** *jumpNestingOkS-mono*:

**assumes** *jumpNestingOk-l'*:  $jumpNestingOkS\ jmps'\ c$   
**and** *subset*:  $jmps' \subseteq jmps$   
**shows**  $jumpNestingOkS\ jmps\ c$

$\langle proof \rangle$

**corollary** *jumpNestingOk-mono*:

**assumes** *jmpOk*:  $jumpNestingOk\ jmps'\ t$   
**and** *subset*:  $jmps' \subseteq jmps$   
**shows**  $jumpNestingOk\ jmps\ t$

$\langle \text{proof} \rangle$

**lemma** *assign-abrupt-propagation*:

**assumes** *f-ok*:  $\text{abrupt } (f \ n \ s) \neq x$   
**and** *ass*:  $\text{abrupt } (\text{assign } f \ n \ s) = x$   
**shows**  $\text{abrupt } s = x$

$\langle \text{proof} \rangle$

**lemma** *wt-init-comp-ty'*:

*is-acc-type* (*prg Env*) (*pid* (*cls Env*))  $T \implies \text{Env} \vdash \text{init-comp-ty } T :: \checkmark$

$\langle \text{proof} \rangle$

**lemma** *fvar-upd-no-jump*:

**assumes** *upd*:  $\text{upd} = \text{snd } (\text{fst } (\text{fvar } \text{statDeclC } \text{stat } \text{fn } a \ s'))$   
**and** *noJmp*:  $\text{abrupt } s \neq \text{Some } (\text{Jump } j)$   
**shows**  $\text{abrupt } (\text{upd } \text{val } s) \neq \text{Some } (\text{Jump } j)$

$\langle \text{proof} \rangle$

**lemma** *avar-state-no-jump*:

**assumes** *jmp*:  $\text{abrupt } (\text{snd } (\text{avar } G \ i \ a \ s)) = \text{Some } (\text{Jump } j)$   
**shows**  $\text{abrupt } s = \text{Some } (\text{Jump } j)$

$\langle \text{proof} \rangle$

**lemma** *avar-upd-no-jump*:

**assumes** *upd*:  $\text{upd} = \text{snd } (\text{fst } (\text{avar } G \ i \ a \ s'))$   
**and** *noJmp*:  $\text{abrupt } s \neq \text{Some } (\text{Jump } j)$   
**shows**  $\text{abrupt } (\text{upd } \text{val } s) \neq \text{Some } (\text{Jump } j)$

$\langle \text{proof} \rangle$

The next theorem expresses: If jumps (breaks, continues, returns) are nested correctly, we won't find an unexpected jump in the result state of the evaluation. For example, a break can't leave its enclosing loop, an return can't leave its enclosing method. To prove this, the method call is critical. Although the wellformedness of the whole program guarantees that the jumps (breaks, continues and returns) are nested correctly in all method bodies, the call rule alone does not guarantee that I will call a method or even a class that is part of the program due to dynamic binding! To be able to ensure this we need a kind of conformance of the state, like in the typesafety proof. But then we will redo the typesafety proof here. It would be nice if we could find an easy precondition that will guarantee that all calls will actually call classes and methods of the current program, which can be instantiated in the typesafety proof later on. To fix this problem, I have instrumented the semantic definition of a call to filter out any breaks in the state and to throw an error instead.

To get an induction hypothesis which is strong enough to perform the proof, we can't just assume *jumpNestingOk* for the empty set and conclude, that no jump at all will be in the resulting state, because the set is altered by the statements *Lab* and *While*.

The wellformedness of the program is used to ensure that for all class initialisations and methods the nesting of jumps is wellformed, too.

**theorem** *jumpNestingOk-eval*:

**assumes** *eval*:  $G \vdash s_0 \multimap \rightarrow (v, s_1)$   
**and** *jmpOk*:  $\text{jumpNestingOk } \text{jmps } t$   
**and** *wt*:  $\text{Env} \vdash t :: T$   
**and** *wf*:  $\text{wf-prog } G$

**and**  $G: \text{prg Env} = G$   
**and**  $\text{no-jmp}: \forall j. \text{abrupt } s0 = \text{Some } (\text{Jump } j) \longrightarrow j \in \text{jmps}$   
 $(\text{is } ?\text{Jmp } \text{jmps } s0)$   
**shows**  $(\forall j. \text{fst } s1 = \text{Some } (\text{Jump } j) \longrightarrow j \in \text{jmps}) \wedge$   
 $(\text{normal } s1 \longrightarrow$   
 $(\forall w \text{ upd}. v = \text{In2 } (w, \text{upd})$   
 $\longrightarrow (\forall s \text{ j val.}$   
 $\text{abrupt } s \neq \text{Some } (\text{Jump } j) \longrightarrow$   
 $\text{abrupt } (\text{upd val } s) \neq \text{Some } (\text{Jump } j))))$   
 $(\text{is } ?\text{Jmp } \text{jmps } s1 \wedge ?\text{Upd } v \text{ } s1)$   
 $\langle \text{proof} \rangle$

**lemmas**  $\text{jumpNestingOk-evalE} = \text{jumpNestingOk-eval } [\text{THEN } \text{conjE}, \text{rule-format}]$

**lemma**  $\text{jumpNestingOk-eval-no-jump}$ :  
**assumes**  $\text{eval}: \text{prg Env} \vdash s0 \dashv t \longrightarrow (v, s1)$  **and**  
 $\text{jmpOk}: \text{jumpNestingOk } \{\} \text{ } t$  **and**  
 $\text{no-jmp}: \text{abrupt } s0 \neq \text{Some } (\text{Jump } j)$  **and**  
 $\text{wt}: \text{Env} \vdash t :: T$  **and**  
 $\text{wf}: \text{wf-prog } (\text{prg Env})$   
**shows**  $\text{abrupt } s1 \neq \text{Some } (\text{Jump } j) \wedge$   
 $(\text{normal } s1 \longrightarrow v = \text{In2 } (w, \text{upd})$   
 $\longrightarrow \text{abrupt } s \neq \text{Some } (\text{Jump } j')$   
 $\longrightarrow \text{abrupt } (\text{upd val } s) \neq \text{Some } (\text{Jump } j'))$   
 $\langle \text{proof} \rangle$

**lemmas**  $\text{jumpNestingOk-eval-no-jumpE}$   
 $= \text{jumpNestingOk-eval-no-jump } [\text{THEN } \text{conjE}, \text{rule-format}]$

**corollary**  $\text{eval-expression-no-jump}$ :  
**assumes**  $\text{eval}: \text{prg Env} \vdash s0 \dashv e \dashv v \longrightarrow s1$  **and**  
 $\text{no-jmp}: \text{abrupt } s0 \neq \text{Some } (\text{Jump } j)$  **and**  
 $\text{wt}: \text{Env} \vdash e :: \neg T$  **and**  
 $\text{wf}: \text{wf-prog } (\text{prg Env})$   
**shows**  $\text{abrupt } s1 \neq \text{Some } (\text{Jump } j)$   
 $\langle \text{proof} \rangle$

**corollary**  $\text{eval-var-no-jump}$ :  
**assumes**  $\text{eval}: \text{prg Env} \vdash s0 \dashv \text{var} \dashv (w, \text{upd}) \longrightarrow s1$  **and**  
 $\text{no-jmp}: \text{abrupt } s0 \neq \text{Some } (\text{Jump } j)$  **and**  
 $\text{wt}: \text{Env} \vdash \text{var} :: T$  **and**  
 $\text{wf}: \text{wf-prog } (\text{prg Env})$   
**shows**  $\text{abrupt } s1 \neq \text{Some } (\text{Jump } j) \wedge$   
 $(\text{normal } s1 \longrightarrow$   
 $(\text{abrupt } s \neq \text{Some } (\text{Jump } j')$   
 $\longrightarrow \text{abrupt } (\text{upd val } s) \neq \text{Some } (\text{Jump } j'))$   
 $\langle \text{proof} \rangle$

**lemmas**  $\text{eval-var-no-jumpE} = \text{eval-var-no-jump } [\text{THEN } \text{conjE}, \text{rule-format}]$

**corollary**  $\text{eval-statement-no-jump}$ :  
**assumes**  $\text{eval}: \text{prg Env} \vdash s0 \dashv c \longrightarrow s1$  **and**  
 $\text{jmpOk}: \text{jumpNestingOkS } \{\} \text{ } c$  **and**  
 $\text{no-jmp}: \text{abrupt } s0 \neq \text{Some } (\text{Jump } j)$  **and**  
 $\text{wt}: \text{Env} \vdash c :: \checkmark$  **and**  
 $\text{wf}: \text{wf-prog } (\text{prg Env})$   
**shows**  $\text{abrupt } s1 \neq \text{Some } (\text{Jump } j)$

$\langle \text{proof} \rangle$

**corollary** *eval-expression-list-no-jump*:

**assumes** *eval*:  $\text{prg } Env \vdash s0 \text{ --es--> } v \rightarrow s1$  **and**  
*no-jmp*:  $\text{abrupt } s0 \neq \text{Some } (\text{Jump } j)$  **and**  
*wt*:  $Env \vdash \text{es} :: \vdash T$  **and**  
*wf*:  $\text{wf-prog } (\text{prg } Env)$   
**shows**  $\text{abrupt } s1 \neq \text{Some } (\text{Jump } j)$

$\langle \text{proof} \rangle$

**lemma** *union-subseteq-elim* [elim]:  $\llbracket A \cup B \subseteq C; \llbracket A \subseteq C; B \subseteq C \rrbracket \implies P \rrbracket \implies P$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-halloc-mono*:

**assumes** *halloc*:  $G \vdash s0 \text{ --halloc } oi \text{ --> } a \rightarrow s1$   
**shows**  $\text{dom } (\text{locals } (\text{store } s0)) \subseteq \text{dom } (\text{locals } (\text{store } s1))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-sxalloc-mono*:

**assumes** *sxalloc*:  $G \vdash s0 \text{ --sxalloc--> } s1$   
**shows**  $\text{dom } (\text{locals } (\text{store } s0)) \subseteq \text{dom } (\text{locals } (\text{store } s1))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-assign-mono*:

**assumes** *f-ok*:  $\text{dom } (\text{locals } (\text{store } s)) \subseteq \text{dom } (\text{locals } (\text{store } (f \ n \ s)))$   
**shows**  $\text{dom } (\text{locals } (\text{store } s)) \subseteq \text{dom } (\text{locals } (\text{store } (\text{assign } f \ n \ s)))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-lvar-mono*:

$\text{dom } (\text{locals } (\text{store } s)) \subseteq \text{dom } (\text{locals } (\text{store } (\text{snd } (\text{lvar } vn \ s') \ \text{val } s)))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-fvar-vvar-mono*:

$\text{dom } (\text{locals } (\text{store } s))$   
 $\subseteq \text{dom } (\text{locals } (\text{store } (\text{snd } (\text{fst } (\text{fvar } \text{statDeclC } \text{stat } fn \ a \ s') \ \text{val } s))))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-fvar-mono*:

$\text{dom } (\text{locals } (\text{store } s))$   
 $\subseteq \text{dom } (\text{locals } (\text{store } (\text{snd } (\text{fvar } \text{statDeclC } \text{stat } fn \ a \ s))))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-avar-vvar-mono*:

$\text{dom } (\text{locals } (\text{store } s))$   
 $\subseteq \text{dom } (\text{locals } (\text{store } (\text{snd } (\text{fst } (\text{avar } G \ i \ a \ s') \ \text{val } s))))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-avar-mono*:

$\text{dom} (\text{locals} (\text{store } s))$   
 $\subseteq \text{dom} (\text{locals} (\text{store} (\text{snd} (\text{avar } G \text{ } i \text{ } a \text{ } s))))$   
 $\langle \text{proof} \rangle$

Since assignments are modelled as functions from states to states, we must take into account these functions. They appear only in the assignment rule and as result from evaluating a variable. That's why we need the complicated second part of the conjunction in the goal. The reason for the very generic way to treat assignments was the aim to omit redundancy. There is only one evaluation rule for each kind of variable (locals, fields, arrays). These rules are used for both accessing variables and updating variables. That's why the evaluation rules for variables result in a pair consisting of a value and an update function. Of course we could also think of a pair of a value and a reference in the store, instead of the generic update function. But as only array updates can cause a special exception (if the types mismatch) and not array reads we then have to introduce two different rules to handle array reads and updates

**lemma** *dom-locals-eval-mono*:

**assumes**  $\text{eval}: G \vdash s0 \rightarrow t \rightarrow (v, s1)$   
**shows**  $\text{dom} (\text{locals} (\text{store } s0)) \subseteq \text{dom} (\text{locals} (\text{store } s1)) \wedge$   
 $(\forall vv. v = \text{In2 } vv \wedge \text{normal } s1$   
 $\rightarrow (\forall s \text{ val. } \text{dom} (\text{locals} (\text{store } s))$   
 $\subseteq \text{dom} (\text{locals} (\text{store} ((\text{snd } vv) \text{ val } s))))$

$\langle \text{proof} \rangle$

**lemma** *dom-locals-eval-mono-elim*:

**assumes**  $\text{eval}: G \vdash s0 \rightarrow t \rightarrow (v, s1)$   
**obtains**  $\text{dom} (\text{locals} (\text{store } s0)) \subseteq \text{dom} (\text{locals} (\text{store } s1))$  **and**  
 $\bigwedge vv \ s \text{ val. } \llbracket v = \text{In2 } vv; \text{normal } s1 \rrbracket$   
 $\implies \text{dom} (\text{locals} (\text{store } s))$   
 $\subseteq \text{dom} (\text{locals} (\text{store} ((\text{snd } vv) \text{ val } s)))$

$\langle \text{proof} \rangle$

**lemma** *halloc-no-abrupt*:

**assumes**  $\text{halloc}: G \vdash s0 \rightarrow \text{halloc } oi \rightarrow a \rightarrow s1$  **and**  
 $\text{normal}: \text{normal } s1$

**shows**  $\text{normal } s0$

$\langle \text{proof} \rangle$

**lemma** *sxalloc-mono-no-abrupt*:

**assumes**  $\text{sxalloc}: G \vdash s0 \rightarrow \text{sxalloc} \rightarrow s1$  **and**  
 $\text{normal}: \text{normal } s1$

**shows**  $\text{normal } s0$

$\langle \text{proof} \rangle$

**lemma** *union-subseteqI*:  $\llbracket A \cup B \subseteq C; A' \subseteq A; B' \subseteq B \rrbracket \implies A' \cup B' \subseteq C$

$\langle \text{proof} \rangle$

**lemma** *union-subseteqII*:  $\llbracket A \cup B \subseteq C; A' \subseteq A \rrbracket \implies A' \cup B \subseteq C$

$\langle \text{proof} \rangle$

**lemma** *union-subseteqIr*:  $\llbracket A \cup B \subseteq C; B' \subseteq B \rrbracket \implies A \cup B' \subseteq C$   
 $\langle \text{proof} \rangle$

**lemma** *subseteq-union-transl* [trans]:  $\llbracket A \subseteq B; B \cup C \subseteq D \rrbracket \implies A \cup C \subseteq D$   
 $\langle \text{proof} \rangle$

**lemma** *subseteq-union-transr* [trans]:  $\llbracket A \subseteq B; C \cup B \subseteq D \rrbracket \implies A \cup C \subseteq D$   
 $\langle \text{proof} \rangle$

**lemma** *union-subseteq-weaken*:  $\llbracket A \cup B \subseteq C; \llbracket A \subseteq C; B \subseteq C \rrbracket \implies P \rrbracket \implies P$   
 $\langle \text{proof} \rangle$

**lemma** *assigns-good-approx*:  
**assumes**  
   *eval*:  $G \vdash s0 \dashv t \succ \rightarrow (v, s1)$  **and**  
   *normal*: *normal* *s1*  
**shows**  $\text{assigns } t \subseteq \text{dom } (\text{locals } (\text{store } s1))$   
 $\langle \text{proof} \rangle$

**corollary** *assignsE-good-approx*:  
**assumes**  
   *eval*:  $\text{prg } Env \vdash s0 \dashv e \dashv v \rightarrow s1$  **and**  
   *normal*: *normal* *s1*  
**shows**  $\text{assignsE } e \subseteq \text{dom } (\text{locals } (\text{store } s1))$   
 $\langle \text{proof} \rangle$

**corollary** *assignsV-good-approx*:  
**assumes**  
   *eval*:  $\text{prg } Env \vdash s0 \dashv v = \succ v f \rightarrow s1$  **and**  
   *normal*: *normal* *s1*  
**shows**  $\text{assignsV } v \subseteq \text{dom } (\text{locals } (\text{store } s1))$   
 $\langle \text{proof} \rangle$

**corollary** *assignsEs-good-approx*:  
**assumes**  
   *eval*:  $\text{prg } Env \vdash s0 \dashv es \dot{=} \succ vs \rightarrow s1$  **and**  
   *normal*: *normal* *s1*  
**shows**  $\text{assignsEs } es \subseteq \text{dom } (\text{locals } (\text{store } s1))$   
 $\langle \text{proof} \rangle$

**lemma** *constVal-eval*:  
**assumes** *const*: *constVal* *e* = *Some* *c* **and**  
   *eval*:  $G \vdash \text{Norm } s0 \dashv e \dashv v \rightarrow s$   
**shows**  $v = c \wedge \text{normal } s$   
 $\langle \text{proof} \rangle$

**lemmas** *constVal-eval-elim* = *constVal-eval* [THEN *conjE*]

**lemma** *eval-unop-type*:  
*typeof* *dt* (*eval-unop* *unop* *v*) = *Some* (*PrimT* (*unop-type* *unop*))  
 $\langle \text{proof} \rangle$

**lemma** *eval-binop-type*:

*typeof* *dt* (*eval-binop binop v1 v2*) = *Some (PrimT (binop-type binop))*  
 ⟨*proof*⟩

**lemma** *constVal-Boolean*:

**assumes** *const*: *constVal e = Some c* **and**  
*wt*: *Env ⊢ e :: -PrimT Boolean*  
**shows** *typeof empty-dt c = Some (PrimT Boolean)*  
 ⟨*proof*⟩

**lemma** *assigns-if-good-approx*:

**assumes**  
*eval*: *prg Env ⊢ s0 -e-> b → s1* **and**  
*normal*: *normal s1* **and**  
*bool*: *Env ⊢ e :: -PrimT Boolean*  
**shows** *assigns-if (the-Bool b) e ⊆ dom (locals (store s1))*  
 ⟨*proof*⟩

**lemma** *assigns-if-good-approx'*:

**assumes** *eval*: *G ⊢ s0 -e-> b → s1*  
**and** *normal*: *normal s1*  
**and** *bool*: *([prg=G, cls=C, lcl=L]) ⊢ e :: - (PrimT Boolean)*  
**shows** *assigns-if (the-Bool b) e ⊆ dom (locals (store s1))*  
 ⟨*proof*⟩

**lemma** *subset-Intl*: *A ⊆ C ⇒ A ∩ B ⊆ C*

⟨*proof*⟩

**lemma** *subset-Intr*: *B ⊆ C ⇒ A ∩ B ⊆ C*

⟨*proof*⟩

**lemma** *da-good-approx*:

**assumes** *eval*: *prg Env ⊢ s0 -t>→ (v, s1)* **and**  
*wt*: *Env ⊢ t :: T* (**is** *?Wt Env t T*) **and**  
*da*: *Env ⊢ dom (locals (store s0)) »t» A* (**is** *?Da Env s0 t A*) **and**  
*wf*: *wf-prog (prg Env)*  
**shows** (*normal s1 ⇒ (nrm A ⊆ dom (locals (store s1)))*) ∧  
 (∀ *l*. *abrupt s1 = Some (Jump (Break l)) ∧ normal s0*  
 $\longrightarrow$  (*brk A l ⊆ dom (locals (store s1))*)) ∧  
 (*abrupt s1 = Some (Jump Ret) ∧ normal s0*  
 $\longrightarrow$  *Result ∈ dom (locals (store s1))*)  
 (**is** *?NormalAssigned s1 A ∧ ?BreakAssigned s0 s1 A ∧ ?ResAssigned s0 s1*)  
 ⟨*proof*⟩

**lemma** *da-good-approxE*:

**assumes**  
*prg Env ⊢ s0 -t>→ (v, s1)* **and** *Env ⊢ t :: T* **and**  
*Env ⊢ dom (locals (store s0)) »t» A* **and** *wf-prog (prg Env)*  
**obtains**  
*normal s1 ⇒ nrm A ⊆ dom (locals (store s1))* **and**  
 ∧ *l*. [*abrupt s1 = Some (Jump (Break l)); normal s0*]



$\implies \text{brk } A \ l \subseteq \text{dom } (\text{locals } (\text{store } s1))$  **and**  
 $\llbracket \text{abrupt } s1 = \text{Some } (\text{Jump Ret}); \text{normal } s0 \rrbracket \implies \text{Result} \in \text{dom } (\text{locals } (\text{store } s1))$   
 $\langle \text{proof} \rangle$

**lemma** *da-good-approxE'*:

**assumes** *eval*:  $G \vdash s0 \dashv t \succ \rightarrow (v, s1)$   
**and** *wt*:  $\langle \text{prg} = G, \text{cls} = C, \text{lcl} = L \rangle \vdash t :: T$   
**and** *da*:  $\langle \text{prg} = G, \text{cls} = C, \text{lcl} = L \rangle \vdash \text{dom } (\text{locals } (\text{store } s0)) \gg t \gg A$   
**and** *wf*: *wf-prog* *G*

**obtains** *normal* *s1*  $\implies \text{nrm } A \subseteq \text{dom } (\text{locals } (\text{store } s1))$  **and**

$\bigwedge l. \llbracket \text{abrupt } s1 = \text{Some } (\text{Jump } (\text{Break } l)); \text{normal } s0 \rrbracket$   
 $\implies \text{brk } A \ l \subseteq \text{dom } (\text{locals } (\text{store } s1))$  **and**

$\llbracket \text{abrupt } s1 = \text{Some } (\text{Jump Ret}); \text{normal } s0 \rrbracket$

$\implies \text{Result} \in \text{dom } (\text{locals } (\text{store } s1))$

$\langle \text{proof} \rangle$

**declare**  $\llbracket \text{simproc } \text{add}: \text{wt-expr wt-var wt-exprs wt-stmt} \rrbracket$

**end**



## Chapter 19

# TypeSafe

## 46 The type soundness proof for Java

**theory** *TypeSafe*

**imports** *DefiniteAssignmentCorrect Conform*

**begin**

**error free**

**hide** *const field*

**lemma** *error-free-halloc*:

**assumes** *halloc*:  $G \vdash s0 \text{ --halloc } oi \succ a \rightarrow s1$  **and**

*error-free-s0*: *error-free s0*

**shows** *error-free s1*

$\langle \text{proof} \rangle$

**lemma** *error-free-sxalloc*:

**assumes** *sxalloc*:  $G \vdash s0 \text{ --sxalloc} \rightarrow s1$  **and** *error-free-s0*: *error-free s0*

**shows** *error-free s1*

$\langle \text{proof} \rangle$

**lemma** *error-free-check-field-access-eq*:

*error-free (check-field-access G accC statDeclC fn stat a s)*

$\implies (check-field-access G accC statDeclC fn stat a s) = s$

$\langle \text{proof} \rangle$

**lemma** *error-free-check-method-access-eq*:

*error-free (check-method-access G accC statT mode sig a' s)*

$\implies (check-method-access G accC statT mode sig a' s) = s$

$\langle \text{proof} \rangle$

**lemma** *error-free-FVar-lemma*:

*error-free s*

$\implies error-free (abupd (if stat then id else np a) s)$

$\langle \text{proof} \rangle$

**lemma** *error-free-init-lvars [simp,intro]*:

*error-free s*  $\implies$

*error-free (init-lvars G C sig mode a pvs s)*

$\langle \text{proof} \rangle$

**lemma** *error-free-LVar-lemma*:

*error-free s*  $\implies error-free (assign (\lambda v. supd lupd(vn \mapsto v)) w s)$

$\langle \text{proof} \rangle$

**lemma** *error-free-throw [simp,intro]*:

*error-free s*  $\implies error-free (abupd (throw x) s)$

$\langle \text{proof} \rangle$

**result conformance**

**constdefs**

$$\begin{aligned}
& \text{assign-conforms} :: st \Rightarrow (val \Rightarrow state \Rightarrow state) \Rightarrow ty \Rightarrow env' \Rightarrow bool \\
& \quad (-\leq | -\leq :: \leq - \quad [71, 71, 71, 71] \ 70) \\
& s \leq | f \leq T :: \leq E \equiv \\
& (\forall s' w. \text{Norm } s' :: \leq E \longrightarrow \text{fst } E, s' \vdash w :: \leq T \longrightarrow s \leq | s' \longrightarrow \text{assign } f \ w \ (\text{Norm } s') :: \leq E) \wedge \\
& (\forall s' w. \text{error-free } s' \longrightarrow (\text{error-free } (\text{assign } f \ w \ s')))
\end{aligned}$$
**constdefs**

$$\begin{aligned}
& rconf :: prog \Rightarrow lenv \Rightarrow st \Rightarrow term \Rightarrow vals \Rightarrow tys \Rightarrow bool \\
& \quad (-, -, +, > :: \leq - \quad [71, 71, 71, 71, 71, 71] \ 70) \\
& G, L, s \vdash t > v :: \leq T \\
& \equiv \text{case } T \text{ of} \\
& \quad \text{Inl } T \Rightarrow \text{if } (\exists \text{ var. } t = \text{In2 var}) \\
& \quad \quad \text{then } (\forall n. (\text{the-In2 } t) = \text{LVar } n \\
& \quad \quad \quad \longrightarrow (\text{fst } (\text{the-In2 } v) = \text{the } (\text{locals } s \ n)) \wedge \\
& \quad \quad \quad (\text{locals } s \ n \neq \text{None} \longrightarrow G, s \vdash \text{fst } (\text{the-In2 } v) :: \leq T)) \wedge \\
& \quad \quad (\neg (\exists n. \text{the-In2 } t = \text{LVar } n) \longrightarrow (G, s \vdash \text{fst } (\text{the-In2 } v) :: \leq T)) \wedge \\
& \quad \quad (s \leq | \text{snd } (\text{the-In2 } v) \leq T :: \leq (G, L)) \\
& \quad \quad \text{else } G, s \vdash \text{the-In1 } v :: \leq T \\
& \quad | \text{Inr } Ts \Rightarrow \text{list-all2 } (conf \ G \ s) \ (\text{the-In3 } v) \ Ts
\end{aligned}$$

With *rconf* we describe the conformance of the result value of a term. This definition gets rather complicated because of the relations between the injections of the different terms, types and values. The main case distinction is between single values and value lists. In case of value lists, every value has to conform to its type. For single values we have to do a further case distinction, between values of variables  $\exists \text{ var. } t = \text{In2 var}$  and ordinary values. Values of variables are modelled as pairs consisting of the current value and an update function which will perform an assignment to the variable. This stems from the decision, that we only have one evaluation rule for each kind of variable. The decision if we read or write to the variable is made by syntactic enclosing rules. So conformance of variable-values must ensure that both the current value and an update will conform to the type. With the introduction of definite assignment of local variables we have to do another case distinction. For the notion of conformance local variables are allowed to be *None*, since the definedness is not ensured by conformance but by definite assignment. Field and array variables must contain a value.

**lemma** *rconf-In1 [simp]*:
$$G, L, s \vdash \text{In1 } ec > \text{In1 } v :: \leq \text{Inl } T = G, s \vdash v :: \leq T$$

*<proof>*

**lemma** *rconf-In2-no-LVar [simp]*:
$$\forall n. va \neq \text{LVar } n \implies$$

$$G, L, s \vdash \text{In2 } va > \text{In2 } vf :: \leq \text{Inl } T = (G, s \vdash \text{fst } vf :: \leq T \wedge s \leq | \text{snd } vf \leq T :: \leq (G, L))$$

*<proof>*

**lemma** *rconf-In2-LVar [simp]*:
$$va = \text{LVar } n \implies$$

$$G, L, s \vdash \text{In2 } va > \text{In2 } vf :: \leq \text{Inl } T$$

$$= ((\text{fst } vf = \text{the } (\text{locals } s \ n)) \wedge$$

$$(\text{locals } s \ n \neq \text{None} \longrightarrow G, s \vdash \text{fst } vf :: \leq T) \wedge s \leq | \text{snd } vf \leq T :: \leq (G, L))$$

*<proof>*

**lemma** *rconf-In3 [simp]*:
$$G, L, s \vdash \text{In3 } es > \text{In3 } vs :: \leq \text{Inr } Ts = \text{list-all2 } (\lambda v \ T. G, s \vdash v :: \leq T) \ vs \ Ts$$

*<proof>*

**fits and conf**

**lemma** *conf-fits*:  $G, s \vdash v :: \preceq T \implies G, s \vdash v \text{ fits } T$   
 $\langle \text{proof} \rangle$

**lemma** *fits-conf*:  
 $\llbracket G, s \vdash v :: \preceq T; G \vdash T \preceq ? T'; G, s \vdash v \text{ fits } T'; \text{ws-prog } G \rrbracket \implies G, s \vdash v :: \preceq T'$   
 $\langle \text{proof} \rangle$

**lemma** *fits-Array*:  
 $\llbracket G, s \vdash v :: \preceq T; G \vdash T'.[] \preceq T.[]; G, s \vdash v \text{ fits } T'; \text{ws-prog } G \rrbracket \implies G, s \vdash v :: \preceq T'$   
 $\langle \text{proof} \rangle$

**gext**

**lemma** *halloc-gext*:  $\bigwedge s1\ s2. G \vdash s1 \text{ --halloc } oi \rightarrow s2 \implies \text{snd } s1 \leq | \text{snd } s2$   
 $\langle \text{proof} \rangle$

**lemma** *sxalloc-gext*:  $\bigwedge s1\ s2. G \vdash s1 \text{ --sxalloc } \rightarrow s2 \implies \text{snd } s1 \leq | \text{snd } s2$   
 $\langle \text{proof} \rangle$

**lemma** *eval-gext-lemma*  $[rule\text{-}format\ (no\text{-}asm)]$ :  
 $G \vdash s \text{ --}t \rightarrow (w, s') \implies \text{snd } s \leq | \text{snd } s' \wedge (\text{case } w \text{ of}$   
 $\quad In1\ v \Rightarrow \text{True}$   
 $\quad | In2\ vf \Rightarrow \text{normal } s \longrightarrow (\forall v\ x\ s. s \leq | \text{snd } (\text{assign } (\text{snd } vf)\ v\ (x, s)))$   
 $\quad | In3\ vs \Rightarrow \text{True})$   
 $\langle \text{proof} \rangle$

**lemma** *eval-gext-f*:  
 $G \vdash \text{Norm } s1 \text{ --}e \rightarrow vf \rightarrow s2 \implies s \leq | \text{snd } (\text{assign } (\text{snd } vf)\ v\ (x, s))$   
 $\langle \text{proof} \rangle$

**lemmas** *eval-gext* = *eval-gext-lemma*  $[THEN\ conjunct1]$

**lemma** *eval-gext'*:  $G \vdash (x1, s1) \text{ --}t \rightarrow (w, (x2, s2)) \implies s1 \leq | s2$   
 $\langle \text{proof} \rangle$

**lemma** *init-yields-initd*:  $G \vdash \text{Norm } s1 \text{ --Init } C \rightarrow s2 \implies \text{initd } C\ s2$   
 $\langle \text{proof} \rangle$

**Lemmas**

**lemma** *obj-ty-obj-class1*:  
 $\llbracket \text{wf-prog } G; \text{is-type } G\ (\text{obj-ty } obj) \rrbracket \implies \text{is-class } G\ (\text{obj-class } obj)$   
 $\langle \text{proof} \rangle$

**lemma** *oconf-init-obj*:  
 $\llbracket \text{wf-prog } G;$   
 $\quad (\text{case } r \text{ of Heap } a \Rightarrow \text{is-type } G\ (\text{obj-ty } obj) \mid \text{Stat } C \Rightarrow \text{is-class } G\ C)$   
 $\rrbracket \implies G, s \vdash obj\ (\text{values} = \text{init-vals } (\text{var-tys } G\ (\text{tag } obj)\ r)) :: \preceq \sqrt{r}$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-newG*:  $\llbracket \text{globs } s \text{ ofref} = \text{None}; (x, s) :: \preceq (G, L);$   
 $\text{wf-prog } G; \text{ case ofref of Heap } a \Rightarrow \text{is-type } G \text{ (obj-ty } (\text{tag}=\text{oi}, \text{values}=\text{vs}))$   
 $\quad | \text{Stat } C \Rightarrow \text{is-class } G \text{ } C \rrbracket \Longrightarrow$   
 $(x, \text{init-obj } G \text{ oi ofref } s) :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *conforms-init-class-obj*:  
 $\llbracket (x, s) :: \preceq (G, L); \text{wf-prog } G; \text{class } G \text{ } C = \text{Some } y; \neg \text{init-ed } C \text{ (globs } s) \rrbracket \Longrightarrow$   
 $(x, \text{init-class-obj } G \text{ } C \text{ } s) :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *fst-init-lvars[simp]*:  
 $\text{fst } (\text{init-lvars } G \text{ } C \text{ sig (invmode } m \text{ } e) \text{ } a' \text{ pvs } (x, s)) =$   
 $(\text{if is-static } m \text{ then } x \text{ else (np } a') \text{ } x)$   
 $\langle \text{proof} \rangle$

**lemma** *halloc-conforms*:  $\bigwedge s1. \llbracket G \vdash s1 \text{ -halloc oi} \succ a \rightarrow s2; \text{wf-prog } G; s1 :: \preceq (G, L);$   
 $\text{is-type } G \text{ (obj-ty } (\text{tag}=\text{oi}, \text{values}=\text{fs})) \rrbracket \Longrightarrow s2 :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *halloc-type-sound*:  
 $\bigwedge s1. \llbracket G \vdash s1 \text{ -halloc oi} \succ a \rightarrow (x, s); \text{wf-prog } G; s1 :: \preceq (G, L);$   
 $T = \text{obj-ty } (\text{tag}=\text{oi}, \text{values}=\text{fs}); \text{is-type } G \text{ } T \rrbracket \Longrightarrow$   
 $(x, s) :: \preceq (G, L) \wedge (x = \text{None} \longrightarrow G, s \vdash \text{Addr } a :: \preceq T)$   
 $\langle \text{proof} \rangle$

**lemma** *sxalloc-type-sound*:  
 $\bigwedge s1 \text{ } s2. \llbracket G \vdash s1 \text{ -sxalloc} \rightarrow s2; \text{wf-prog } G \rrbracket \Longrightarrow$   
 $\text{case fst } s1 \text{ of}$   
 $\quad \text{None} \Rightarrow s2 = s1$   
 $\quad | \text{Some } \text{abr} \Rightarrow (\text{case } \text{abr} \text{ of}$   
 $\quad \quad \text{Xcpt } x \Rightarrow (\exists a. \text{fst } s2 = \text{Some}(\text{Xcpt } (\text{Loc } a)) \wedge$   
 $\quad \quad \quad (\forall L. s1 :: \preceq (G, L) \longrightarrow s2 :: \preceq (G, L)))$   
 $\quad \quad | \text{Jump } j \Rightarrow s2 = s1$   
 $\quad \quad | \text{Error } e \Rightarrow s2 = s1)$   
 $\langle \text{proof} \rangle$

**lemma** *wt-init-comp-ty*:  
 $\text{is-acc-type } G \text{ (pid } C) \text{ } T \Longrightarrow (\text{prg}=\text{G}, \text{cls}=\text{C}, \text{lcl}=\text{L}) \vdash \text{init-comp-ty } T :: \checkmark$   
 $\langle \text{proof} \rangle$

**declare** *fun-upd-same* [simp]

**declare** *fun-upd-apply* [simp del]

**constdefs**  
 $\text{DynT-prop} :: [\text{prog}, \text{inv-mode}, \text{qtname}, \text{ref-ty}] \Rightarrow \text{bool}$   
 $(\vdash \longrightarrow \preceq - [71, 71, 71, 71] \text{ } 70)$

$$G \vdash \text{mode} \rightarrow D \preceq t \equiv \text{mode} = \text{IntVir} \longrightarrow \text{is-class } G \ D \wedge \\ (\text{if } (\exists T. t = \text{ArrayT } T) \text{ then } D = \text{Object} \text{ else } G \vdash \text{Class } D \preceq \text{RefT } t)$$

**lemma** *DynT-propI*:

$$\llbracket (x, s) :: \preceq (G, L); G, s \vdash a' :: \preceq \text{RefT } \text{statT}; \text{wf-prog } G; \text{mode} = \text{IntVir} \longrightarrow a' \neq \text{Null} \rrbracket \\ \implies G \vdash \text{mode} \rightarrow \text{invocation-class mode } s \ a' \ \text{statT} \preceq \text{statT} \\ \langle \text{proof} \rangle$$

**lemma** *invocation-methd*:

$$\llbracket \text{wf-prog } G; \text{statT} \neq \text{NullT}; \\ (\forall \text{ statC}. \text{statT} = \text{ClassT } \text{statC} \longrightarrow \text{is-class } G \ \text{statC}); \\ (\forall I. \text{statT} = \text{IfaceT } I \longrightarrow \text{is-iface } G \ I \wedge \text{mode} \neq \text{SuperM}); \\ (\forall T. \text{statT} = \text{ArrayT } T \longrightarrow \text{mode} \neq \text{SuperM}); \\ G \vdash \text{mode} \rightarrow \text{invocation-class mode } s \ a' \ \text{statT} \preceq \text{statT}; \\ \text{dynlookup } G \ \text{statT} \ (\text{invocation-class mode } s \ a' \ \text{statT}) \ \text{sig} = \text{Some } m \rrbracket \\ \implies \text{methd } G \ (\text{invocation-declclass } G \ \text{mode } s \ a' \ \text{statT} \ \text{sig}) \ \text{sig} = \text{Some } m \\ \langle \text{proof} \rangle$$

**lemma** *DynT-mheadsD*:

$$\llbracket G \vdash \text{invmode } sm \ e \rightarrow \text{invC} \preceq \text{statT}; \\ \text{wf-prog } G; (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash e :: -\text{RefT } \text{statT}; \\ (\text{statDeclT}, sm) \in \text{mheads } G \ C \ \text{statT} \ \text{sig}; \\ \text{invC} = \text{invocation-class } (\text{invmode } sm \ e) \ s \ a' \ \text{statT}; \\ \text{declC} = \text{invocation-declclass } G \ (\text{invmode } sm \ e) \ s \ a' \ \text{statT} \ \text{sig} \rrbracket \\ \implies \\ \exists dm. \\ \text{methd } G \ \text{declC} \ \text{sig} = \text{Some } dm \wedge \text{dynlookup } G \ \text{statT} \ \text{invC} \ \text{sig} = \text{Some } dm \wedge \\ G \vdash \text{resTy } (\text{methd } dm) \preceq \text{resTy } sm \wedge \\ \text{wf-mdecl } G \ \text{declC} \ (\text{sig}, \text{methd } dm) \wedge \\ \text{declC} = \text{declclass } dm \wedge \\ \text{is-static } dm = \text{is-static } sm \wedge \\ \text{is-class } G \ \text{invC} \wedge \text{is-class } G \ \text{declC} \wedge G \vdash \text{invC} \preceq_C \ \text{declC} \wedge \\ (\text{if } \text{invmode } sm \ e = \text{IntVir} \\ \text{then } (\forall \text{ statC}. \text{statT} = \text{ClassT } \text{statC} \longrightarrow G \vdash \text{invC} \preceq_C \ \text{statC}) \\ \text{else } ( (\exists \text{ statC}. \text{statT} = \text{ClassT } \text{statC} \wedge G \vdash \text{statC} \preceq_C \ \text{declC}) \\ \vee (\forall \text{ statC}. \text{statT} \neq \text{ClassT } \text{statC} \wedge \text{declC} = \text{Object})) \wedge \\ \text{statDeclT} = \text{ClassT } (\text{declclass } dm)) \\ \langle \text{proof} \rangle$$

**corollary** *DynT-mheadsE* [consumes 7]:

— Same as *DynT-mheadsD* but better suited for application in typesafety proof

**assumes** *invC-compatible*:  $G \vdash \text{mode} \rightarrow \text{invC} \preceq \text{statT}$

**and** *wf*:  $\text{wf-prog } G$

**and** *wt-e*:  $(\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash e :: -\text{RefT } \text{statT}$

**and** *mheads*:  $(\text{statDeclT}, sm) \in \text{mheads } G \ C \ \text{statT} \ \text{sig}$

**and** *mode*:  $\text{mode} = \text{invmode } sm \ e$

**and** *invC*:  $\text{invC} = \text{invocation-class mode } s \ a' \ \text{statT}$

**and** *declC*:  $\text{declC} = \text{invocation-declclass } G \ \text{mode } s \ a' \ \text{statT} \ \text{sig}$

**and** *dm*:  $\bigwedge dm. \llbracket \text{methd } G \ \text{declC} \ \text{sig} = \text{Some } dm;$

$\text{dynlookup } G \ \text{statT} \ \text{invC} \ \text{sig} = \text{Some } dm;$

$G \vdash \text{resTy } (\text{methd } dm) \preceq \text{resTy } sm;$

$\text{wf-mdecl } G \ \text{declC} \ (\text{sig}, \text{methd } dm);$

$\text{declC} = \text{declclass } dm;$

$\text{is-static } dm = \text{is-static } sm;$

$\text{is-class } G \ \text{invC}; \text{is-class } G \ \text{declC}; G \vdash \text{invC} \preceq_C \ \text{declC};$

$(\text{if } \text{invmode } sm \ e = \text{IntVir}$



then  $(\forall \text{ statC}. \text{statT} = \text{ClassT statC} \longrightarrow G \vdash \text{invC} \preceq_C \text{statC})$   
 else  $(\exists \text{ statC}. \text{statT} = \text{ClassT statC} \wedge G \vdash \text{statC} \preceq_C \text{declC})$   
 $\vee (\forall \text{ statC}. \text{statT} \neq \text{ClassT statC} \wedge \text{declC} = \text{Object})) \wedge$   
 $\text{statDeclT} = \text{ClassT} (\text{declclass dm})) \implies P$

**shows**  $P$

$\langle \text{proof} \rangle$

**lemma** *DynT-conf*:  $\llbracket G \vdash \text{invocation-class mode } s \text{ } a' \text{ statT} \preceq_C \text{declC}; \text{wf-prog } G;$   
 $\text{isrtype } G (\text{statT});$   
 $G, s \vdash a' :: \preceq_{\text{RefT}} \text{statT}; \text{mode} = \text{IntVir} \longrightarrow a' \neq \text{Null};$   
 $\text{mode} \neq \text{IntVir} \longrightarrow (\exists \text{ statC}. \text{statT} = \text{ClassT statC} \wedge G \vdash \text{statC} \preceq_C \text{declC})$   
 $\vee (\forall \text{ statC}. \text{statT} \neq \text{ClassT statC} \wedge \text{declC} = \text{Object}) \rrbracket$   
 $\implies G, s \vdash a' :: \preceq_{\text{Class}} \text{declC}$   
 $\langle \text{proof} \rangle$

**lemma** *Ass-lemma*:

$\llbracket G \vdash \text{Norm } s0 \text{ } -\text{var} \Rightarrow (w, f) \rightarrow \text{Norm } s1; G \vdash \text{Norm } s1 \text{ } -e \rightarrow v \rightarrow \text{Norm } s2;$   
 $G, s2 \vdash v :: \preceq_{eT}; s1 \leq |s2 \longrightarrow \text{assign } f \text{ } v (\text{Norm } s2) :: \preceq (G, L) \rrbracket$   
 $\implies \text{assign } f \text{ } v (\text{Norm } s2) :: \preceq (G, L) \wedge$   
 $(\text{normal } (\text{assign } f \text{ } v (\text{Norm } s2)) \longrightarrow G, \text{store } (\text{assign } f \text{ } v (\text{Norm } s2)) \vdash v :: \preceq_{eT})$   
 $\langle \text{proof} \rangle$

**lemma** *Throw-lemma*:  $\llbracket G \vdash \text{tn} \preceq_C \text{SXcpt Throwable}; \text{wf-prog } G; (x1, s1) :: \preceq (G, L);$   
 $x1 = \text{None} \longrightarrow G, s1 \vdash a' :: \preceq_{\text{Class}} \text{tn} \rrbracket \implies (\text{throw } a' \text{ } x1, s1) :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *Try-lemma*:  $\llbracket G \vdash \text{obj-ty } (\text{the } (\text{globs } s1' (\text{Heap } a))) \preceq_{\text{Class}} \text{tn};$   
 $(\text{Some } (\text{Xcpt } (\text{Loc } a)), s1') :: \preceq (G, L); \text{wf-prog } G \rrbracket$   
 $\implies \text{Norm } (\text{lupd}(vn \mapsto \text{Addr } a) \text{ } s1') :: \preceq (G, L(vn \mapsto \text{Class } \text{tn}))$   
 $\langle \text{proof} \rangle$

**lemma** *Fin-lemma*:

$\llbracket G \vdash \text{Norm } s1 \text{ } -c2 \rightarrow (x2, s2); \text{wf-prog } G; (\text{Some } a, s1) :: \preceq (G, L); (x2, s2) :: \preceq (G, L);$   
 $\text{dom } (\text{locals } s1) \subseteq \text{dom } (\text{locals } s2) \rrbracket$   
 $\implies (\text{abrupt-if True } (\text{Some } a) \text{ } x2, s2) :: \preceq (G, L)$   
 $\langle \text{proof} \rangle$

**lemma** *FVar-lemma1*:

$\llbracket \text{table-of } (\text{DeclConcepts.fields } G \text{ statC}) (\text{fn}, \text{statDeclC}) = \text{Some } f ;$   
 $x2 = \text{None} \longrightarrow G, s2 \vdash a :: \preceq_{\text{Class}} \text{statC}; \text{wf-prog } G; G \vdash \text{statC} \preceq_C \text{statDeclC};$   
 $\text{statDeclC} \neq \text{Object};$   
 $\text{class } G \text{ statDeclC} = \text{Some } y; (x2, s2) :: \preceq (G, L); s1 \leq |s2;$   
 $\text{initd statDeclC } (\text{globs } s1);$   
 $(\text{if static } f \text{ then id else np } a) \text{ } x2 = \text{None} \rrbracket$   
 $\implies$   
 $\exists \text{ obj. globs } s2 (\text{if static } f \text{ then Inr statDeclC else Inl } (\text{the-Addr } a))$   
 $= \text{Some obj} \wedge$   
 $\text{var-tys } G (\text{tag obj}) (\text{if static } f \text{ then Inr statDeclC else Inl } (\text{the-Addr } a))$   
 $(\text{Inl}(\text{fn}, \text{statDeclC})) = \text{Some } (\text{type } f)$   
 $\langle \text{proof} \rangle$

**lemma** *FVar-lemma2: error-free state*

$\Rightarrow$  *error-free*  
 (assign  
   ( $\lambda v$ . supd  
     (upd-gobj  
       (if static field then Inr statDeclC  
         else Inl (the-Addr a))  
       (Inl (fn, statDeclC)) v))  
   w state)  
 <proof>

**declare** *split-paired-All* [simp del] *split-paired-Ex* [simp del]

**declare** *split-if* [split del] *split-if-asm* [split del]  
*option.split* [split del] *option.split-asm* [split del]

<ML>

**lemma** *FVar-lemma:*

$\llbracket ((v, f), \text{Norm } s2') = \text{fvar statDeclC (static field) fn a (x2, s2)}; \\ G \vdash \text{statC} \preceq_C \text{statDeclC}; \\ \text{table-of (DeclConcepts.fields } G \text{ statC) (fn, statDeclC) = Some field}; \\ \text{wf-prog } G; \\ x2 = \text{None} \longrightarrow G, s2 \vdash a :: \preceq \text{Class statC}; \\ \text{statDeclC} \neq \text{Object}; \text{class } G \text{ statDeclC} = \text{Some } y; \\ (x2, s2) :: \preceq (G, L); s1 \leq |s2; \text{inited statDeclC (globs } s1) \rrbracket \Longrightarrow \\ G, s2 \upharpoonright v :: \preceq \text{type field} \wedge s2' \leq |f \preceq \text{type field} :: \preceq (G, L)$

<proof>

**declare** *split-paired-All* [simp] *split-paired-Ex* [simp]

**declare** *split-if* [split] *split-if-asm* [split]  
*option.split* [split] *option.split-asm* [split]

<ML>

**lemma** *AVar-lemma1:*  $\llbracket \text{globs } s \text{ (Inl } a) = \text{Some obj}; \text{tag obj} = \text{Arr } ty \text{ } i; \\ \text{the-Intg } i' \text{ in-bounds } i; \text{wf-prog } G; G \vdash ty.[] \preceq Tb.[]; \text{Norm } s :: \preceq (G, L) \\ \rrbracket \Longrightarrow G, s \vdash \text{the } ((\text{values obj}) (\text{Inr (the-Intg } i')) :: \preceq Tb$

<proof>

**lemma** *obj-split:*  $\exists t \text{ vs. obj} = \langle \text{tag} = t, \text{values} = \text{vs} \rangle$

<proof>

**lemma** *AVar-lemma2: error-free state*

$\Rightarrow$  *error-free*  
 (assign  
   ( $\lambda v$  (x, s').  
     ((raise-if ( $\neg G, s \upharpoonright v \text{ fits } T$ ) ArrStore) x,  
       upd-gobj (Inl a) (Inr (the-Intg i)) v s')  
   w state)  
 <proof>

**lemma** *AVar-lemma:*  $\llbracket \text{wf-prog } G; G \vdash (x1, s1) -e2 \rightarrow i \rightarrow (x2, s2); \\ ((v, f), \text{Norm } s2') = \text{avar } G \text{ } i \text{ } a (x2, s2); x1 = \text{None} \longrightarrow G, s1 \vdash a :: \preceq Ta.[]; \\ (x2, s2) :: \preceq (G, L); s1 \leq |s2 \rrbracket \Longrightarrow G, s2 \upharpoonright v :: \preceq Ta \wedge s2' \leq |f \preceq Ta :: \preceq (G, L)$

<proof>

**Call**

**lemma** *conforms-init-lvars-lemma*:  $\llbracket \text{wf-prog } G; \text{wf-mhead } G \text{ } P \text{ sig } mh; \text{list-all2 } (\text{conf } G \text{ } s) \text{ pvs } pTsa; G \vdash pTsa[\preceq](\text{parTs sig}) \rrbracket \implies$   
 $G, s \vdash \text{empty } (\text{pars } mh[\mapsto] \text{pvs})$   
 $[\sim::\preceq] \text{table-of lvars}(\text{pars } mh[\mapsto] \text{parTs sig})$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-map-lname* [simp]:  
 $G, s \vdash (\text{lname-case } l1 \text{ } l2)[::\preceq](\text{lname-case } L1 \text{ } L2)$   
 $=$   
 $(G, s \vdash l1[::\preceq] L1 \wedge G, s \vdash (\lambda x::\text{unit} . l2)[::\preceq](\lambda x::\text{unit} . L2))$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-map-lname* [simp]:  
 $G, s \vdash (\text{lname-case } l1 \text{ } l2)[\sim::\preceq](\text{lname-case } L1 \text{ } L2)$   
 $=$   
 $(G, s \vdash l1[\sim::\preceq] L1 \wedge G, s \vdash (\lambda x::\text{unit} . l2)[\sim::\preceq](\lambda x::\text{unit} . L2))$   
 $\langle \text{proof} \rangle$

**lemma** *lconf-map-ename* [simp]:  
 $G, s \vdash (\text{ename-case } l1 \text{ } l2)[::\preceq](\text{ename-case } L1 \text{ } L2)$   
 $=$   
 $(G, s \vdash l1[::\preceq] L1 \wedge G, s \vdash (\lambda x::\text{unit} . l2)[::\preceq](\lambda x::\text{unit} . L2))$   
 $\langle \text{proof} \rangle$

**lemma** *wlconf-map-ename* [simp]:  
 $G, s \vdash (\text{ename-case } l1 \text{ } l2)[\sim::\preceq](\text{ename-case } L1 \text{ } L2)$   
 $=$   
 $(G, s \vdash l1[\sim::\preceq] L1 \wedge G, s \vdash (\lambda x::\text{unit} . l2)[\sim::\preceq](\lambda x::\text{unit} . L2))$   
 $\langle \text{proof} \rangle$

**lemma** *defval-conf1* [rule-format (no-asm), elim]:  
 $\text{is-type } G \text{ } T \longrightarrow (\exists v \in \text{Some } (\text{default-val } T): G, s \vdash v::\preceq T)$   
 $\langle \text{proof} \rangle$

**lemma** *np-no-jump*:  $x \neq \text{Some } (\text{Jump } j) \implies (\text{np } a') \text{ } x \neq \text{Some } (\text{Jump } j)$   
 $\langle \text{proof} \rangle$

**declare** *split-paired-All* [simp del] *split-paired-Ex* [simp del]  
**declare** *split-if* [split del] *split-if-asm* [split del]  
 $\text{option.split}$  [split del]  $\text{option.split-asm}$  [split del]  
 $\langle \text{ML} \rangle$

**lemma** *conforms-init-lvars*:  
 $\llbracket \text{wf-mhead } G \text{ } (\text{pid decl } C) \text{ sig } (\text{mhead } (\text{mthd } dm)); \text{wf-prog } G; \text{list-all2 } (\text{conf } G \text{ } s) \text{ pvs } pTsa; G \vdash pTsa[\preceq](\text{parTs sig});$   
 $(x, s)::\preceq(G, L);$

```

methd G declC sig = Some dm;
isrtype G statT;
G ⊢ invC ≤C declC;
G, s ⊢ a' :: ≤RefT statT;
invmode (mhd sm) e = IntVir ⟶ a' ≠ Null;
invmode (mhd sm) e ≠ IntVir ⟶
  (∃ statC. statT = ClassT statC ∧ G ⊢ statC ≤C declC)
  ∨ (∀ statC. statT ≠ ClassT statC ∧ declC = Object);
invC = invocation-class (invmode (mhd sm) e) s a' statT;
declC = invocation-declclass G (invmode (mhd sm) e) s a' statT sig;
x ≠ Some (Jump Ret)
] ⟹
init-lvars G declC sig (invmode (mhd sm) e) a'
pvs (x, s) :: ≤(G, λ k.
  (case k of
    EName e ⇒ (case e of
      VName v
        ⇒ (table-of (lcls (mbody (methd dm)))
          (pars (methd dm) [↦] parTs sig)) v
      | Res ⇒ Some (resTy (methd dm)))
    | This ⇒ if (is-static (methd sm))
      then None else Some (Class declC)))
⟨proof⟩
declare split-paired-All [simp] split-paired-Ex [simp]
declare split-if [split] split-if-asm [split]
          option.split [split] option.split-asm [split]
⟨ML⟩

```

## 47 accessibility

**theorem** *dynamic-field-access-ok*:

```

assumes wf: wf-prog G and
  not-Null: ¬ stat ⟶ a ≠ Null and
  conform-a: G, (store s) ⊢ a :: ≤ Class statC and
  conform-s: s :: ≤(G, L) and
  normal-s: normal s and
  wt-e: (⟦prg = G, cls = accC, lcl = L⟧) ⊢ e :: − Class statC and
  f: accfield G accC statC fn = Some f and
  dynC: if stat then dynC = declclass f
        else dynC = obj-class (lookup-obj (store s) a) and
  stat: if stat then (is-static f) else (¬ is-static f)
shows table-of (DeclConcepts.fields G dynC) (fn, declclass f) = Some (fld f) ∧
  G ⊢ Field fn f in dynC dyn-accessible-from accC
⟨proof⟩

```

**lemma** *error-free-field-access*:

```

assumes accfield: accfield G accC statC fn = Some (statDeclC, f) and
  wt-e: (⟦prg = G, cls = accC, lcl = L⟧) ⊢ e :: − Class statC and
  eval-init: G ⊢ Norm s0 − Init statDeclC ⟶ s1 and
  eval-e: G ⊢ s1 − e ⟶ a ⟶ s2 and
  conf-s2: s2 :: ≤(G, L) and
  conf-a: normal s2 ⟹ G, store s2 ⊢ a :: ≤ Class statC and
  fvar: (v, s2') = fvar statDeclC (is-static f) fn a s2 and
  wf: wf-prog G
shows check-field-access G accC statDeclC fn (is-static f) a s2' = s2'
⟨proof⟩

```

**lemma** *call-access-ok*:

**assumes** *invC-prop*:  $G \vdash \text{invmode } \text{statM } e \rightarrow \text{invC} \preceq \text{statT}$   
**and** *wf*: *wf-prog* *G*  
**and** *wt-e*:  $(\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash e :: \neg \text{RefT } \text{statT}$   
**and** *statM*:  $(\text{statDeclT}, \text{statM}) \in \text{mheads } G \text{ accC } \text{statT } \text{sig}$   
**and** *invC*: *invC* = *invocation-class* (*invmode statM e*) *s a statT*  
**shows**  $\exists \text{ dynM}. \text{dynlookup } G \text{ statT } \text{invC } \text{sig} = \text{Some } \text{dynM} \wedge$   
 $G \vdash \text{Method } \text{sig } \text{dynM} \text{ in } \text{invC } \text{dyn-accessible-from } \text{accC}$   
 $\langle \text{proof} \rangle$

**lemma** *error-free-call-access*:

**assumes**  
*eval-args*:  $G \vdash s1 \text{ --args} \Rightarrow vs \rightarrow s2$  **and**  
*wt-e*:  $(\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash e :: \neg (\text{RefT } \text{statT})$  **and**  
*statM*:  $\text{max-spec } G \text{ accC } \text{statT } (\text{name} = mn, \text{parTs} = pTs)$   
 $= \{((\text{statDeclT}, \text{statM}), pTs')\}$  **and**  
*conf-s2*:  $s2 :: \preceq (G, L)$  **and**  
*conf-a*:  $\text{normal } s1 \Rightarrow G, \text{store } s1 \vdash a :: \preceq \text{RefT } \text{statT}$  **and**  
*invProp*:  $\text{normal } s3 \Rightarrow$   
 $G \vdash \text{invmode } \text{statM } e \rightarrow \text{invC} \preceq \text{statT}$  **and**  
*s3*:  $s3 = \text{init-lvars } G \text{ invDeclC } (\text{name} = mn, \text{parTs} = pTs')$   
 $(\text{invmode } \text{statM } e) \text{ a vs } s2$  **and**  
*invC*: *invC* = *invocation-class* (*invmode statM e*) (*store s2*) *a statT* **and**  
*invDeclC*: *invDeclC* = *invocation-declclass* *G* (*invmode statM e*) (*store s2*)  
 $\text{a statT } (\text{name} = mn, \text{parTs} = pTs')$  **and**  
*wf*: *wf-prog* *G*  
**shows**  $\text{check-method-access } G \text{ accC } \text{statT } (\text{invmode } \text{statM } e) (\text{name} = mn, \text{parTs} = pTs') \text{ a } s3$   
 $= s3$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-eq-length-append-simp*:

$\bigwedge \text{tab } qs. \text{length } ps = \text{length } qs \Rightarrow \text{tab}(ps[\mapsto]qs @ zs) = \text{tab}(ps[\mapsto]qs)$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-upd-eq-length-simp*:

$\bigwedge \text{tab } qs \text{ x y}. \text{length } ps = \text{length } qs$   
 $\Rightarrow \text{tab}(ps[\mapsto]qs)(x \mapsto y) = \text{tab}(ps @ [x][\mapsto]qs @ [y])$   
 $\langle \text{proof} \rangle$

**lemma** *map-upd-cong*:  $\text{tab} = \text{tab}' \Rightarrow \text{tab}(x \mapsto y) = \text{tab}'(x \mapsto y)$

$\langle \text{proof} \rangle$

**lemma** *map-upd-cong-ext*:  $\text{tab } z = \text{tab}' z \Rightarrow (\text{tab}(x \mapsto y)) z = (\text{tab}'(x \mapsto y)) z$

$\langle \text{proof} \rangle$

**lemma** *map-upds-cong*:  $\text{tab} = \text{tab}' \Rightarrow \text{tab}(xs[\mapsto]ys) = \text{tab}'(xs[\mapsto]ys)$

$\langle \text{proof} \rangle$

**lemma** *map-upds-cong-ext*:

$\bigwedge \text{tab } \text{tab}' \text{ ys}. \text{tab } z = \text{tab}' z \Rightarrow (\text{tab}(xs[\mapsto]ys)) z = (\text{tab}'(xs[\mapsto]ys)) z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upd-override*:  $(\text{tab}(x \mapsto y))\ x = (\text{tab}'(x \mapsto y))\ x$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-eq-length-suffix*:  $\bigwedge\ \text{tab}\ qs.$   
 $\text{length}\ ps = \text{length}\ qs \implies \text{tab}(ps @ xs[\mapsto]qs) = \text{tab}(ps[\mapsto]qs)(xs[\mapsto][])$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-upds-eq-length-prefix-simp*:  
 $\bigwedge\ \text{tab}\ qs.\ \text{length}\ ps = \text{length}\ qs$   
 $\implies \text{tab}(ps[\mapsto]qs)(xs[\mapsto]ys) = \text{tab}(ps @ xs[\mapsto]qs @ ys)$   
 $\langle \text{proof} \rangle$

**lemma** *map-upd-cut-irrelevant*:  
 $\llbracket (\text{tab}(x \mapsto y))\ vn = \text{Some}\ el;\ (\text{tab}'(x \mapsto y))\ vn = \text{None} \rrbracket$   
 $\implies \text{tab}\ vn = \text{Some}\ el$   
 $\langle \text{proof} \rangle$

**lemma** *map-upd-Some-expand*:  
 $\llbracket \text{tab}\ vn = \text{Some}\ z \rrbracket$   
 $\implies \exists\ z.\ (\text{tab}(x \mapsto y))\ vn = \text{Some}\ z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-Some-expand*:  
 $\bigwedge\ \text{tab}\ ys\ z.\ \llbracket \text{tab}\ vn = \text{Some}\ z \rrbracket$   
 $\implies \exists\ z.\ (\text{tab}(xs[\mapsto]ys))\ vn = \text{Some}\ z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upd-Some-swap*:  
 $(\text{tab}(r \mapsto w)(u \mapsto v))\ vn = \text{Some}\ z \implies \exists\ z.\ (\text{tab}(u \mapsto v)(r \mapsto w))\ vn = \text{Some}\ z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upd-None-swap*:  
 $(\text{tab}(r \mapsto w)(u \mapsto v))\ vn = \text{None} \implies (\text{tab}(u \mapsto v)(r \mapsto w))\ vn = \text{None}$   
 $\langle \text{proof} \rangle$

**lemma** *map-eq-upd-eq*:  $\text{tab}\ vn = \text{tab}'\ vn \implies (\text{tab}(x \mapsto y))\ vn = (\text{tab}'(x \mapsto y))\ vn$   
 $\langle \text{proof} \rangle$

**lemma** *map-upd-in-expansion-map-swap*:  
 $\llbracket (\text{tab}(x \mapsto y))\ vn = \text{Some}\ z;\ \text{tab}\ vn \neq \text{Some}\ z \rrbracket$   
 $\implies (\text{tab}'(x \mapsto y))\ vn = \text{Some}\ z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-in-expansion-map-swap*:

$\bigwedge \text{tab } \text{tab}' \text{ } \text{ys } z. \llbracket (\text{tab}(xs[\mapsto]ys)) \text{ } vn = \text{Some } z; \text{tab } vn \neq \text{Some } z \rrbracket$   
 $\implies (\text{tab}'(xs[\mapsto]ys)) \text{ } vn = \text{Some } z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-Some-swap*:

**assumes**  $r\text{-}u$ :  $(\text{tab}(r[\mapsto]w)(u[\mapsto]v)(xs[\mapsto]ys)) \text{ } vn = \text{Some } z$   
**shows**  $\exists z. (\text{tab}(u[\mapsto]v)(r[\mapsto]w)(xs[\mapsto]ys)) \text{ } vn = \text{Some } z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-Some-insert*:

**assumes**  $z$ :  $(\text{tab}(xs[\mapsto]ys)) \text{ } vn = \text{Some } z$   
**shows**  $\exists z. (\text{tab}(u[\mapsto]v)(xs[\mapsto]ys)) \text{ } vn = \text{Some } z$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-None-cut*:

**assumes** *expand-None*:  $(\text{tab}(xs[\mapsto]ys)) \text{ } vn = \text{None}$   
**shows**  $\text{tab } vn = \text{None}$   
 $\langle \text{proof} \rangle$

**lemma** *map-upds-cut-irrelevant*:

$\bigwedge \text{tab } \text{tab}' \text{ } \text{ys}. \llbracket (\text{tab}(xs[\mapsto]ys)) \text{ } vn = \text{Some } \text{el}; (\text{tab}'(xs[\mapsto]ys)) \text{ } vn = \text{None} \rrbracket$   
 $\implies \text{tab } vn = \text{Some } \text{el}$   
 $\langle \text{proof} \rangle$

**lemma** *dom-vname-split*:

$\text{dom } (\text{lname-case } (\text{ename-case } (\text{tab}(x[\mapsto]y)(xs[\mapsto]ys)) \text{ } a) \text{ } b)$   
 $= \text{dom } (\text{lname-case } (\text{ename-case } (\text{tab}(x[\mapsto]y)) \text{ } a) \text{ } b) \cup$   
 $\text{dom } (\text{lname-case } (\text{ename-case } (\text{tab}(xs[\mapsto]ys)) \text{ } a) \text{ } b)$   
 $(\text{is } ?\text{List } x \text{ } xs \text{ } y \text{ } ys = ?\text{Hd } x \text{ } y \cup ?\text{Tl } xs \text{ } ys)$   
 $\langle \text{proof} \rangle$

**lemma** *dom-map-upd*:  $\bigwedge \text{tab}. \text{dom } (\text{tab}(x[\mapsto]y)) = \text{dom } \text{tab} \cup \{x\}$

$\langle \text{proof} \rangle$

**lemma** *dom-map-upds*:  $\bigwedge \text{tab } \text{ys}. \text{length } xs = \text{length } ys$

$\implies \text{dom } (\text{tab}(xs[\mapsto]ys)) = \text{dom } \text{tab} \cup \text{set } xs$

$\langle \text{proof} \rangle$

**lemma** *dom-ename-case-None-simp*:

$\text{dom } (\text{ename-case } \text{vname-tab } \text{None}) = \text{VName } ' (\text{dom } \text{vname-tab})$   
 $\langle \text{proof} \rangle$

**lemma** *dom-ename-case-Some-simp*:

$\text{dom } (\text{ename-case } \text{vname-tab } (\text{Some } a)) = \text{VName } ' (\text{dom } \text{vname-tab}) \cup \{\text{Res}\}$   
 $\langle \text{proof} \rangle$

**lemma** *dom-lname-case-None-simp*:

$\text{dom } (\text{lname-case } \text{ename-tab } \text{None}) = \text{EName } ' (\text{dom } \text{ename-tab})$   
 $\langle \text{proof} \rangle$

**lemma** *dom-lname-case-Some-simp*:

$\text{dom } (\text{lname-case } \text{ename-tab } (\text{Some } a)) = \text{EName } ' (\text{dom } \text{ename-tab}) \cup \{\text{This}\}$   
 $\langle \text{proof} \rangle$

**lemmas** *dom-lname-ename-case-simps* =

*dom-ename-case-None-simp* *dom-ename-case-Some-simp*  
*dom-lname-case-None-simp* *dom-lname-case-Some-simp*

**lemma** *image-comp*:

$f ' g ' A = (f \circ g) ' A$   
 $\langle \text{proof} \rangle$

**lemma** *dom-locals-init-lvars*:

**assumes** *m*:  $m = (\text{methd } (\text{the } (\text{methd } G \ C \ \text{sig})))$   
**assumes** *len*:  $\text{length } (\text{pars } m) = \text{length } \text{pvs}$   
**shows**  $\text{dom } (\text{locals } (\text{store } (\text{init-lvars } G \ C \ \text{sig } (\text{invmode } m \ e) \ a \ \text{pvs } s)))$   
 $= \text{parameters } m$

$\langle \text{proof} \rangle$

**lemma** *da-e2-BinOp*:

**assumes** *da*:  $(\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash \text{dom } (\text{locals } (\text{store } s0)) \gg \langle \text{BinOp } \text{binop } e1 \ e2 \rangle_e \gg A$   
**and** *wt-e1*:  $(\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash e1 :: -e1T$   
**and** *wt-e2*:  $(\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash e2 :: -e2T$   
**and** *wt-binop*:  $\text{wt-binop } G \ \text{binop } e1T \ e2T$   
**and** *conf-s0*:  $s0 :: \preceq (G, L)$   
**and** *normal-s1*:  $\text{normal } s1$   
**and** *eval-e1*:  $G \vdash s0 \ -e1 \multimap v1 \rightarrow s1$   
**and** *conf-v1*:  $G, \text{store } s1 \vdash v1 :: \preceq e1T$   
**and** *wf*:  $\text{wf-prog } G$   
**shows**  $\exists \ E2. (\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash \text{dom } (\text{locals } (\text{store } s1))$   
 $\gg (\text{if need-second-arg } \text{binop } v1 \text{ then } \langle e2 \rangle_e \text{ else } \langle \text{Skip} \rangle_s) \gg E2$

$\langle \text{proof} \rangle$

## main proof of type safety

**lemma** *eval-type-sound*:

**assumes** *eval*:  $G \vdash s0 \ -t \multimap (v, s1)$   
**and** *wt*:  $(\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash t :: T$   
**and** *da*:  $(\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash \text{dom } (\text{locals } (\text{store } s0)) \gg t \gg A$   
**and** *wf*:  $\text{wf-prog } G$   
**and** *conf-s0*:  $s0 :: \preceq (G, L)$   
**shows**  $s1 :: \preceq (G, L) \wedge (\text{normal } s1 \rightarrow G, L, \text{store } s1 \vdash t \multimap v :: \preceq T) \wedge$   
 $(\text{error-free } s0 = \text{error-free } s1)$

$\langle \text{proof} \rangle$

**corollary** *eval-type-soundE* [consumes 5]:

**assumes** *eval*:  $G \vdash s0 \ -t \multimap (v, s1)$   
**and** *conf*:  $s0 :: \preceq (G, L)$   
**and** *wt*:  $(\text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L) \vdash t :: T$



**and**  $da: (\text{prg} = G, \text{cls} = \text{acc}C, \text{lcl} = L) \vdash \text{dom} (\text{locals} (\text{snd } s0)) \gg t \gg A$   
**and**  $wf: wf\text{-prog } G$   
**and**  $\text{elim}: \llbracket s1 :: \preceq (G, L); \text{normal } s1 \implies G, L, \text{snd } s1 \vdash t \gg v :: \preceq T; \text{error-free } s0 = \text{error-free } s1 \rrbracket \implies P$   
**shows**  $P$   
 $\langle \text{proof} \rangle$

**corollary** *eval-ts*:

$\llbracket G \vdash s - e \gg v \rightarrow s'; wf\text{-prog } G; s :: \preceq (G, L); (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash e :: - T; (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash \text{dom} (\text{locals} (\text{store } s)) \gg \text{In1 } e \gg A \rrbracket$   
 $\implies s' :: \preceq (G, L) \wedge (\text{normal } s' \longrightarrow G, \text{store } s \vdash v :: \preceq T) \wedge (\text{error-free } s = \text{error-free } s')$   
 $\langle \text{proof} \rangle$

**corollary** *evals-ts*:

$\llbracket G \vdash s - es \gg vs \rightarrow s'; wf\text{-prog } G; s :: \preceq (G, L); (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash es :: \dot{=} Ts; (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash \text{dom} (\text{locals} (\text{store } s)) \gg \text{In3 } es \gg A \rrbracket$   
 $\implies s' :: \preceq (G, L) \wedge (\text{normal } s' \longrightarrow \text{list-all2 } (\text{conf } G (\text{store } s')) \text{ vs } Ts) \wedge (\text{error-free } s = \text{error-free } s')$   
 $\langle \text{proof} \rangle$

**corollary** *evar-ts*:

$\llbracket G \vdash s - v \gg vf \rightarrow s'; wf\text{-prog } G; s :: \preceq (G, L); (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash v :: = T; (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash \text{dom} (\text{locals} (\text{store } s)) \gg \text{In2 } v \gg A \rrbracket \implies$   
 $s' :: \preceq (G, L) \wedge (\text{normal } s' \longrightarrow G, L, (\text{store } s') \vdash \text{In2 } v \gg \text{In2 } vf :: \preceq \text{In1 } T) \wedge (\text{error-free } s = \text{error-free } s')$   
 $\langle \text{proof} \rangle$

**theorem** *exec-ts*:

$\llbracket G \vdash s - c \rightarrow s'; wf\text{-prog } G; s :: \preceq (G, L); (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash c :: \surd; (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash \text{dom} (\text{locals} (\text{store } s)) \gg \text{In1r } c \gg A \rrbracket$   
 $\implies s' :: \preceq (G, L) \wedge (\text{error-free } s \longrightarrow \text{error-free } s')$   
 $\langle \text{proof} \rangle$

**lemma** *wf-eval-Fin*:

**assumes**  $wf: wf\text{-prog } G$   
**and**  $wt\text{-}c1: (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash \text{In1r } c1 :: \text{In1 } (\text{PrimT } \text{Void})$   
**and**  $da\text{-}c1: (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash \text{dom} (\text{locals} (\text{store } (\text{Norm } s0))) \gg \text{In1r } c1 \gg A$   
**and**  $\text{conf}\text{-}s0: \text{Norm } s0 :: \preceq (G, L)$   
**and**  $\text{eval}\text{-}c1: G \vdash \text{Norm } s0 - c1 \rightarrow (x1, s1)$   
**and**  $\text{eval}\text{-}c2: G \vdash \text{Norm } s1 - c2 \rightarrow s2$   
**and**  $s3: s3 = \text{abupd } (\text{abrupt-if } (x1 \neq \text{None}) \ x1) \ s2$   
**shows**  $G \vdash \text{Norm } s0 - c1 \text{ Finally } c2 \rightarrow s3$   
 $\langle \text{proof} \rangle$

## 48 Ideas for the future

In the type soundness proof and the correctness proof of definite assignment we perform induction on the evaluation relation with the further preconditions that the term is welltyped and definitely assigned. During the proofs we have to establish the welltypedness and definite assignment of the subterms to be able to apply the induction hypothesis. So large parts of both proofs are the same work in propagating welltypedness and definite assignment. So we can derive a new induction rule for induction on the evaluation of a wellformed term, were these propagations is already done, once and forever. Then we can do the proofs with this rule and can enjoy the time we have saved. Here is a first and incomplete sketch of such a rule.

**theorem** *wellformed-eval-induct* [consumes 4, case-names *Abrupt Skip Expr Lab Comp If*]:

**assumes** *eval*:  $G \vdash s0 \rightarrow t \rightarrow (v, s1)$   
**and** *wt*:  $(\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \vdash t :: T$   
**and** *da*:  $(\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \vdash \text{dom} (\text{locals} (\text{store } s0)) \gg t \gg A$   
**and** *wf*: *wf-prog*  $G$   
**and** *abrupt*:  $\bigwedge s \ t \ \text{abr} \ L \ \text{acc}C \ T \ A.$   

$$\begin{aligned} & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash t :: T; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash \text{dom} (\text{locals} (\text{store} (\text{Some } \text{abr}, s))) \gg t \gg A \\ & \rrbracket \implies P \ L \ \text{acc}C \ (\text{Some } \text{abr}, s) \ t \ (\text{arbitrary3 } t) \ (\text{Some } \text{abr}, s) \end{aligned}$$
  
**and** *skip*:  $\bigwedge s \ L \ \text{acc}C. P \ L \ \text{acc}C \ (\text{Norm } s) \ \langle \text{Skip} \rangle_s \diamond (\text{Norm } s)$   
**and** *expr*:  $\bigwedge e \ s0 \ s1 \ v \ L \ \text{acc}C \ eT \ E.$   

$$\begin{aligned} & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash e :: -eT; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \\ & \quad \vdash \text{dom} (\text{locals} (\text{store} ((\text{Norm } s0)::\text{state}))) \gg \langle e \rangle_e \gg E; \\ & P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle e \rangle_e \ [v]_e \ s1 \rrbracket \\ & \implies P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle \text{Expr } e \rangle_s \diamond s1 \end{aligned}$$
  
**and** *lab*:  $\bigwedge c \ l \ s0 \ s1 \ L \ \text{acc}C \ C.$   

$$\begin{aligned} & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash c :: \sqrt{}; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \\ & \quad \vdash \text{dom} (\text{locals} (\text{store} ((\text{Norm } s0)::\text{state}))) \gg \langle c \rangle_s \gg C; \\ & P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle c \rangle_s \diamond s1 \rrbracket \\ & \implies P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle l \cdot c \rangle_s \diamond (\text{abupd} (\text{absorb } l) \ s1) \end{aligned}$$
  
**and** *comp*:  $\bigwedge c1 \ c2 \ s0 \ s1 \ s2 \ L \ \text{acc}C \ C1.$   

$$\begin{aligned} & \llbracket G \vdash \text{Norm } s0 \rightarrow c1 \rightarrow s1; G \vdash s1 \rightarrow c2 \rightarrow s2; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash c1 :: \sqrt{}; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash c2 :: \sqrt{}; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash \\ & \quad \text{dom} (\text{locals} (\text{store} ((\text{Norm } s0)::\text{state}))) \gg \langle c1 \rangle_s \gg C1; \\ & P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle c1 \rangle_s \diamond s1; \\ & \bigwedge Q. \llbracket \text{normal } s1; \\ & \quad \bigwedge C2. \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \\ & \quad \quad \vdash \text{dom} (\text{locals} (\text{store } s1)) \gg \langle c2 \rangle_s \gg C2; \\ & \quad P \ L \ \text{acc}C \ s1 \ \langle c2 \rangle_s \diamond s2 \rrbracket \implies Q \\ & \rrbracket \implies Q \\ & \rrbracket \implies P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle c1;; c2 \rangle_s \diamond s2 \end{aligned}$$
  
**and** *if*:  $\bigwedge b \ c1 \ c2 \ e \ s0 \ s1 \ s2 \ L \ \text{acc}C \ E.$   

$$\begin{aligned} & \llbracket G \vdash \text{Norm } s0 \rightarrow e \rightarrow b \rightarrow s1; \\ & G \vdash s1 \rightarrow (\text{if the-Bool } b \text{ then } c1 \text{ else } c2) \rightarrow s2; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash e :: -\text{PrimT Boolean}; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash (\text{if the-Bool } b \text{ then } c1 \text{ else } c2) :: \sqrt{}; \\ & \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash \\ & \quad \text{dom} (\text{locals} (\text{store} ((\text{Norm } s0)::\text{state}))) \gg \langle e \rangle_e \gg E; \\ & P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle e \rangle_e \ [b]_e \ s1; \\ & \bigwedge Q. \llbracket \text{normal } s1; \\ & \quad \bigwedge C. \llbracket (\text{prg}=G, \text{cls}=\text{acc}C, \text{lcl}=L) \rrbracket \vdash (\text{dom} (\text{locals} (\text{store } s1))) \\ & \quad \quad \gg \langle \text{if the-Bool } b \text{ then } c1 \text{ else } c2 \rangle_s \gg C; \\ & \quad P \ L \ \text{acc}C \ s1 \ \langle \text{if the-Bool } b \text{ then } c1 \text{ else } c2 \rangle_s \diamond s2 \\ & \rrbracket \implies Q \\ & \rrbracket \implies Q \\ & \rrbracket \implies P \ L \ \text{acc}C \ (\text{Norm } s0) \ \langle \text{If}(e) \ c1 \ \text{Else } c2 \rangle_s \diamond s2 \end{aligned}$$
  
**shows**  $P \ L \ \text{acc}C \ s0 \ t \ v \ s1$

*<proof>*

**end**

## Chapter 20

## Evaln

## 49 Operational evaluation (big-step) semantics of Java expressions and statements

theory *Evaln* imports *TypeSafe* begin

Variant of *eval* relation with counter for bounded recursive depth. In principal *evaln* could replace *eval*.

Validity of the axiomatic semantics builds on *evaln*. For recursive method calls the axiomatic semantics rule assumes the method ok to derive a proof for the body. To prove the method rule sound we need to perform induction on the recursion depth. For the completeness proof of the axiomatic semantics the notion of the most general formula is used. The most general formula right now builds on the ordinary evaluation relation *eval*. So sometimes we have to switch between *evaln* and *eval* and vice versa. To make this switch easy *evaln* also does all the technical accessibility tests *check-field-access* and *check-method-access* like *eval*. If it would omit them *evaln* and *eval* would only be equivalent for welltyped, and definitely assigned terms.

inductive

```

evaln :: [prog, state, term, nat, vals, state] ⇒ bool
  (⊢- -->---> '(-, -') [61,61,80,61,0,0] 60)
and evaln :: [prog, state, var, vvar, nat, state] ⇒ bool
  (⊢- --=>---> - [61,61,90,61,61,61] 60)
and evaln :: [prog, state, expr, val, nat, state] ⇒ bool
  (⊢- --->---> - [61,61,80,61,61,61] 60)
and evaln :: [prog, state, expr list, val list, nat, state] ⇒ bool
  (⊢- --≡>---> - [61,61,61,61,61,61] 60)
and execn :: [prog, state, stmt, nat, state] ⇒ bool
  (⊢- ----> - [61,61,65, 61,61] 60)
for G :: prog

```

where

```

G⊢s -c -n→ s' ≡ G⊢s -In1r c>-n→ (◇ , s')
| G⊢s -e->v -n→ s' ≡ G⊢s -In1l e>-n→ (In1 v , s')
| G⊢s -e=>vf -n→ s' ≡ G⊢s -In2 e>-n→ (In2 vf , s')
| G⊢s -e≡>v -n→ s' ≡ G⊢s -In3 e>-n→ (In3 v , s')

```

— propagation of abrupt completion

```

| Abrupt: G⊢(Some xc,s) -t>-n→ (arbitrary3 t,(Some xc,s))

```

— evaluation of variables

```

| LVar: G⊢Norm s -LVar vn=>lvar vn s-n→ Norm s

| FVar: [G⊢Norm s0 -Init statDeclC-n→ s1; G⊢s1 -e->a-n→ s2;
  (v,s2') = fvar statDeclC stat fn a s2;
  s3 = check-field-access G accC statDeclC fn stat a s2'] ⇒
  G⊢Norm s0 -{accC,statDeclC,stat}e..fn=>v-n→ s3

| AVar: [G⊢Norm s0 -e1->a-n→ s1 ; G⊢s1 -e2->i-n→ s2;
  (v,s2') = avar G i a s2'] ⇒
  G⊢Norm s0 -e1.[e2]=>v-n→ s2'

```

— evaluation of expressions

```

| NewC: [G⊢Norm s0 -Init C-n→ s1;

```

- $$\begin{array}{l}
G \vdash \quad s1 \text{ --halloc } (CInst \ C) \succ a \rightarrow s2 \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --NewC } C \text{ --}\succ Addr \ a \text{ --}n \rightarrow s2 \\
| \text{ NewA: } \parallel G \vdash Norm \ s0 \text{ --init-comp-ty } T \text{ --}n \rightarrow s1; \ G \vdash s1 \text{ --e--}\succ i' \text{ --}n \rightarrow s2; \\
\quad G \vdash abupd \ (check\text{-neg } i') \ s2 \text{ --halloc } (Arr \ T \ (the\text{-Intg } i')) \succ a \rightarrow s3 \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --New } T[e] \text{ --}\succ Addr \ a \text{ --}n \rightarrow s3 \\
| \text{ Cast: } \parallel G \vdash Norm \ s0 \text{ --e--}\succ v \text{ --}n \rightarrow s1; \\
\quad s2 = abupd \ (raise\text{-if } (\neg G, snd \ s1 \vdash v \text{ fits } T) \ ClassCast) \ s1 \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --Cast } T \ e \text{ --}\succ v \text{ --}n \rightarrow s2 \\
| \text{ Inst: } \parallel G \vdash Norm \ s0 \text{ --e--}\succ v \text{ --}n \rightarrow s1; \\
\quad b = (v \neq Null \wedge G, store \ s1 \vdash v \text{ fits } RefT \ T) \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --e } InstOf \ T \text{ --}\succ Bool \ b \text{ --}n \rightarrow s1 \\
| \text{ Lit: } \quad G \vdash Norm \ s \text{ --Lit } v \text{ --}\succ v \text{ --}n \rightarrow Norm \ s \\
| \text{ UnOp: } \parallel G \vdash Norm \ s0 \text{ --e--}\succ v \text{ --}n \rightarrow s1 \parallel \\
\implies G \vdash Norm \ s0 \text{ --UnOp } unop \ e \text{ --}\succ (eval\text{-unop } unop \ v) \text{ --}n \rightarrow s1 \\
| \text{ BinOp: } \parallel G \vdash Norm \ s0 \text{ --e1--}\succ v1 \text{ --}n \rightarrow s1; \\
\quad G \vdash s1 \text{ --(if need-second-arg binop v1 then (In1l e2) else (In1r Skip))} \\
\quad \text{ --}\succ \text{ --}n \rightarrow (In1 \ v2, s2) \parallel \\
\implies G \vdash Norm \ s0 \text{ --BinOp } binop \ e1 \ e2 \text{ --}\succ (eval\text{-binop } binop \ v1 \ v2) \text{ --}n \rightarrow s2 \\
| \text{ Super: } \quad G \vdash Norm \ s \text{ --Super--}\succ val\text{-this } s \text{ --}n \rightarrow Norm \ s \\
| \text{ Acc: } \parallel G \vdash Norm \ s0 \text{ --va=}\succ (v, f) \text{ --}n \rightarrow s1 \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --Acc } va \text{ --}\succ v \text{ --}n \rightarrow s1 \\
| \text{ Ass: } \parallel G \vdash Norm \ s0 \text{ --va=}\succ (w, f) \text{ --}n \rightarrow s1; \\
\quad G \vdash \quad s1 \text{ --e--}\succ v \quad \text{ --}n \rightarrow s2 \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --va:=e--}\succ v \text{ --}n \rightarrow assign \ f \ v \ s2 \\
| \text{ Cond: } \parallel G \vdash Norm \ s0 \text{ --e0--}\succ b \text{ --}n \rightarrow s1; \\
\quad G \vdash \quad s1 \text{ --(if the-Bool b then e1 else e2)--}\succ v \text{ --}n \rightarrow s2 \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --e0 ? e1 : e2--}\succ v \text{ --}n \rightarrow s2 \\
| \text{ Call: } \\
\parallel G \vdash Norm \ s0 \text{ --e--}\succ a' \text{ --}n \rightarrow s1; \ G \vdash s1 \text{ --args=}\succ vs \text{ --}n \rightarrow s2; \\
\quad D = invocation\text{-declclass } G \ mode \ (store \ s2) \ a' \ statT \ (\llbracket name=mn, parTs=pTs \rrbracket); \\
\quad s3 = init\text{-lvars } G \ D \ (\llbracket name=mn, parTs=pTs \rrbracket) \ mode \ a' \ vs \ s2; \\
\quad s3' = check\text{-method-access } G \ accC \ statT \ mode \ (\llbracket name=mn, parTs=pTs \rrbracket) \ a' \ s3; \\
\quad G \vdash s3' \text{ --Methd } D \ (\llbracket name=mn, parTs=pTs \rrbracket) \text{ --}\succ v \text{ --}n \rightarrow s4 \\
\parallel \\
\implies \\
\quad G \vdash Norm \ s0 \text{ --}\{accC, statT, mode\}e.mn(\{pTs\}args)\text{ --}\succ v \text{ --}n \rightarrow (restore\text{-lvars } s2 \ s4) \\
| \text{ Methd: } \parallel G \vdash Norm \ s0 \text{ --body } G \ D \ sig \text{ --}\succ v \text{ --}n \rightarrow s1 \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --Methd } D \ sig \text{ --}\succ v \text{ --}Suc \ n \rightarrow s1 \\
| \text{ Body: } \parallel G \vdash Norm \ s0 \text{ --Init } D \text{ --}n \rightarrow s1; \ G \vdash s1 \text{ --c--}n \rightarrow s2; \\
\quad s3 = (if \ (\exists \ l. abrupt \ s2 = Some \ (Jump \ (Break \ l))) \vee \\
\quad \quad abrupt \ s2 = Some \ (Jump \ (Cont \ l))) \\
\quad \quad then \ abupd \ (\lambda \ x. Some \ (Error \ CrossMethodJump)) \ s2 \\
\quad \quad else \ s2) \parallel \implies \\
\quad G \vdash Norm \ s0 \text{ --Body } D \ c \\
\quad \text{ --}\succ the \ (locals \ (store \ s2) \ Result) \text{ --}n \rightarrow abupd \ (absorb \ Ret) \ s3
\end{array}$$

— evaluation of expression lists

| *Nil*:

$$G \vdash \text{Norm } s0 \text{ --} [\dot{=} \dot{\succ}] \text{ --} n \rightarrow \text{Norm } s0$$

| *Cons*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} e \dot{\succ} v \text{ --} n \rightarrow s1;$

$$G \vdash s1 \text{ --} es \dot{=} \dot{\succ} vs \text{ --} n \rightarrow s2 \rrbracket \implies G \vdash \text{Norm } s0 \text{ --} e \# es \dot{=} \dot{\succ} v \# vs \text{ --} n \rightarrow s2$$

— execution of statements

| *Skip*:

$$G \vdash \text{Norm } s \text{ --} \text{Skip} \text{ --} n \rightarrow \text{Norm } s$$

| *Expr*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} e \dot{\succ} v \text{ --} n \rightarrow s1 \rrbracket \implies$

$$G \vdash \text{Norm } s0 \text{ --} \text{Expr } e \text{ --} n \rightarrow s1$$

| *Lab*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} c \text{ --} n \rightarrow s1 \rrbracket \implies$

$$G \vdash \text{Norm } s0 \text{ --} l \cdot c \text{ --} n \rightarrow \text{abupd } (\text{absorb } l) s1$$

| *Comp*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} c1 \text{ --} n \rightarrow s1;$

$$G \vdash s1 \text{ --} c2 \text{ --} n \rightarrow s2 \rrbracket \implies G \vdash \text{Norm } s0 \text{ --} c1;; c2 \text{ --} n \rightarrow s2$$

| *If*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} e \dot{\succ} b \text{ --} n \rightarrow s1;$

$$G \vdash s1 \text{ --} (\text{if the-Bool } b \text{ then } c1 \text{ else } c2) \text{ --} n \rightarrow s2 \rrbracket \implies G \vdash \text{Norm } s0 \text{ --} \text{If}(e) c1 \text{ Else } c2 \text{ --} n \rightarrow s2$$

| *Loop*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} e \dot{\succ} b \text{ --} n \rightarrow s1;$

$$\begin{aligned} & \text{if the-Bool } b \\ & \text{then } (G \vdash s1 \text{ --} c \text{ --} n \rightarrow s2 \wedge \\ & \quad G \vdash (\text{abupd } (\text{absorb } (\text{Cont } l)) s2) \text{ --} l \cdot \text{While}(e) c \text{ --} n \rightarrow s3) \\ & \text{else } s3 = s1 \rrbracket \implies G \vdash \text{Norm } s0 \text{ --} l \cdot \text{While}(e) c \text{ --} n \rightarrow s3 \end{aligned}$$

| *Jmp*:  $G \vdash \text{Norm } s \text{ --} \text{Jmp } j \text{ --} n \rightarrow (\text{Some } (\text{Jump } j), s)$

| *Throw*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} e \dot{\succ} a' \text{ --} n \rightarrow s1 \rrbracket \implies$

$$G \vdash \text{Norm } s0 \text{ --} \text{Throw } e \text{ --} n \rightarrow \text{abupd } (\text{throw } a') s1$$

| *Try*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} c1 \text{ --} n \rightarrow s1; G \vdash s1 \text{ --} \text{salloc} \rightarrow s2;$

$$\begin{aligned} & \text{if } G, s2 \vdash \text{catch } tn \text{ then } G \vdash \text{new-xcpt-var } vn \text{ } s2 \text{ --} c2 \text{ --} n \rightarrow s3 \text{ else } s3 = s2 \rrbracket \\ & \implies G \vdash \text{Norm } s0 \text{ --} \text{Try } c1 \text{ Catch}(tn \text{ } vn) \text{ } c2 \text{ --} n \rightarrow s3 \end{aligned}$$

| *Fin*:  $\llbracket G \vdash \text{Norm } s0 \text{ --} c1 \text{ --} n \rightarrow (x1, s1);$

$$\begin{aligned} & G \vdash \text{Norm } s1 \text{ --} c2 \text{ --} n \rightarrow s2; \\ & s3 = (\text{if } (\exists \text{ err. } x1 = \text{Some } (\text{Error } \text{err})) \\ & \quad \text{then } (x1, s1) \\ & \quad \text{else } \text{abupd } (\text{abrupt-if } (x1 \neq \text{None}) x1) s2) \rrbracket \implies \\ & G \vdash \text{Norm } s0 \text{ --} c1 \text{ Finally } c2 \text{ --} n \rightarrow s3 \end{aligned}$$

| *Init*:  $\llbracket \text{the } (\text{class } G \text{ } C) = c;$

$$\begin{aligned} & \text{if inited } C \text{ (globs } s0) \text{ then } s3 = \text{Norm } s0 \\ & \text{else } (G \vdash \text{Norm } (\text{init-class-obj } G \text{ } C \text{ } s0) \\ & \quad \text{--} (\text{if } C = \text{Object then Skip else Init } (\text{super } c)) \text{ --} n \rightarrow s1 \wedge \\ & \quad G \vdash \text{set-lvars empty } s1 \text{ --} \text{init } c \text{ --} n \rightarrow s2 \wedge \\ & \quad s3 = \text{restore-lvars } s1 \text{ } s2) \rrbracket \\ & \implies \end{aligned}$$

$$G \vdash \text{Norm } s0 \text{ --Init } C \text{ --}n \rightarrow s3$$
**monos***if-bool-eq-conj*

**declare** *split-if* [*split del*] *split-if-asm* [*split del*]  
*option.split* [*split del*] *option.split-asm* [*split del*]  
*not-None-eq* [*simp del*]  
*split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]  
 ⟨*ML*⟩

**inductive-cases** *evaln-cases*:  $G \vdash s \text{ --}t \text{ --}n \rightarrow (v, s')$

**inductive-cases** *evaln-elim-cases*:

$G \vdash (\text{Some } xc, s) \text{ --}t$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1r Skip}$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1r (Jmp } j)$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1r (l. } c)$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In3 } ([\ ])$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In3 } (e \# es)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (Lit } w)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (UnOp unop } e)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (BinOp binop } e1 \ e2)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In2 (LVar } vn)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (Cast } T \ e)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (e InstOf } T)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (Super)}$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (Acc } va)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1r (Expr } e)$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1r (c1;; c2)}$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1l (Methd } C \ \text{sig})$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1l (Body } D \ c)$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1l (e0 ? e1 : e2)}$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1r (If(e) c1 Else c2)}$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1r (l. While(e) c)}$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1r (c1 Finally c2)}$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1r (Throw } e)$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1l (NewC } C)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (New } T[e])$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l (Ass } va \ e)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1r (Try c1 Catch(tn vn) c2)}$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In2 } (\{accC, statDeclC, stat\}e..fn)$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In2 } (e1.[e2])$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1l } (\{accC, statT, mode\}e.mn(\{pT\}p))$	$\succ \text{--}n \rightarrow (v, s')$
$G \vdash \text{Norm } s \text{ --In1r (Init } C)$	$\succ \text{--}n \rightarrow (x, s')$
$G \vdash \text{Norm } s \text{ --In1r (Init } C)$	$\succ \text{--}n \rightarrow (x, s')$

**declare** *split-if* [*split*] *split-if-asm* [*split*]  
*option.split* [*split*] *option.split-asm* [*split*]  
*not-None-eq* [*simp*]  
*split-paired-All* [*simp*] *split-paired-Ex* [*simp*]  
 ⟨*ML*⟩

**lemma** *evaln-Inj-elim*:  $G \vdash s \text{ --}t \text{ --}n \rightarrow (w, s') \implies \text{case } t \text{ of In1 } ec \Rightarrow$   
 (case *ec* of *Inl* *e*  $\Rightarrow (\exists v. w = \text{In1 } v) \mid \text{Inr } c \Rightarrow w = \Diamond$ )  
 $\mid \text{In2 } e \Rightarrow (\exists v. w = \text{In2 } v) \mid \text{In3 } e \Rightarrow (\exists v. w = \text{In3 } v)$   
 ⟨*proof*⟩

The following simplification procedures set up the proper injections of terms and their corresponding values in the evaluation relation: E.g. an expression (injection *In1l* into terms) always evaluates to ordinary values (injection *In1* into generalised values *vals*).

**lemma** *evaln-expr-eq*:  $G \vdash s - \text{In1l } t \succ - n \rightarrow (w, s') = (\exists v. w = \text{In1 } v \wedge G \vdash s - t \dot{=} \succ v - n \rightarrow s')$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-var-eq*:  $G \vdash s - \text{In2 } t \succ - n \rightarrow (w, s') = (\exists vf. w = \text{In2 } vf \wedge G \vdash s - t = \succ vf - n \rightarrow s')$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-exprs-eq*:  $G \vdash s - \text{In3 } t \succ - n \rightarrow (w, s') = (\exists vs. w = \text{In3 } vs \wedge G \vdash s - t \dot{=} \succ vs - n \rightarrow s')$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-stmt-eq*:  $G \vdash s - \text{In1r } t \succ - n \rightarrow (w, s') = (w = \Diamond \wedge G \vdash s - t - n \rightarrow s')$   
 $\langle \text{proof} \rangle$

$\langle \text{ML} \rangle$

**declare** *evaln-AbruptIs* [intro!]

**lemma** *evaln-Callee*:  $G \vdash \text{Norm } s - \text{In1l } (\text{Callee } l \ e) \succ - n \rightarrow (v, s') = \text{False}$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-InsInitE*:  $G \vdash \text{Norm } s - \text{In1l } (\text{InsInitE } c \ e) \succ - n \rightarrow (v, s') = \text{False}$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-InsInitV*:  $G \vdash \text{Norm } s - \text{In2 } (\text{InsInitV } c \ w) \succ - n \rightarrow (v, s') = \text{False}$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-FinA*:  $G \vdash \text{Norm } s - \text{In1r } (\text{FinA } a \ c) \succ - n \rightarrow (v, s') = \text{False}$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-abrupt-lemma*:  $G \vdash s - e \succ - n \rightarrow (v, s') \implies$   
 $\text{fst } s = \text{Some } xc \longrightarrow s' = s \wedge v = \text{arbitrary3 } e$   
 $\langle \text{proof} \rangle$

**lemma** *evaln-abrupt*:  
 $\bigwedge s'. G \vdash (\text{Some } xc, s) - e \succ - n \rightarrow (w, s') = (s' = (\text{Some } xc, s) \wedge$   
 $w = \text{arbitrary3 } e \wedge G \vdash (\text{Some } xc, s) - e \succ - n \rightarrow (\text{arbitrary3 } e, (\text{Some } xc, s)))$   
 $\langle \text{proof} \rangle$

$\langle \text{ML} \rangle$

**lemma** *evaln-LitI*:  $G \vdash s - \text{Lit } v \dot{=} \succ (\text{if normal } s \text{ then } v \text{ else arbitrary}) - n \rightarrow s$   
 $\langle \text{proof} \rangle$

**lemma** *CondI*:  
 $\bigwedge s1. \llbracket G \vdash s - e \dot{=} \succ b - n \rightarrow s1; G \vdash s1 - (\text{if the-Bool } b \text{ then } e1 \text{ else } e2) \dot{=} \succ v - n \rightarrow s2 \rrbracket \implies$   
 $G \vdash s - e \ ? \ e1 : e2 \dot{=} \succ (\text{if normal } s1 \text{ then } v \text{ else arbitrary}) - n \rightarrow s2$



$\langle \text{proof} \rangle$

**lemma** *evaln-SkipI* [intro!]:  $G \vdash s - \text{Skip} - n \rightarrow s$

$\langle \text{proof} \rangle$

**lemma** *evaln-ExprI*:  $G \vdash s - e - \succ v - n \rightarrow s' \implies G \vdash s - \text{Expr } e - n \rightarrow s'$

$\langle \text{proof} \rangle$

**lemma** *evaln-CompI*:  $\llbracket G \vdash s - c1 - n \rightarrow s1; G \vdash s1 - c2 - n \rightarrow s2 \rrbracket \implies G \vdash s - c1;; c2 - n \rightarrow s2$

$\langle \text{proof} \rangle$

**lemma** *evaln-IfI*:

$\llbracket G \vdash s - e - \succ v - n \rightarrow s1; G \vdash s1 - (\text{if the-Bool } v \text{ then } c1 \text{ else } c2) - n \rightarrow s2 \rrbracket \implies$   
 $G \vdash s - \text{If}(e) \ c1 \ \text{Else } c2 - n \rightarrow s2$

$\langle \text{proof} \rangle$

**lemma** *evaln-SkipD* [dest!]:  $G \vdash s - \text{Skip} - n \rightarrow s' \implies s' = s$

$\langle \text{proof} \rangle$

**lemma** *evaln-Skip-eq* [simp]:  $G \vdash s - \text{Skip} - n \rightarrow s' = (s = s')$

$\langle \text{proof} \rangle$

## evaln implies eval

**lemma** *evaln-eval*:

**assumes** *evaln*:  $G \vdash s0 - t \succ - n \rightarrow (v, s1)$

**shows**  $G \vdash s0 - t \succ \rightarrow (v, s1)$

$\langle \text{proof} \rangle$

**lemma** *Suc-le-D-lemma*:  $\llbracket \text{Suc } n \leq m'; (\bigwedge m. n \leq m \implies P (\text{Suc } m)) \rrbracket \implies P m'$

$\langle \text{proof} \rangle$

**lemma** *evaln-nonstrict* [rule-format (no-asm), elim]:

$G \vdash s - t \succ - n \rightarrow (w, s') \implies \forall m. n \leq m \longrightarrow G \vdash s - t \succ - m \rightarrow (w, s')$

$\langle \text{proof} \rangle$

**lemmas** *evaln-nonstrict-Suc* = *evaln-nonstrict* [OF - le-refl [THEN le-SucI]]

**lemma** *evaln-max2*:  $\llbracket G \vdash s1 - t1 \succ - n1 \rightarrow (w1, s1'); G \vdash s2 - t2 \succ - n2 \rightarrow (w2, s2') \rrbracket \implies$

$G \vdash s1 - t1 \succ - \max n1 \ n2 \rightarrow (w1, s1') \wedge G \vdash s2 - t2 \succ - \max n1 \ n2 \rightarrow (w2, s2')$

$\langle \text{proof} \rangle$

**corollary** *evaln-max2E* [consumes 2]:

$\llbracket G \vdash s1 - t1 \succ - n1 \rightarrow (w1, s1'); G \vdash s2 - t2 \succ - n2 \rightarrow (w2, s2') \rrbracket$

$\llbracket G \vdash s1 - t1 \succ - \max n1 \ n2 \rightarrow (w1, s1'); G \vdash s2 - t2 \succ - \max n1 \ n2 \rightarrow (w2, s2') \rrbracket \implies P \rrbracket \implies P$

$\langle \text{proof} \rangle$

**lemma** *evaln-max3*:

$\llbracket G \vdash s1 -t1 \succ -n1 \rightarrow (w1, s1'); G \vdash s2 -t2 \succ -n2 \rightarrow (w2, s2'); G \vdash s3 -t3 \succ -n3 \rightarrow (w3, s3') \rrbracket \implies$   
 $G \vdash s1 -t1 \succ -\max (\max n1 n2) n3 \rightarrow (w1, s1') \wedge$   
 $G \vdash s2 -t2 \succ -\max (\max n1 n2) n3 \rightarrow (w2, s2') \wedge$   
 $G \vdash s3 -t3 \succ -\max (\max n1 n2) n3 \rightarrow (w3, s3')$   
 $\langle \text{proof} \rangle$

**corollary** *evaln-max3E*:

$\llbracket G \vdash s1 -t1 \succ -n1 \rightarrow (w1, s1'); G \vdash s2 -t2 \succ -n2 \rightarrow (w2, s2'); G \vdash s3 -t3 \succ -n3 \rightarrow (w3, s3');$   
 $\llbracket G \vdash s1 -t1 \succ -\max (\max n1 n2) n3 \rightarrow (w1, s1');$   
 $G \vdash s2 -t2 \succ -\max (\max n1 n2) n3 \rightarrow (w2, s2');$   
 $G \vdash s3 -t3 \succ -\max (\max n1 n2) n3 \rightarrow (w3, s3') \rrbracket \implies P$   
 $\llbracket \implies P$   
 $\langle \text{proof} \rangle$

**lemma** *le-max3I1*:  $(n2::nat) \leq \max n1 (\max n2 n3)$   
 $\langle \text{proof} \rangle$

**lemma** *le-max3I2*:  $(n3::nat) \leq \max n1 (\max n2 n3)$   
 $\langle \text{proof} \rangle$

**declare**  $\llbracket [\text{simproc del: wt-expr wt-var wt-exprs wt-stmt}] \rrbracket$

**eval implies evaln**

**lemma** *eval-evaln*:

**assumes** *eval*:  $G \vdash s0 -t \succ \rightarrow (v, s1)$   
**shows**  $\exists n. G \vdash s0 -t \succ -n \rightarrow (v, s1)$   
 $\langle \text{proof} \rangle$

**end**

# Chapter 21

## Trans

**theory** *Trans* **imports** *Evaln* **begin**

**constdefs** *groundVar*:: *var*  $\Rightarrow$  *bool*  
*groundVar* *v*  $\equiv$  (case *v* of  
  *LVar* *ln*  $\Rightarrow$  *True*  
  | {*accC*,*statDeclC*,*stat*}*e*..*fn*  $\Rightarrow$   $\exists$  *a*. *e*=*Lit* *a*  
  | *e1*..*e2*  $\Rightarrow$   $\exists$  *a* *i*. *e1* = *Lit* *a*  $\wedge$  *e2* = *Lit* *i*  
  | *InsInitV* *c* *v*  $\Rightarrow$  *False*)

**lemma** *groundVar-cases* [consumes 1, case-names *LVar FVar AVar*]:

**assumes** *ground*: *groundVar* *v* **and**  
    *LVar*:  $\bigwedge$  *ln*.  $\llbracket v = \text{LVar } ln \rrbracket \Longrightarrow P$  **and**  
    *FVar*:  $\bigwedge$  *accC statDeclC stat a fn*.  
       $\llbracket v = \{accC, statDeclC, stat\} (Lit\ a) .. fn \rrbracket \Longrightarrow P$  **and**  
    *AVar*:  $\bigwedge$  *a i*.  $\llbracket v = (Lit\ a) .. [Lit\ i] \rrbracket \Longrightarrow P$   
  **shows** *P*  
*<proof>*

**constdefs** *groundExprs*:: *expr list*  $\Rightarrow$  *bool*  
*groundExprs* *es*  $\equiv$  *list-all* ( $\lambda$  *e*.  $\exists$  *v*. *e*=*Lit* *v*) *es*

**consts** *the-val*:: *expr*  $\Rightarrow$  *val*

**primrec**

*the-val* (*Lit* *v*) = *v*

**consts** *the-var*:: *prog*  $\Rightarrow$  *state*  $\Rightarrow$  *var*  $\Rightarrow$  (*vvar*  $\times$  *state*)

**primrec**

*the-var* *G* *s* (*LVar* *ln*) = (*lvar* *ln* (*store* *s*), *s*)

*the-var-FVar-def*:

*the-var* *G* *s* ({*accC*,*statDeclC*,*stat*}*a*..*fn*) = *fvar* *statDeclC* *stat* *fn* (*the-val* *a*) *s*

*the-var-AVar-def*:

*the-var* *G* *s* (*a*..*i*) = *avar* *G* (*the-val* *i*) (*the-val* *a*) *s*

**lemma** *the-var-FVar-simp* [*simp*]:

*the-var* *G* *s* ({*accC*,*statDeclC*,*stat*}(*Lit* *a*)..*fn*) = *fvar* *statDeclC* *stat* *fn* *a* *s*

*<proof>*

**declare** *the-var-FVar-def* [*simp del*]

**lemma** *the-var-AVar-simp*:

*the-var*  $G\ s\ ((Lit\ a).[Lit\ i]) = avar\ G\ i\ a\ s$   
 $\langle proof \rangle$

**declare** *the-var-AVar-def* [*simp del*]

**syntax** (*xsymbols*)

*Ref*  $:: loc \Rightarrow expr$

*SKIP*  $:: expr$

**translations**

*Ref*  $a == Lit\ (Addr\ a)$

*SKIP*  $== Lit\ Unit$

**inductive**

*step*  $:: [prog, term \times state, term \times state] \Rightarrow bool\ (\vdash \mapsto 1\ [61, 82, 82]\ 81)$

**for**  $G :: prog$

**where**

*Abrupt*:  
 $\llbracket \forall v. t \neq \langle Lit\ v \rangle;$   
 $\forall t. t \neq \langle l \cdot Skip \rangle;$   
 $\forall C\ vn\ c. t \neq \langle Try\ Skip\ Catch(C\ vn)\ c \rangle;$   
 $\forall x\ c. t \neq \langle Skip\ Finally\ c \rangle \wedge xc \neq Xcpt\ x;$   
 $\forall a\ c. t \neq \langle FinA\ a\ c \rangle \rrbracket$   
 $\implies$   
 $G \vdash (t, Some\ xc, s) \mapsto 1\ (\langle Lit\ arbitrary \rangle, Some\ xc, s)$

| *InsInitE*:  $\llbracket G \vdash (\langle c \rangle, Norm\ s) \mapsto 1\ (\langle c^\wedge, s' \rangle) \rrbracket$   
 $\implies$   
 $G \vdash (\langle InsInitE\ c\ e \rangle, Norm\ s) \mapsto 1\ (\langle InsInitE\ c' e \rangle, s')$

| *NewC*:  $G \vdash (\langle NewC\ C \rangle, Norm\ s) \mapsto 1\ (\langle InsInitE\ (Init\ C)\ (NewC\ C) \rangle, Norm\ s)$   
| *NewCInitd*:  $\llbracket G \vdash Norm\ s -halloc\ (CInst\ C) \succ a \rightarrow s' \rrbracket$   
 $\implies$   
 $G \vdash (\langle InsInitE\ Skip\ (NewC\ C) \rangle, Norm\ s) \mapsto 1\ (\langle Ref\ a \rangle, s')$

| *NewA*:  
 $G \vdash (\langle New\ T[e] \rangle, Norm\ s) \mapsto 1\ (\langle InsInitE\ (init-comp-ty\ T)\ (New\ T[e]) \rangle, Norm\ s)$   
| *InsInitNewAIdx*:  
 $\llbracket G \vdash (\langle e \rangle, Norm\ s) \mapsto 1\ (\langle e^\wedge, s' \rangle) \rrbracket$   
 $\implies$   
 $G \vdash (\langle InsInitE\ Skip\ (New\ T[e]) \rangle, Norm\ s) \mapsto 1\ (\langle InsInitE\ Skip\ (New\ T[e^\wedge]) \rangle, s')$   
| *InsInitNewA*:  
 $\llbracket G \vdash abupd\ (check-neg\ i)\ (Norm\ s) -halloc\ (Arr\ T\ (the-Intg\ i)) \succ a \rightarrow s' \rrbracket$   
 $\implies$   
 $G \vdash (\langle InsInitE\ Skip\ (New\ T[Lit\ i]) \rangle, Norm\ s) \mapsto 1\ (\langle Ref\ a \rangle, s')$

| *CastE*:  
 $\llbracket G \vdash (\langle e \rangle, Norm\ s) \mapsto 1\ (\langle e^\wedge, s' \rangle) \rrbracket$   
 $\implies$

$$\begin{array}{l}
G \vdash (\langle \text{Cast } T \ e \rangle, \text{None}, s) \mapsto 1 \ (\langle \text{Cast } T \ e \rangle, s') \\
| \text{Cast:} \quad \llbracket s' = \text{abupd } (\text{raise-if } (\neg G, s \vdash v \text{ fits } T) \ \text{ClassCast}) \ (\text{Norm } s) \rrbracket \\
\quad \implies \\
G \vdash (\langle \text{Cast } T \ (\text{Lit } v) \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Lit } v \rangle, s') \\
\\
| \text{InstE:} \quad \llbracket G \vdash (\langle e \rangle, \text{Norm } s) \mapsto 1 \ (\langle e'::\text{expr} \rangle, s') \rrbracket \\
\quad \implies \\
G \vdash (\langle e \ \text{InstOf } T \rangle, \text{Norm } s) \mapsto 1 \ (\langle e \rangle, s') \\
| \text{Inst:} \quad \llbracket b = (v \neq \text{Null} \wedge G, s \vdash v \text{ fits RefT } T) \rrbracket \\
\quad \implies \\
G \vdash (\langle (\text{Lit } v) \ \text{InstOf } T \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Lit } (\text{Bool } b) \rangle, s') \\
\\
| \text{UnOpE:} \quad \llbracket G \vdash (\langle e \rangle, \text{Norm } s) \mapsto 1 \ (\langle e \rangle, s') \rrbracket \\
\quad \implies \\
G \vdash (\langle \text{UnOp unop } e \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{UnOp unop } e \rangle, s') \\
| \text{UnOp:} \quad G \vdash (\langle \text{UnOp unop } (\text{Lit } v) \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Lit } (\text{eval-unop unop } v) \rangle, \text{Norm } s) \\
\\
| \text{BinOpE1:} \quad \llbracket G \vdash (\langle e1 \rangle, \text{Norm } s) \mapsto 1 \ (\langle e1 \rangle, s') \rrbracket \\
\quad \implies \\
G \vdash (\langle \text{BinOp binop } e1 \ e2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{BinOp binop } e1' \ e2 \rangle, s') \\
| \text{BinOpE2:} \quad \llbracket \text{need-second-arg binop } v1; G \vdash (\langle e2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle e2 \rangle, s') \rrbracket \\
\quad \implies \\
G \vdash (\langle \text{BinOp binop } (\text{Lit } v1) \ e2 \rangle, \text{Norm } s) \\
\quad \mapsto 1 \ (\langle \text{BinOp binop } (\text{Lit } v1) \ e2 \rangle, s') \\
| \text{BinOpTerm:} \quad \llbracket \neg \text{need-second-arg binop } v1 \rrbracket \\
\quad \implies \\
G \vdash (\langle \text{BinOp binop } (\text{Lit } v1) \ e2 \rangle, \text{Norm } s) \\
\quad \mapsto 1 \ (\langle \text{Lit } v1 \rangle, \text{Norm } s) \\
| \text{BinOp:} \quad G \vdash (\langle \text{BinOp binop } (\text{Lit } v1) \ (\text{Lit } v2) \rangle, \text{Norm } s) \\
\quad \mapsto 1 \ (\langle \text{Lit } (\text{eval-binop binop } v1 \ v2) \rangle, \text{Norm } s) \\
\\
| \text{Super:} \quad G \vdash (\langle \text{Super} \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Lit } (\text{val-this } s) \rangle, \text{Norm } s) \\
\\
| \text{AccVA:} \quad \llbracket G \vdash (\langle va \rangle, \text{Norm } s) \mapsto 1 \ (\langle va \rangle, s') \rrbracket \\
\quad \implies \\
G \vdash (\langle \text{Acc } va \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Acc } va \rangle, s') \\
| \text{Acc:} \quad \llbracket \text{groundVar } va; ((v, vf), s') = \text{the-var } G \ (\text{Norm } s) \ va \rrbracket \\
\quad \implies \\
G \vdash (\langle \text{Acc } va \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Lit } v \rangle, s') \\
\\
| \text{AssVA:} \quad \llbracket G \vdash (\langle va \rangle, \text{Norm } s) \mapsto 1 \ (\langle va \rangle, s') \rrbracket \\
\quad \implies \\
G \vdash (\langle va := e \rangle, \text{Norm } s) \mapsto 1 \ (\langle va' := e \rangle, s') \\
| \text{AssE:} \quad \llbracket \text{groundVar } va; G \vdash (\langle e \rangle, \text{Norm } s) \mapsto 1 \ (\langle e \rangle, s') \rrbracket \\
\quad \implies \\
G \vdash (\langle va := e \rangle, \text{Norm } s) \mapsto 1 \ (\langle va := e \rangle, s') \\
| \text{Ass:} \quad \llbracket \text{groundVar } va; ((w, f), s') = \text{the-var } G \ (\text{Norm } s) \ va \rrbracket \\
\quad \implies \\
G \vdash (\langle va := (\text{Lit } v) \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Lit } v \rangle, \text{assign } f \ v \ s') \\
\\
| \text{CondC:} \quad \llbracket G \vdash (\langle e0 \rangle, \text{Norm } s) \mapsto 1 \ (\langle e0 \rangle, s') \rrbracket
\end{array}$$

	$\begin{aligned} &\Rightarrow \\ &G \vdash (\langle e0? e1:e2 \rangle, Norm\ s) \mapsto 1\ (\langle e0'? e1:e2 \rangle, s') \\   \text{Cond: } &G \vdash (\langle Lit\ b? e1:e2 \rangle, Norm\ s) \mapsto 1\ (\langle if\ the\text{-}Bool\ b\ then\ e1\ else\ e2 \rangle, Norm\ s) \\ \\   \text{CallTarget: } &\llbracket G \vdash (\langle e \rangle, Norm\ s) \mapsto 1\ (\langle e' \rangle, s') \rrbracket \\ &\Rightarrow \\ &G \vdash (\langle \{accC, statT, mode\} e \cdot mn(\{pTs\} args) \rangle, Norm\ s) \\ &\mapsto 1\ (\langle \{accC, statT, mode\} e' \cdot mn(\{pTs\} args) \rangle, s') \\   \text{CallArgs: } &\llbracket G \vdash (\langle args \rangle, Norm\ s) \mapsto 1\ (\langle args' \rangle, s') \rrbracket \\ &\Rightarrow \\ &G \vdash (\langle \{accC, statT, mode\} Lit\ a \cdot mn(\{pTs\} args) \rangle, Norm\ s) \\ &\mapsto 1\ (\langle \{accC, statT, mode\} Lit\ a \cdot mn(\{pTs\} args') \rangle, s') \\   \text{Call: } &\llbracket groundExprs\ args; vs = map\ the\text{-}val\ args; \\ &D = invocation\text{-}declclass\ G\ mode\ s\ a\ statT\ (\llbracket name=mn, parTs=pTs \rrbracket); \\ &s' = init\text{-}lwars\ G\ D\ (\llbracket name=mn, parTs=pTs \rrbracket)\ mode\ a'\ vs\ (Norm\ s) \rrbracket \\ &\Rightarrow \\ &G \vdash (\langle \{accC, statT, mode\} Lit\ a \cdot mn(\{pTs\} args) \rangle, Norm\ s) \\ &\mapsto 1\ (\langle Callee\ (locals\ s)\ (Methd\ D\ (\llbracket name=mn, parTs=pTs \rrbracket)) \rangle, s') \\ \\   \text{Callee: } &\llbracket G \vdash (\langle e \rangle, Norm\ s) \mapsto 1\ (\langle e'::expr \rangle, s') \rrbracket \\ &\Rightarrow \\ &G \vdash (\langle Callee\ lcls\text{-}caller\ e \rangle, Norm\ s) \mapsto 1\ (\langle e' \rangle, s') \\ \\   \text{CalleeRet: } &G \vdash (\langle Callee\ lcls\text{-}caller\ (Lit\ v) \rangle, Norm\ s) \\ &\mapsto 1\ (\langle Lit\ v \rangle, (set\text{-}lwars\ lcls\text{-}caller\ (Norm\ s))) \\ \\   \text{Methd: } &G \vdash (\langle Methd\ D\ sig \rangle, Norm\ s) \mapsto 1\ (\langle body\ G\ D\ sig \rangle, Norm\ s) \\ \\   \text{Body: } &G \vdash (\langle Body\ D\ c \rangle, Norm\ s) \mapsto 1\ (\langle InsInitE\ (Init\ D)\ (Body\ D\ c) \rangle, Norm\ s) \\ \\   \text{InsInitBody: } &\llbracket G \vdash (\langle c \rangle, Norm\ s) \mapsto 1\ (\langle c' \rangle, s') \rrbracket \\ &\Rightarrow \\ &G \vdash (\langle InsInitE\ Skip\ (Body\ D\ c) \rangle, Norm\ s) \mapsto 1\ (\langle InsInitE\ Skip\ (Body\ D\ c') \rangle, s') \\   \text{InsInitBodyRet: } &G \vdash (\langle InsInitE\ Skip\ (Body\ D\ Skip) \rangle, Norm\ s) \\ &\mapsto 1\ (\langle Lit\ (the\ ((locals\ s)\ Result)) \rangle, abupd\ (absorb\ Ret)\ (Norm\ s)) \\ \\   \text{FVar: } &\llbracket \neg\ initied\ statDeclC\ (globs\ s) \rrbracket \\ &\Rightarrow \\ &G \vdash (\langle \{accC, statDeclC, stat\} e \cdot fn \rangle, Norm\ s) \\ &\mapsto 1\ (\langle InsInitV\ (Init\ statDeclC)\ (\{accC, statDeclC, stat\} e \cdot fn) \rangle, Norm\ s) \\   \text{InsInitFVarE: } &\llbracket G \vdash (\langle e \rangle, Norm\ s) \mapsto 1\ (\langle e' \rangle, s') \rrbracket \\ &\Rightarrow \\ &G \vdash (\langle InsInitV\ Skip\ (\{accC, statDeclC, stat\} e \cdot fn) \rangle, Norm\ s) \\ &\mapsto 1\ (\langle InsInitV\ Skip\ (\{accC, statDeclC, stat\} e' \cdot fn) \rangle, s') \\   \text{InsInitFVar: } &G \vdash (\langle InsInitV\ Skip\ (\{accC, statDeclC, stat\} Lit\ a \cdot fn) \rangle, Norm\ s) \\ &\mapsto 1\ (\langle \{accC, statDeclC, stat\} Lit\ a \cdot fn \rangle, Norm\ s) \end{aligned}$
--	--

— Notice, that we do not have literal values for *vars*. The rules for accessing variables (*Acc*) and assigning to variables (*Ass*), test this with the predicate *groundVar*. After initialisation is done and the *FVar* is evaluated, we can't just throw away the *InsInitFVar* term and return a literal value, as in the cases of *New* or *NewC*. Instead we just return the evaluated *FVar* and test for initialisation in the rule *FVar*.

$$\begin{aligned}
| \text{ AVarE1: } & \llbracket G \vdash (\langle e1 \rangle, \text{Norm } s) \mapsto 1 \ (\langle e1' \rangle, s') \rrbracket \\
& \implies \\
& G \vdash (\langle e1.[e2] \rangle, \text{Norm } s) \mapsto 1 \ (\langle e1'.[e2] \rangle, s')
\end{aligned}$$

$$\begin{aligned}
| \text{ AVarE2: } & G \vdash (\langle e2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle e2' \rangle, s') \\
& \implies \\
& G \vdash (\langle \text{Lit } a.[e2] \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Lit } a.[e2'] \rangle, s')
\end{aligned}$$

— *Nil* is fully evaluated

$$\begin{aligned}
| \text{ ConsHd: } & \llbracket G \vdash (\langle e::\text{expr} \rangle, \text{Norm } s) \mapsto 1 \ (\langle e'::\text{expr} \rangle, s') \rrbracket \\
& \implies \\
& G \vdash (\langle e\#es \rangle, \text{Norm } s) \mapsto 1 \ (\langle e'\#es \rangle, s')
\end{aligned}$$

$$\begin{aligned}
| \text{ ConsTl: } & \llbracket G \vdash (\langle es \rangle, \text{Norm } s) \mapsto 1 \ (\langle es' \rangle, s') \rrbracket \\
& \implies \\
& G \vdash (\langle (\text{Lit } v)\#es \rangle, \text{Norm } s) \mapsto 1 \ (\langle (\text{Lit } v)\#es' \rangle, s')
\end{aligned}$$

$$| \text{ Skip: } G \vdash (\langle \text{Skip} \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{SKIP} \rangle, \text{Norm } s)$$

$$\begin{aligned}
| \text{ ExprE: } & \llbracket G \vdash (\langle e \rangle, \text{Norm } s) \mapsto 1 \ (\langle e' \rangle, s') \rrbracket \\
& \implies \\
& G \vdash (\langle \text{Expr } e \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Expr } e' \rangle, s') \\
| \text{ Expr: } & G \vdash (\langle \text{Expr } (\text{Lit } v) \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Skip} \rangle, \text{Norm } s)
\end{aligned}$$

$$\begin{aligned}
| \text{ LabC: } & \llbracket G \vdash (\langle c \rangle, \text{Norm } s) \mapsto 1 \ (\langle c' \rangle, s') \rrbracket \\
& \implies \\
& G \vdash (\langle l \cdot c \rangle, \text{Norm } s) \mapsto 1 \ (\langle l \cdot c' \rangle, s') \\
| \text{ Lab: } & G \vdash (\langle l \cdot \text{Skip} \rangle, s) \mapsto 1 \ (\langle \text{Skip} \rangle, \text{abupd } (\text{absorb } l) \ s)
\end{aligned}$$

$$\begin{aligned}
| \text{ CompC1: } & \llbracket G \vdash (\langle c1 \rangle, \text{Norm } s) \mapsto 1 \ (\langle c1' \rangle, s') \rrbracket \\
& \implies \\
& G \vdash (\langle c1;; c2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle c1';; c2 \rangle, s')
\end{aligned}$$

$$| \text{ Comp: } G \vdash (\langle \text{Skip};; c2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle c2 \rangle, \text{Norm } s)$$

$$\begin{aligned}
| \text{ IfE: } & \llbracket G \vdash (\langle e \rangle, \text{Norm } s) \mapsto 1 \ (\langle e' \rangle, s') \rrbracket \\
& \implies \\
& G \vdash (\langle \text{If}(e) \ s1 \ \text{Else } s2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{If}(e') \ s1 \ \text{Else } s2 \rangle, s') \\
| \text{ If: } & G \vdash (\langle \text{If}(\text{Lit } v) \ s1 \ \text{Else } s2 \rangle, \text{Norm } s) \\
& \mapsto 1 \ (\langle \text{if the-Bool } v \text{ then } s1 \ \text{else } s2 \rangle, \text{Norm } s)
\end{aligned}$$

$$\begin{aligned}
| \text{ Loop: } & G \vdash (\langle l \cdot \text{While}(e) \ c \rangle, \text{Norm } s) \\
& \mapsto 1 \ (\langle \text{If}(e) \ (\text{Cont } l \cdot c;; l \cdot \text{While}(e) \ c) \ \text{Else } \text{Skip} \rangle, \text{Norm } s)
\end{aligned}$$

$$| \text{ Jmp: } G \vdash (\langle \text{Jmp } j \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Skip} \rangle, (\text{Some } (\text{Jump } j), s))$$

$$| \text{ ThrowE: } \llbracket G \vdash (\langle e \rangle, \text{Norm } s) \mapsto 1 \ (\langle e' \rangle, s') \rrbracket$$

$$\begin{array}{l}
\Rightarrow \\
G \vdash (\langle \text{Throw } e \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Throw } e' \rangle, s') \\
| \text{ Throw: } G \vdash (\langle \text{Throw } (\text{Lit } a) \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Skip} \rangle, \text{abupd } (\text{throw } a) (\text{Norm } s)) \\
| \text{ TryC1: } \llbracket G \vdash (\langle c1 \rangle, \text{Norm } s) \mapsto 1 \ (\langle c1' \rangle, s') \rrbracket \\
\Rightarrow \\
G \vdash (\langle \text{Try } c1 \text{ Catch } (C \text{ vn}) \ c2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Try } c1' \text{ Catch } (C \text{ vn}) \ c2 \rangle, s') \\
| \text{ Try: } \llbracket G \vdash s \text{ --salloc--} \rightarrow s' \rrbracket \\
\Rightarrow \\
G \vdash (\langle \text{Try Skip Catch } (C \text{ vn}) \ c2 \rangle, s) \\
\mapsto 1 \ (\text{if } G, s \vdash \text{catch } C \text{ then } (\langle c2 \rangle, \text{new-xcpt-var } \text{vn } s') \\
\text{else } (\langle \text{Skip} \rangle, s')) \\
| \text{ FinC1: } \llbracket G \vdash (\langle c1 \rangle, \text{Norm } s) \mapsto 1 \ (\langle c1' \rangle, s') \rrbracket \\
\Rightarrow \\
G \vdash (\langle c1 \text{ Finally } c2 \rangle, \text{Norm } s) \mapsto 1 \ (\langle c1' \text{ Finally } c2 \rangle, s') \\
| \text{ Fin: } G \vdash (\langle \text{Skip Finally } c2 \rangle, (a, s)) \mapsto 1 \ (\langle \text{FinA } a \ c2 \rangle, \text{Norm } s) \\
| \text{ FinAC: } \llbracket G \vdash (\langle c \rangle, s) \mapsto 1 \ (\langle c' \rangle, s') \rrbracket \\
\Rightarrow \\
G \vdash (\langle \text{FinA } a \ c \rangle, s) \mapsto 1 \ (\langle \text{FinA } a \ c' \rangle, s') \\
| \text{ FinA: } G \vdash (\langle \text{FinA } a \text{ Skip} \rangle, s) \mapsto 1 \ (\langle \text{Skip} \rangle, \text{abupd } (\text{abrupt-if } (a \neq \text{None}) \ a) \ s) \\
| \text{ InitI: } \llbracket \text{inited } C \text{ (globs } s) \rrbracket \\
\Rightarrow \\
G \vdash (\langle \text{Init } C \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{Skip} \rangle, \text{Norm } s) \\
| \text{ Init: } \llbracket \text{the } (\text{class } G \ C) = c; \neg \text{inited } C \text{ (globs } s) \rrbracket \\
\Rightarrow \\
G \vdash (\langle \text{Init } C \rangle, \text{Norm } s) \\
\mapsto 1 \ ((\text{if } C = \text{Object then Skip else } (\text{Init } (\text{super } c))) ; \\
\text{Expr } (\text{Callee } (\text{locals } s) (\text{InsInitE } (\text{init } c) \text{ SKIP}))) \\
, \text{Norm } (\text{init-class-obj } G \ C \ s)) \\
\text{--- InsInitE is just used as trick to embed the statement } \text{init } c \text{ into an expression} \\
| \text{ InsInitESKIP: } \\
G \vdash (\langle \text{InsInitE Skip SKIP} \rangle, \text{Norm } s) \mapsto 1 \ (\langle \text{SKIP} \rangle, \text{Norm } s)
\end{array}$$
**abbreviation**

*stepn*::  $[prog, term \times state, nat, term \times state] \Rightarrow bool \ (\vdash - \mapsto - \text{ [61,82,82] } 81)$   
**where**  $G \vdash p \mapsto^n p' \equiv (p, p') \in \{(x, y). \text{step } G \ x \ y\}^n$

**abbreviation**

*steptr*::  $[prog, term \times state, term \times state] \Rightarrow bool \ (\vdash - \mapsto^* - \text{ [61,82,82] } 81)$   
**where**  $G \vdash p \mapsto^* p' \equiv (p, p') \in \{(x, y). \text{step } G \ x \ y\}^*$

**lemma** *rtranc1-imp-rel-pow*:  $p \in R^* \Rightarrow \exists n. p \in R^n$   
 $\langle \text{proof} \rangle$

**end**



## Chapter 22

### AxSem

## 50 Axiomatic semantics of Java expressions and statements (see also Eval.thy)

**theory** *AxSem* **imports** *Evaln TypeSafe* **begin**

design issues:

- a strong version of validity for triples with premises, namely one that takes the recursive depth needed to complete execution, enables correctness proof
- auxiliary variables are handled first-class (-i Thomas Kleymann)
- expressions not flattened to elementary assignments (as usual for axiomatic semantics) but treated first-class =i explicit result value handling
- intermediate values not on triple, but on assertion level (with result entry)
- multiple results with semantical substitution mechanism not requiring a stack
- because of dynamic method binding, terms need to be dependent on state. this is also useful for conditional expressions and statements
- result values in triples exactly as in eval relation (also for xcpt states)
- validity: additional assumption of state conformance and well-typedness, which is required for soundness and thus rule hazard required of completeness

restrictions:

- all triples in a derivation are of the same type (due to weak polymorphism)

**types** *res = vals* — result entry

**syntax**

*Val* :: *val*  $\Rightarrow$  *res*

*Var* :: *var*  $\Rightarrow$  *res*

*Vals* :: *val list*  $\Rightarrow$  *res*

**translations**

*Val* *x*  $\Rightarrow$  (*In1* *x*)

*Var* *x*  $\Rightarrow$  (*In2* *x*)

*Vals* *x*  $\Rightarrow$  (*In3* *x*)

**syntax**

-*Val* :: [*pttrn*]  $\Rightarrow$  *pttrn* (*Val*:- [951] 950)

-*Var* :: [*pttrn*]  $\Rightarrow$  *pttrn* (*Var*:- [951] 950)

-*Vals* :: [*pttrn*]  $\Rightarrow$  *pttrn* (*Vals*:- [951] 950)

**translations**

$\lambda Val:v . b == (\lambda v. b) \circ the-In1$

$\lambda Var:v . b == (\lambda v. b) \circ the-In2$

$\lambda Vals:v. b == (\lambda v. b) \circ the-In3$

— relation on result values, state and auxiliary variables

**types** '*a assn* = *res*  $\Rightarrow$  *state*  $\Rightarrow$  '*a*  $\Rightarrow$  *bool*

**translations**

*res*  $\leq$  (*type*) *AxSem.res*

*a assn*  $\leq$  (*type*) *vals*  $\Rightarrow$  *state*  $\Rightarrow$  *a*  $\Rightarrow$  *bool*

**constdefs**

*assn-imp* :: '*a assn*  $\Rightarrow$  '*a assn*  $\Rightarrow$  *bool* (infixr  $\Rightarrow$  25)

$P \Rightarrow Q \equiv \forall Y s Z. P Y s Z \longrightarrow Q Y s Z$

**lemma** *assn-imp-def2* [iff]:  $(P \Rightarrow Q) = (\forall Y\ s\ Z. P\ Y\ s\ Z \longrightarrow Q\ Y\ s\ Z)$   
 ⟨proof⟩

## assertion transformers

### 51 peek-and

#### constdefs

*peek-and* ::  $'a\ assn \Rightarrow (state \Rightarrow bool) \Rightarrow 'a\ assn$  (**infixl**  $\wedge$ . 13)  
 $P \wedge. p \equiv \lambda Y\ s\ Z. P\ Y\ s\ Z \wedge p\ s$

**lemma** *peek-and-def2* [simp]:  $peek\ and\ P\ p\ Y\ s = (\lambda Z. (P\ Y\ s\ Z \wedge p\ s))$   
 ⟨proof⟩

**lemma** *peek-and-Not* [simp]:  $(P \wedge. (\lambda s. \neg f\ s)) = (P \wedge. Not \circ f)$   
 ⟨proof⟩

**lemma** *peek-and-and* [simp]:  $peek\ and\ (peek\ and\ P\ p)\ p = peek\ and\ P\ p$   
 ⟨proof⟩

**lemma** *peek-and-commut*:  $(P \wedge. p \wedge. q) = (P \wedge. q \wedge. p)$   
 ⟨proof⟩

#### syntax

*Normal* ::  $'a\ assn \Rightarrow 'a\ assn$

#### translations

*Normal*  $P == P \wedge. normal$

**lemma** *peek-and-Normal* [simp]:  $peek\ and\ (Normal\ P)\ p = Normal\ (peek\ and\ P\ p)$   
 ⟨proof⟩

### 52 assn-supd

#### constdefs

*assn-supd* ::  $'a\ assn \Rightarrow (state \Rightarrow state) \Rightarrow 'a\ assn$  (**infixl**  $;$ . 13)  
 $P ;. f \equiv \lambda Y\ s'\ Z. \exists s. P\ Y\ s\ Z \wedge s' = f\ s$

**lemma** *assn-supd-def2* [simp]:  $assn\ supd\ P\ f\ Y\ s'\ Z = (\exists s. P\ Y\ s\ Z \wedge s' = f\ s)$   
 ⟨proof⟩

### 53 supd-assn

#### constdefs

*supd-assn* ::  $(state \Rightarrow state) \Rightarrow 'a\ assn \Rightarrow 'a\ assn$  (**infixr**  $;$ . 13)  
 $f ;. P \equiv \lambda Y\ s. P\ Y\ (f\ s)$

**lemma** *supd-assn-def2* [simp]:  $(f ;. P)\ Y\ s = P\ Y\ (f\ s)$   
 ⟨proof⟩

**lemma** *supd-assn-supdD* [elim]:  $((f ;. Q) ;. f)\ Y\ s\ Z \Longrightarrow Q\ Y\ s\ Z$

$\langle \text{proof} \rangle$

**lemma** *supd-assn-supdI* [elim]:  $Q \ Y \ s \ Z \implies (f \ .; (Q \ ;. f)) \ Y \ s \ Z$

$\langle \text{proof} \rangle$

## 54 subst-res

**constdefs**

*subst-res*  $:: 'a \ \text{assn} \Rightarrow \text{res} \Rightarrow 'a \ \text{assn}$  ( $\leftarrow$ - [60,61] 60)  
 $P \leftarrow w \equiv \lambda Y. P \ w$

**lemma** *subst-res-def2* [simp]:  $(P \leftarrow w) \ Y = P \ w$

$\langle \text{proof} \rangle$

**lemma** *subst-subst-res* [simp]:  $P \leftarrow w \leftarrow v = P \leftarrow w$

$\langle \text{proof} \rangle$

**lemma** *peek-and-subst-res* [simp]:  $(P \ \wedge. \ p) \leftarrow w = (P \leftarrow w \ \wedge. \ p)$

$\langle \text{proof} \rangle$

## 55 subst-Bool

**constdefs**

*subst-Bool*  $:: 'a \ \text{assn} \Rightarrow \text{bool} \Rightarrow 'a \ \text{assn}$  ( $\leftarrow$ =- [60,61] 60)  
 $P \leftarrow = b \equiv \lambda Y \ s \ Z. \exists v. P \ (\text{Val } v) \ s \ Z \ \wedge \ (\text{normal } s \longrightarrow \text{the-Bool } v = b)$

**lemma** *subst-Bool-def2* [simp]:

$(P \leftarrow = b) \ Y \ s \ Z = (\exists v. P \ (\text{Val } v) \ s \ Z \ \wedge \ (\text{normal } s \longrightarrow \text{the-Bool } v = b))$

$\langle \text{proof} \rangle$

**lemma** *subst-Bool-the-BoolI*:  $P \ (\text{Val } b) \ s \ Z \implies (P \leftarrow = \text{the-Bool } b) \ Y \ s \ Z$

$\langle \text{proof} \rangle$

## 56 peek-res

**constdefs**

*peek-res*  $:: (\text{res} \Rightarrow 'a \ \text{assn}) \Rightarrow 'a \ \text{assn}$   
 $\text{peek-res } Pf \equiv \lambda Y. Pf \ Y \ Y$

**syntax**

@*peek-res*  $:: \text{pttrn} \Rightarrow 'a \ \text{assn} \Rightarrow 'a \ \text{assn}$  ( $\lambda$ :-. - [0,3] 3)

**translations**

$\lambda w. P \ == \ \text{peek-res} \ (\lambda w. P)$

**lemma** *peek-res-def2* [simp]:  $\text{peek-res } P \ Y = P \ Y \ Y$

$\langle \text{proof} \rangle$

**lemma** *peek-res-subst-res* [simp]:  $\text{peek-res } P \leftarrow w = P \ w \leftarrow w$

$\langle \text{proof} \rangle$

**lemma** *peek-subst-res-allI*:

$(\bigwedge a. T\ a\ (P\ (f\ a) \leftarrow f\ a)) \implies \forall a. T\ a\ (peek\text{-}res\ P \leftarrow f\ a)$   
 $\langle proof \rangle$

## 57 ign-res

**constdefs**

$ign\text{-}res \quad :: \quad 'a\ assn \Rightarrow 'a\ assn \quad (-\downarrow [1000]\ 1000)$   
 $P\downarrow \quad \equiv \lambda Y\ s\ Z. \exists Y. P\ Y\ s\ Z$

**lemma** *ign-res-def2* [simp]:  $P\downarrow\ Y\ s\ Z = (\exists Y. P\ Y\ s\ Z)$   
 $\langle proof \rangle$

**lemma** *ign-ign-res* [simp]:  $P\downarrow\downarrow = P\downarrow$   
 $\langle proof \rangle$

**lemma** *ign-subst-res* [simp]:  $P\downarrow \leftarrow w = P\downarrow$   
 $\langle proof \rangle$

**lemma** *peek-and-ign-res* [simp]:  $(P \wedge. p)\downarrow = (P\downarrow \wedge. p)$   
 $\langle proof \rangle$

## 58 peek-st

**constdefs**

$peek\text{-}st \quad :: \quad (st \Rightarrow 'a\ assn) \Rightarrow 'a\ assn$   
 $peek\text{-}st\ P \equiv \lambda Y\ s. P\ (store\ s)\ Y\ s$

**syntax**

$@peek\text{-}st \quad :: \quad pttrn \Rightarrow 'a\ assn \Rightarrow 'a\ assn \quad (\lambda\text{-}.. - [0,3]\ 3)$

**translations**

$\lambda s.. P \quad == \quad peek\text{-}st\ (\lambda s. P)$

**lemma** *peek-st-def2* [simp]:  $(\lambda s.. Pf\ s)\ Y\ s = Pf\ (store\ s)\ Y\ s$   
 $\langle proof \rangle$

**lemma** *peek-st-triv* [simp]:  $(\lambda s.. P) = P$   
 $\langle proof \rangle$

**lemma** *peek-st-st* [simp]:  $(\lambda s.. \lambda s'.. P\ s\ s') = (\lambda s.. P\ s\ s)$   
 $\langle proof \rangle$

**lemma** *peek-st-split* [simp]:  $(\lambda s.. \lambda Y\ s'. P\ s\ Y\ s') = (\lambda Y\ s. P\ (store\ s)\ Y\ s)$   
 $\langle proof \rangle$

**lemma** *peek-st-subst-res* [simp]:  $(\lambda s.. P\ s) \leftarrow w = (\lambda s.. P\ s \leftarrow w)$   
 $\langle proof \rangle$

**lemma** *peek-st-Normal* [simp]:  $(\lambda s..(Normal (P s))) = Normal (\lambda s.. P s)$   
 ⟨proof⟩

## 59 ign-res-eq

**constdefs**

$ign-res-eq :: 'a\ assn \Rightarrow res \Rightarrow 'a\ assn$  ( $\downarrow = -$  [60,61] 60)  
 $P \downarrow = w \equiv \lambda Y.. P \downarrow \wedge. (\lambda s.. Y = w)$

**lemma** *ign-res-eq-def2* [simp]:  $(P \downarrow = w) Y s Z = ((\exists Y.. P Y s Z) \wedge Y = w)$   
 ⟨proof⟩

**lemma** *ign-ign-res-eq* [simp]:  $(P \downarrow = w) \downarrow = P \downarrow$   
 ⟨proof⟩

**lemma** *ign-res-eq-subst-res*:  $P \downarrow = w \leftarrow w = P \downarrow$   
 ⟨proof⟩

**lemma** *subst-Bool-ign-res-eq*:  $((P \leftarrow = b) \downarrow = x) Y s Z = ((P \leftarrow = b) Y s Z \wedge Y = x)$   
 ⟨proof⟩

## 60 RefVar

**constdefs**

$RefVar :: (state \Rightarrow vvar \times state) \Rightarrow 'a\ assn \Rightarrow 'a\ assn$  (**infixr**  $..$ ; 13)  
 $vf ..; P \equiv \lambda Y s.. let (v, s') = vf s in P (Var v) s'$

**lemma** *RefVar-def2* [simp]:  $(vf ..; P) Y s =$   
 $P (Var (fst (vf s))) (snd (vf s))$   
 ⟨proof⟩

## 61 allocation

**constdefs**

$Alloc :: prog \Rightarrow obj-tag \Rightarrow 'a\ assn \Rightarrow 'a\ assn$   
 $Alloc G otag P \equiv \lambda Y s Z..$   
 $\forall s' a.. G \vdash s -halloc\ otag \succ a \rightarrow s' \longrightarrow P (Val (Addr a)) s' Z$

$SXAlloc :: prog \Rightarrow 'a\ assn \Rightarrow 'a\ assn$   
 $SXAlloc G P \equiv \lambda Y s Z.. \forall s'.. G \vdash s -salloc \rightarrow s' \longrightarrow P Y s' Z$

**lemma** *Alloc-def2* [simp]:  $Alloc G otag P Y s Z =$   
 $(\forall s' a.. G \vdash s -halloc\ otag \succ a \rightarrow s' \longrightarrow P (Val (Addr a)) s' Z)$   
 ⟨proof⟩

**lemma** *SXAlloc-def2* [simp]:  
 $SXAlloc G P Y s Z = (\forall s'.. G \vdash s -salloc \rightarrow s' \longrightarrow P Y s' Z)$   
 ⟨proof⟩

**validity****constdefs**

$type-ok :: prog \Rightarrow term \Rightarrow state \Rightarrow bool$   
 $type-ok\ G\ t\ s \equiv$   
 $\exists L\ T\ C\ A. (normal\ s \longrightarrow (\llbracket prg=G, cls=C, lcl=L \rrbracket) \vdash t :: T \wedge$   
 $\quad (\llbracket prg=G, cls=C, lcl=L \rrbracket) \vdash_{dom} (locals\ (store\ s)) \gg t \gg A) \wedge s :: \preceq(G, L)$

**datatype**  $'a\ triple = triple\ ('a\ assn)\ term\ ('a\ assn)$   
 $(\{(1-)\} / -> / \{(1-)\}) \quad [3, 65, 3] 75$

**types**  $'a\ triples = 'a\ triple\ set$

**syntax**

$var-triple :: ['a\ assn, var, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / -=> / \{(1-)\}) \quad [3, 80, 3] 75$   
 $expr-triple :: ['a\ assn, expr, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / --> / \{(1-)\}) \quad [3, 80, 3] 75$   
 $exprs-triple :: ['a\ assn, expr\ list, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / -\#> / \{(1-)\}) \quad [3, 65, 3] 75$   
 $stmt-triple :: ['a\ assn, stmt, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / \dot{-}> / \{(1-)\}) \quad [3, 65, 3] 75$

**syntax** (*xsymbols*)

$triple :: ['a\ assn, term, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / -\succ / \{(1-)\}) \quad [3, 65, 3] 75$   
 $var-triple :: ['a\ assn, var, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / -\succ / \{(1-)\}) \quad [3, 80, 3] 75$   
 $expr-triple :: ['a\ assn, expr, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / --\succ / \{(1-)\}) \quad [3, 80, 3] 75$   
 $exprs-triple :: ['a\ assn, expr\ list, 'a\ assn] \Rightarrow 'a\ triple$   
 $(\{(1-)\} / -\dot{\succ} / \{(1-)\}) \quad [3, 65, 3] 75$

**translations**

$\{P\}\ e \dashv \succ \{Q\} == \{P\}\ In1l\ e \succ \{Q\}$   
 $\{P\}\ e = \succ \{Q\} == \{P\}\ In2\ e \succ \{Q\}$   
 $\{P\}\ e \dot{=} \succ \{Q\} == \{P\}\ In3\ e \succ \{Q\}$   
 $\{P\}\ .c. \{Q\} == \{P\}\ In1r\ c \succ \{Q\}$

**lemma** *inj-triple*:  $inj\ (\lambda(P, t, Q). \{P\}\ t \succ \{Q\})$   
 $\langle proof \rangle$

**lemma** *triple-inj-eq*:  $(\{P\}\ t \succ \{Q\} = \{P'\}\ t' \succ \{Q'\}) = (P = P' \wedge t = t' \wedge Q = Q')$   
 $\langle proof \rangle$

**constdefs**

$mtriples :: ('c \Rightarrow 'sig \Rightarrow 'a\ assn) \Rightarrow ('c \Rightarrow 'sig \Rightarrow expr) \Rightarrow$   
 $(('c \Rightarrow 'sig \Rightarrow 'a\ assn) \Rightarrow ('c \times 'sig)\ set \Rightarrow 'a\ triples)$   
 $(\{(\{(1-)\} / --\succ / \{(1-)\} \mid -)\} [3, 65, 3, 65] 75)$   
 $\{\{P\}\ tf \dashv \succ \{Q\} \mid ms\} \equiv (\lambda(C, sig). \{Normal(P\ C\ sig)\}\ tf\ C\ sig \dashv \succ \{Q\}\ C\ sig)\ 'ms$

**consts**

$triple-valid :: prog \Rightarrow nat \Rightarrow 'a\ triple \Rightarrow bool$   
 $(\dashv \models \dot{-} [61, 0, 58] 57)$

$ax\text{-}valids :: prog \Rightarrow 'b \text{ triples} \Rightarrow 'a \text{ triples} \Rightarrow bool$   
 $(-,|-|- [61,58,58] 57)$

### syntax

$triples\text{-}valid :: prog \Rightarrow nat \Rightarrow 'a \text{ triples} \Rightarrow bool$   
 $(-|-|- [61,0, 58] 57)$   
 $ax\text{-}valid :: prog \Rightarrow 'b \text{ triples} \Rightarrow 'a \text{ triple} \Rightarrow bool$   
 $(-,|-|- [61,58,58] 57)$

### syntax (*xsymbols*)

$triples\text{-}valid :: prog \Rightarrow nat \Rightarrow 'a \text{ triples} \Rightarrow bool$   
 $(-|-|- [61,0, 58] 57)$   
 $ax\text{-}valid :: prog \Rightarrow 'b \text{ triples} \Rightarrow 'a \text{ triple} \Rightarrow bool$   
 $(-,|-|- [61,58,58] 57)$

**defs**  $triple\text{-}valid\text{-}def: G \models n:t \equiv case\ t\ of\ \{P\}\ t \succ \{Q\} \Rightarrow$   
 $\forall Y\ s\ Z. P\ Y\ s\ Z \longrightarrow type\text{-}ok\ G\ t\ s \longrightarrow$   
 $(\forall Y'\ s'. G \vdash s - t \succ - n \longrightarrow (Y',s') \longrightarrow Q\ Y'\ s'\ Z)$   
**translations**  $G \models n:ts == Ball\ ts\ (triple\text{-}valid\ G\ n)$   
**defs**  $ax\text{-}valids\text{-}def: G, A \models ts \equiv \forall n. G \models n:A \longrightarrow G \models n:ts$   
**translations**  $G, A \models t == G, A \models \{t\}$

**lemma**  $triple\text{-}valid\text{-}def2: G \models n:\{P\}\ t \succ \{Q\} =$   
 $(\forall Y\ s\ Z. P\ Y\ s\ Z$   
 $\longrightarrow (\exists L. (normal\ s \longrightarrow (\exists C\ T\ A. (\llbracket prg=G, cls=C, lcl=L \rrbracket) \vdash t :: T \wedge$   
 $(\llbracket prg=G, cls=C, lcl=L \rrbracket) \vdash_{dom}\ (locals\ (store\ s)) \gg t \gg A)) \wedge$   
 $s :: \preceq (G, L))$   
 $\longrightarrow (\forall Y'\ s'. G \vdash s - t \succ - n \longrightarrow (Y',s') \longrightarrow Q\ Y'\ s'\ Z))$   
 $\langle proof \rangle$

**declare**  $split\text{-}paired\text{-}All\ [simp\ del]\ split\text{-}paired\text{-}Ex\ [simp\ del]$   
**declare**  $split\text{-}if\ [split\ del]\ split\text{-}if\text{-}asm\ [split\ del]$   
 $option.\text{split}\ [split\ del]\ option.\text{split}\text{-}asm\ [split\ del]$   
 $\langle ML \rangle$

### inductive

$ax\text{-}derivs :: prog \Rightarrow 'a \text{ triples} \Rightarrow 'a \text{ triples} \Rightarrow bool\ (-,|-|- [61,58,58] 57)$   
**and**  $ax\text{-}deriv :: prog \Rightarrow 'a \text{ triples} \Rightarrow 'a \text{ triple} \Rightarrow bool\ (-,|-|- [61,58,58] 57)$   
**for**  $G :: prog$   
**where**

$G, A \vdash t \equiv G, A \models \{t\}$

|  $empty: G, A \models \{\}$   
|  $insert: \llbracket G, A \vdash t; G, A \models ts \rrbracket \Longrightarrow$   
 $G, A \models insert\ t\ ts$

|  $asm: ts \subseteq A \Longrightarrow G, A \models ts$

|  $weaken: \llbracket G, A \models ts'; ts \subseteq ts' \rrbracket \Longrightarrow G, A \models ts$

|  $conseq: \forall Y\ s\ Z. P\ Y\ s\ Z \longrightarrow (\exists P'\ Q'. G, A \models \{P'\}\ t \succ \{Q'\} \wedge (\forall Y'\ s'\ Z'. P'\ Y'\ s'\ Z' \longrightarrow Q'\ Y'\ s'\ Z')) \longrightarrow$   
 $Q\ Y'\ s'\ Z))$



$$\Longrightarrow G, A \vdash \{P\} t \triangleright \{Q\}$$

$$| \text{hazard}: G, A \vdash \{P \wedge. \text{Not} \circ \text{type-ok } G \ t\} t \triangleright \{Q\}$$

$$| \text{Abrupt}: G, A \vdash \{P \leftarrow (\text{arbitrary3 } t) \wedge. \text{Not} \circ \text{normal}\} t \triangleright \{P\}$$

— variables

$$| \text{LVar}: G, A \vdash \{\text{Normal } (\lambda s.. P \leftarrow \text{Var } (\text{lvar } vn \ s))\} \text{LVar } vn \triangleright \{P\}$$

$$| \text{FVar}: \llbracket G, A \vdash \{\text{Normal } P\} . \text{Init } C. \{Q\}; \\ G, A \vdash \{Q\} e \triangleright \{\lambda \text{Val}:a.. \text{fvar } C \text{ stat } \text{fn } a \ ..; R\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} \{\text{acc } C, C, \text{stat}\} e.. \text{fn} \triangleright \{R\}$$

$$| \text{AVar}: \llbracket G, A \vdash \{\text{Normal } P\} e1 \triangleright \{Q\}; \\ \forall a. G, A \vdash \{Q \leftarrow \text{Val } a\} e2 \triangleright \{\lambda \text{Val}:i.. \text{avar } G \ i \ a \ ..; R\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} e1.[e2] \triangleright \{R\}$$

— expressions

$$| \text{NewC}: \llbracket G, A \vdash \{\text{Normal } P\} . \text{Init } C. \{\text{Alloc } G \ (C \text{Inst } C) \ Q\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} \text{NewC } C \triangleright \{Q\}$$

$$| \text{NewA}: \llbracket G, A \vdash \{\text{Normal } P\} . \text{init-comp-ty } T. \{Q\}; G, A \vdash \{Q\} e \triangleright \\ \{\lambda \text{Val}:i.. \text{abupd } (\text{check-neg } i) \ .; \text{Alloc } G \ (\text{Arr } T \ (\text{the-Intg } i)) \ R\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} \text{New } T[e] \triangleright \{R\}$$

$$| \text{Cast}: \llbracket G, A \vdash \{\text{Normal } P\} e \triangleright \{\lambda \text{Val}:v.. \lambda s.. \\ \text{abupd } (\text{raise-if } (\neg G, s \vdash v \text{ fits } T) \ \text{ClassCast}) \ .; Q \leftarrow \text{Val } v\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} \text{Cast } T \ e \triangleright \{Q\}$$

$$| \text{Inst}: \llbracket G, A \vdash \{\text{Normal } P\} e \triangleright \{\lambda \text{Val}:v.. \lambda s.. \\ Q \leftarrow \text{Val } (\text{Bool } (v \neq \text{Null} \wedge G, s \vdash v \text{ fits } \text{RefT } T))\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} e \text{InstOf } T \triangleright \{Q\}$$

$$| \text{Lit}: G, A \vdash \{\text{Normal } (P \leftarrow \text{Val } v)\} \text{Lit } v \triangleright \{P\}$$

$$| \text{UnOp}: \llbracket G, A \vdash \{\text{Normal } P\} e \triangleright \{\lambda \text{Val}:v.. Q \leftarrow \text{Val } (\text{eval-unop } \text{unop } v)\} \rrbracket \\ \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} \text{UnOp } \text{unop } e \triangleright \{Q\}$$

$$| \text{BinOp}: \\ \llbracket G, A \vdash \{\text{Normal } P\} e1 \triangleright \{Q\}; \\ \forall v1. G, A \vdash \{Q \leftarrow \text{Val } v1\} \\ (\text{if need-second-arg binop } v1 \text{ then } (\text{In1l } e2) \text{ else } (\text{In1r Skip})) \triangleright \\ \{\lambda \text{Val}:v2.. R \leftarrow \text{Val } (\text{eval-binop } \text{binop } v1 \ v2)\} \rrbracket \\ \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} \text{BinOp } \text{binop } e1 \ e2 \triangleright \{R\}$$

$$| \text{Super}: G, A \vdash \{\text{Normal } (\lambda s.. P \leftarrow \text{Val } (\text{val-this } s))\} \text{Super} \triangleright \{P\}$$

$$| \text{Acc}: \llbracket G, A \vdash \{\text{Normal } P\} va \triangleright \{\lambda \text{Var}:(v,f).. Q \leftarrow \text{Val } v\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} \text{Acc } va \triangleright \{Q\}$$

$$| \text{Ass}: \llbracket G, A \vdash \{\text{Normal } P\} va \triangleright \{Q\}; \\ \forall vf. G, A \vdash \{Q \leftarrow \text{Var } vf\} e \triangleright \{\lambda \text{Val}:v.. \text{assign } (\text{snd } vf) \ v \ .; R\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} va := e \triangleright \{R\}$$

$$| \text{Cond}: \llbracket G, A \vdash \{\text{Normal } P\} e0 \triangleright \{P'\}; \\ \forall b. G, A \vdash \{P' \leftarrow b\} (\text{if } b \text{ then } e1 \text{ else } e2) \triangleright \{Q\} \rrbracket \Longrightarrow \\ G, A \vdash \{\text{Normal } P\} e0 \ ? \ e1 : e2 \triangleright \{Q\}$$

<i>Call</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ e \multimap \{Q\}; \forall a. G, A \vdash \{Q \leftarrow Val\ a\} \ args \doteq \{R\ a\}; \\ & \forall a\ vs\ invC\ declC\ l. G, A \vdash \{(R\ a \leftarrow Vals\ vs \wedge. \\ & (\lambda s. declC = invocation-declclass\ G\ mode\ (store\ s)\ a\ statT\ (\llbracket name = mn, parTs = pTs \rrbracket) \wedge \\ & invC = invocation-class\ mode\ (store\ s)\ a\ statT \wedge \\ & l = locals\ (store\ s))\ ;. \\ & init-lvars\ G\ declC\ (\llbracket name = mn, parTs = pTs \rrbracket)\ mode\ a\ vs) \wedge. \\ & (\lambda s. normal\ s \longrightarrow G \vdash mode \rightarrow invC \preceq statT)\} \\ & Methd\ declC\ (\llbracket name = mn, parTs = pTs \rrbracket) \multimap \{set-lvars\ l\ ;\ S\} \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \{accC, statT, mode\} e \cdot mn(\{pTs\} args) \multimap \{S\} \end{aligned}$
<i>Methd</i> :	$\begin{aligned} & \llbracket G, A \cup \{\{P\}\} \ Methd \multimap \{Q\} \mid ms \rrbracket \vdash \{\{P\}\ body\ G \multimap \{Q\} \mid ms\} \rrbracket \implies \\ & G, A \vdash \{\{P\}\ Methd \multimap \{Q\} \mid ms\} \end{aligned}$
<i>Body</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ .Init\ D. \{Q\}; \\ & G, A \vdash \{Q\} \ .c. \{\lambda s.. abupd\ (absorb\ Ret) \ ;\ R \leftarrow (In1\ (the\ (locals\ s\ Result)))\} \rrbracket \\ & \implies \\ & G, A \vdash \{Normal\ P\} \ Body\ D\ c \multimap \{R\} \end{aligned}$
— expression lists	
<i>Nil</i> :	$G, A \vdash \{Normal\ (P \leftarrow Vals\ [])\} [] \doteq \{P\}$
<i>Cons</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ e \multimap \{Q\}; \\ & \forall v. G, A \vdash \{Q \leftarrow Val\ v\} \ es \doteq \{\lambda Vals:vs.. R \leftarrow Vals\ (v \# vs)\} \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \ e \# es \doteq \{R\} \end{aligned}$
— statements	
<i>Skip</i> :	$G, A \vdash \{Normal\ (P \leftarrow \Diamond)\} \ .Skip. \{P\}$
<i>Expr</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ e \multimap \{Q \leftarrow \Diamond\} \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \ .Expr\ e. \{Q\} \end{aligned}$
<i>Lab</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ .c. \{abupd\ (absorb\ l) \ ;\ Q\} \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \ .l. c. \{Q\} \end{aligned}$
<i>Comp</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ .c1. \{Q\}; \\ & G, A \vdash \{Q\} \ .c2. \{R\} \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \ .c1;;c2. \{R\} \end{aligned}$
<i>If</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ e \multimap \{P'\}; \\ & \forall b. G, A \vdash \{P' \leftarrow b\} \ .(if\ b\ then\ c1\ else\ c2). \{Q\} \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \ .If(e)\ c1\ Else\ c2. \{Q\} \end{aligned}$
<i>Loop</i> :	$\begin{aligned} & \llbracket G, A \vdash \{P\} \ e \multimap \{P'\}; \\ & G, A \vdash \{Normal\ (P' \leftarrow True)\} \ .c. \{abupd\ (absorb\ (Cont\ l)) \ ;\ P\} \rrbracket \implies \\ & G, A \vdash \{P\} \ .l. While(e)\ c. \{(P' \leftarrow False) \downarrow = \Diamond\} \end{aligned}$
<i>Jmp</i> :	$G, A \vdash \{Normal\ (abupd\ (\lambda a. (Some\ (Jump\ j)))) \ ;\ P \leftarrow \Diamond\} \ .Jmp\ j. \{P\}$
<i>Throw</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ e \multimap \{\lambda Val:a.. abupd\ (throw\ a) \ ;\ Q \leftarrow \Diamond\} \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \ .Throw\ e. \{Q\} \end{aligned}$
<i>Try</i> :	$\begin{aligned} & \llbracket G, A \vdash \{Normal\ P\} \ .c1. \{SXAlloc\ G\ Q\}; \\ & G, A \vdash \{Q \wedge. (\lambda s. G, s \vdash catch\ C) \ ;. new-xcpt-var\ vn\} \ .c2. \{R\}; \\ & (Q \wedge. (\lambda s. \neg G, s \vdash catch\ C)) \Rightarrow R \rrbracket \implies \\ & G, A \vdash \{Normal\ P\} \ .Try\ c1\ Catch(C\ vn)\ c2. \{R\} \end{aligned}$

| *Fin*:  $\llbracket G, A \vdash \{ \text{Normal } P \} .c1. \{ Q \};$   
 $\forall x. G, A \vdash \{ Q \wedge (\lambda s. x = \text{fst } s) ;. \text{abupd } (\lambda x. \text{None}) \}$   
 $.c2. \{ \text{abupd } (\text{abrupt-if } (x \neq \text{None}) x) ;. R \} \rrbracket \implies$   
 $G, A \vdash \{ \text{Normal } P \} .c1 \text{ Finally } c2. \{ R \}$

| *Done*:  $G, A \vdash \{ \text{Normal } (P \leftarrow \Diamond \wedge \text{initd } C) \} .\text{Init } C. \{ P \}$

| *Init*:  $\llbracket \text{the } (\text{class } G \ C) = c;$   
 $G, A \vdash \{ \text{Normal } ((P \wedge \text{Not } \circ \text{initd } C) ;. \text{supd } (\text{init-class-obj } G \ C)) \}$   
 $.(\text{if } C = \text{Object then Skip else Init } (\text{super } c)). \{ Q \};$   
 $\forall l. G, A \vdash \{ Q \wedge (\lambda s. l = \text{locals } (\text{store } s)) ;. \text{set-lvars empty} \}$   
 $.\text{init } c. \{ \text{set-lvars } l ;. R \} \rrbracket \implies$   
 $G, A \vdash \{ \text{Normal } (P \wedge \text{Not } \circ \text{initd } C) \} .\text{Init } C. \{ R \}$

— Some dummy rules for the intermediate terms *Callee*, *InsInitE*, *InsInitV*, *FinA* only used by the smallstep semantics.

| *InsInitV*:  $G, A \vdash \{ \text{Normal } P \} \text{ InsInitV } c \ v \multimap \{ Q \}$   
| *InsInitE*:  $G, A \vdash \{ \text{Normal } P \} \text{ InsInitE } c \ e \multimap \{ Q \}$   
| *Callee*:  $G, A \vdash \{ \text{Normal } P \} \text{ Callee } l \ e \multimap \{ Q \}$   
| *FinA*:  $G, A \vdash \{ \text{Normal } P \} .\text{FinA } a \ c. \{ Q \}$

### constdefs

*adapt-pre* :: 'a assn  $\Rightarrow$  'a assn  $\Rightarrow$  'a assn  $\Rightarrow$  'a assn  
*adapt-pre*  $P \ Q \ Q' \equiv \lambda Y \ s \ Z. \forall Y' \ s'. \exists Z'. P \ Y \ s \ Z' \wedge (Q \ Y' \ s' \ Z' \longrightarrow Q' \ Y' \ s' \ Z)$

### rules derived by induction

**lemma** *cut-valid*:  $\llbracket G, A' \models ts; G, A \models A \rrbracket \implies G, A \models ts$   
 $\langle \text{proof} \rangle$

**lemma** *ax-thin* [*rule-format* (*no-asm*)]:

$G, (A :: 'a \text{ triple set}) \vdash (ts :: 'a \text{ triple set}) \implies \forall A. A' \subseteq A \longrightarrow G, A \vdash ts$   
 $\langle \text{proof} \rangle$

**lemma** *ax-thin-insert*:  $G, (A :: 'a \text{ triple set}) \vdash (t :: 'a \text{ triple}) \implies G, \text{insert } x \ A \vdash t$   
 $\langle \text{proof} \rangle$

**lemma** *subset-mtriples-iff*:

$ts \subseteq \{ \{ P \} \text{ mb-} \multimap \{ Q \} \mid ms \} = (\exists ms'. ms' \subseteq ms \wedge ts = \{ \{ P \} \text{ mb-} \multimap \{ Q \} \mid ms' \})$   
 $\langle \text{proof} \rangle$

**lemma** *weaken*:

$G, (A :: 'a \text{ triple set}) \vdash (ts :: 'a \text{ triple set}) \implies !ts. ts \subseteq ts' \longrightarrow G, A \vdash ts$   
 $\langle \text{proof} \rangle$

### rules derived from conseq

In the following rules we often have to give some type annotations like:  $G, A \vdash \{ P \} \ t \multimap \{ Q \}$ . Given only the term above without annotations, Isabelle would infer a more general type were we could have different types of auxiliary variables in the assumption set (*A*) and in the triple itself (*P* and *Q*). But *ax-derivs.Methd* enforces the same type in the inductive definition of the derivation. So we

have to restrict the types to be able to apply the rules.

**lemma** *conseq12*:  $\llbracket G, (A::'a \text{ triple set}) \vdash \{P'::'a \text{ assn}\} t \succ \{Q'\};$   
 $\forall Y s Z. P Y s Z \longrightarrow (\forall Y' s'. (\forall Y Z'. P' Y s Z' \longrightarrow Q' Y' s' Z') \longrightarrow$   
 $Q Y' s' Z) \rrbracket$   
 $\implies G, A \vdash \{P::'a \text{ assn}\} t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *conseq12'*:  $\llbracket G, (A::'a \text{ triple set}) \vdash \{P'::'a \text{ assn}\} t \succ \{Q'\}; \forall s Y' s'.$   
 $(\forall Y Z. P' Y s Z \longrightarrow Q' Y' s' Z) \longrightarrow$   
 $(\forall Y Z. P Y s Z \longrightarrow Q Y' s' Z) \rrbracket$   
 $\implies G, A \vdash \{P::'a \text{ assn}\} t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *conseq12-from-conseq12'*:  $\llbracket G, (A::'a \text{ triple set}) \vdash \{P'::'a \text{ assn}\} t \succ \{Q'\};$   
 $\forall Y s Z. P Y s Z \longrightarrow (\forall Y' s'. (\forall Y Z'. P' Y s Z' \longrightarrow Q' Y' s' Z') \longrightarrow$   
 $Q Y' s' Z) \rrbracket$   
 $\implies G, A \vdash \{P::'a \text{ assn}\} t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *conseq1*:  $\llbracket G, (A::'a \text{ triple set}) \vdash \{P'::'a \text{ assn}\} t \succ \{Q\}; P \Rightarrow P' \rrbracket$   
 $\implies G, A \vdash \{P::'a \text{ assn}\} t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *conseq2*:  $\llbracket G, (A::'a \text{ triple set}) \vdash \{P::'a \text{ assn}\} t \succ \{Q'\}; Q' \Rightarrow Q \rrbracket$   
 $\implies G, A \vdash \{P::'a \text{ assn}\} t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-escape*:  
 $\llbracket \forall Y s Z. P Y s Z$   
 $\longrightarrow G, (A::'a \text{ triple set}) \vdash \{\lambda Y' s' (Z'::'a). (Y', s') = (Y, s)\}$   
 $t \succ$   
 $\{\lambda Y s Z'. Q Y s Z\}$   
 $\rrbracket \implies G, A \vdash \{P::'a \text{ assn}\} t \succ \{Q::'a \text{ assn}\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-constant*:  $\llbracket C \implies G, (A::'a \text{ triple set}) \vdash \{P::'a \text{ assn}\} t \succ \{Q\} \rrbracket$   
 $\implies G, A \vdash \{\lambda Y s Z. C \wedge P Y s Z\} t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-impossible [intro]*:  
 $G, (A::'a \text{ triple set}) \vdash \{\lambda Y s Z. \text{False}\} t \succ \{Q::'a \text{ assn}\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-nochange-lemma*:  $\llbracket P Y s; \text{All } (op = w) \rrbracket \implies P w s$   
 $\langle \text{proof} \rangle$

**lemma** *ax-nochange*:

$$G, (A::(\text{res} \times \text{state}) \text{ triple set}) \vdash \{\lambda Y s Z. (Y, s) = Z\} \text{ } t \succ \{\lambda Y s Z. (Y, s) = Z\} \\ \implies G, A \vdash \{P::(\text{res} \times \text{state}) \text{ assn}\} \text{ } t \succ \{P\} \\ \langle \text{proof} \rangle$$

**lemma** *ax-trivial*:  $G, (A::'a \text{ triple set}) \vdash \{P::'a \text{ assn}\} \text{ } t \succ \{\lambda Y s Z. \text{True}\}$

$\langle \text{proof} \rangle$

**lemma** *ax-disj*:

$$\llbracket G, (A::'a \text{ triple set}) \vdash \{P1::'a \text{ assn}\} \text{ } t \succ \{Q1\}; G, A \vdash \{P2::'a \text{ assn}\} \text{ } t \succ \{Q2\} \rrbracket \\ \implies G, A \vdash \{\lambda Y s Z. P1 Y s Z \vee P2 Y s Z\} \text{ } t \succ \{\lambda Y s Z. Q1 Y s Z \vee Q2 Y s Z\} \\ \langle \text{proof} \rangle$$

**lemma** *ax-supd-shuffle*:

$$(\exists Q. G, (A::'a \text{ triple set}) \vdash \{P::'a \text{ assn}\} \text{ } .c1. \{Q\} \wedge G, A \vdash \{Q ;. f\} \text{ } .c2. \{R\}) = \\ (\exists Q'. G, A \vdash \{P\} \text{ } .c1. \{f ;. Q'\} \wedge G, A \vdash \{Q'\} \text{ } .c2. \{R\}) \\ \langle \text{proof} \rangle$$

**lemma** *ax-cases*:

$$\llbracket G, (A::'a \text{ triple set}) \vdash \{P \wedge. C\} \text{ } t \succ \{Q::'a \text{ assn}\}; \\ G, A \vdash \{P \wedge. \text{Not} \circ C\} \text{ } t \succ \{Q\} \rrbracket \implies G, A \vdash \{P\} \text{ } t \succ \{Q\} \\ \langle \text{proof} \rangle$$

**lemma** *ax-adapt*:  $G, (A::'a \text{ triple set}) \vdash \{P::'a \text{ assn}\} \text{ } t \succ \{Q\}$

$$\implies G, A \vdash \{\text{adapt-pre } P \ Q \ Q'\} \text{ } t \succ \{Q'\} \\ \langle \text{proof} \rangle$$

**lemma** *adapt-pre-adapts*:  $G, (A::'a \text{ triple set}) \models \{P::'a \text{ assn}\} \text{ } t \succ \{Q\}$

$$\longrightarrow G, A \models \{\text{adapt-pre } P \ Q \ Q'\} \text{ } t \succ \{Q'\} \\ \langle \text{proof} \rangle$$

**lemma** *adapt-pre-weakest*:

$$\forall G \ (A::'a \text{ triple set}) \ t. \ G, A \models \{P\} \text{ } t \succ \{Q\} \longrightarrow G, A \models \{P'\} \text{ } t \succ \{Q'\} \implies \\ P' \Rightarrow \text{adapt-pre } P \ Q \ (Q'::'a \text{ assn}) \\ \langle \text{proof} \rangle$$

**lemma** *peek-and-forget1-Normal*:

$$G, (A::'a \text{ triple set}) \vdash \{\text{Normal } P\} \text{ } t \succ \{Q::'a \text{ assn}\} \\ \implies G, A \vdash \{\text{Normal } (P \wedge. p)\} \text{ } t \succ \{Q\} \\ \langle \text{proof} \rangle$$

**lemma** *peek-and-forget1*:

$$G, (A::'a \text{ triple set}) \vdash \{P::'a \text{ assn}\} \text{ } t \succ \{Q\} \\ \implies G, A \vdash \{P \wedge. p\} \text{ } t \succ \{Q\} \\ \langle \text{proof} \rangle$$

**lemmas** *ax-NormalD* = *peek-and-forget1* [*of* - - - - *normal*]

**lemma** *peek-and-forget2*:

$G, (A :: 'a \text{ triple set}) \vdash \{P :: 'a \text{ assn}\} \text{ } t \succ \{Q \wedge p\}$

$\implies G, A \vdash \{P\} \text{ } t \succ \{Q\}$

*<proof>*

**lemma** *ax-subst-Val-allI*:

$\forall v. G, (A :: 'a \text{ triple set}) \vdash \{(P' \quad v) \leftarrow \text{Val } v\} \text{ } t \succ \{(Q \text{ } v) :: 'a \text{ assn}\}$

$\implies \forall v. G, A \vdash \{(\lambda w. P' (\text{the-In1 } w)) \leftarrow \text{Val } v\} \text{ } t \succ \{Q \text{ } v\}$

*<proof>*

**lemma** *ax-subst-Var-allI*:

$\forall v. G, (A :: 'a \text{ triple set}) \vdash \{(P' \quad v) \leftarrow \text{Var } v\} \text{ } t \succ \{(Q \text{ } v) :: 'a \text{ assn}\}$

$\implies \forall v. G, A \vdash \{(\lambda w. P' (\text{the-In2 } w)) \leftarrow \text{Var } v\} \text{ } t \succ \{Q \text{ } v\}$

*<proof>*

**lemma** *ax-subst-Vals-allI*:

$(\forall v. G, (A :: 'a \text{ triple set}) \vdash \{(P' \quad v) \leftarrow \text{Vals } v\} \text{ } t \succ \{(Q \text{ } v) :: 'a \text{ assn}\})$

$\implies \forall v. G, A \vdash \{(\lambda w. P' (\text{the-In3 } w)) \leftarrow \text{Vals } v\} \text{ } t \succ \{Q \text{ } v\}$

*<proof>*

## alternative axioms

**lemma** *ax-Lit2*:

$G, (A :: 'a \text{ triple set}) \vdash \{\text{Normal } P :: 'a \text{ assn}\} \text{ } \text{Lit } v \multimap \{\text{Normal } (P \downarrow = \text{Val } v)\}$

*<proof>*

**lemma** *ax-Lit2-test-complete*:

$G, (A :: 'a \text{ triple set}) \vdash \{\text{Normal } (P \leftarrow \text{Val } v) :: 'a \text{ assn}\} \text{ } \text{Lit } v \multimap \{P\}$

*<proof>*

**lemma** *ax-LVar2*:  $G, (A :: 'a \text{ triple set}) \vdash \{\text{Normal } P :: 'a \text{ assn}\} \text{ } \text{LVar } vn \multimap \{\text{Normal } (\lambda s. P \downarrow = \text{Var } (\text{lvar } vn \text{ } s))\}$

*<proof>*

**lemma** *ax-Super2*:  $G, (A :: 'a \text{ triple set}) \vdash$

$\{\text{Normal } P :: 'a \text{ assn}\} \text{ } \text{Super} \multimap \{\text{Normal } (\lambda s. P \downarrow = \text{Val } (\text{val-this } s))\}$

*<proof>*

**lemma** *ax-Nil2*:

$G, (A :: 'a \text{ triple set}) \vdash \{\text{Normal } P :: 'a \text{ assn}\} \text{ } [] \multimap \{\text{Normal } (P \downarrow = \text{Vals } [])\}$

*<proof>*

## misc derived structural rules

**lemma** *ax-finite-mtriples-lemma*:  $\llbracket F \subseteq ms; \text{finite } ms; \forall (C, sig) \in ms.$

$G, (A :: 'a \text{ triple set}) \vdash \{\text{Normal } (P \text{ } C \text{ } sig) :: 'a \text{ assn}\} \text{ } mb \text{ } C \text{ } sig \multimap \{Q \text{ } C \text{ } sig\} \rrbracket \implies$

$G, A \vdash \{\{P\} \text{ } mb \multimap \{Q\} \mid F\}$

*<proof>*

**lemmas** *ax-finite-mtriples* = *ax-finite-mtriples-lemma* [*OF subset-refl*]

**lemma** *ax-derivs-insertD*:

$G, (A::'a \text{ triple set}) \vdash \text{insert } (t::'a \text{ triple}) \text{ } ts \implies G, A \vdash t \wedge G, A \vdash ts$   
 $\langle \text{proof} \rangle$

**lemma** *ax-methods-spec*:

$\llbracket G, (A::'a \text{ triple set}) \vdash \text{split } f \text{ ' } ms; (C, sig) \in ms \rrbracket \implies G, A \vdash ((f \ C \ sig)::'a \text{ triple})$   
 $\langle \text{proof} \rangle$

**lemma** *ax-finite-pointwise-lemma* [rule-format]:  $\llbracket F \subseteq ms; \text{finite } ms \rrbracket \implies$

$((\forall (C, sig) \in F. G, (A::'a \text{ triple set}) \vdash (f \ C \ sig)::'a \text{ triple})) \longrightarrow (\forall (C, sig) \in ms. G, A \vdash (g \ C \ sig)::'a \text{ triple})) \longrightarrow$   
 $G, A \vdash \text{split } f \text{ ' } F \longrightarrow G, A \vdash \text{split } g \text{ ' } F$   
 $\langle \text{proof} \rangle$

**lemmas** *ax-finite-pointwise* = *ax-finite-pointwise-lemma* [OF subset-refl]

**lemma** *ax-no-hazard*:

$G, (A::'a \text{ triple set}) \vdash \{P \wedge. \text{type-ok } G \ t\} \ t \succ \{Q::'a \text{ assn}\} \implies G, A \vdash \{P\} \ t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-free-wt*:

$(\exists T \ L \ C. (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash t::T) \longrightarrow G, (A::'a \text{ triple set}) \vdash \{\text{Normal } P\} \ t \succ \{Q::'a \text{ assn}\} \implies$   
 $G, A \vdash \{\text{Normal } P\} \ t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

$\langle \text{ML} \rangle$

**declare** *ax-Abrupts* [intro!]

**lemmas** *ax-Normal-cases* = *ax-cases* [of - - normal]

**lemma** *ax-Skip* [intro!]:  $G, (A::'a \text{ triple set}) \vdash \{P \leftarrow \Diamond\} . \text{Skip}. \{P::'a \text{ assn}\}$

$\langle \text{proof} \rangle$

**lemmas** *ax-SkipI* = *ax-Skip* [THEN conseq1, standard]

## derived rules for methd call

**lemma** *ax-Call-known-DynT*:

$\llbracket G \vdash \text{IntVir} \rightarrow C \preceq \text{statT};$   
 $\forall a \text{ vs } l. G, A \vdash \{(R \leftarrow \text{Vals } vs \wedge. (\lambda s. l = \text{locals } (store \ s))) \};$   
 $\text{init-lvars } G \ C \ (\text{name} = mn, \text{parTs} = pTs) \ \text{IntVir } a \text{ vs}\}$   
 $\text{Methd } C \ (\text{name} = mn, \text{parTs} = pTs) \dashv \succ \{\text{set-lvars } l \ .; S\};$   
 $\forall a. G, A \vdash \{Q \leftarrow \text{Val } a\} \ \text{args} \dashv \succ$   
 $\{R \ a \wedge. (\lambda s. C = \text{obj-class } (the \ (heap \ (store \ s) \ (the \ \text{Addr } a)))) \wedge$   
 $C = \text{invocation-declclass}$   
 $G \ \text{IntVir } (store \ s) \ a \ \text{statT } (\text{name} = mn, \text{parTs} = pTs) \ \};$   
 $G, (A::'a \text{ triple set}) \vdash \{\text{Normal } P\} \ e \dashv \succ \{Q::'a \text{ assn}\}$   
 $\implies G, A \vdash \{\text{Normal } P\} \ \{\text{accC}, \text{statT}, \text{IntVir}\} e \cdot mn(\{pTs\} \text{args}) \dashv \succ \{S\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-Call-Static*:

$\llbracket \forall a \text{ vs } l. G, A \vdash \{R \leftarrow \text{Vals } vs \wedge. (\lambda s. l = \text{locals } (store \ s)) \};$   
 $\quad \text{init-lvars } G \ C \ (\llbracket name=mn, parTs=pTs \rrbracket \text{ Static any-Addr } vs) \}$   
 $\quad \text{Methd } C \ (\llbracket name=mn, parTs=pTs \rrbracket) \multimap \{set-lvars \ l \ ; \ S\};$   
 $G, A \vdash \{Normal \ P\} \ e \multimap \{Q\};$   
 $\forall a. G, (A::'a \text{ triple set}) \vdash \{Q \leftarrow \text{Val } a\} \text{ args} \dot{\multimap} \{(R::val \Rightarrow 'a \text{ assn}) \ a$   
 $\wedge. (\lambda s. C = \text{invocation-declclass}$   
 $\quad G \text{ Static } (store \ s) \ a \text{ statT } (\llbracket name=mn, parTs=pTs \rrbracket))\}$   
 $\rrbracket \implies G, A \vdash \{Normal \ P\} \{accC, statT, Static\} e \cdot mn(\{pTs\}args) \multimap \{S\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-Methd1*:

$\llbracket G, A \cup \{\{P\} \text{ Methd} \multimap \{Q\} \mid ms\} \vdash \{\{P\} \text{ body } G \multimap \{Q\} \mid ms\}; (C, sig) \in ms \rrbracket \implies$   
 $G, A \vdash \{Normal \ (P \ C \ sig)\} \text{ Methd } C \ sig \multimap \{Q \ C \ sig\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-MethdN*:

$G, \text{insert}(\{Normal \ P\} \text{ Methd } C \ sig \multimap \{Q\}) \ A \vdash$   
 $\{Normal \ P\} \text{ body } G \ C \ sig \multimap \{Q\} \implies$   
 $G, A \vdash \{Normal \ P\} \text{ Methd } C \ sig \multimap \{Q\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-StatRef*:

$G, (A::'a \text{ triple set}) \vdash \{Normal \ (P \leftarrow \text{Val } Null)\} \text{ StatRef } rt \multimap \{P::'a \text{ assn}\}$   
 $\langle \text{proof} \rangle$

## rules derived from Init and Done

**lemma** *ax-InitS*:  $\llbracket the \ (class \ G \ C) = c; C \neq \text{Object};$

$\forall l. G, A \vdash \{Q \wedge. (\lambda s. l = \text{locals } (store \ s)) \}; \text{ set-lvars empty}\}$   
 $\quad \text{init } c. \{set-lvars \ l \ ; \ R\};$   
 $G, A \vdash \{Normal \ ((P \wedge. \text{Not} \circ \text{initd } C) \ ; \ \text{supd } (\text{init-class-obj } G \ C))\}$   
 $\quad \text{Init } (\text{super } c). \{Q\} \rrbracket \implies$   
 $G, (A::'a \text{ triple set}) \vdash \{Normal \ (P \wedge. \text{Not} \circ \text{initd } C)\} \text{ Init } C. \{R::'a \text{ assn}\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-Init-Skip-lemma*:

$\forall l. G, (A::'a \text{ triple set}) \vdash \{P \leftarrow \Diamond \wedge. (\lambda s. l = \text{locals } (store \ s)) \}; \text{ set-lvars } l'\}$   
 $\quad \text{Skip}. \{(set-lvars \ l \ ; \ P)::'a \text{ assn}\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-triv-InitS*:  $\llbracket the \ (class \ G \ C) = c; \text{init } c = \text{Skip}; C \neq \text{Object};$

$P \leftarrow \Diamond \implies (\text{supd } (\text{init-class-obj } G \ C) \ ; \ P);$   
 $G, A \vdash \{Normal \ (P \wedge. \text{initd } C)\} \text{ Init } (\text{super } c). \{(P \wedge. \text{initd } C) \leftarrow \Diamond\} \rrbracket \implies$   
 $G, (A::'a \text{ triple set}) \vdash \{Normal \ P \leftarrow \Diamond\} \text{ Init } C. \{(P \wedge. \text{initd } C)::'a \text{ assn}\}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-Init-Object*:  $wf\text{-prog } G \implies G, (A::'a \text{ triple set}) \vdash$

$\{Normal \ ((\text{supd } (\text{init-class-obj } G \ \text{Object}) \ ; \ P \leftarrow \Diamond) \wedge. \text{Not} \circ \text{initd } \text{Object})\}$   
 $\quad \text{Init } \text{Object}. \{(P \wedge. \text{initd } \text{Object})::'a \text{ assn}\}$   
 $\langle \text{proof} \rangle$



**lemma** *ax-triv-Init-Object*:  $\llbracket \text{wf-prog } G; (P::'a \text{ assn}) \Rightarrow (\text{supd } (\text{init-class-obj } G \text{ Object}) .; P) \rrbracket \Longrightarrow$   
 $G, (A::'a \text{ triple set}) \vdash \{ \text{Normal } P \leftarrow \Diamond \} . \text{Init Object} . \{ P \wedge . \text{initd Object} \}$   
 $\langle \text{proof} \rangle$

### introduction rules for Alloc and SXAlloc

**lemma** *ax-SXAlloc-Normal*:  
 $G, (A::'a \text{ triple set}) \vdash \{ P::'a \text{ assn} \} .c. \{ \text{Normal } Q \}$   
 $\Longrightarrow G, A \vdash \{ P \} .c. \{ \text{SXAlloc } G \text{ } Q \}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-Alloc*:  
 $G, (A::'a \text{ triple set}) \vdash \{ P::'a \text{ assn} \} t \succ$   
 $\{ \text{Normal } (\lambda Y (x,s) Z. (\forall a. \text{new-Addr } (\text{heap } s) = \text{Some } a \longrightarrow$   
 $Q (\text{Val } (\text{Addr } a)) (\text{Norm}(\text{init-obj } G (\text{CInst } C) (\text{Heap } a) s)) Z)) \wedge .$   
 $\text{heap-free } (\text{Suc } (\text{Suc } 0))) \}$   
 $\Longrightarrow G, A \vdash \{ P \} t \succ \{ \text{Alloc } G (\text{CInst } C) \text{ } Q \}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-Alloc-Arr*:  
 $G, (A::'a \text{ triple set}) \vdash \{ P::'a \text{ assn} \} t \succ$   
 $\{ \lambda \text{Val}:i. \text{Normal } (\lambda Y (x,s) Z. \neg \text{the-Intg } i < 0 \wedge$   
 $(\forall a. \text{new-Addr } (\text{heap } s) = \text{Some } a \longrightarrow$   
 $Q (\text{Val } (\text{Addr } a)) (\text{Norm } (\text{init-obj } G (\text{Arr } T (\text{the-Intg } i)) (\text{Heap } a) s)) Z)) \wedge .$   
 $\text{heap-free } (\text{Suc } (\text{Suc } 0))) \}$   
 $\Longrightarrow$   
 $G, A \vdash \{ P \} t \succ \{ \lambda \text{Val}:i. \text{abupd } (\text{check-neg } i) .; \text{Alloc } G (\text{Arr } T(\text{the-Intg } i)) \text{ } Q \}$   
 $\langle \text{proof} \rangle$

**lemma** *ax-SXAlloc-catch-SXcpt*:  
 $\llbracket G, (A::'a \text{ triple set}) \vdash \{ P::'a \text{ assn} \} t \succ$   
 $\{ (\lambda Y (x,s) Z. x = \text{Some } (\text{Xcpt } (\text{Std } xn)) \wedge$   
 $(\forall a. \text{new-Addr } (\text{heap } s) = \text{Some } a \longrightarrow$   
 $Q Y (\text{Some } (\text{Xcpt } (\text{Loc } a)), \text{init-obj } G (\text{CInst } (\text{SXcpt } xn)) (\text{Heap } a) s) Z))$   
 $\wedge . \text{heap-free } (\text{Suc } (\text{Suc } 0))) \rrbracket$   
 $\Longrightarrow$   
 $G, A \vdash \{ P \} t \succ \{ \text{SXAlloc } G (\lambda Y s Z. Q Y s Z \wedge G, s \vdash \text{catch } \text{SXcpt } xn) \}$   
 $\langle \text{proof} \rangle$

**end**



## Chapter 23

# AxSound

## 62 Soundness proof for Axiomatic semantics of Java expressions and statements

theory *AxSound* imports *AxSem* begin

validity

consts

*triple-valid2*:: *prog*  $\Rightarrow$  *nat*  $\Rightarrow$  *'a triple*  $\Rightarrow$  *bool*  
 ( *-*  $\models$  ::- [61,0, 58] 57)  
*ax-valids2*:: *prog*  $\Rightarrow$  *'a triples*  $\Rightarrow$  *'a triples*  $\Rightarrow$  *bool*  
 ( *-*  $\models$  ::- [61,58,58] 57)

**defs** *triple-valid2-def*:  $G \models n::t \equiv \text{case } t \text{ of } \{P\} \triangleright \{Q\} \Rightarrow$   
 $\forall Y \ s \ Z. P \ Y \ s \ Z \longrightarrow (\forall L. s::\preceq(G,L)$   
 $\longrightarrow (\forall T \ C \ A. (\text{normal } s \longrightarrow ((\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash t::T \wedge$   
 $(\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash \text{dom } (\text{locals } (\text{store } s)) \triangleright t \triangleright A)) \longrightarrow$   
 $(\forall Y' \ s'. G \vdash s -t \triangleright -n \longrightarrow (Y', s') \longrightarrow Q \ Y' \ s' \ Z \wedge s'::\preceq(G,L))))$

This definition differs from the ordinary *triple-valid-def* manly in the conclusion: We also ensures conformance of the result state. So we don't have to apply the type soundness lemma all the time during induction. This definition is only introduced for the soundness proof of the axiomatic semantics, in the end we will conclude to the ordinary definition.

**defs** *ax-valids2-def*:  $G, A \models ts \equiv \forall n. (\forall t \in A. G \models n::t) \longrightarrow (\forall t \in ts. G \models n::t)$

**lemma** *triple-valid2-def2*:  $G \models n::\{P\} \triangleright \{Q\} =$   
 $(\forall Y \ s \ Z. P \ Y \ s \ Z \longrightarrow (\forall Y' \ s'. G \vdash s -t \triangleright -n \longrightarrow (Y', s') \longrightarrow$   
 $(\forall L. s::\preceq(G,L) \longrightarrow (\forall T \ C \ A. (\text{normal } s \longrightarrow ((\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash t::T \wedge$   
 $(\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash \text{dom } (\text{locals } (\text{store } s)) \triangleright t \triangleright A)) \longrightarrow$   
 $Q \ Y' \ s' \ Z \wedge s'::\preceq(G,L))))$   
 <proof>

**lemma** *triple-valid2-eq* [rule-format (no-asm)]:  
 $\text{wf-prog } G \implies \text{triple-valid2 } G = \text{triple-valid } G$   
 <proof>

**lemma** *ax-valids2-eq*:  $\text{wf-prog } G \implies G, A \models ts = G, A \models ts$   
 <proof>

**lemma** *triple-valid2-Suc* [rule-format (no-asm)]:  $G \models \text{Suc } n::t \longrightarrow G \models n::t$   
 <proof>

**lemma** *Methd-triple-valid2-0*:  $G \models 0::\{\text{Normal } P\} \text{ Methd } C \text{ sig} \triangleright \{Q\}$   
 <proof>

**lemma** *Methd-triple-valid2-SucI*:  
 $\llbracket G \models n::\{\text{Normal } P\} \text{ body } G \ C \ \text{sig} \triangleright \{Q\} \rrbracket$   
 $\implies G \models \text{Suc } n::\{\text{Normal } P\} \text{ Methd } C \ \text{sig} \triangleright \{Q\}$   
 <proof>

**lemma** *triples-valid2-Suc*:

*Ball ts (triple-valid2 G (Suc n))  $\implies$  Ball ts (triple-valid2 G n)*  
*<proof>*

**lemma**  $G \models n::\text{insert } t \ A = (G \models n:t \wedge G \models n:A)$   
*<proof>*

### soundness

**lemma** *Method-sound*:

**assumes** *recursive*:  $G, A \cup \{\{P\} \text{ Method} \multimap \{Q\} \mid ms\} \models \{\{P\} \text{ body } G \multimap \{Q\} \mid ms\}$   
**shows**  $G, A \models \{\{P\} \text{ Method} \multimap \{Q\} \mid ms\}$   
*<proof>*

**lemma** *valids2-inductI*:  $\forall s \ t \ n \ Y' \ s'. \ G \vdash s \multimap t \multimap n \rightarrow (Y', s') \longrightarrow t = c \longrightarrow$   
 $\text{Ball } A \ (\text{triple-valid2 } G \ n) \longrightarrow (\forall Y \ Z. \ P \ Y \ s \ Z \longrightarrow$   
 $(\forall L. \ s::\preceq(G, L) \longrightarrow$   
 $(\forall T \ C \ A. \ (\text{normal } s \longrightarrow (\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = L \rrbracket \vdash t::T) \wedge$   
 $\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = L \rrbracket \vdash \text{dom } (\text{locals } (\text{store } s)) \gg t \gg A) \longrightarrow$   
 $Q \ Y' \ s' \ Z \wedge s'::\preceq(G, L))) \implies$   
 $G, A \models \{\{P\} \ c \multimap \{Q\}\}$   
*<proof>*

**lemma** *da-good-approx-evalnE [consumes 4]*:

**assumes** *evaln*:  $G \vdash s0 \multimap t \multimap n \rightarrow (v, s1)$   
**and** *wt*:  $\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = L \rrbracket \vdash t::T$   
**and** *da*:  $\llbracket \text{prg} = G, \text{cls} = C, \text{lcl} = L \rrbracket \vdash \text{dom } (\text{locals } (\text{store } s0)) \gg t \gg A$   
**and** *wf*: *wf-prog*  $G$   
**and** *elim*:  $\llbracket \text{normal } s1 \implies \text{nrm } A \subseteq \text{dom } (\text{locals } (\text{store } s1));$   
 $\wedge l. \llbracket \text{abrupt } s1 = \text{Some } (\text{Jump } (\text{Break } l)); \text{normal } s0 \rrbracket$   
 $\implies \text{brk } A \ l \subseteq \text{dom } (\text{locals } (\text{store } s1));$   
 $\llbracket \text{abrupt } s1 = \text{Some } (\text{Jump } \text{Ret}); \text{normal } s0 \rrbracket$   
 $\implies \text{Result} \in \text{dom } (\text{locals } (\text{store } s1))$   
 $\rrbracket \implies P$   
**shows**  $P$   
*<proof>*

**lemma** *validI*:

**assumes**  $I: \bigwedge n \ s0 \ L \ \text{accC} \ T \ C \ v \ s1 \ Y \ Z.$   
 $\llbracket \forall t \in A. \ G \models n::t; \ s0::\preceq(G, L);$   
 $\text{normal } s0 \implies \llbracket \text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L \rrbracket \vdash t::T;$   
 $\text{normal } s0 \implies \llbracket \text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L \rrbracket \vdash \text{dom } (\text{locals } (\text{store } s0)) \gg t \gg C;$   
 $G \vdash s0 \multimap t \multimap n \rightarrow (v, s1); \ P \ Y \ s0 \ Z \rrbracket \implies Q \ v \ s1 \ Z \wedge s1::\preceq(G, L)$   
**shows**  $G, A \models \{\{P\} \ t \multimap \{Q\}\}$   
*<proof>*

**declare**  $[[\text{simproc } \text{add}: \text{wt-expr } \text{wt-var } \text{wt-exprs } \text{wt-stmt}]]$

**lemma** *valid-stmtI*:

**assumes**  $I: \bigwedge n \ s0 \ L \ \text{accC} \ C \ s1 \ Y \ Z.$   
 $\llbracket \forall t \in A. \ G \models n::t; \ s0::\preceq(G, L);$   
 $\text{normal } s0 \implies \llbracket \text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L \rrbracket \vdash c::\checkmark;$   
 $\text{normal } s0 \implies \llbracket \text{prg} = G, \text{cls} = \text{accC}, \text{lcl} = L \rrbracket \vdash \text{dom } (\text{locals } (\text{store } s0)) \gg \langle c \rangle_s \gg C;$

$$G \vdash s0 \text{ } -c-n \rightarrow s1; P \ Y \ s0 \ Z \Longrightarrow Q \ \Diamond \ s1 \ Z \wedge s1::\preceq(G,L)$$
**shows**  $G, A \models::\{ \{P\} \ \langle c \rangle_s \succ \{Q\} \}$   
 <proof>

**lemma** *valid-stmt-NormalI*:

**assumes**  $I: \bigwedge n \ s0 \ L \ accC \ C \ s1 \ Y \ Z.$   

$$\begin{aligned} & \llbracket \forall t \in A. \ G \models n::t; \ s0::\preceq(G,L); \ normal \ s0; \ (\langle prg=G, cls=accC, lcl=L \rangle) \vdash c::\surd; \\ & (\langle prg=G, cls=accC, lcl=L \rangle) \vdash dom \ (locals \ (store \ s0)) \rangle \langle c \rangle_s \rangle C; \\ & G \vdash s0 \text{ } -c-n \rightarrow s1; (Normal \ P) \ Y \ s0 \ Z \Longrightarrow Q \ \Diamond \ s1 \ Z \wedge s1::\preceq(G,L) \end{aligned}$$
**shows**  $G, A \models::\{ \{Normal \ P\} \ \langle c \rangle_s \succ \{Q\} \}$   
 <proof>

**lemma** *valid-var-NormalI*:

**assumes**  $I: \bigwedge n \ s0 \ L \ accC \ T \ C \ vf \ s1 \ Y \ Z.$   

$$\begin{aligned} & \llbracket \forall t \in A. \ G \models n::t; \ s0::\preceq(G,L); \ normal \ s0; \\ & (\langle prg=G, cls=accC, lcl=L \rangle) \vdash t::=T; \\ & (\langle prg=G, cls=accC, lcl=L \rangle) \vdash dom \ (locals \ (store \ s0)) \rangle \langle t \rangle_v \rangle C; \\ & G \vdash s0 \text{ } -t=\succ vf-n \rightarrow s1; (Normal \ P) \ Y \ s0 \ Z \Longrightarrow Q \ (In2 \ vf) \ s1 \ Z \wedge s1::\preceq(G,L) \end{aligned}$$
**shows**  $G, A \models::\{ \{Normal \ P\} \ \langle t \rangle_v \succ \{Q\} \}$   
 <proof>

**lemma** *valid-expr-NormalI*:

**assumes**  $I: \bigwedge n \ s0 \ L \ accC \ T \ C \ v \ s1 \ Y \ Z.$   

$$\begin{aligned} & \llbracket \forall t \in A. \ G \models n::t; \ s0::\preceq(G,L); \ normal \ s0; \\ & (\langle prg=G, cls=accC, lcl=L \rangle) \vdash t::=T; \\ & (\langle prg=G, cls=accC, lcl=L \rangle) \vdash dom \ (locals \ (store \ s0)) \rangle \langle t \rangle_e \rangle C; \\ & G \vdash s0 \text{ } -t=\succ v-n \rightarrow s1; (Normal \ P) \ Y \ s0 \ Z \Longrightarrow Q \ (In1 \ v) \ s1 \ Z \wedge s1::\preceq(G,L) \end{aligned}$$
**shows**  $G, A \models::\{ \{Normal \ P\} \ \langle t \rangle_e \succ \{Q\} \}$   
 <proof>

**lemma** *valid-expr-list-NormalI*:

**assumes**  $I: \bigwedge n \ s0 \ L \ accC \ T \ C \ vs \ s1 \ Y \ Z.$   

$$\begin{aligned} & \llbracket \forall t \in A. \ G \models n::t; \ s0::\preceq(G,L); \ normal \ s0; \\ & (\langle prg=G, cls=accC, lcl=L \rangle) \vdash t::=T; \\ & (\langle prg=G, cls=accC, lcl=L \rangle) \vdash dom \ (locals \ (store \ s0)) \rangle \langle t \rangle_l \rangle C; \\ & G \vdash s0 \text{ } -t=\succ vs-n \rightarrow s1; (Normal \ P) \ Y \ s0 \ Z \Longrightarrow Q \ (In3 \ vs) \ s1 \ Z \wedge s1::\preceq(G,L) \end{aligned}$$
**shows**  $G, A \models::\{ \{Normal \ P\} \ \langle t \rangle_l \succ \{Q\} \}$   
 <proof>

**lemma** *validE* [consumes 5]:

**assumes** *valid*:  $G, A \models::\{ \{P\} \ t \succ \{Q\} \}$   
**and**  $P: P \ Y \ s0 \ Z$   
**and** *valid-A*:  $\forall t \in A. \ G \models n::t$   
**and** *conf*:  $s0::\preceq(G,L)$   
**and** *eval*:  $G \vdash s0 \text{ } -t=\succ -n \rightarrow (v, s1)$   
**and** *wt*:  $normal \ s0 \Longrightarrow (\langle prg=G, cls=accC, lcl=L \rangle) \vdash t::T$   
**and** *da*:  $normal \ s0 \Longrightarrow (\langle prg=G, cls=accC, lcl=L \rangle) \vdash dom \ (locals \ (store \ s0)) \rangle t \rangle C$   
**and** *elim*:  $\llbracket Q \ v \ s1 \ Z; \ s1::\preceq(G,L) \rrbracket \Longrightarrow concl$   
**shows** *concl*  
 <proof>

**lemma** *all-empty*:  $(!x. P) = P$   
 ⟨proof⟩

**corollary** *evaln-type-sound*:

**assumes** *evaln*:  $G \vdash s0 \rightarrow -t \rightarrow -n \rightarrow (v, s1)$  **and**  
     *wt*:  $(\text{prg} = G, \text{cls} = \text{acc}C, \text{lcl} = L) \vdash t :: T$  **and**  
     *da*:  $(\text{prg} = G, \text{cls} = \text{acc}C, \text{lcl} = L) \vdash \text{dom} (\text{locals} (\text{store } s0)) \gg t \gg A$  **and**  
     *conf-s0*:  $s0 :: \preceq (G, L)$  **and**  
     *wf*: *wf-prog*  $G$   
**shows**  $s1 :: \preceq (G, L) \wedge (\text{normal } s1 \longrightarrow G, L, \text{store } s1 \vdash t \rightarrow v :: \preceq T) \wedge$   
      $(\text{error-free } s0 = \text{error-free } s1)$   
 ⟨proof⟩

**corollary** *dom-locals-evaln-mono-elim* [*consumes 1*]:

**assumes**  
     *evaln*:  $G \vdash s0 \rightarrow -t \rightarrow -n \rightarrow (v, s1)$  **and**  
     *hyps*:  $\llbracket \text{dom} (\text{locals} (\text{store } s0)) \subseteq \text{dom} (\text{locals} (\text{store } s1)) \rrbracket$   
      $\bigwedge v \ s \ \text{val}. \llbracket v = \text{In2 } vv; \text{normal } s1 \rrbracket$   
      $\implies \text{dom} (\text{locals} (\text{store } s))$   
      $\subseteq \text{dom} (\text{locals} (\text{store } ((\text{snd } vv) \ \text{val } s))) \rrbracket \implies P$   
**shows**  $P$   
 ⟨proof⟩

**lemma** *evaln-no-abrupt*:

$\bigwedge s \ s'. \llbracket G \vdash s \rightarrow -t \rightarrow -n \rightarrow (w, s') ; \text{normal } s' \rrbracket \implies \text{normal } s$   
 ⟨proof⟩

**declare** *inj-term-simps* [*simp*]

**lemma** *ax-sound2*:

**assumes** *wf*: *wf-prog*  $G$   
**and** *deriv*:  $G, A \vdash ts$   
**shows**  $G, A \models ts$   
 ⟨proof⟩

**declare** *inj-term-simps* [*simp del*]

**theorem** *ax-sound*:

*wf-prog*  $G \implies G, (A :: 'a \text{ triple set}) \vdash (ts :: 'a \text{ triple set}) \implies G, A \models ts$   
 ⟨proof⟩

**lemma** *sound-valid2-lemma*:

$\llbracket \forall v \ n. \text{Ball } A (\text{triple-valid2 } G \ n) \longrightarrow P \ v \ n ; \text{Ball } A (\text{triple-valid2 } G \ n) \rrbracket$   
 $\implies P \ v \ n$   
 ⟨proof⟩

**end**





## Chapter 24

### AxCompl

### 63 Completeness proof for Axiomatic semantics of Java expressions and statements

**theory** *AxCompl* **imports** *AxSem* **begin**

design issues:

- proof structured by Most General Formulas (-i Thomas Kleymann)

**set of not yet initialized classes**

**constdefs**

*nyinitcls* :: *prog*  $\Rightarrow$  *state*  $\Rightarrow$  *qname* *set*  
*nyinitcls* *G* *s*  $\equiv$  { *C*. *is-class* *G* *C*  $\wedge$   $\neg$  *initd* *C* *s* }

**lemma** *nyinitcls-subset-class*: *nyinitcls* *G* *s*  $\subseteq$  { *C*. *is-class* *G* *C* }  
 $\langle$ *proof* $\rangle$

**lemmas** *finite-nyinitcls* [*simp*] =  
*finite-is-class* [*THEN nyinitcls-subset-class* [*THEN finite-subset*], *standard*]

**lemma** *card-nyinitcls-bound*: *card* (*nyinitcls* *G* *s*)  $\leq$  *card* { *C*. *is-class* *G* *C* }  
 $\langle$ *proof* $\rangle$

**lemma** *nyinitcls-set-locals-cong* [*simp*]:  
*nyinitcls* *G* (*x*, *set-locals* *l* *s*) = *nyinitcls* *G* (*x*, *s*)  
 $\langle$ *proof* $\rangle$

**lemma** *nyinitcls-abrupt-cong* [*simp*]: *nyinitcls* *G* (*f* *x*, *y*) = *nyinitcls* *G* (*x*, *y*)  
 $\langle$ *proof* $\rangle$

**lemma** *nyinitcls-abupd-cong* [*simp*]:!!*s*. *nyinitcls* *G* (*abupd* *f* *s*) = *nyinitcls* *G* *s*  
 $\langle$ *proof* $\rangle$

**lemma** *card-nyinitcls-abrupt-congE* [*elim*!]:  
*card* (*nyinitcls* *G* (*x*, *s*))  $\leq$  *n*  $\Longrightarrow$  *card* (*nyinitcls* *G* (*y*, *s*))  $\leq$  *n*  
 $\langle$ *proof* $\rangle$

**lemma** *nyinitcls-new-xcpt-var* [*simp*]:  
*nyinitcls* *G* (*new-xcpt-var* *vn* *s*) = *nyinitcls* *G* *s*  
 $\langle$ *proof* $\rangle$

**lemma** *nyinitcls-init-lvars* [*simp*]:  
*nyinitcls* *G* ((*init-lvars* *G* *C* *sig* *mode* *a'* *pvs*) *s*) = *nyinitcls* *G* *s*  
 $\langle$ *proof* $\rangle$

**lemma** *nyinitcls-emptyD*:  $\llbracket$ *nyinitcls* *G* *s* = {} $\rrbracket$ ; *is-class* *G* *C*  $\Longrightarrow$  *initd* *C* *s*  
 $\langle$ *proof* $\rangle$

**lemma** *card-Suc-lemma*:

$\llbracket \text{card } (\text{insert } a \ A) \leq \text{Suc } n; a \notin A; \text{finite } A \rrbracket \implies \text{card } A \leq n$   
 $\langle \text{proof} \rangle$

**lemma** *nyinitcls-le-SucD*:

$\llbracket \text{card } (\text{nyinitcls } G \ (x,s)) \leq \text{Suc } n; \neg \text{initd } C \ (\text{globs } s); \text{class } G \ C = \text{Some } y \rrbracket \implies$   
 $\text{card } (\text{nyinitcls } G \ (x, \text{init-class-obj } G \ C \ s)) \leq n$   
 $\langle \text{proof} \rangle$

**lemma** *initd-gext'*:  $\llbracket s \leq |s'; \text{initd } C \ (\text{globs } s) \rrbracket \implies \text{initd } C \ (\text{globs } s')$

$\langle \text{proof} \rangle$

**lemma** *nyinitcls-gext*:  $\text{snd } s \leq | \text{snd } s' \implies \text{nyinitcls } G \ s' \subseteq \text{nyinitcls } G \ s$

$\langle \text{proof} \rangle$

**lemma** *card-nyinitcls-gext*:

$\llbracket \text{snd } s \leq | \text{snd } s'; \text{card } (\text{nyinitcls } G \ s) \leq n \rrbracket \implies \text{card } (\text{nyinitcls } G \ s') \leq n$   
 $\langle \text{proof} \rangle$

**init-le**

**constdefs**

$\text{init-le} :: \text{prog} \Rightarrow \text{nat} \Rightarrow \text{state} \Rightarrow \text{bool} \quad (\text{-+init}\leq- \ [51,51] \ 50)$   
 $G \vdash \text{init}\leq n \equiv \lambda s. \text{card } (\text{nyinitcls } G \ s) \leq n$

**lemma** *init-le-def2* [simp]:  $(G \vdash \text{init}\leq n) \ s = (\text{card } (\text{nyinitcls } G \ s) \leq n)$

$\langle \text{proof} \rangle$

**lemma** *All-init-leD*:

$\forall n::\text{nat}. G, (A::'a \ \text{triple set}) \vdash \{P \ \wedge. \ G \vdash \text{init}\leq n\} \ t \succ \{Q::'a \ \text{assn}\}$   
 $\implies G, A \vdash \{P\} \ t \succ \{Q\}$   
 $\langle \text{proof} \rangle$

## Most General Triples and Formulas

**constdefs**

$\text{remember-init-state} :: \text{state assn} \quad (\doteq)$   
 $\doteq \equiv \lambda Y \ s \ Z. s = Z$

**lemma** *remember-init-state-def2* [simp]:  $\doteq \ Y = \text{op} =$

$\langle \text{proof} \rangle$

**consts**

$\text{MGF} :: [\text{state assn}, \text{term}, \text{prog}] \Rightarrow \text{state triple} \ (\{-\} \rightarrow \{-\rightarrow\} [3,65,3] 62)$   
 $\text{MGFn} :: [\text{nat} \quad , \text{term}, \text{prog}] \Rightarrow \text{state triple} \ (\{=-\} \rightarrow \{-\rightarrow\} [3,65,3] 62)$

**defs**

*MGF-def:*

$$\{P\} t \succ \{G \rightarrow\} \equiv \{P\} t \succ \{\lambda Y s' s. G \vdash s -t \rightarrow (Y, s')\}$$

*MGFn-def:*

$$\{=:n\} t \succ \{G \rightarrow\} \equiv \{\dot{=} \wedge. G \vdash \text{init} \leq n\} t \succ \{G \rightarrow\}$$

**lemma** *MGF-valid: wf-prog*  $G \implies G, \{\dot{=}\} \vdash \{=:n\} t \succ \{G \rightarrow\}$

*<proof>*

**lemma** *MGF-res-eq-lemma [simp]:*

$$(\forall Y' Y s. Y = Y' \wedge P s \longrightarrow Q s) = (\forall s. P s \longrightarrow Q s)$$

*<proof>*

**lemma** *MGFn-def2:*

$$\begin{aligned} G, A \vdash \{=:n\} t \succ \{G \rightarrow\} &= G, A \vdash \{\dot{=} \wedge. G \vdash \text{init} \leq n\} \\ &\quad t \succ \{\lambda Y s' s. G \vdash s -t \rightarrow (Y, s')\} \end{aligned}$$

*<proof>*

**lemma** *MGF-MGFn-iff:*

$$G, (A :: \text{state triple set}) \vdash \{\dot{=}\} t \succ \{G \rightarrow\} = (\forall n. G, A \vdash \{=:n\} t \succ \{G \rightarrow\})$$

*<proof>*

**lemma** *MGFnD:*

$$\begin{aligned} G, (A :: \text{state triple set}) \vdash \{=:n\} t \succ \{G \rightarrow\} &\implies \\ G, A \vdash \{(\lambda Y' s' s. s' = s \quad \wedge \quad P s) \wedge. G \vdash \text{init} \leq n\} \\ t \succ \{(\lambda Y' s' s. G \vdash s -t \rightarrow (Y', s') \wedge P s) \wedge. G \vdash \text{init} \leq n\} \end{aligned}$$

*<proof>*

**lemmas**  $MGFnD' = MGFnD$  [of - - -  $\lambda x. \text{True}$ ]

To derive the most general formula, we can always assume a normal state in the precondition, since abrupt cases can be handled uniformly by the abrupt rule.

**lemma** *MGFNormalI:*  $G, A \vdash \{\text{Normal} \dot{=}\} t \succ \{G \rightarrow\} \implies$

$$G, (A :: \text{state triple set}) \vdash \{\dot{=} :: \text{state assn}\} t \succ \{G \rightarrow\}$$

*<proof>*

**lemma** *MGFNormalD:*

$$G, (A :: \text{state triple set}) \vdash \{\dot{=}\} t \succ \{G \rightarrow\} \implies G, A \vdash \{\text{Normal} \dot{=}\} t \succ \{G \rightarrow\}$$

*<proof>*

Additionally to *MGFNormalI*, we also expand the definition of the most general formula here

**lemma** *MGFn-NormalI:*

$$\begin{aligned} G, (A :: \text{state triple set}) \vdash \{\text{Normal}((\lambda Y' s' s. s' = s \wedge \text{normal } s) \wedge. G \vdash \text{init} \leq n)\} t \succ \\ \{\lambda Y s' s. G \vdash s -t \rightarrow (Y, s')\} \implies G, A \vdash \{=:n\} t \succ \{G \rightarrow\} \end{aligned}$$

*<proof>*

To derive the most general formula, we can restrict ourselves to welltyped terms, since all others can be uniformly handled by the hazard rule.

**lemma** *MGFn-free-wt:*

$$\begin{aligned} (\exists T L C. (\text{prg} = G, \text{cls} = C, \text{lcl} = L) \vdash t :: T) \\ \longrightarrow G, (A :: \text{state triple set}) \vdash \{=:n\} t \succ \{G \rightarrow\} \end{aligned}$$

$$\implies G, A \vdash \{=:n\} \ t \succ \{G \rightarrow\}$$

*<proof>*

To derive the most general formula, we can restrict ourselves to welltyped terms and assume that the state in the precondition conforms to the environment. All type violations can be uniformly handled by the hazard rule.

**lemma** *MGFn-free-wt-NormalConformI*:

$$\begin{aligned} & (\forall \ T \ L \ C \ . \ (\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash t :: T \\ & \longrightarrow G, (A :: \text{state triple set}) \\ & \quad \vdash \{ \text{Normal}((\lambda Y' \ s' \ s. \ s'=s \wedge \text{normal } s) \wedge. G \vdash \text{init} \leq n) \wedge. (\lambda s. s :: \preceq(G, L)) \} \\ & \quad t \succ \\ & \quad \{ \lambda Y \ s' \ s. G \vdash s - t \succ \rightarrow (Y, s') \} \\ & \implies G, A \vdash \{=:n\} t \succ \{G \rightarrow\} \end{aligned}$$

*<proof>*

To derive the most general formula, we can restrict ourselves to welltyped terms and assume that the state in the precondition conforms to the environment and that the term is definitely assigned with respect to this state. All type violations can be uniformly handled by the hazard rule.

**lemma** *MGFn-free-wt-da-NormalConformI*:

$$\begin{aligned} & (\forall \ T \ L \ C \ B. \ (\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash t :: T \\ & \longrightarrow G, (A :: \text{state triple set}) \\ & \quad \vdash \{ \text{Normal}((\lambda Y' \ s' \ s. \ s'=s \wedge \text{normal } s) \wedge. G \vdash \text{init} \leq n) \wedge. (\lambda s. s :: \preceq(G, L)) \\ & \quad \wedge. (\lambda s. (\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash \text{dom } (\text{locals } (\text{store } s)) \gg t \gg B) \} \\ & \quad t \succ \\ & \quad \{ \lambda Y \ s' \ s. G \vdash s - t \succ \rightarrow (Y, s') \} \\ & \implies G, A \vdash \{=:n\} t \succ \{G \rightarrow\} \end{aligned}$$

*<proof>*

## main lemmas

**lemma** *MGFn-Init*:

**assumes** *mgf-hyp*:  $\forall m. \text{Suc } m \leq n \longrightarrow (\forall t. G, A \vdash \{=:m\} \ t \succ \{G \rightarrow\})$   
**shows**  $G, (A :: \text{state triple set}) \vdash \{=:n\} \ (\text{Init } C) \succ \{G \rightarrow\}$   
*<proof>*

**lemmas**  $\text{MGFn-InitD} = \text{MGFn-Init} \ [\text{THEN } \text{MGFnD}, \text{ THEN } \text{ax-NormalD}]$

**lemma** *MGFn-Call*:

**assumes** *mgf-methods*:  
 $\forall C \ \text{sig}. \ G, (A :: \text{state triple set}) \vdash \{=:n\} \ \langle (\text{Methd } C \ \text{sig}) \rangle_e \succ \{G \rightarrow\}$   
**and** *mgf-e*:  $G, A \vdash \{=:n\} \ \langle e \rangle_e \succ \{G \rightarrow\}$   
**and** *mgf-ps*:  $G, A \vdash \{=:n\} \ \langle ps \rangle_t \succ \{G \rightarrow\}$   
**and** *wf*: *wf-prog*  $G$   
**shows**  $G, A \vdash \{=:n\} \ \langle \{ \text{acc } C, \text{stat } T, \text{mode} \} e \cdot \text{mn}(\{ pTs \} ps) \rangle_e \succ \{G \rightarrow\}$   
*<proof>*

**lemma** *eval-expression-no-jump'*:

**assumes** *eval*:  $G \vdash s0 - e \rightarrow v \rightarrow s1$   
**and** *no-jmp*:  $\text{abrupt } s0 \neq \text{Some } (\text{Jump } j)$   
**and** *wt*:  $(\text{prg}=G, \text{cls}=C, \text{lcl}=L) \vdash e :: -T$   
**and** *wf*: *wf-prog*  $G$   
**shows**  $\text{abrupt } s1 \neq \text{Some } (\text{Jump } j)$   
*<proof>*

To derive the most general formula for the loop statement, we need to come up with a proper loop invariant, which intuitively states that we are currently inside the evaluation of the loop. To define

such an invariant, we unroll the loop in iterated evaluations of the expression and evaluations of the loop body.

**constdefs**

$unroll :: prog \Rightarrow label \Rightarrow expr \Rightarrow stmt \Rightarrow (state \times state) set$

$unroll\ G\ l\ e\ c \equiv \{(s, t). \exists\ v\ s1\ s2. \\ G \vdash s -e-\succ v \rightarrow s1 \wedge the-Bool\ v \wedge normal\ s1 \wedge \\ G \vdash s1 -c\rightarrow s2 \wedge t = (abupd\ (absorb\ (Cont\ l))\ s2)\}$

**lemma unroll-while:**

**assumes**  $unroll: (s, t) \in (unroll\ G\ l\ e\ c)^*$   
**and**  $eval-e: G \vdash t -e-\succ v \rightarrow s'$   
**and**  $normal-termination: normal\ s' \longrightarrow \neg the-Bool\ v$   
**and**  $wt: (\langle prg = G, cls = C, lcl = L \rangle) \vdash e :: -T$   
**and**  $wf: wf-prog\ G$   
**shows**  $G \vdash s -l\cdot While(e)\ c \rightarrow s'$

$\langle proof \rangle$

**lemma MGFn-Loop:**

**assumes**  $mfg-e: G, (A :: state\ triple\ set) \vdash \{=:n\} \langle e \rangle_e \succ \{G \rightarrow\}$   
**and**  $mfg-c: G, A \vdash \{=:n\} \langle c \rangle_s \succ \{G \rightarrow\}$   
**and**  $wf: wf-prog\ G$   
**shows**  $G, A \vdash \{=:n\} \langle l\cdot While(e)\ c \rangle_s \succ \{G \rightarrow\}$

$\langle proof \rangle$

**lemma MGFn-FVar:**

**fixes**  $A :: state\ triple\ set$   
**assumes**  $mfg-init: G, A \vdash \{=:n\} \langle Init\ statDeclC \rangle_s \succ \{G \rightarrow\}$   
**and**  $mfg-e: G, A \vdash \{=:n\} \langle e \rangle_e \succ \{G \rightarrow\}$   
**and**  $wf: wf-prog\ G$   
**shows**  $G, A \vdash \{=:n\} \langle \{accC, statDeclC, stat\} e..fn \rangle_v \succ \{G \rightarrow\}$

$\langle proof \rangle$

**lemma MGFn-Fin:**

**assumes**  $wf: wf-prog\ G$   
**and**  $mfg-c1: G, A \vdash \{=:n\} \langle c1 \rangle_s \succ \{G \rightarrow\}$   
**and**  $mfg-c2: G, A \vdash \{=:n\} \langle c2 \rangle_s \succ \{G \rightarrow\}$   
**shows**  $G, (A :: state\ triple\ set) \vdash \{=:n\} \langle c1\ Finally\ c2 \rangle_s \succ \{G \rightarrow\}$

$\langle proof \rangle$

**lemma Body-no-break:**

**assumes**  $eval-init: G \vdash Norm\ s0 -Init\ D \rightarrow s1$   
**and**  $eval-c: G \vdash s1 -c \rightarrow s2$   
**and**  $jmpOk: jumpNestingOkS\ \{Ret\}\ c$   
**and**  $wt-c: (\langle prg = G, cls = C, lcl = L \rangle) \vdash c :: \surd$   
**and**  $clsD: class\ G\ D = Some\ d$   
**and**  $wf: wf-prog\ G$   
**shows**  $\forall\ l. abrupt\ s2 \neq Some\ (Jump\ (Break\ l)) \wedge \\ abrupt\ s2 \neq Some\ (Jump\ (Cont\ l))$

$\langle proof \rangle$

**lemma** *MGFn-Body*:

**assumes** *wf*: *wf-prog* *G*  
**and** *mgf-init*:  $G, A \vdash \{=:n\} \langle \text{Init } D \rangle_s \succ \{G \rightarrow\}$   
**and** *mgf-c*:  $G, A \vdash \{=:n\} \langle c \rangle_s \succ \{G \rightarrow\}$   
**shows**  $G, (A :: \text{state triple set}) \vdash \{=:n\} \langle \text{Body } D \ c \rangle_e \succ \{G \rightarrow\}$   
 $\langle \text{proof} \rangle$

**lemma** *MGFn-lemma*:

**assumes** *mgf-methods*:  
 $\bigwedge n. \forall C \text{ sig}. G, (A :: \text{state triple set}) \vdash \{=:n\} \langle \text{Methd } C \text{ sig} \rangle_e \succ \{G \rightarrow\}$   
**and** *wf*: *wf-prog* *G*  
**shows**  $\bigwedge t. G, A \vdash \{=:n\} t \succ \{G \rightarrow\}$   
 $\langle \text{proof} \rangle$

**lemma** *MGF-asm*:

$\llbracket \forall C \text{ sig}. \text{is-methd } G \ C \text{ sig} \longrightarrow G, A \vdash \{\doteq\} \text{In1l } (\text{Methd } C \text{ sig}) \succ \{G \rightarrow\}; \text{wf-prog } G \rrbracket$   
 $\implies G, (A :: \text{state triple set}) \vdash \{\doteq\} t \succ \{G \rightarrow\}$   
 $\langle \text{proof} \rangle$

## nested version

**lemma** *nesting-lemma'* [*rule-format* (*no-asm*)]:

**assumes** *ax-derivs-asm*:  $\bigwedge A \text{ ts}. \text{ts} \subseteq A \implies P \ A \ \text{ts}$   
**and** *MGF-nested-Methd*:  $\bigwedge A \text{ pn}. \forall b \in \text{bdy } \text{pn}. P \ (\text{insert } (\text{mgf-call } \text{pn}) \ A) \ \{\text{mgf } b\}$   
 $\implies P \ A \ \{\text{mgf-call } \text{pn}\}$   
**and** *MGF-asm*:  $\bigwedge A \ t. \forall \text{pn} \in U. P \ A \ \{\text{mgf-call } \text{pn}\} \implies P \ A \ \{\text{mgf } t\}$   
**and** *finU*: *finite* *U*  
**and** *uA*:  $uA = \text{mgf-call}' U$   
**shows**  $\forall A. A \subseteq uA \longrightarrow n \leq \text{card } uA \longrightarrow \text{card } A = \text{card } uA - n$   
 $\longrightarrow (\forall t. P \ A \ \{\text{mgf } t\})$   
 $\langle \text{proof} \rangle$

**lemma** *nesting-lemma* [*rule-format* (*no-asm*)]:

**assumes** *ax-derivs-asm*:  $\bigwedge A \text{ ts}. \text{ts} \subseteq A \implies P \ A \ \text{ts}$   
**and** *MGF-nested-Methd*:  $\bigwedge A \text{ pn}. \forall b \in \text{bdy } \text{pn}. P \ (\text{insert } (\text{mgf } (f \text{ pn})) \ A) \ \{\text{mgf } b\}$   
 $\implies P \ A \ \{\text{mgf } (f \text{ pn})\}$   
**and** *MGF-asm*:  $\bigwedge A \ t. \forall \text{pn} \in U. P \ A \ \{\text{mgf } (f \text{ pn})\} \implies P \ A \ \{\text{mgf } t\}$   
**and** *finU*: *finite* *U*  
**shows**  $P \ \{\} \ \{\text{mgf } t\}$   
 $\langle \text{proof} \rangle$

**lemma** *MGF-nested-Methd*:  $\llbracket$

$G, \text{insert } (\{\text{Normal } \doteq\} \langle \text{Methd } C \text{ sig} \rangle_e \succ \{G \rightarrow\}) \ A$   
 $\vdash \{\text{Normal } \doteq\} \langle \text{body } G \ C \text{ sig} \rangle_e \succ \{G \rightarrow\}$   
 $\rrbracket \implies G, A \vdash \{\text{Normal } \doteq\} \langle \text{Methd } C \text{ sig} \rangle_e \succ \{G \rightarrow\}$   
 $\langle \text{proof} \rangle$

**lemma** *MGF-deriv*: *wf-prog* *G*  $\implies G, (\{\} :: \text{state triple set}) \vdash \{\doteq\} t \succ \{G \rightarrow\}$

$\langle \text{proof} \rangle$

**simultaneous version**

**lemma** *MGF-simult-Methd-lemma: finite ms  $\implies$*   
 $G, A \cup (\lambda(C, sig). \{Normal \doteq\} \langle Methd \ C \ sig \rangle_e \succ \{G \rightarrow\}) \text{ ' } ms$   
 $\vdash (\lambda(C, sig). \{Normal \doteq\} \langle body \ G \ C \ sig \rangle_e \succ \{G \rightarrow\}) \text{ ' } ms \implies$   
 $G, A \vdash (\lambda(C, sig). \{Normal \doteq\} \langle Methd \ C \ sig \rangle_e \succ \{G \rightarrow\}) \text{ ' } ms$   
*<proof>*

**lemma** *MGF-simult-Methd: wf-prog G  $\implies$*   
 $G, (\{\} :: state \ triple \ set) \vdash (\lambda(C, sig). \{Normal \doteq\} \langle Methd \ C \ sig \rangle_e \succ \{G \rightarrow\})$   
 $\text{ ' } Collect \ (split \ (is-methd \ G))$   
*<proof>*

**corollaries**

**lemma** *eval-to-evaln:  $\llbracket G \vdash s \rightarrow t \succ \rightarrow (Y', s'); type-ok \ G \ t \ s; wf-prog \ G \rrbracket$*   
 $\implies \exists n. \ G \vdash s \rightarrow t \succ \rightarrow n \rightarrow (Y', s')$   
*<proof>*

**lemma** *MGF-complete:*  
**assumes** *valid:*  $G, \{\} \models \{P\} \ t \succ \{Q\}$   
**and** *mgf:*  $G, (\{\} :: state \ triple \ set) \vdash \{\doteq\} \ t \succ \{G \rightarrow\}$   
**and** *wf:* *wf-prog G*  
**shows**  $G, (\{\} :: state \ triple \ set) \vdash \{P :: state \ assn\} \ t \succ \{Q\}$   
*<proof>*

**theorem** *ax-complete:*  
**assumes** *wf:* *wf-prog G*  
**and** *valid:*  $G, \{\} \models \{P :: state \ assn\} \ t \succ \{Q\}$   
**shows**  $G, (\{\} :: state \ triple \ set) \vdash \{P\} \ t \succ \{Q\}$   
*<proof>*

**end**



## Chapter 25

### AxExample

## 64 Example of a proof based on the Bali axiomatic semantics

**theory** *AxExample* **imports** *AxSem Example* **begin**

**constdefs**

*arr-inv* :: *st*  $\Rightarrow$  *bool*  
*arr-inv*  $\equiv \lambda s. \exists \text{obj } a \ T \text{ el. } \text{globs } s \ (\text{Stat } \text{Base}) = \text{Some } \text{obj} \wedge$   
 $\text{values } \text{obj} \ (\text{Inl } (\text{arr}, \text{Base})) = \text{Some } (\text{Addr } a) \wedge$   
 $\text{heap } s \ a = \text{Some } (\text{tag}=\text{Arr } T \ 2, \text{values}=\text{el})$

**lemma** *arr-inv-new-obj*:

$\bigwedge a. \llbracket \text{arr-inv } s; \text{new-Addr } (\text{heap } s) = \text{Some } a \rrbracket \Longrightarrow \text{arr-inv } (\text{gupd}(\text{Inl } a \mapsto x) \ s)$   
 $\langle \text{proof} \rangle$

**lemma** *arr-inv-set-locals* [*simp*]: *arr-inv* (*set-locals* *l s*) = *arr-inv* *s*  
 $\langle \text{proof} \rangle$

**lemma** *arr-inv-gupd-Stat* [*simp*]:

*Base*  $\neq$  *C*  $\Longrightarrow \text{arr-inv } (\text{gupd}(\text{Stat } C \mapsto \text{obj}) \ s) = \text{arr-inv } s$   
 $\langle \text{proof} \rangle$

**lemma** *ax-inv-lupd* [*simp*]: *arr-inv* (*lupd*(*x*  $\mapsto$  *y*) *s*) = *arr-inv* *s*  
 $\langle \text{proof} \rangle$

**declare** *split-if-asm* [*split del*]

**declare** *lvar-def* [*simp*]

$\langle \text{ML} \rangle$

**theorem** *ax-test*: *tprg*,( $\{\}::'a$  triple set) $\vdash$

$\{\text{Normal } (\lambda Y \ s \ Z::'a. \text{heap-free four } s \wedge \neg \text{initd } \text{Base } s \wedge \neg \text{initd } \text{Ext } s)\}$

*.test* [*Class* *Base*].

$\{\lambda Y \ s \ Z. \text{abrupt } s = \text{Some } (\text{Xcpt } (\text{Std } \text{IndOutBound}))\}$

$\langle \text{proof} \rangle$

**lemma** *Loop-Xcpt-benchmark*:

$Q = (\lambda Y \ (x,s) \ Z. x \neq \text{None} \longrightarrow \text{the-Bool } (\text{the } (\text{locals } s \ i))) \Longrightarrow$

$G,(\{\}::'a \text{ triple set}) \vdash \{\text{Normal } (\lambda Y \ s \ Z::'a. \text{True})\}$

*.lab1*• *While*(*Lit* (*Bool* *True*)) (*If*(*Acc* (*LVar* *i*)) (*Throw* (*Acc* (*LVar* *xcpt*))) *Else*  
 $(\text{Expr } (\text{Ass } (\text{LVar } i) (\text{Acc } (\text{LVar } j))))). \{Q\}$

$\langle \text{proof} \rangle$

**end**