Java Source and Bytecode Formalizations in Isabelle: Bali

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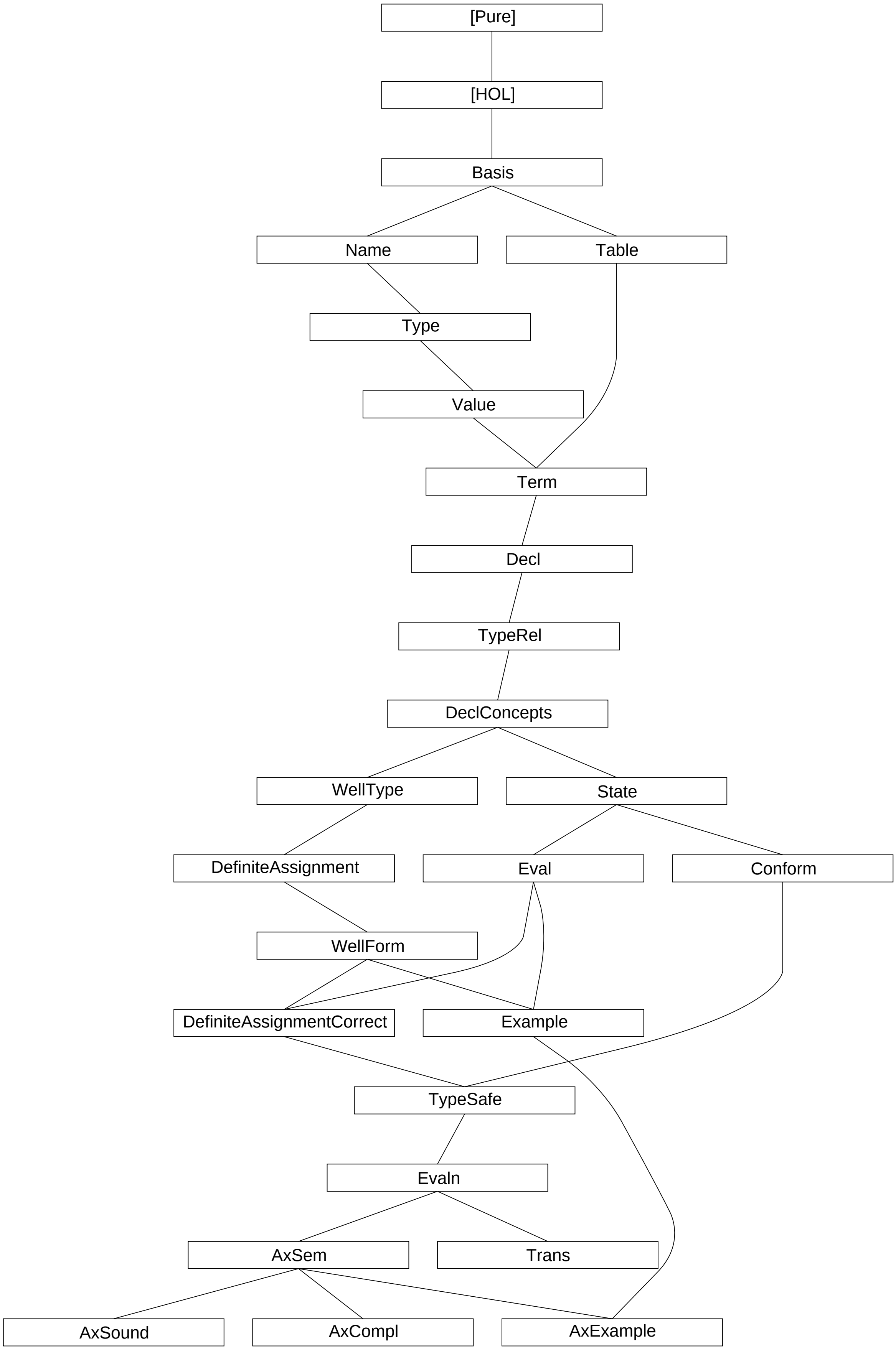
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Chapter 1

Overview

These theories, called Bali, model and analyse different aspects of the JavaCard **source language**. The basis is an abstract model of the JavaCard source language. On it, a type system, an operational semantics and an axiomatic semantics (Hoare logic) are built. The execution of a wellformed program (with respect to the type system) according to the operational semantics is proved to be typesafe. The axiomatic semantics is proved to be sound and relative complete with respect to the operational semantics.
We have modelled large parts of the original JavaCard source language. It models features such as:

- The basic “primitive types” of Java
- Classes and related concepts
- Class fields and methods
- Instance fields and methods
- Interfaces and related concepts
- Arrays
- Static initialisation
- Static overloading of fields and methods
- Inheritance, overriding and hiding of methods, dynamic binding
- All cases of abrupt termination
  - Exception throwing and handling
  - `break`, `continue` and `return`
- Packages
- Access Modifiers (`private`, `protected`, `public`)
- A “definite assignment” check

The following features are missing in Bali wrt. JavaCard:

- Some primitive types (`byte, short`)
- Syntactic variants of statements (`do`-loop, `for`-loop)
- Interface fields

- Inner Classes

In addition, features are missing that are not part of the JavaCard language, such as multithreading and garbage collection. No attempt has been made to model peculiarities of JavaCard such as the applet firewall or the transaction mechanism.
Overview of the theories:

Basis Some basic definitions and settings not specific to JavaCard but missing in HOL.

Table Definition and some properties of a lookup table to map various names (like class names or method names) to some content (like classes or methods).

Name Definition of various names (class names, variable names, package names,...)

Value JavaCard expression values (Boolean, Integer, Addresses,...)

Type JavaCard types. Primitive types (Boolean, Integer,...) and reference types (Classes, Interfaces, Arrays,...)

Term JavaCard terms. Variables, expressions and statements.

Decl Class, interface and program declarations. Recursion operators for the class and the interface hierarchy.

TypeRel Various relations on types like the subclass-, subinterface-, widening-, narrowing- and casting-relation.

DeclConcepts Advanced concepts on the class and interface hierarchy like inheritance, overriding, hiding, accessibility of types and members according to the access modifiers, method lookup.

WellType Typesystem on the JavaCard term level.

DefiniteAssignment The definite assignment analysis on the JavaCard term level.

WellForm Typesystem on the JavaCard class, interface and program level.

State The program state (like object store) for the execution of JavaCard. Abrupt completion (exceptions, break, continue, return) is modelled as flag inside the state.

Eval Operational (big step) semantics for JavaCard.

Example An concrete example of a JavaCard program to validate the typesystem and the operational semantics.

Conform Conformance predicate for states. When does an execution state conform to the static types of the program given by the typesystem.

DefiniteAssignmentCorrect Correctness of the definite assignment analysis. If the analysis regards a variable as definitely assigned at a certain program point, the variable will actually be assigned there during execution.

TypeSafe Typesafety proof of the execution of JavaCard. "Welltyped programs don't go wrong" or more technical: The execution of a welltyped JavaCard program preserves the conformance of execution states.

Evaln Copy of the operational semantics given in theory Eval expanded with an annotation for the maximal recursive depth. The semantics is not altered. The annotation is needed for the soundness proof of the axiomatic semantics.

Trans A smallstep operational semantics for JavaCard.

AxSem An axiomatic semantics (Hoare logic) for JavaCard.

AxSound The soundness proof of the axiomatic semantics with respect to the operational semantics.

AxCompl The proof of (relative) completeness of the axiomatic semantics with respect to the operational semantics.

AxExample An concrete example of the axiomatic semantics at work, applied to prove some properties of the JavaCard example given in theory Example.

Chapter 2

Basis

1 Definitions extending HOL as logical basis of Bali

theory *Basis* **imports** *Main* **begin**

declare [[*unify-search-bound* = *40*, *unify-trace-bound* = *40*]]

misc

declare *same-fstI* [*intro*!]

declare *split-if-asm* [*split*] *option.split* [*split*] *option.split-asm* [*split*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*))) ⟫
declare *if-weak-cong* [*cong del*] *option.weak-case-cong* [*cong del*]
declare *length-Suc-conv* [*iff*]

lemma *Collect-split-eq*: {*p*. *P* (*split f p*)} = {(*a*,*b*). *P* (*f a b*)}
apply *auto*
done

lemma *subset-insertD*:
A <= *insert x B* ==> *A* <= *B* & *x* ~: *A* | (*EX B'*. *A* = *insert x B'* & *B'* <= *B*)
apply (*case-tac x*:*A*)
apply (*rule disjI2*)
apply (*rule-tac x* = *A*−{*x*} **in** *exI*)
apply *fast*+
done

syntax
3 :: *nat* (*3*)
4 :: *nat* (*4*)
translations
3 == *Suc 2*
4 == *Suc 3*

lemma *range-bool-domain*: *range f* = {*f True*, *f False*}
apply *auto*
apply (*case-tac xa*)
apply *auto*
done

lemma *irrefl-tranclI'*: *r*^−*1 Int r*^+ = {} ==> !*x*. (*x*, *x*) ~: *r*^+
by(*blast elim*: *tranclE dest*: *trancl-into-rtrancl*)

lemma *trancl-rtrancl-trancl*:
⟦(*x*,*y*)∈*r*^+; (*y*,*z*)∈*r*^∗⟧ ⟹ (*x*,*z*)∈*r*^+
by (*auto dest*: *tranclD rtrancl-trans rtrancl-into-trancl2*)

lemma *rtrancl-into-trancl3*:
⟦(*a*,*b*)∈*r*^∗; *a*≠*b*⟧ ⟹ (*a*,*b*)∈*r*^+
apply (*drule rtranclD*)

```
apply auto
done


lemma rtrancl-into-rtrancl2:
  ⟦ (a, b) ∈  r; (b, c) ∈ r^* ⟧ ⟹ (a, c) ∈  r^*
by (auto intro: r-into-rtrancl rtrancl-trans)


lemma triangle-lemma:
 ⟦ ⋀ a b c. ⟦(a,b)∈r; (a,c)∈r⟧ ⟹ b=c; (a,x)∈r*; (a,y)∈r*⟧
 ⟹ (x,y)∈r* ∨ (y,x)∈r*
proof -
  note converse-rtrancl-induct = converse-rtrancl-induct [consumes 1]
  note converse-rtranclE = converse-rtranclE [consumes 1]
  assume unique: ⋀ a b c. ⟦(a,b)∈r; (a,c)∈r⟧ ⟹ b=c
  assume (a,x)∈r*
  then show (a,y)∈r* ⟹ (x,y)∈r* ∨ (y,x)∈r*
  proof (induct rule: converse-rtrancl-induct)
    assume (x,y)∈r*
    then show ?thesis
      by blast
  next
    fix a v
    assume a-v-r: (a, v) ∈ r and
           v-x-rt: (v, x) ∈ r* and
           a-y-rt: (a, y) ∈ r*  and
              hyp: (v, y) ∈ r* ⟹ (x, y) ∈ r* ∨ (y, x) ∈ r*
    from a-y-rt
    show (x, y) ∈ r* ∨ (y, x) ∈ r*
    proof (cases rule: converse-rtranclE)
      assume a=y
      with a-v-r v-x-rt have (y,x) ∈ r*
        by (auto intro: r-into-rtrancl rtrancl-trans)
      then show ?thesis
        by blast
    next
      fix w
      assume a-w-r: (a, w) ∈ r and
             w-y-rt: (w, y) ∈ r*
      from a-v-r a-w-r unique
      have v=w
        by auto
      with w-y-rt hyp
      show ?thesis
        by blast
    qed
  qed
qed


lemma rtrancl-cases [consumes 1, case-names Refl Trancl]:
 ⟦(a,b)∈r*;  a = b ⟹ P; (a,b)∈r⁺ ⟹ P⟧ ⟹ P
apply (erule rtranclE)
apply (auto dest: rtrancl-into-trancl1)
done
```

theorems *converse-rtrancl-induct*
= *converse-rtrancl-induct* [*consumes 1*,*case-names Id Step*]

theorems *converse-trancl-induct*
= *converse-trancl-induct* [*consumes 1*,*case-names Single Step*]

lemma *Ball-weaken*:$[\![Ball\ s\ P;\bigwedge\ x.\ P\ x \longrightarrow Q\ x]\!] \Longrightarrow Ball\ s\ Q$
by *auto*

lemma *finite-SetCompr2*: [| *finite* (*Collect P*); !*y*. *P y* −−> *finite* (*range* (*f y*)) |] ==>
finite {*f y x* |*x y*. *P y*}
apply (*subgoal-tac* {*f y x* |*x y*. *P y*} = *UNION* (*Collect P*) (%*y*. *range* (*f y*)))
prefer *2* **apply** *fast*
apply (*erule ssubst*)
apply (*erule finite-UN-I*)
apply *fast*
done

lemma *list-all2-trans*: $\forall\ a\ b\ c.\ P1\ a\ b \longrightarrow P2\ b\ c \longrightarrow P3\ a\ c \Longrightarrow$
$\forall xs2\ xs3.\ list\text{-}all2\ P1\ xs1\ xs2 \longrightarrow list\text{-}all2\ P2\ xs2\ xs3 \longrightarrow list\text{-}all2\ P3\ xs1\ xs3$
apply (*induct-tac xs1*)
apply *simp*
apply (*rule allI*)
apply (*induct-tac xs2*)
apply *simp*
apply (*rule allI*)
apply (*induct-tac xs3*)
apply *auto*
done

pairs

lemma *surjective-pairing5*: *p* = (*fst p*, *fst* (*snd p*), *fst* (*snd* (*snd p*)), *fst* (*snd* (*snd* (*snd p*))),
snd (*snd* (*snd* (*snd p*))))
apply *auto*
done

lemma *fst-splitE* [*elim*!]:
[| *fst s′* = *x′*; !!*x s*. [| *s′* = (*x*,*s*); *x* = *x′* |] ==> *Q* |] ==> *Q*
apply (*cut-tac p* = *s′* **in** *surjective-pairing*)
apply *auto*
done

lemma *fst-in-set-lemma* [*rule-format* (*no-asm*)]: (*x*, *y*) : *set l* −−> *x* : *fst* ‘ *set l*
apply (*induct-tac l*)
apply *auto*
done

quantifiers

lemma *All-Ex-refl-eq2* [*simp*]:
$(!x.\ (?\ b.\ x = f\ b\ \&\ Q\ b) \longrightarrow P\ x) = (!b.\ Q\ b\ {-}{-}{>}\ P\ (f\ b))$
apply *auto*
done

lemma *ex-ex-miniscope1* [*simp*]:
$(EX\ w\ v.\ P\ w\ v\ \&\ Q\ v) = (EX\ v.\ (EX\ w.\ P\ w\ v)\ \&\ Q\ v)$
apply *auto*
done

lemma *ex-miniscope2* [*simp*]:
$(EX\ v.\ P\ v\ \&\ Q\ \&\ R\ v) = (Q\ \&\ (EX\ v.\ P\ v\ \&\ R\ v))$
apply *auto*
done

lemma *ex-reorder31*: $(\exists\, z\ x\ y.\ P\ x\ y\ z) = (\exists\, x\ y\ z.\ P\ x\ y\ z)$
apply *auto*
done

lemma *All-Ex-refl-eq1* [*simp*]: $(!x.\ (?\ b.\ x = f\ b)\ {-}{-}{>}\ P\ x) = (!b.\ P\ (f\ b))$
apply *auto*
done

sums

hide *const In0 In1*

syntax
fun-sum :: $('a => 'c) => ('b => 'c) => (('a{+}'b) => 'c)$ (**infixr** $'(+')80$)
translations
fun-sum == *CONST sum-case*

consts *the-Inl* :: $'a + 'b \Rightarrow 'a$
the-Inr :: $'a + 'b \Rightarrow 'b$
primrec *the-Inl* (*Inl a*) = *a*
primrec *the-Inr* (*Inr b*) = *b*

datatype $('a, 'b, 'c)$ *sum3* = *In1* $'a$ | *In2* $'b$ | *In3* $'c$

consts *the-In1* :: $('a, 'b, 'c)$ *sum3* $\Rightarrow 'a$
the-In2 :: $('a, 'b, 'c)$ *sum3* $\Rightarrow 'b$
the-In3 :: $('a, 'b, 'c)$ *sum3* $\Rightarrow 'c$
primrec *the-In1* (*In1 a*) = *a*
primrec *the-In2* (*In2 b*) = *b*
primrec *the-In3* (*In3 c*) = *c*

syntax
In1l :: $'al \Rightarrow ('al + 'ar, 'b, 'c)$ *sum3*
In1r :: $'ar \Rightarrow ('al + 'ar, 'b, 'c)$ *sum3*
translations
In1l e == *In1* (*Inl e*)
In1r c == *In1* (*Inr c*)

syntax *the-In1l* :: ('*al* + '*ar*, '*b*, '*c*) *sum3* ⇒ '*al*
the-In1r :: ('*al* + '*ar*, '*b*, '*c*) *sum3* ⇒ '*ar*
translations
the-In1l == *the-Inl* ∘ *the-In1*
the-In1r == *the-Inr* ∘ *the-In1*

ML ⟪
fun sum3-instantiate thm = *map* (*fn s* => *simplify*(*simpset*()*delsimps*[@{*thm not-None-eq*}])
(*read-instantiate* [(*t*,*In*^*s*^ *?x*)] *thm*)) [*1l*,*2*,*3*,*1r*]
⟫

translations
option<= (*type*) *Datatype.option*
list <= (*type*) *List.list*
sum3 <= (*type*) *Basis.sum3*

quantifiers for option type

syntax
Oall :: [*pttrn*, '*a option*, *bool*] => *bool* ((*3*! -:-:/ -) [*0*,*0*,*10*] *10*)
Oex :: [*pttrn*, '*a option*, *bool*] => *bool* ((*3?* -:-:/ -) [*0*,*0*,*10*] *10*)

syntax (*symbols*)
Oall :: [*pttrn*, '*a option*, *bool*] => *bool* ((*3*∀ -∈-:/ -) [*0*,*0*,*10*] *10*)
Oex :: [*pttrn*, '*a option*, *bool*] => *bool* ((*3*∃ -∈-:/ -) [*0*,*0*,*10*] *10*)

translations
! *x*:*A*: *P* == ! *x*:*o2s A*. *P*
? *x*:*A*: *P* == *?* *x*:*o2s A*. *P*

Special map update

Deemed too special for theory Map.

constdefs
chg-map :: ('*b* => '*b*) => '*a* => ('*a* ~=> '*b*) => ('*a* ~=> '*b*)
chg-map f a m == *case m a of None* => *m* | *Some b* => *m*(*a*|−>*f b*)

lemma *chg-map-new*[*simp*]: *m a* = *None* ==> *chg-map f a m* = *m*
by (*unfold chg-map-def*, *auto*)

lemma *chg-map-upd*[*simp*]: *m a* = *Some b* ==> *chg-map f a m* = *m*(*a*|−>*f b*)
by (*unfold chg-map-def*, *auto*)

lemma *chg-map-other* [*simp*]: *a* ≠ *b* ⟹ *chg-map f a m b* = *m b*
by (*auto simp*: *chg-map-def split add*: *option.split*)

unique association lists

constdefs
unique :: ('*a* × '*b*) *list* ⇒ *bool*
unique ≡ *distinct* ∘ *map fst*

lemma *uniqueD* [*rule-format* (*no-asm*)]:
unique l−−> (!*x y*. (*x*,*y*):*set l* −−> (!*x*' *y*'. (*x*',*y*'):*set l* −−> *x*=*x*'−−> *y*=*y*'))

```
apply (unfold unique-def o-def)
apply (induct-tac l)
apply  (auto dest: fst-in-set-lemma)
done


lemma unique-Nil [simp]: unique []
apply (unfold unique-def)
apply (simp (no-asm))
done


lemma unique-Cons [simp]: unique ((x,y)#l) = (unique l & (!y. (x,y) ~: set l))
apply (unfold unique-def)
apply  (auto dest: fst-in-set-lemma)
done

lemmas unique-ConsI = conjI [THEN unique-Cons [THEN iffD2], standard]


lemma unique-single [simp]: !!p. unique [p]
apply auto
done


lemma unique-ConsD: unique (x#xs) ==> unique xs
apply (simp add: unique-def)
done


lemma unique-append [rule-format (no-asm)]: unique l' ==> unique l -->
  (!(x,y):set l. !(x',y'):set l'. x' ~= x) --> unique (l @ l')
apply (induct-tac l)
apply  (auto dest: fst-in-set-lemma)
done


lemma unique-map-inj [rule-format (no-asm)]: unique l --> inj f --> unique (map (%(k,x). (f k, g k
x)) l)
apply (induct-tac l)
apply  (auto dest: fst-in-set-lemma simp add: inj-eq)
done


lemma map-of-SomeI [rule-format (no-asm)]: unique l --> (k, x) : set l --> map-of l k = Some x
apply (induct-tac l)
apply auto
done
```

list patterns

```
consts
  lsplit        :: [['a, 'a list] => 'b, 'a list] => 'b
defs
  lsplit-def:    lsplit == %f l. f (hd l) (tl l)

syntax
  -lpttrn    :: [pttrn,pttrn] => pttrn     (-#/- [901,900] 900)
translations
```

```
%y#x#xs. b  == lsplit (%y x#xs. b)
%x#xs  . b  == lsplit (%x xs  . b)
```

lemma *lsplit* [*simp*]: *lsplit c* (*x*#*xs*) = *c x xs*
apply (*unfold lsplit-def*)
apply (*simp* (*no-asm*))
done

lemma *lsplit2* [*simp*]: *lsplit P* (*x*#*xs*) *y z* = *P x xs y z*
apply (*unfold lsplit-def*)
apply *simp*
done

end

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Chapter 3

Table

2 Abstract tables and their implementation as lists

theory *Table* **imports** *Basis* **begin**

design issues:

- definition of table: infinite map vs. list vs. finite set list chosen, because:
  - + a priori finite
  - + lookup is more operational than for finite set
  - - not very abstract, but function table converts it to abstract mapping
- coding of lookup result: Some/None vs. value/arbitrary Some/None chosen, because:
  - ++ makes definedness check possible (applies also to finite set), which is important for the type standard, hiding/overriding, etc. (though it may perhaps be possible at least for the operational semantics to treat programs as infinite, i.e. where classes, fields, methods etc. of any name are considered to be defined)
  - - sometimes awkward case distinctions, alleviated by operator 'the'

types $('a,\ 'b)\ table$ — table with key type 'a and contents type 'b
$\quad = 'a \rightharpoonup 'b$
$\quad ('a,\ 'b)\ tables$ — non-unique table with key 'a and contents 'b
$\quad = 'a \Rightarrow 'b\ set$

map of / table of

syntax
$\quad table\text{-}of :: ('a \times 'b)\ list \Rightarrow ('a,\ 'b)\ table$ — concrete table

translations
$\quad table\text{-}of == map\text{-}of$

$\quad (type)'a \rightharpoonup 'b <= (type)'a \Rightarrow 'b\ Datatype.option$
$\quad (type)('a,\ 'b)\ table <= (type)'a \rightharpoonup 'b$

lemma *map-add-find-left*[*simp*]:
$n\ k = None \Longrightarrow (m\ ++\ n)\ k = m\ k$
by (*simp add*: *map-add-def*)

Conditional Override

constdefs
cond-override::
$\quad ('b \Rightarrow 'b \Rightarrow bool) \Rightarrow ('a,\ 'b)table \Rightarrow ('a,\ 'b)table \Rightarrow ('a,\ 'b)\ table$

— when merging tables old and new, only override an entry of table old when the condition cond holds
cond-override cond old new ≡
 λ *k*.
 (*case new k of*
  None ⇒ *old k*
 | *Some new-val* ⇒ (*case old k of*
   None ⇒ *Some new-val*
  | *Some old-val* ⇒ (*if cond new-val old-val*
   then Some new-val
   else Some old-val)))

lemma *cond-override-empty1*[*simp*]: *cond-override c empty t* = *t*
by (*simp add*: *cond-override-def expand-fun-eq*)

lemma *cond-override-empty2*[*simp*]: *cond-override c t empty* = *t*
by (*simp add*: *cond-override-def expand-fun-eq*)

lemma *cond-override-None*[*simp*]:
old k = *None* $\Longrightarrow$ (*cond-override c old new*) *k* = *new k*
by (*simp add*: *cond-override-def*)

lemma *cond-override-override*:
$[\![$*old k* = *Some ov*;*new k* = *Some nv*; *C nv ov*$]\!]$
$\Longrightarrow$ (*cond-override C old new*) *k* = *Some nv*
by (*auto simp add*: *cond-override-def*)

lemma *cond-override-noOverride*:
$[\![$*old k* = *Some ov*;*new k* = *Some nv*; $\neg$ (*C nv ov*)$]\!]$
$\Longrightarrow$ (*cond-override C old new*) *k* = *Some ov*
by (*auto simp add*: *cond-override-def*)

lemma *dom-cond-override*: *dom* (*cond-override C s t*) $\subseteq$ *dom s* $\cup$ *dom t*
by (*auto simp add*: *cond-override-def dom-def*)

lemma *finite-dom-cond-override*:
$[\![$ *finite* (*dom s*); *finite* (*dom t*) $]\!]$ $\Longrightarrow$ *finite* (*dom* (*cond-override C s t*))
apply (*rule-tac B*=*dom s* $\cup$ *dom t* **in** *finite-subset*)
apply (*rule dom-cond-override*)
by (*rule finite-UnI*)

Filter on Tables

constdefs
filter-tab:: ('*a* $\Rightarrow$ '*b* $\Rightarrow$ *bool*) $\Rightarrow$ ('*a*, '*b*) *table* $\Rightarrow$ ('*a*, '*b*) *table*
filter-tab c t $\equiv$ λ *k*. (*case t k of*
  None $\Rightarrow$ *None*
  | *Some x* $\Rightarrow$ *if c k x then Some x else None*)

lemma *filter-tab-empty*[*simp*]: *filter-tab c empty* = *empty*
by (*simp add*: *filter-tab-def empty-def*)

lemma *filter-tab-True*[*simp*]: *filter-tab* (λ*x y*. *True*) *t* = *t*
by (*simp add*: *expand-fun-eq filter-tab-def*)

lemma *filter-tab-False*[*simp*]: *filter-tab* (λ*x y*. *False*) *t* = *empty*
by (*simp add*: *expand-fun-eq filter-tab-def empty-def*)

lemma *filter-tab-ran-subset*: *ran* (*filter-tab c t*) $\subseteq$ *ran t*

by (*auto simp add*: *filter-tab-def ran-def*)

lemma *filter-tab-range-subset*: *range* (*filter-tab c t*) $\subseteq$ *range t* $\cup$ {*None*}
apply (*auto simp add*: *filter-tab-def*)
apply (*drule sym*, *blast*)
done

lemma *finite-range-filter-tab*:
finite (*range t*) $\Longrightarrow$ *finite* (*range* (*filter-tab c t*))
apply (*rule-tac B*=*range t* $\cup$ {*None*} **in** *finite-subset*)
apply (*rule filter-tab-range-subset*)
apply (*auto intro*: *finite-UnI*)
done

lemma *filter-tab-SomeD*[*dest*!]:
filter-tab c t k = *Some x* $\Longrightarrow$ (*t k* = *Some x*) $\wedge$ *c k x*
by (*auto simp add*: *filter-tab-def*)

lemma *filter-tab-SomeI*: $[\![$*t k* = *Some x*;*C k x*$]\!]$ $\Longrightarrow$*filter-tab C t k* = *Some x*
by (*simp add*: *filter-tab-def*)

lemma *filter-tab-all-True*:
$\forall$ *k y*. *t k* = *Some y* $\longrightarrow$ *p k y* $\Longrightarrow$*filter-tab p t* = *t*
apply (*auto simp add*: *filter-tab-def expand-fun-eq*)
done

lemma *filter-tab-all-True-Some*:
$[\![$$\forall$ *k y*. *t k* = *Some y* $\longrightarrow$ *p k y*; *t k* = *Some v*$]\!]$ $\Longrightarrow$ *filter-tab p t k* = *Some v*
by (*auto simp add*: *filter-tab-def expand-fun-eq*)

lemma *filter-tab-all-False*:
$\forall$ *k y*. *t k* = *Some y* $\longrightarrow$ $\neg$ *p k y* $\Longrightarrow$*filter-tab p t* = *empty*
by (*auto simp add*: *filter-tab-def expand-fun-eq*)

lemma *filter-tab-None*: *t k* = *None* $\Longrightarrow$ *filter-tab p t k* = *None*
apply (*simp add*: *filter-tab-def expand-fun-eq*)
done

lemma *filter-tab-dom-subset*: *dom* (*filter-tab C t*) $\subseteq$ *dom t*
by (*auto simp add*: *filter-tab-def dom-def*)

lemma *filter-tab-eq*: $[\![$*a*=*b*$]\!]$ $\Longrightarrow$ *filter-tab C a* = *filter-tab C b*
by (*auto simp add*: *expand-fun-eq filter-tab-def*)

lemma *finite-dom-filter-tab*:
finite (*dom t*) $\Longrightarrow$ *finite* (*dom* (*filter-tab C t*))
apply (*rule-tac B*=*dom t* **in** *finite-subset*)
by (*rule filter-tab-dom-subset*)

lemma *filter-tab-weaken*:
$\llbracket\forall\ a \in t\ k\colon \exists\ b \in s\ k\colon P\ a\ b;$
$\ \bigwedge k\ x\ y.\ \llbracket t\ k = Some\ x; s\ k = Some\ y\rrbracket \Longrightarrow cond\ k\ x \longrightarrow cond\ k\ y$
$\rrbracket \Longrightarrow \forall\ a \in filter\text{-}tab\ cond\ t\ k\colon \exists\ b \in filter\text{-}tab\ cond\ s\ k\colon P\ a\ b$
apply (*force simp add*: *filter-tab-def*)
done

lemma *cond-override-filter*:
$\llbracket\bigwedge k\ old\ new.\ \llbracket s\ k = Some\ new;\ t\ k = Some\ old\rrbracket$
$\quad\Longrightarrow (\neg\ overC\ new\ old \longrightarrow\ \neg\ filterC\ k\ new) \wedge$
$\qquad(overC\ new\ old \longrightarrow filterC\ k\ old \longrightarrow filterC\ k\ new)$
$\rrbracket \Longrightarrow$
$\quad cond\text{-}override\ overC\ (filter\text{-}tab\ filterC\ t)\ (filter\text{-}tab\ filterC\ s)$
$\quad = filter\text{-}tab\ filterC\ (cond\text{-}override\ overC\ t\ s)$
by (*auto simp add*: *expand-fun-eq cond-override-def filter-tab-def*)

Misc.

lemma *Ball-set-table*: $(\forall\ (x,y)\in set\ l.\ P\ x\ y) \Longrightarrow \forall\ x.\ \forall\ y\in map\text{-}of\ l\ x\colon P\ x\ y$
apply (*erule rev-mp*)
apply (*induct l*)
apply *simp*
apply (*simp* (*no-asm*))
apply *auto*
done

lemma *Ball-set-tableD*:
$\llbracket(\forall\ (x,y)\in set\ l.\ P\ x\ y);\ x \in o2s\ (table\text{-}of\ l\ xa)\rrbracket \Longrightarrow P\ xa\ x$
apply (*frule Ball-set-table*)
by *auto*

declare *map-of-SomeD* [*elim*]

lemma *table-of-Some-in-set*:
$table\text{-}of\ l\ k = Some\ x \Longrightarrow (k,x) \in set\ l$
by *auto*

lemma *set-get-eq*:
$unique\ l \Longrightarrow (k,\ the\ (table\text{-}of\ l\ k)) \in set\ l = (table\text{-}of\ l\ k \neq None)$
by (*auto dest!*: *weak-map-of-SomeI*)

lemma *inj-Pair-const2*: $inj\ (\lambda k.\ (k,\ C))$
apply (*rule inj-onI*)
apply *auto*
done

lemma *table-of-mapconst-SomeI*:
$\llbracket table\text{-}of\ t\ k = Some\ y';\ snd\ y{=}y';\ fst\ y{=}c\rrbracket \Longrightarrow$
$\quad table\text{-}of\ (map\ (\lambda(k,x).\ (k,c,x))\ t)\ k = Some\ y$
apply (*induct t*)

apply *auto*
done

lemma *table-of-mapconst-NoneI*:
$[\![$*table-of t k* = *None*$]\!] \Longrightarrow$
table-of (*map* (λ(*k*,*x*). (*k*,*c*,*x*)) *t*) *k* = *None*
apply (*induct t*)
apply *auto*
done

lemmas *table-of-map2-SomeI* = *inj-Pair-const2* [*THEN map-of-mapk-SomeI*, *standard*]

lemma *table-of-map-SomeI* [*rule-format* (*no-asm*)]: *table-of t k* = *Some x* $\longrightarrow$
table-of (*map* (λ(*k*,*x*). (*k*, *f x*)) *t*) *k* = *Some* (*f x*)
apply (*induct-tac t*)
apply *auto*
done

lemma *table-of-remap-SomeD* [*rule-format* (*no-asm*)]:
table-of (*map* (λ((*k*,*k'*),*x*). (*k*,(*k'*,*x*))) *t*) *k* = *Some* (*k'*,*x*) $\longrightarrow$
table-of t (*k*, *k'*) = *Some x*
apply (*induct-tac t*)
apply *auto*
done

lemma *table-of-mapf-Some* [*rule-format* (*no-asm*)]: $\forall$ *x y*. *f x* = *f y* $\longrightarrow$ *x* = *y* $\Longrightarrow$
table-of (*map* (λ(*k*,*x*). (*k*,*f x*)) *t*) *k* = *Some* (*f x*) $\longrightarrow$ *table-of t k* = *Some x*
apply (*induct-tac t*)
apply *auto*
done

lemma *table-of-mapf-SomeD* [*rule-format* (*no-asm*), *dest*!]:
table-of (*map* (λ(*k*,*x*). (*k*, *f x*)) *t*) *k* = *Some z* $\longrightarrow$ ($\exists$ *y*$\in$*table-of t k*: *z*=*f y*)
apply (*induct-tac t*)
apply *auto*
done

lemma *table-of-mapf-NoneD* [*rule-format* (*no-asm*), *dest*!]:
table-of (*map* (λ(*k*,*x*). (*k*, *f x*)) *t*) *k* = *None* $\longrightarrow$ (*table-of t k* = *None*)
apply (*induct-tac t*)
apply *auto*
done

lemma *table-of-mapkey-SomeD* [*rule-format* (*no-asm*), *dest*!]:
table-of (*map* (λ(*k*,*x*). ((*k*,*C*),*x*)) *t*) (*k*,*D*) = *Some x* $\longrightarrow$ *C* = *D* $\wedge$ *table-of t k* = *Some x*
apply (*induct-tac t*)
apply *auto*
done

lemma *table-of-mapkey-SomeD2* [*rule-format* (*no-asm*), *dest*!]:
table-of (*map* (λ(*k*,*x*). ((*k*,*C*),*x*)) *t*) *ek* = *Some x*
$\longrightarrow$ *C* = *snd ek* $\wedge$ *table-of t* (*fst ek*) = *Some x*

apply (*induct-tac t*)
apply *auto*
done

lemma *table-append-Some-iff*: *table-of* (*xs@ys*) *k* = *Some z* =
(*table-of xs k* = *Some z* ∨ (*table-of xs k* = *None* ∧ *table-of ys k* = *Some z*))
apply (*simp*)
apply (*rule map-add-Some-iff*)
done

lemma *table-of-filter-unique-SomeD* [*rule-format* (*no-asm*)]:
table-of (*filter P xs*) *k* = *Some z* ⟹ *unique xs* ⟶ *table-of xs k* = *Some z*
apply (*induct xs*)
apply (*auto del*: *map-of-SomeD intro*!: *map-of-SomeD*)
done

consts
Un-tables :: ('*a*, '*b*) *tables set* ⇒ ('*a*, '*b*) *tables*
overrides-t :: ('*a*, '*b*) *tables* ⇒ ('*a*, '*b*) *tables* ⇒
('*a*, '*b*) *tables* (**infixl** ⊕⊕ *100*)
hidings-entails:: ('*a*, '*b*) *tables* ⇒ ('*a*, '*c*) *tables* ⇒
('*b* ⇒ '*c* ⇒ *bool*) ⇒ *bool* (- *hidings* - *entails* - *20*)
— variant for unique table:
hiding-entails :: ('*a*, '*b*) *table* ⇒ ('*a*, '*c*) *table* ⇒
('*b* ⇒ '*c* ⇒ *bool*) ⇒ *bool* (- *hiding* - *entails* - *20*)
— variant for a unique table and conditional overriding:
cond-hiding-entails :: ('*a*, '*b*) *table* ⇒ ('*a*, '*c*) *table*
⇒ ('*b* ⇒ '*c* ⇒ *bool*) ⇒ ('*b* ⇒ '*c* ⇒ *bool*) ⇒ *bool*
(- *hiding* - *under* - *entails* - *20*)

defs
Un-tables-def: *Un-tables ts* ≡ λ*k*. ⋃*t*∈*ts*. *t k*
overrides-t-def: *s* ⊕⊕ *t* ≡ λ*k*. *if t k* = {} *then s k else t k*
hidings-entails-def: *t hidings s entails R* ≡ ∀ *k*. ∀ *x*∈*t k*. ∀ *y*∈*s k*. *R x y*
hiding-entails-def: *t hiding s entails R* ≡ ∀ *k*. ∀ *x*∈*t k*: ∀ *y*∈*s k*: *R x y*
cond-hiding-entails-def: *t hiding s under C entails R*
≡ ∀ *k*. ∀ *x*∈*t k*: ∀ *y*∈*s k*: *C x y* ⟶ *R x y*

Untables

lemma *Un-tablesI* [*intro*]: ⋀*x*. ⟦*t* ∈ *ts*; *x* ∈ *t k*⟧ ⟹ *x* ∈ *Un-tables ts k*
apply (*simp add*: *Un-tables-def*)
apply *auto*
done

lemma *Un-tablesD* [*dest*!]: ⋀*x*. *x* ∈ *Un-tables ts k* ⟹ ∃ *t*. *t* ∈ *ts* ∧ *x* ∈ *t k*
apply (*simp add*: *Un-tables-def*)
apply *auto*
done

lemma *Un-tables-empty* [*simp*]: *Un-tables* {} = (λ*k*. {})
apply (*unfold Un-tables-def*)
apply (*simp* (*no-asm*))
done

overrides

lemma *empty-overrides-t* [*simp*]: $(\lambda k.\ \{\}) \oplus\oplus\ m = m$
apply (*unfold overrides-t-def*)
apply (*simp* (*no-asm*))
done

lemma *overrides-empty-t* [*simp*]: $m \oplus\oplus (\lambda k.\ \{\}) = m$
apply (*unfold overrides-t-def*)
apply (*simp* (*no-asm*))
done

lemma *overrides-t-Some-iff*:
$(x \in (s \oplus\oplus\ t)\ k) = (x \in t\ k \vee t\ k = \{\} \wedge x \in s\ k)$
by (*simp add*: *overrides-t-def*)

lemmas *overrides-t-SomeD* = *overrides-t-Some-iff* [*THEN iffD1*, *dest*!]

lemma *overrides-t-right-empty* [*simp*]: $n\ k = \{\} \Longrightarrow (m \oplus\oplus\ n)\ k = m\ k$
by (*simp add*: *overrides-t-def*)

lemma *overrides-t-find-right* [*simp*]: $n\ k \neq \{\} \Longrightarrow (m \oplus\oplus\ n)\ k = n\ k$
by (*simp add*: *overrides-t-def*)

hiding entails

lemma *hiding-entailsD*:
$[\![t\ \textit{hiding}\ s\ \textit{entails}\ R;\ t\ k = \textit{Some}\ x;\ s\ k = \textit{Some}\ y]\!] \Longrightarrow R\ x\ y$
by (*simp add*: *hiding-entails-def*)

lemma *empty-hiding-entails*: *empty hiding s entails R*
by (*simp add*: *hiding-entails-def*)

lemma *hiding-empty-entails*: *t hiding empty entails R*
by (*simp add*: *hiding-entails-def*)
declare *empty-hiding-entails* [*simp*] *hiding-empty-entails* [*simp*]

cond hiding entails

lemma *cond-hiding-entailsD*:
$[\![t\ \textit{hiding}\ s\ \textit{under}\ C\ \textit{entails}\ R;\ t\ k = \textit{Some}\ x;\ s\ k = \textit{Some}\ y;\ C\ x\ y]\!] \Longrightarrow R\ x\ y$
by (*simp add*: *cond-hiding-entails-def*)

lemma *empty-cond-hiding-entails*[*simp*]: *empty hiding s under C entails R*
by (*simp add*: *cond-hiding-entails-def*)

lemma *cond-hiding-empty-entails*[*simp*]: *t hiding empty under C entails R*
by (*simp add*: *cond-hiding-entails-def*)

lemma *hidings-entailsD*: $[\![t\ \textit{hidings}\ s\ \textit{entails}\ R;\ x \in t\ k;\ y \in s\ k]\!] \Longrightarrow R\ x\ y$
by (*simp add*: *hidings-entails-def*)

lemma *hidings-empty-entails*: *t hidings* $(\lambda k.\ \{\})$ *entails R*
apply (*unfold hidings-entails-def*)
apply (*simp* (*no-asm*))
done

lemma *empty-hidings-entails*:
$(\lambda k.\ \{\})$ *hidings s entails R***apply** (*unfold hidings-entails-def*)
by (*simp* (*no-asm*))
declare *empty-hidings-entails* [*intro*!] *hidings-empty-entails* [*intro*!]

consts
atleast-free :: ('*a* ~=> '*b*) => *nat* => *bool*
primrec
atleast-free m 0 = *True*
atleast-free-Suc:
atleast-free m (*Suc n*) = (*? a. m a* = *None* & (!*b. atleast-free* (*m*(*a*|−>*b*)) *n*))

lemma *atleast-free-weaken* [*rule-format* (*no-asm*)]:
!*m. atleast-free m* (*Suc n*) ⟶ *atleast-free m n*
apply (*induct-tac n*)
apply (*simp* (*no-asm*))
apply *clarify*
apply (*simp* (*no-asm*))
apply (*drule atleast-free-Suc* [*THEN iffD1*])
apply *fast*
done

lemma *atleast-free-SucI*:
[| *h a* = *None*; !*obj. atleast-free* (*h*(*a*|−>*obj*)) *n* |] ==> *atleast-free h* (*Suc n*)
by *force*

declare *fun-upd-apply* [*simp del*]

lemma *atleast-free-SucD-lemma* [*rule-format* (*no-asm*)]:
!*m a. m a* = *None* −−> (!*c. atleast-free* (*m*(*a*|−>*c*)) *n*) −−>
(!*b d. a* ~= *b* −−> *atleast-free* (*m*(*b*|−>*d*)) *n*)
apply (*induct-tac n*)
apply *auto*
apply (*rule-tac x* = *a* **in** *exI*)
apply (*rule conjI*)
apply (*force simp add*: *fun-upd-apply*)
apply (*erule-tac V* = *m a* = *None* **in** *thin-rl*)
apply *clarify*
apply (*subst fun-upd-twist*)
apply (*erule not-sym*)
apply (*rename-tac ba*)
apply (*drule-tac x* = *ba* **in** *spec*)
apply *clarify*
apply (*tactic smp-tac 2 1*)
apply (*erule* (*1*) *notE impE*)
apply (*case-tac aa* = *b*)
apply *fast*+

done
declare *fun-upd-apply* [*simp*]

lemma *atleast-free-SucD* [*rule-format* (*no-asm*)]: *atleast-free h* (*Suc n*) ==> *atleast-free* (*h*(*a*|−>*b*)) *n*
apply *auto*
apply (*case-tac aa* = *a*)
apply *auto*
apply (*erule atleast-free-SucD-lemma*)
apply *auto*
done

declare *atleast-free-Suc* [*simp del*]
end

Chapter 4

Name

3 Java names

theory *Name* **imports** *Basis* **begin**

typedecl *tnam* — ordinary type name, i.e. class or interface name
typedecl *pname* — package name
typedecl *mname* — method name
typedecl *vname* — variable or field name
typedecl *label* — label as destination of break or continue

datatype *ename* — expression name
= *VNam vname*
| *Res* — special name to model the return value of methods

datatype *lname* — names for local variables and the This pointer
= *EName ename*
| *This*
syntax
VName :: *vname* ⇒ *lname*
Result :: *lname*

translations
VName n == *EName* (*VNam n*)
Result == *EName Res*

datatype *xname* — names of standard exceptions
= *Throwable*
| *NullPointer* | *OutOfMemory* | *ClassCast*
| *NegArrSize* | *IndOutBound* | *ArrStore*

lemma *xn-cases*:
$xn = Throwable \lor xn = NullPointer \lor$
$xn = OutOfMemory \lor xn = ClassCast \lor$
$xn = NegArrSize \lor xn = IndOutBound \lor xn = ArrStore$
apply (*induct xn*)
apply *auto*
done

datatype *tname* — type names for standard classes and other type names
= *Object'*
| *SXcpt'* *xname*
| *TName* *tnam*

record *qtname* = — qualified tname cf. 6.5.3, 6.5.4
pid :: *pname*
tid :: *tname*

axclass *has-pname* < *type*
consts *pname*::'*a*::*has-pname* ⇒ *pname*

instance *pname*::*has-pname* **..**

defs (**overloaded**)
pname-pname-def: *pname* (*p*::*pname*) ≡ *p*

axclass *has-tname* < *type*

consts $tname::'a::has\text{-}tname \Rightarrow tname$

instance $tname::has\text{-}tname$ **..**

defs (**overloaded**)
$tname\text{-}tname\text{-}def$: $tname\ (t::tname) \equiv t$

axclass $has\text{-}qtname < type$
consts $qtname::\ 'a::has\text{-}qtname \Rightarrow qtname$

instance $qtname\text{-}ext\text{-}type :: (type)\ has\text{-}qtname$ **..**

defs (**overloaded**)
$qtname\text{-}qtname\text{-}def$: $qtname\ (q::qtname) \equiv q$

translations
$mname\ <= Name.mname$
$xname\ <= Name.xname$
$tname\ <= Name.tname$
$ename\ <= Name.ename$
$qtname <= (type)\ (\!|pid::pname,tid::tname|\!)$
$(type)\ 'a\ qtname\text{-}scheme <= (type)\ (\!|pid::pname,tid::tname,\ldots::'a|\!)$

axiomatization $java\text{-}lang::pname$ — package java.lang

consts
$Object :: qtname$
$SXcpt\ :: xname \Rightarrow qtname$
defs
$Object\text{-}def$: $Object\ \equiv (\!|pid = java\text{-}lang,\ tid = Object'|\!)$
$SXcpt\text{-}def$: $SXcpt\ \equiv \lambda x.\ (\!|pid = java\text{-}lang,\ tid = SXcpt'\ x|\!)$

lemma $Object\text{-}neq\text{-}SXcpt\ [simp]$: $Object \neq SXcpt\ xn$
by $(simp\ add\text{: } Object\text{-}def\ SXcpt\text{-}def)$

lemma $SXcpt\text{-}inject\ [simp]$: $(SXcpt\ xn = SXcpt\ xm) = (xn = xm)$
by $(simp\ add\text{: } SXcpt\text{-}def)$
end

Chapter 5

Value

4 Java values

theory *Value* **imports** *Type* **begin**

typedecl *loc* — locations, i.e. abstract references on objects

datatype *val*
  = *Unit* — dummy result value of void methods
  | *Bool bool* — Boolean value
  | *Intg int* — integer value
  | *Null* — null reference
  | *Addr loc* — addresses, i.e. locations of objects

translations *val* <= (*type*) *Term.val*
  loc <= (*type*) *Term.loc*

consts *the-Bool* :: *val* ⇒ *bool*
primrec *the-Bool* (*Bool b*) = *b*
consts *the-Intg* :: *val* ⇒ *int*
primrec *the-Intg* (*Intg i*) = *i*
consts *the-Addr* :: *val* ⇒ *loc*
primrec *the-Addr* (*Addr a*) = *a*

types *dyn-ty* = *loc* ⇒ *ty option*
consts
  typeof :: *dyn-ty* ⇒ *val* ⇒ *ty option*
  defpval :: *prim-ty* ⇒ *val* — default value for primitive types
  default-val :: *ty* ⇒ *val* — default value for all types

primrec *typeof dt Unit* = *Some* (*PrimT Void*)
  typeof dt (*Bool b*) = *Some* (*PrimT Boolean*)
  typeof dt (*Intg i*) = *Some* (*PrimT Integer*)
  typeof dt Null = *Some NT*
  typeof dt (*Addr a*) = *dt a*

primrec *defpval Void* = *Unit*
  defpval Boolean = *Bool False*
  defpval Integer = *Intg 0*
primrec *default-val* (*PrimT pt*) = *defpval pt*
  default-val (*RefT r*) = *Null*

end

Chapter 6

Type

5 Java types

theory *Type* **imports** *Name* **begin**

simplifications:

- only the most important primitive types
- the null type is regarded as reference type

datatype *prim-ty* — primitive type, cf. 4.2
= *Void* — result type of void methods
| *Boolean*
| *Integer*

datatype *ref-ty* — reference type, cf. 4.3
= *NullT* — null type, cf. 4.1
| *IfaceT qtname* — interface type
| *ClassT qtname* — class type
| *ArrayT ty* — array type

and *ty* — any type, cf. 4.1
= *PrimT prim-ty* — primitive type
| *RefT ref-ty* — reference type

translations
prim-ty <= (*type*) *Type.prim-ty*
ref-ty <= (*type*) *Type.ref-ty*
ty <= (*type*) *Type.ty*

syntax
NT :: *ty*
Iface :: *qtname* $\Rightarrow$ *ty*
Class :: *qtname* $\Rightarrow$ *ty*
Array :: *ty* $\Rightarrow$ *ty* (-.[] [*90*] *90*)

translations
NT == *RefT NullT*
Iface I == *RefT* (*IfaceT I*)
Class C == *RefT* (*ClassT C*)
T.[] == *RefT* (*ArrayT T*)

constdefs
the-Class :: *ty* $\Rightarrow$ *qtname*
the-Class T $\equiv$ *SOME C*. *T* = *Class C*

lemma *the-Class-eq* [*simp*]: *the-Class* (*Class C*)= *C*
by (*auto simp add*: *the-Class-def*)

end

Chapter 7

Term

6 Java expressions and statements

theory *Term* **imports** *Value Table* **begin**

design issues:

- invocation frames for local variables could be reduced to special static objects (one per method). This would reduce redundancy, but yield a rather non-standard execution model more difficult to understand.
- method bodies separated from calls to handle assumptions in axiomat. semantics NB: Body is intended to be in the environment of the called method.
- class initialization is regarded as (auxiliary) statement (required for AxSem)
- result expression of method return is handled by a special result variable result variable is treated uniformly with local variables
  - \+ welltypedness and existence of the result/return expression is ensured without extra efford

simplifications:

- expression statement allowed for any expression
- This is modeled as a special non-assignable local variable
- Super is modeled as a general expression with the same value as This
- access to field x in current class via This.x
- NewA creates only one-dimensional arrays; initialization of further subarrays may be simulated with nested NewAs
- The 'Lit' constructor is allowed to contain a reference value. But this is assumed to be prohibited in the input language, which is enforced by the type-checking rules.
- a call of a static method via a type name may be simulated by a dummy variable
- no nested blocks with inner local variables
- no synchronized statements
- no secondary forms of if, while (e.g. no for) (may be easily simulated)
- no switch (may be simulated with if)
- the *try-catch-finally* statement is divided into the *try-catch* statement and a finally statement, which may be considered as try..finally with empty catch
- the *try-catch* statement has exactly one catch clause; multiple ones can be simulated with instanceof
- the compiler is supposed to add the annotations - during type-checking. This transformation is left out as its result is checked by the type rules anyway

types *locals* = (*lname*, *val*) *table* — local variables

datatype *jump*
  = *Break label* — break

| *Cont label* — continue
| *Ret* — return from method

datatype *xcpt* — exception
= *Loc loc* — location of allocated execption object
| *Std xname* — intermediate standard exception, see Eval.thy

datatype *error*
= *AccessViolation* — Access to a member that isn't permitted
| *CrossMethodJump* — Method exits with a break or continue

datatype *abrupt* — abrupt completion
= *Xcpt xcpt* — exception
| *Jump jump* — break, continue, return
| *Error error* — runtime errors, we wan't to detect and proof absent in welltyped programms
types
abopt = *abrupt option*

Local variable store and exception. Anticipation of State.thy used by smallstep semantics. For a method call, we save the local variables of the caller in the term Callee to restore them after method return. Also an exception must be restored after the finally statement

translations
locals <= (*type*) (*lname*, *val*) *table*

datatype *inv-mode* — invocation mode for method calls
= *Static* — static
| *SuperM* — super
| *IntVir* — interface or virtual

record *sig* = — signature of a method, cf. 8.4.2
name ::*mname* — acutally belongs to Decl.thy
parTs::*ty list*

translations
sig <= (*type*) (|*name*::*mname*,*parTs*::*ty list*|)
sig <= (*type*) (|*name*::*mname*,*parTs*::*ty list*,...::'*a*|)

— function codes for unary operations
datatype *unop* = *UPlus* — `+` unary plus
| *UMinus* — `-` unary minus
| *UBitNot* — bitwise NOT
| *UNot* — `!` logical complement

— function codes for binary operations
datatype *binop* = *Mul* — `*` multiplication
| *Div* — `/` division
| *Mod* — `%` remainder
| *Plus* — `+` addition
| *Minus* — `-` subtraction
| *LShift* — `<<` left shift
| *RShift* — `>>` signed right shift
| *RShiftU* — `>>>` unsigned right shift
| *Less* — `<` less than
| *Le* — `<=` less than or equal
| *Greater* — `>` greater than
| *Ge* — `>=` greater than or equal
| *Eq* — `==` equal
| *Neq* — `!=` not equal

| *BitAnd* — `&` bitwise AND
| *And* — `&` boolean AND
| *BitXor* — `^` bitwise Xor
| *Xor* — `^` boolean Xor
| *BitOr* — `|` bitwise Or
| *Or* — `|` boolean Or
| *CondAnd* — `&&` conditional And
| *CondOr* — `||` conditional Or

The boolean operators `&` and `|` strictly evaluate both of their arguments. The conditional operators `&&` and `||` only evaluate the second argument if the value of the whole expression isn't allready determined by the first argument. e.g.: `false && e` e is not evaluated; `true || e` e is not evaluated;

datatype *var*
= *LVar lname* — local variable (incl. parameters)
| *FVar qtname qtname bool expr vname* ({-,-,-}-..-[*10*,*10*,*10*,*85*,*99*]*90*)
— class field
— {*accC*,*statDeclC*,*stat*}*e*..*fn*
— *accC*: accessing class (static class were
— the code is declared. Annotation only needed for
— evaluation to check accessibility)
— *statDeclC*: static declaration class of field
— *stat*: static or instance field?
— *e*: reference to object
— *fn*: field name
| *AVar expr expr* (-.[-][*90*,*10*]*90*)
— array component
— *e1*.[*e2*]: e1 array reference; e2 index
| *InsInitV stmt var*
— insertion of initialization before evaluation
— of var (technical term for smallstep semantics.)

and *expr*
= *NewC qtname* — class instance creation
| *NewA ty expr* (*New* -[-][*99*,*10*]*85*)
— array creation
| *Cast ty expr* — type cast
| *Inst expr ref-ty* (- *InstOf* -[*85*,*99*] *85*)
— instanceof
| *Lit val* — literal value, references not allowed
| *UnOp unop expr* — unary operation
| *BinOp binop expr expr* — binary operation

| *Super* — special Super keyword
| *Acc var* — variable access
| *Ass var expr* (-:=- [*90*,*85*]*85*)
— variable assign
| *Cond expr expr expr* (- *?* - : - [*85*,*85*,*80*]*80*) — conditional
| *Call qtname ref-ty inv-mode expr mname* (*ty list*) (*expr list*)
({-,-,-}-·-′({-}-′)[*10*,*10*,*10*,*85*,*99*,*10*,*10*]*85*)
— method call
— {*accC*,*statT*,*mode*}*e*·*mn*({*pTs*}*args*) ”
— *accC*: accessing class (static class were
— the call code is declared. Annotation only needed for
— evaluation to check accessibility)
— *statT*: static declaration class/interface of
— method
— *mode*: invocation mode
— *e*: reference to object

— *mn*: field name
— *pTs*: types of parameters
— *args*: the actual parameters/arguments
| *Methd qtname sig* — (folded) method (see below)
| *Body qtname stmt* — (unfolded) method body
| *InsInitE stmt expr*
— insertion of initialization before
— evaluation of expr (technical term for smallstep sem.)
| *Callee locals expr* — save callers locals in callee-Frame
— (technical term for smallstep semantics)

and *stmt*
= *Skip* — empty statement
| *Expr expr* — expression statement
| *Lab jump stmt* (-· - [*99,66*] *66*)
— labeled statement; handles break
| *Comp stmt stmt* (-;; - [*66,65*] *65*)
| *If′ expr stmt stmt* (*If′*(-′) - *Else* - [*80,79,79*] *70*)
| *Loop label expr stmt* (-· *While′*(-′) - [*99,80,79*] *70*)
| *Jmp jump* — break, continue, return
| *Throw expr*
| *TryC stmt qtname vname stmt* (*Try* - *Catch′*(- -′) - [*79,99,80,79*] *70*)
— *Try c1 Catch*(*C vn*) *c2*
— *c1*: block were exception may be thrown
— *C*: execption class to catch
— *vn*: local name for exception used in *c2*
— *c2*: block to execute when exception is cateched
| *Fin stmt stmt* (- *Finally* - [*79,79*] *70*)
| *FinA abopt stmt* — Save abruption of first statement
— technical term for smallstep sem.)
| *Init qtname* — class initialization

The expressions Methd and Body are artificial program constructs, in the sense that they are not used to define a concrete Bali program. In the operational semantic's they are "generated on the fly" to decompose the task to define the behaviour of the Call expression. They are crucial for the axiomatic semantics to give a syntactic hook to insert some assertions (cf. AxSem.thy, Eval.thy). The Init statement (to initialize a class on its first use) is inserted in various places by the semantics. Callee, InsInitV, InsInitE,FinA are only needed as intermediate steps in the smallstep (transition) semantics (cf. Trans.thy). Callee is used to save the local variables of the caller for method return. So ve avoid modelling a frame stack. The InsInitV/E terms are only used by the smallstep semantics to model the intermediate steps of class-initialisation.

types *term* = (*expr*+*stmt*,*var*,*expr list*) *sum3*
translations
sig <= (*type*) *mname* × *ty list*
var <= (*type*) *Term.var*
expr <= (*type*) *Term.expr*
stmt <= (*type*) *Term.stmt*
term <= (*type*) (*expr*+*stmt*,*var*,*expr list*) *sum3*

syntax

this :: *expr*
LAcc :: *vname* ⇒ *expr* (!!)
LAss :: *vname* ⇒ *expr* ⇒*stmt* (-:==- [*90,85*] *85*)
Return :: *expr* ⇒ *stmt*
StatRef :: *ref-ty* ⇒ *expr*

translations

$\mathit{this} \quad == \mathit{Acc}\ (\mathit{LVar}\ \mathit{This})$
$!!v \quad == \mathit{Acc}\ (\mathit{LVar}\ (\mathit{EName}\ (\mathit{VNam}\ v)))$
$v{:}{=}{=}e \quad == \mathit{Expr}\ (\mathit{Ass}\ (\mathit{LVar}\ (\mathit{EName}\ (\mathit{VNam}\ v)))\ e)$
$\mathit{Return}\ e \quad == \mathit{Expr}\ (\mathit{Ass}\ (\mathit{LVar}\ (\mathit{EName}\ \mathit{Res}))\ e);;\ \mathit{Jmp}\ \mathit{Ret}$
— `Res := e;; Jmp Ret`
$\mathit{StatRef}\ rt == \mathit{Cast}\ (\mathit{RefT}\ rt)\ (\mathit{Lit}\ \mathit{Null})$

constdefs

$\mathit{is\text{-}stmt} :: \mathit{term} \Rightarrow \mathit{bool}$
$\mathit{is\text{-}stmt}\ t \equiv \exists\, c.\ t{=}\mathit{In1r}\ c$

ML-setup $\langle\!\langle$ *bind-thms* (*is-stmt-rews*, *sum3-instantiate* @{*thm is-stmt-def*}) $\rangle\!\rangle$

declare *is-stmt-rews* [*simp*]

Here is some syntactic stuff to handle the injections of statements, expressions, variables and expression lists into general terms.

syntax
$\mathit{expr\text{-}inj\text{-}term} :: \mathit{expr} \Rightarrow \mathit{term}\ (\langle\text{-}\rangle_e\ 1000)$
$\mathit{stmt\text{-}inj\text{-}term} :: \mathit{stmt} \Rightarrow \mathit{term}\ (\langle\text{-}\rangle_s\ 1000)$
$\mathit{var\text{-}inj\text{-}term} :: \mathit{var} \Rightarrow \mathit{term}\ (\langle\text{-}\rangle_v\ 1000)$
$\mathit{lst\text{-}inj\text{-}term} :: \mathit{expr}\ \mathit{list} \Rightarrow \mathit{term}\ (\langle\text{-}\rangle_l\ 1000)$

translations
$\langle e\rangle_e \rightharpoonup \mathit{In1l}\ e$
$\langle c\rangle_s \rightharpoonup \mathit{In1r}\ c$
$\langle v\rangle_v \rightharpoonup \mathit{In2}\ v$
$\langle es\rangle_l \rightharpoonup \mathit{In3}\ es$

It seems to be more elegant to have an overloaded injection like the following.

axclass *inj-term* < *type*
consts $\mathit{inj\text{-}term} :: {'a}{::}\mathit{inj\text{-}term} \Rightarrow \mathit{term}\ (\langle\text{-}\rangle\ 1000)$

How this overloaded injections work can be seen in the theory *DefiniteAssignment*. Other big inductive relations on terms defined in theories *WellType*, *Eval*, *Evaln* and *AxSem* don't follow this convention right now, but introduce subtle syntactic sugar in the relations themselves to make a distinction on expressions, statements and so on. So unfortunately you will encounter a mixture of dealing with these injections. The translations above are used as bridge between the different conventions.

instance *stmt*::*inj-term* **..**

defs (**overloaded**)
stmt-inj-term-def: $\langle c{::}\mathit{stmt}\rangle \equiv \mathit{In1r}\ c$

lemma *stmt-inj-term-simp*: $\langle c{::}\mathit{stmt}\rangle = \mathit{In1r}\ c$
by (*simp add*: *stmt-inj-term-def*)

lemma *stmt-inj-term* [*iff*]: $\langle x{::}\mathit{stmt}\rangle = \langle y\rangle \equiv x = y$
by (*simp add*: *stmt-inj-term-simp*)

instance *expr*::*inj-term* **..**

defs (**overloaded**)
expr-inj-term-def: $\langle e{::}\mathit{expr}\rangle \equiv \mathit{In1l}\ e$

lemma *expr-inj-term-simp*: $\langle e{::}expr\rangle = In1l\ e$
by (*simp add*: *expr-inj-term-def*)

lemma *expr-inj-term* [*iff*]: $\langle x{::}expr\rangle = \langle y\rangle \equiv x = y$
by (*simp add*: *expr-inj-term-simp*)

instance *var*::*inj-term* **..**

defs (**overloaded**)
var-inj-term-def: $\langle v{::}var\rangle \equiv In2\ v$

lemma *var-inj-term-simp*: $\langle v{::}var\rangle = In2\ v$
by (*simp add*: *var-inj-term-def*)

lemma *var-inj-term* [*iff*]: $\langle x{::}var\rangle = \langle y\rangle \equiv x = y$
by (*simp add*: *var-inj-term-simp*)

instance *list*:: (*type*) *inj-term* **..**

defs (**overloaded**)
expr-list-inj-term-def: $\langle es{::}expr\ list\rangle \equiv In3\ es$

lemma *expr-list-inj-term-simp*: $\langle es{::}expr\ list\rangle = In3\ es$
by (*simp add*: *expr-list-inj-term-def*)

lemma *expr-list-inj-term* [*iff*]: $\langle x{::}expr\ list\rangle = \langle y\rangle \equiv x = y$
by (*simp add*: *expr-list-inj-term-simp*)

lemmas *inj-term-simps* = *stmt-inj-term-simp expr-inj-term-simp var-inj-term-simp*
expr-list-inj-term-simp
lemmas *inj-term-sym-simps* = *stmt-inj-term-simp* [*THEN sym*]
expr-inj-term-simp [*THEN sym*]
var-inj-term-simp [*THEN sym*]
expr-list-inj-term-simp [*THEN sym*]

lemma *stmt-expr-inj-term* [*iff*]: $\langle t{::}stmt\rangle \neq \langle w{::}expr\rangle$
by (*simp add*: *inj-term-simps*)

lemma *expr-stmt-inj-term* [*iff*]: $\langle t{::}expr\rangle \neq \langle w{::}stmt\rangle$
by (*simp add*: *inj-term-simps*)

lemma *stmt-var-inj-term* [*iff*]: $\langle t{::}stmt\rangle \neq \langle w{::}var\rangle$
by (*simp add*: *inj-term-simps*)

lemma *var-stmt-inj-term* [*iff*]: $\langle t{::}var\rangle \neq \langle w{::}stmt\rangle$
by (*simp add*: *inj-term-simps*)

lemma *stmt-elist-inj-term* [*iff*]: $\langle t{::}stmt\rangle \neq \langle w{::}expr\ list\rangle$
by (*simp add*: *inj-term-simps*)

lemma *elist-stmt-inj-term* [*iff*]: $\langle t{::}expr\ list\rangle \neq \langle w{::}stmt\rangle$

by (*simp add*: *inj-term-simps*)

lemma *expr-var-inj-term* [*iff*]: $\langle t::expr\rangle \neq \langle w::var\rangle$
by (*simp add*: *inj-term-simps*)

lemma *var-expr-inj-term* [*iff*]: $\langle t::var\rangle \neq \langle w::expr\rangle$
by (*simp add*: *inj-term-simps*)

lemma *expr-elist-inj-term* [*iff*]: $\langle t::expr\rangle \neq \langle w::expr\ list\rangle$
by (*simp add*: *inj-term-simps*)

lemma *elist-expr-inj-term* [*iff*]: $\langle t::expr\ list\rangle \neq \langle w::expr\rangle$
by (*simp add*: *inj-term-simps*)

lemma *var-elist-inj-term* [*iff*]: $\langle t::var\rangle \neq \langle w::expr\ list\rangle$
by (*simp add*: *inj-term-simps*)

lemma *elist-var-inj-term* [*iff*]: $\langle t::expr\ list\rangle \neq \langle w::var\rangle$
by (*simp add*: *inj-term-simps*)

lemma *term-cases*:
$[\![\bigwedge\ v.\ P\ \langle v\rangle_v;\ \bigwedge\ e.\ P\ \langle e\rangle_e;\bigwedge\ c.\ P\ \langle c\rangle_s;\bigwedge\ l.\ P\ \langle l\rangle_l]\!]$
$\Longrightarrow P\ t$
apply (*cases t*)
apply (*case-tac a*)
apply *auto*
done

Evaluation of unary operations

consts *eval-unop* :: *unop* ⇒ *val* ⇒ *val*
primrec
eval-unop UPlus v = *Intg* (*the-Intg v*)
eval-unop UMinus v = *Intg* (− (*the-Intg v*))
eval-unop UBitNot v = *Intg 42* — FIXME: Not yet implemented
eval-unop UNot v = *Bool* (¬ *the-Bool v*)

Evaluation of binary operations

consts *eval-binop* :: *binop* ⇒ *val* ⇒ *val* ⇒ *val*
primrec
eval-binop Mul v1 v2 = *Intg* ((*the-Intg v1*) ∗ (*the-Intg v2*))
eval-binop Div v1 v2 = *Intg* ((*the-Intg v1*) *div* (*the-Intg v2*))
eval-binop Mod v1 v2 = *Intg* ((*the-Intg v1*) *mod* (*the-Intg v2*))
eval-binop Plus v1 v2 = *Intg* ((*the-Intg v1*) + (*the-Intg v2*))
eval-binop Minus v1 v2 = *Intg* ((*the-Intg v1*) − (*the-Intg v2*))

— Be aware of the explicit coercion of the shift distance to nat
eval-binop LShift v1 v2 = *Intg* ((*the-Intg v1*) ∗ (*2*ˆ(*nat* (*the-Intg v2*))))
eval-binop RShift v1 v2 = *Intg* ((*the-Intg v1*) *div* (*2*ˆ(*nat* (*the-Intg v2*))))
eval-binop RShiftU v1 v2 = *Intg 42* — FIXME: Not yet implemented

eval-binop Less v1 v2 = *Bool* ((*the-Intg v1*) < (*the-Intg v2*))
eval-binop Le v1 v2 = *Bool* ((*the-Intg v1*) ≤ (*the-Intg v2*))
eval-binop Greater v1 v2 = *Bool* ((*the-Intg v2*) < (*the-Intg v1*))
eval-binop Ge v1 v2 = *Bool* ((*the-Intg v2*) ≤ (*the-Intg v1*))

eval-binop Eq v1 v2 = *Bool* (*v1*=*v2*)

$eval\text{-}binop\ Neq\quad v1\ v2 = Bool\ (v1 \neq v2)$
$eval\text{-}binop\ BitAnd\ \ v1\ v2 = Intg\ 42$ — FIXME: Not yet implemented
$eval\text{-}binop\ And\quad v1\ v2 = Bool\ ((the\text{-}Bool\ v1) \wedge (the\text{-}Bool\ v2))$
$eval\text{-}binop\ BitXor\ \ v1\ v2 = Intg\ 42$ — FIXME: Not yet implemented
$eval\text{-}binop\ Xor\quad v1\ v2 = Bool\ ((the\text{-}Bool\ v1) \neq (the\text{-}Bool\ v2))$
$eval\text{-}binop\ BitOr\ \ v1\ v2 = Intg\ 42$ — FIXME: Not yet implemented
$eval\text{-}binop\ Or\quad v1\ v2 = Bool\ ((the\text{-}Bool\ v1) \vee (the\text{-}Bool\ v2))$
$eval\text{-}binop\ CondAnd\ v1\ v2 = Bool\ ((the\text{-}Bool\ v1) \wedge (the\text{-}Bool\ v2))$
$eval\text{-}binop\ CondOr\ \ v1\ v2 = Bool\ ((the\text{-}Bool\ v1) \vee (the\text{-}Bool\ v2))$

constdefs $need\text{-}second\text{-}arg :: binop \Rightarrow val \Rightarrow bool$
$need\text{-}second\text{-}arg\ binop\ v1 \equiv \neg\ ((binop{=}CondAnd \wedge\ \neg\ the\text{-}Bool\ v1) \vee$
$(binop{=}CondOr\ \ \wedge\ the\text{-}Bool\ v1))$

CondAnd and *CondOr* only evalulate the second argument if the value isn't already determined by the first argument

lemma $need\text{-}second\text{-}arg\text{-}CondAnd\ [simp]:\ need\text{-}second\text{-}arg\ CondAnd\ (Bool\ b) = b$
by $(simp\ add:\ need\text{-}second\text{-}arg\text{-}def)$

lemma $need\text{-}second\text{-}arg\text{-}CondOr\ [simp]:\ need\text{-}second\text{-}arg\ CondOr\ (Bool\ b) = (\neg\ b)$
by $(simp\ add:\ need\text{-}second\text{-}arg\text{-}def)$

lemma $need\text{-}second\text{-}arg\text{-}strict[simp]$:
$[\![binop \neq CondAnd;\ binop \neq CondOr]\!] \Longrightarrow need\text{-}second\text{-}arg\ binop\ b$
by $(cases\ binop)$
$(simp\text{-}all\ add:\ need\text{-}second\text{-}arg\text{-}def)$
end

Chapter 8

Decl

7 Field, method, interface, and class declarations, whole Java programs

theory *Decl* **imports** *Term Table* **begin**

improvements:

- clarification and correction of some aspects of the package/access concept (Also submitted as bug report to the Java Bug Database: Bug Id: 4485402 and Bug Id: 4493343 http://developer.java.su
)

simplifications:

- the only field and method modifiers are static and the access modifiers
- no constructors, which may be simulated by new + suitable methods
- there is just one global initializer per class, which can simulate all others
- no throws clause
- a void method is replaced by one that returns Unit (of dummy type Void)
- no interface fields
- every class has an explicit superclass (unused for Object)
- the (standard) methods of Object and of standard exceptions are not specified
- no main method

8 Modifier

Access modifier

datatype *acc-modi*
  $= Private \mid Package \mid Protected \mid Public$

We can define a linear order for the access modifiers. With Private yielding the most restrictive access and public the most liberal access policy: Private ¡ Package ¡ Protected ¡ Public

instance *acc-modi*:: *ord* **..**

defs (**overloaded**)
less-acc-def:
$a < (b{::}acc\text{-}modi)$
  $\equiv (case\ a\ of$
    $Private \Rightarrow (b{=}Package \lor b{=}Protected \lor b{=}Public)$
   $\mid Package \Rightarrow (b{=}Protected \lor b{=}Public)$
   $\mid Protected \Rightarrow (b{=}Public)$
   $\mid Public \Rightarrow False)$
le-acc-def:
$a \le (b{::}acc\text{-}modi) \equiv (a = b) \lor (a < b)$

instance *acc-modi*:: *order*
proof
 fix $x\ y\ z{::}acc\text{-}modi$
 {
 show $x \le x$ — reflexivity
  by (*auto simp add*: *le-acc-def*)
 next

assume $x \le y$ $y \le z$ — transitivity
thus $x \le z$
by (*auto simp add*: *le-acc-def less-acc-def split add*: *acc-modi.split*)
next
assume $x \le y$ $y \le x$ — antisymmetry
thus $x = y$
proof −
have $\forall$ x y. $x < (y$::*acc-modi*$) \wedge y < x \longrightarrow$ *False*
by (*auto simp add*: *less-acc-def split add*: *acc-modi.split*)
with *prems* **show** *?thesis*
by (*unfold le-acc-def*) *iprover*
qed
next
show $(x < y) = (x \le y \wedge x \neq y)$
by (*auto simp add*: *le-acc-def less-acc-def split add*: *acc-modi.split*)
}
qed

instance *acc-modi*:: *linorder*
proof
fix *x y*:: *acc-modi*
show $x \le y \vee y \le x$
by (*auto simp add*: *less-acc-def le-acc-def split add*: *acc-modi.split*)
qed

lemma *acc-modi-top* [*simp*]: *Public* $\le$ *a* $\Longrightarrow$ *a* = *Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-top1* [*simp*, *intro*!]: *a* $\le$ *Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-le-Public*:
a $\le$ *Public* $\Longrightarrow$ *a*=*Private* $\vee$ *a* = *Package* $\vee$ *a*=*Protected* $\vee$ *a*=*Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-bottom*: *a* $\le$ *Private* $\Longrightarrow$ *a* = *Private*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-Private-le*:
Private $\le$ *a* $\Longrightarrow$ *a*=*Private* $\vee$ *a* = *Package* $\vee$ *a*=*Protected* $\vee$ *a*=*Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-Package-le*:
Package $\le$ *a* $\Longrightarrow$ *a* = *Package* $\vee$ *a*=*Protected* $\vee$ *a*=*Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.split*)

lemma *acc-modi-le-Package*:
a $\le$ *Package* $\Longrightarrow$ *a*=*Private* $\vee$ *a* = *Package*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-Protected-le*:

Protected $\leq$ *a* $\Longrightarrow$ *a*=*Protected* $\vee$ *a*=*Public*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemma *acc-modi-le-Protected*:
a $\leq$ *Protected* $\Longrightarrow$ *a*=*Private* $\vee$ *a* = *Package* $\vee$ *a* = *Protected*
by (*auto simp add*: *le-acc-def less-acc-def split*: *acc-modi.splits*)

lemmas *acc-modi-le-Dests* = *acc-modi-top* *acc-modi-le-Public*
acc-modi-Private-le *acc-modi-bottom*
acc-modi-Package-le *acc-modi-le-Package*
acc-modi-Protected-le *acc-modi-le-Protected*

lemma *acc-modi-Package-le-cases*
[*consumes 1*,*case-names Package Protected Public*]:
Package $\leq$ *m* $\Longrightarrow$ (*m* = *Package* $\Longrightarrow$ *P m*) $\Longrightarrow$ (*m*=*Protected* $\Longrightarrow$ *P m*) $\Longrightarrow$
(*m*=*Public* $\Longrightarrow$ *P m*) $\Longrightarrow$ *P m*
by (*auto dest*: *acc-modi-Package-le*)

Static Modifier

types *stat-modi* = *bool*

9 Declaration (base "class" for member,interface and class declarations

record *decl* =
access :: *acc-modi*

translations
decl <= (*type*) (|*access*::*acc-modi*|)
decl <= (*type*) (|*access*::*acc-modi*,...::'*a*|)

10 Member (field or method)

record *member* = *decl* +
static :: *stat-modi*

translations
member <= (*type*) (|*access*::*acc-modi*,*static*::*bool*|)
member <= (*type*) (|*access*::*acc-modi*,*static*::*bool*,...::'*a*|)

11 Field

record *field* = *member* +
type :: *ty*
translations
field <= (*type*) (|*access*::*acc-modi*, *static*::*bool*, *type*::*ty*|)
field <= (*type*) (|*access*::*acc-modi*, *static*::*bool*, *type*::*ty*,...::'*a*|)

types
fdecl
= *vname* × *field*

translations
fdecl <= (*type*) *vname* × *field*

12 Method

record *mhead* = *member* +
  pars ::*vname list*
  resT ::*ty*

record *mbody* =
  lcls:: (*vname* × *ty*) *list*
  stmt:: *stmt*

record *methd* = *mhead* +
  mbody::*mbody*

types *mdecl* = *sig* × *methd*

translations
mhead <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
  pars::*vname list*, *resT*::*ty*|)
mhead <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
  pars::*vname list*, *resT*::*ty*,...::'*a*|)
mbody <= (*type*) (|*lcls*::(*vname* × *ty*) *list*,*stmt*::*stmt*|)
mbody <= (*type*) (|*lcls*::(*vname* × *ty*) *list*,*stmt*::*stmt*,...::'*a*|)
methd <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
  pars::*vname list*, *resT*::*ty*,*mbody*::*mbody*|)
methd <= (*type*) (|*access*::*acc-modi*, *static*::*bool*,
  pars::*vname list*, *resT*::*ty*,*mbody*::*mbody*,...::'*a*|)
mdecl <= (*type*) *sig* × *methd*

constdefs
mhead::*methd* ⇒ *mhead*
mhead m ≡ (|*access*=*access m*, *static*=*static m*, *pars*=*pars m*, *resT*=*resT m*|)

lemma *access-mhead* [*simp*]:*access* (*mhead m*) = *access m*
by (*simp add*: *mhead-def*)

lemma *static-mhead* [*simp*]:*static* (*mhead m*) = *static m*
by (*simp add*: *mhead-def*)

lemma *pars-mhead* [*simp*]:*pars* (*mhead m*) = *pars m*
by (*simp add*: *mhead-def*)

lemma *resT-mhead* [*simp*]:*resT* (*mhead m*) = *resT m*
by (*simp add*: *mhead-def*)

To be able to talk uniformaly about field and method declarations we introduce the notion of a member declaration (e.g. useful to define accessiblity)

datatype *memberdecl* = *fdecl fdecl* | *mdecl mdecl*

datatype *memberid* = *fid vname* | *mid sig*

axclass *has-memberid* < *type*
consts
memberid :: '*a*::*has-memberid* ⇒ *memberid*

instance *memberdecl*::*has-memberid* **..**

defs (**overloaded**)
memberdecl-memberid-def:
  memberid m ≡ (*case m of*
      fdecl (*vn*,*f*) ⇒ *fid vn*
    | *mdecl* (*sig*,*m*) ⇒ *mid sig*)

lemma *memberid-fdecl-simp*[*simp*]: *memberid* (*fdecl* (*vn*,*f*)) = *fid vn*
by (*simp add*: *memberdecl-memberid-def*)

lemma *memberid-fdecl-simp1*: *memberid* (*fdecl f*) = *fid* (*fst f*)
by (*cases f*) (*simp add*: *memberdecl-memberid-def*)

lemma *memberid-mdecl-simp*[*simp*]: *memberid* (*mdecl* (*sig*,*m*)) = *mid sig*
by (*simp add*: *memberdecl-memberid-def*)

lemma *memberid-mdecl-simp1*: *memberid* (*mdecl m*) = *mid* (*fst m*)
by (*cases m*) (*simp add*: *memberdecl-memberid-def*)

instance * :: (*type*, *has-memberid*) *has-memberid* **..**

defs (**overloaded**)
pair-memberid-def:
  memberid p ≡ *memberid* (*snd p*)

lemma *memberid-pair-simp*[*simp*]: *memberid* (*c*,*m*) = *memberid m*
by (*simp add*: *pair-memberid-def*)

lemma *memberid-pair-simp1*: *memberid p* = *memberid* (*snd p*)
by (*simp add*: *pair-memberid-def*)

constdefs *is-field* :: *qtname* × *memberdecl* ⇒ *bool*
is-field m ≡ ∃ *declC f*. *m*=(*declC*,*fdecl f*)

lemma *is-fieldD*: *is-field m* ⟹ ∃ *declC f*. *m*=(*declC*,*fdecl f*)
by (*simp add*: *is-field-def*)

lemma *is-fieldI*: *is-field* (*C*,*fdecl f*)
by (*simp add*: *is-field-def*)

constdefs *is-method* :: *qtname* × *memberdecl* ⇒ *bool*
is-method membr ≡ ∃ *declC m*. *membr*=(*declC*,*mdecl m*)

lemma *is-methodD*: *is-method membr* ⟹ ∃ *declC m*. *membr*=(*declC*,*mdecl m*)
by (*simp add*: *is-method-def*)

lemma *is-methodI*: *is-method* (*C*,*mdecl m*)

by (*simp add*: *is-method-def*)

13 Interface

record *ibody* = *decl* + — interface body
imethods :: (*sig* × *mhead*) *list* — method heads

record *iface* = *ibody* + — interface
isuperIfs:: *qtname list* — superinterface list
types
idecl — interface declaration, cf. 9.1
= *qtname* × *iface*

translations
ibody <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*|)
ibody <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*,...::'*a*|)
iface <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*,
isuperIfs::*qtname list*|)
iface <= (*type*) (|*access*::*acc-modi*,*imethods*::(*sig* × *mhead*) *list*,
isuperIfs::*qtname list*,...::'*a*|)
idecl <= (*type*) *qtname* × *iface*

constdefs
ibody :: *iface* ⇒ *ibody*
ibody i ≡ (|*access*=*access i*,*imethods*=*imethods i*|)

lemma *access-ibody* [*simp*]: (*access* (*ibody i*)) = *access i*
by (*simp add*: *ibody-def*)

lemma *imethods-ibody* [*simp*]: (*imethods* (*ibody i*)) = *imethods i*
by (*simp add*: *ibody-def*)

14 Class

record *cbody* = *decl* + — class body
cfields:: *fdecl list*
methods:: *mdecl list*
init :: *stmt* — initializer

record *class* = *cbody* + — class
super :: *qtname* — superclass
superIfs:: *qtname list* — implemented interfaces
types
cdecl — class declaration, cf. 8.1
= *qtname* × *class*

translations
cbody <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
methods::*mdecl list*,*init*::*stmt*|)
cbody <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
methods::*mdecl list*,*init*::*stmt*,...::'*a*|)
class <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
methods::*mdecl list*,*init*::*stmt*,
super::*qtname*,*superIfs*::*qtname list*|)
class <= (*type*) (|*access*::*acc-modi*,*cfields*::*fdecl list*,
methods::*mdecl list*,*init*::*stmt*,
super::*qtname*,*superIfs*::*qtname list*,...::'*a*|)

cdecl <= (*type*) *qtname* × *class*

constdefs
cbody :: *class* ⇒ *cbody*
cbody c ≡ (|*access*=*access c*, *cfields*=*cfields c*,*methods*=*methods c*,*init*=*init c*|)

lemma *access-cbody* [*simp*]:*access* (*cbody c*) = *access c*
by (*simp add*: *cbody-def*)

lemma *cfields-cbody* [*simp*]:*cfields* (*cbody c*) = *cfields c*
by (*simp add*: *cbody-def*)

lemma *methods-cbody* [*simp*]:*methods* (*cbody c*) = *methods c*
by (*simp add*: *cbody-def*)

lemma *init-cbody* [*simp*]:*init* (*cbody c*) = *init c*
by (*simp add*: *cbody-def*)

standard classes

consts

Object-mdecls :: *mdecl list* — methods of Object
SXcpt-mdecls :: *mdecl list* — methods of SXcpts
ObjectC :: *cdecl* — declaration of root class
SXcptC ::*xname* ⇒ *cdecl* — declarations of throwable classes

defs

ObjectC-def:*ObjectC* ≡ (*Object*,(|*access*=*Public*,*cfields*=[],*methods*=*Object-mdecls*,
init=*Skip*,*super*=*arbitrary*,*superIfs*=[]|))
SXcptC-def:*SXcptC xn*≡ (*SXcpt xn*,(|*access*=*Public*,*cfields*=[],*methods*=*SXcpt-mdecls*,
init=*Skip*,
super=*if xn* = *Throwable then Object*
else SXcpt Throwable,
superIfs=[]|))

lemma *ObjectC-neq-SXcptC* [*simp*]: *ObjectC* ≠ *SXcptC xn*
by (*simp add*: *ObjectC-def SXcptC-def Object-def SXcpt-def*)

lemma *SXcptC-inject* [*simp*]: (*SXcptC xn* = *SXcptC xm*) = (*xn* = *xm*)
by (*simp add*: *SXcptC-def*)

constdefs *standard-classes* :: *cdecl list*
standard-classes ≡ [*ObjectC*, *SXcptC Throwable*,
SXcptC NullPointer, *SXcptC OutOfMemory*, *SXcptC ClassCast*,
SXcptC NegArrSize , *SXcptC IndOutBound*, *SXcptC ArrStore*]

programs

record *prog* =
ifaces ::*idecl list*

classes::*cdecl list*

translations
prog<= (*type*) (|*ifaces*::*idecl list*,*classes*::*cdecl list*|)
prog<= (*type*) (|*ifaces*::*idecl list*,*classes*::*cdecl list*,...::'*a*|)

syntax
iface :: *prog* ⇒ (*qtname*, *iface*) *table*
class :: *prog* ⇒ (*qtname*, *class*) *table*
is-iface :: *prog* ⇒ *qtname* ⇒ *bool*
is-class :: *prog* ⇒ *qtname* ⇒ *bool*

translations
iface G I == *table-of* (*ifaces G*) *I*
class G C == *table-of* (*classes G*) *C*
is-iface G I == *iface G I* ≠ *None*
is-class G C == *class G C* ≠ *None*

is type

consts
is-type :: *prog* ⇒ *ty* ⇒ *bool*
isrtype :: *prog* ⇒ *ref-ty* ⇒ *bool*

primrec *is-type G* (*PrimT pt*) = *True*
is-type G (*RefT rt*) = *isrtype G rt*
isrtype G (*NullT*) = *True*
isrtype G (*IfaceT tn*) = *is-iface G tn*
isrtype G (*ClassT tn*) = *is-class G tn*
isrtype G (*ArrayT T*) = *is-type G T*

lemma *type-is-iface*: *is-type G* (*Iface I*) ⟹ *is-iface G I*
by *auto*

lemma *type-is-class*: *is-type G* (*Class C*) ⟹ *is-class G C*
by *auto*

subinterface and subclass relation, in anticipation of TypeRel.thy

consts
subint1 :: *prog* ⇒ (*qtname* × *qtname*) *set* — direct subinterface
subcls1 :: *prog* ⇒ (*qtname* × *qtname*) *set* — direct subclass

defs
subint1-def: *subint1 G* ≡ {(*I*,*J*). ∃ *i*∈*iface G I*: *J*∈*set* (*isuperIfs i*)}
subcls1-def: *subcls1 G* ≡ {(*C*,*D*). *C*≠*Object* ∧ (∃ *c*∈*class G C*: *super c* = *D*)}

syntax
-subcls1 :: *prog* => [*qtname*, *qtname*] => *bool* (-|−-<:*C1*- [*71*,*71*,*71*] *70*)
-subclseq:: *prog* => [*qtname*, *qtname*] => *bool* (-|−-<=:*C* -[*71*,*71*,*71*] *70*)
-subcls :: *prog* => [*qtname*, *qtname*] => *bool* (-|−-<:*C* -[*71*,*71*,*71*] *70*)

syntax (*xsymbols*)
-subcls1 :: *prog* ⇒ [*qtname*, *qtname*] ⇒ *bool* (-⊢-$\prec_{C1}$- [*71*,*71*,*71*] *70*)
-subclseq:: *prog* ⇒ [*qtname*, *qtname*] ⇒ *bool* (-⊢-$\preceq_{C}$ - [*71*,*71*,*71*] *70*)
-subcls :: *prog* ⇒ [*qtname*, *qtname*] ⇒ *bool* (-⊢-$\prec_{C}$ - [*71*,*71*,*71*] *70*)

translations

$G \vdash C \prec_{C1} D$ == $(C,D) \in$ *subcls1 G*
$G \vdash C \preceq_C D$ == $(C,D) \in$(*subcls1 G*)ˆ*
$G \vdash C \prec_C D$ == $(C,D) \in$(*subcls1 G*)ˆ+

lemma *subint1I*: ⟦*iface G I* = *Some i*; $J \in$ *set* (*isuperIfs i*)⟧
$\Longrightarrow (I,J) \in$ *subint1 G*
apply (*simp add*: *subint1-def*)
done

lemma *subcls1I*:⟦*class G C* = *Some c*; $C \neq$ *Object*⟧ $\Longrightarrow$ (*C*,(*super c*)) $\in$ *subcls1 G*
apply (*simp add*: *subcls1-def*)
done

lemma *subint1D*: $(I,J) \in$*subint1 G*$\Longrightarrow \exists\, i \in$*iface G I*: $J \in$*set* (*isuperIfs i*)
by (*simp add*: *subint1-def*)

lemma *subcls1D*:
$(C,D) \in$*subcls1 G* $\Longrightarrow$ $C \neq$*Object* $\wedge$ ($\exists\, c$. *class G C* = *Some c* $\wedge$ (*super c* = *D*))
apply (*simp add*: *subcls1-def*)
apply *auto*
done

lemma *subint1-def2*:
subint1 G = (*SIGMA I*: {*I*. *is-iface G I*}. *set* (*isuperIfs* (*the* (*iface G I*))))
apply (*unfold subint1-def*)
apply *auto*
done

lemma *subcls1-def2*:
subcls1 G =
(*SIGMA C*: {*C*. *is-class G C*}. {*D*. $C \neq$*Object* $\wedge$ *super* (*the*(*class G C*))=*D*})
apply (*unfold subcls1-def*)
apply *auto*
done

lemma *subcls-is-class*:
⟦$G \vdash C \prec_C D$⟧ $\Longrightarrow \exists$ *c*. *class G C* = *Some c*
by (*auto simp add*: *subcls1-def dest*: *tranclD*)

lemma *no-subcls1-Object*:$G \vdash$*Object*$\prec_{C1}$ $D \Longrightarrow P$
by (*auto simp add*: *subcls1-def*)

lemma *no-subcls-Object*: $G \vdash$*Object*$\prec_C$ $D \Longrightarrow P$
apply (*erule trancl-induct*)
apply (*auto intro*: *no-subcls1-Object*)
done

well-structured programs

constdefs
ws-idecl :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname list* $\Rightarrow$ *bool*
ws-idecl G I si $\equiv$ $\forall$ *J*$\in$*set si*. *is-iface G J* $\wedge$ (*J*,*I*)$\notin$(*subint1 G*)ˆ+

ws-cdecl :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname* $\Rightarrow$ *bool*
ws-cdecl G C sc $\equiv$ *C*$\neq$*Object* $\longrightarrow$ *is-class G sc* $\wedge$ (*sc*,*C*)$\notin$(*subcls1 G*)ˆ+

ws-prog :: *prog* $\Rightarrow$ *bool*
ws-prog G $\equiv$ ($\forall$(*I*,*i*)$\in$*set* (*ifaces G*). *ws-idecl G I* (*isuperIfs i*)) $\wedge$
($\forall$(*C*,*c*)$\in$*set* (*classes G*). *ws-cdecl G C* (*super c*))

lemma *ws-progI*:
⟦$\forall$(*I*,*i*)$\in$*set* (*ifaces G*). $\forall$ *J*$\in$*set* (*isuperIfs i*). *is-iface G J* $\wedge$
(*J*,*I*) $\notin$ (*subint1 G*)ˆ+;
$\forall$(*C*,*c*)$\in$*set* (*classes G*). *C*$\neq$*Object* $\longrightarrow$ *is-class G* (*super c*) $\wedge$
((*super c*),*C*) $\notin$ (*subcls1 G*)ˆ+
⟧ $\Longrightarrow$ *ws-prog G*
apply (*unfold ws-prog-def ws-idecl-def ws-cdecl-def*)
apply (*erule-tac conjI*)
apply *blast*
done

lemma *ws-prog-ideclD*:
⟦*iface G I* = *Some i*; *J*$\in$*set* (*isuperIfs i*); *ws-prog G*⟧ $\Longrightarrow$
is-iface G J $\wedge$ (*J*,*I*)$\notin$(*subint1 G*)ˆ+
apply (*unfold ws-prog-def ws-idecl-def*)
apply *clarify*
apply (*drule-tac map-of-SomeD*)
apply *auto*
done

lemma *ws-prog-cdeclD*:
⟦*class G C* = *Some c*; *C*$\neq$*Object*; *ws-prog G*⟧ $\Longrightarrow$
is-class G (*super c*) $\wedge$ (*super c*,*C*)$\notin$(*subcls1 G*)ˆ+
apply (*unfold ws-prog-def ws-cdecl-def*)
apply *clarify*
apply (*drule-tac map-of-SomeD*)
apply *auto*
done

well-foundedness

lemma *finite-is-iface*: *finite* {*I*. *is-iface G I*}
apply (*fold dom-def*)
apply (*rule-tac finite-dom-map-of*)
done

lemma *finite-is-class*: *finite* {*C*. *is-class G C*}
apply (*fold dom-def*)
apply (*rule-tac finite-dom-map-of*)
done

lemma *finite-subint1*: *finite* (*subint1 G*)
apply (*subst subint1-def2*)
apply (*rule finite-SigmaI*)
apply (*rule finite-is-iface*)
apply (*simp* (*no-asm*))
done

lemma *finite-subcls1*: *finite* (*subcls1 G*)
apply (*subst subcls1-def2*)
apply (*rule finite-SigmaI*)
apply (*rule finite-is-class*)
apply (*rule-tac B* = {*super* (*the* (*class G C*))} **in** *finite-subset*)
apply *auto*
done

lemma *subint1-irrefl-lemma1*:
ws-prog G $\Longrightarrow$ (*subint1 G*) ^−*1* $\cap$ (*subint1 G*) ^+ = {}
apply (*force dest*: *subint1D ws-prog-ideclD conjunct2*)
done

lemma *subcls1-irrefl-lemma1*:
ws-prog G $\Longrightarrow$ (*subcls1 G*) ^−*1* $\cap$ (*subcls1 G*) ^+ = {}
apply (*force dest*: *subcls1D ws-prog-cdeclD conjunct2*)
done

lemmas *subint1-irrefl-lemma2* = *subint1-irrefl-lemma1* [*THEN irrefl-tranclI'*]
lemmas *subcls1-irrefl-lemma2* = *subcls1-irrefl-lemma1* [*THEN irrefl-tranclI'*]

lemma *subint1-irrefl*: $[\![(x,\ y) \in$ *subint1 G*; *ws-prog G*$]\!] \Longrightarrow x \neq y$
apply (*rule irrefl-trancl-rD*)
apply (*rule subint1-irrefl-lemma2*)
apply *auto*
done

lemma *subcls1-irrefl*: $[\![(x,\ y) \in$ *subcls1 G*; *ws-prog G*$]\!] \Longrightarrow x \neq y$
apply (*rule irrefl-trancl-rD*)
apply (*rule subcls1-irrefl-lemma2*)
apply *auto*
done

lemmas *subint1-acyclic* = *subint1-irrefl-lemma2* [*THEN acyclicI*, *standard*]
lemmas *subcls1-acyclic* = *subcls1-irrefl-lemma2* [*THEN acyclicI*, *standard*]

lemma *wf-subint1*: *ws-prog G* $\Longrightarrow$ *wf* ((*subint1 G*)$^{-1}$)
by (*auto intro*: *finite-acyclic-wf-converse finite-subint1 subint1-acyclic*)

lemma *wf-subcls1*: *ws-prog G* $\Longrightarrow$ *wf* ((*subcls1 G*)$^{-1}$)
by (*auto intro*: *finite-acyclic-wf-converse finite-subcls1 subcls1-acyclic*)

lemma *subint1-induct*:
$[\![ws\text{-}prog\ G;\ \bigwedge x.\ \forall y.\ (x,\ y) \in subint1\ G \longrightarrow P\ y \Longrightarrow P\ x]\!] \Longrightarrow P\ a$
apply (*frule wf-subint1*)
apply (*erule wf-induct*)
apply (*simp* (*no-asm-use*) *only*: *converse-iff*)
apply *blast*
done

lemma *subcls1-induct* [*consumes 1*]:
$[\![ws\text{-}prog\ G;\ \bigwedge x.\ \forall y.\ (x,\ y) \in subcls1\ G \longrightarrow P\ y \Longrightarrow P\ x]\!] \Longrightarrow P\ a$
apply (*frule wf-subcls1*)
apply (*erule wf-induct*)
apply (*simp* (*no-asm-use*) *only*: *converse-iff*)
apply *blast*
done

lemma *ws-subint1-induct*:
$[\![is\text{-}iface\ G\ I;\ ws\text{-}prog\ G;\ \bigwedge I\ i.\ [\![iface\ G\ I = Some\ i \wedge$
$(\forall J \in set\ (isuperIfs\ i).\ (I,J) \in subint1\ G \wedge P\ J \wedge is\text{-}iface\ G\ J)]\!] \Longrightarrow P\ I$
$]\!] \Longrightarrow P\ I$
apply (*erule rev-mp*)
apply (*rule subint1-induct*)
apply *assumption*
apply (*simp* (*no-asm*))
apply *safe*
apply (*blast dest*: *subint1I ws-prog-ideclD*)
done

lemma *ws-subcls1-induct*: $[\![is\text{-}class\ G\ C;\ ws\text{-}prog\ G;$
$\bigwedge C\ c.\ [\![class\ G\ C = Some\ c;$
$(C \neq Object \longrightarrow (C,(super\ c)) \in subcls1\ G \wedge$
$P\ (super\ c) \wedge is\text{-}class\ G\ (super\ c))]\!] \Longrightarrow P\ C$
$]\!] \Longrightarrow P\ C$
apply (*erule rev-mp*)
apply (*rule subcls1-induct*)
apply *assumption*
apply (*simp* (*no-asm*))
apply *safe*
apply (*fast dest*: *subcls1I ws-prog-cdeclD*)
done

lemma *ws-class-induct* [*consumes 2*, *case-names Object Subcls*]:
$[\![class\ G\ C = Some\ c;\ ws\text{-}prog\ G;$
$\bigwedge co.\ class\ G\ Object = Some\ co \Longrightarrow P\ Object;$
$\bigwedge C\ c.\ [\![class\ G\ C = Some\ c;\ C \neq Object;\ P\ (super\ c)]\!] \Longrightarrow P\ C$
$]\!] \Longrightarrow P\ C$
proof −
  assume *clsC*: $class\ G\ C = Some\ c$
  and *init*: $\bigwedge co.\ class\ G\ Object = Some\ co \Longrightarrow P\ Object$
  and *step*: $\bigwedge C\ c.\ [\![class\ G\ C = Some\ c;\ C \neq Object;\ P\ (super\ c)]\!] \Longrightarrow P\ C$
  assume *ws*: $ws\text{-}prog\ G$
  then have $is\text{-}class\ G\ C \Longrightarrow P\ C$
  proof (*induct rule*: *subcls1-induct*)

fix C
assume $hyp{:}\forall\ S.\ G{\vdash}C \prec_{C1} S \longrightarrow \mathit{is\text{-}class}\ G\ S \longrightarrow P\ S$
and $\mathit{iscls}{:}\mathit{is\text{-}class}\ G\ C$
show $P\ C$
proof $(\mathit{cases}\ C{=}\mathit{Object})$
case *True* **with** *iscls init* **show** $P\ C$ **by** *auto*
next
case *False* **with** *ws step hyp iscls*
show $P\ C$ **by** (*auto dest*: *subcls1I ws-prog-cdeclD*)
qed
qed
with *clsC* **show** *?thesis* **by** *simp*
qed

lemma *ws-class-induct′* [*consumes 2*, *case-names Object Subcls*]:
$[\![\mathit{is\text{-}class}\ G\ C;\ \mathit{ws\text{-}prog}\ G;$
$\bigwedge co.\ \mathit{class}\ G\ \mathit{Object} = \mathit{Some}\ co \Longrightarrow P\ \mathit{Object};$
$\bigwedge C\ c.\ [\![\mathit{class}\ G\ C = \mathit{Some}\ c;\ C \neq \mathit{Object};\ P\ (\mathit{super}\ c)]\!] \Longrightarrow P\ C$
$]\!] \Longrightarrow P\ C$
by (*auto intro*: *ws-class-induct*)

lemma *ws-class-induct″* [*consumes 2*, *case-names Object Subcls*]:
$[\![\mathit{class}\ G\ C = \mathit{Some}\ c;\ \mathit{ws\text{-}prog}\ G;$
$\bigwedge co.\ \mathit{class}\ G\ \mathit{Object} = \mathit{Some}\ co \Longrightarrow P\ \mathit{Object}\ co;$
$\bigwedge\ C\ c\ sc.\ [\![\mathit{class}\ G\ C = \mathit{Some}\ c;\ \mathit{class}\ G\ (\mathit{super}\ c) = \mathit{Some}\ sc;$
$C \neq \mathit{Object};\ P\ (\mathit{super}\ c)\ sc]\!] \Longrightarrow P\ C\ c$
$]\!] \Longrightarrow P\ C\ c$
proof −
assume clsC: $\mathit{class}\ G\ C = \mathit{Some}\ c$
and init: $\bigwedge co.\ \mathit{class}\ G\ \mathit{Object} = \mathit{Some}\ co \Longrightarrow P\ \mathit{Object}\ co$
and step: $\bigwedge\ C\ c\ sc\ .\ [\![\mathit{class}\ G\ C = \mathit{Some}\ c;\ \mathit{class}\ G\ (\mathit{super}\ c) = \mathit{Some}\ sc;$
$C \neq \mathit{Object};\ P\ (\mathit{super}\ c)\ sc]\!] \Longrightarrow P\ C\ c$
assume *ws*: *ws-prog G*
then have $\bigwedge\ c.\ \mathit{class}\ G\ C = \mathit{Some}\ c \Longrightarrow P\ C\ c$
proof (*induct rule*: *subcls1-induct*)
fix $C\ c$
assume $hyp{:}\forall\ S.\ G{\vdash}C \prec_{C1} S \longrightarrow (\forall\ s.\ \mathit{class}\ G\ S = \mathit{Some}\ s \longrightarrow P\ S\ s)$
and $\mathit{iscls}{:}\mathit{class}\ G\ C = \mathit{Some}\ c$
show $P\ C\ c$
proof $(\mathit{cases}\ C{=}\mathit{Object})$
case *True* **with** *iscls init* **show** $P\ C\ c$ **by** *auto*
next
case *False*
with *ws iscls* **obtain** *sc* **where**
sc: $\mathit{class}\ G\ (\mathit{super}\ c) = \mathit{Some}\ sc$
by (*auto dest*: *ws-prog-cdeclD*)
from *iscls False* **have** $G{\vdash}C \prec_{C1} (\mathit{super}\ c)$ **by** (*rule subcls1I*)
with *False ws step hyp iscls sc*
show $P\ C\ c$
by (*auto*)
qed
qed
with *clsC* **show** $P\ C\ c$ **by** *auto*
qed

lemma *ws-interface-induct* [*consumes 2*, *case-names Step*]:

assumes *is-if-I*: *is-iface G I* **and**
ws: *ws-prog G* **and**
hyp-sub: $\bigwedge I\ i.\ [\![$*iface G I = Some i*;
$\forall\ J \in$ *set* (*isuperIfs i*).
$(I,J) \in$ *subint1 G* $\wedge$ *P J* $\wedge$ *is-iface G J*$]\!] \Longrightarrow$ *P I*
shows *P I*
proof −
from *is-if-I ws*
show *P I*
proof (*rule ws-subint1-induct*)
fix *I i*
assume *hyp*: *iface G I = Some i* $\wedge$
($\forall$ *J*$\in$*set* (*isuperIfs i*). (*I*,*J*) $\in$*subint1 G* $\wedge$ *P J* $\wedge$ *is-iface G J*)
then have *if-I*: *iface G I = Some i*
by *blast*
show *P I*
proof (*cases isuperIfs i*)
case *Nil*
with *if-I hyp-sub*
show *P I*
by *auto*
next
case (*Cons hd tl*)
with *hyp if-I hyp-sub*
show *P I*
by *auto*
qed
qed
qed

general recursion operators for the interface and class hiearchies

consts
iface-rec :: *prog* × *qtname* ⇒ (*qtname* ⇒ *iface* ⇒ ′*a set* ⇒ ′*a*) ⇒ ′*a*
class-rec :: *prog* × *qtname* ⇒ ′*a* ⇒ (*qtname* ⇒ *class* ⇒ ′*a* ⇒ ′*a*) ⇒ ′*a*

recdef *iface-rec same-fst ws-prog* (λ*G*. (*subint1 G*)ˆ−*1*)
iface-rec (*G*,*I*) =
(λ*f*. *case iface G I of*
None ⇒ *arbitrary*
| *Some i* ⇒ *if ws-prog G*
then f I i
((λ*J*. *iface-rec* (*G*,*J*) *f*)‘*set* (*isuperIfs i*))
else arbitrary)
(**hints** *recdef-wf*: *wf-subint1 intro*: *subint1I*)
declare *iface-rec.simps* [*simp del*]

lemma *iface-rec*:
$[\![$*iface G I = Some i*; *ws-prog G*$]\!] \Longrightarrow$
iface-rec (*G*,*I*) *f = f I i* ((λ*J*. *iface-rec* (*G*,*J*) *f*)‘*set* (*isuperIfs i*))
apply (*subst iface-rec.simps*)
apply *simp*
done

recdef *class-rec same-fst ws-prog* (λ*G*. (*subcls1 G*)ˆ−*1*)
class-rec(*G*,*C*) =
(λ*t f*. *case class G C of*
None ⇒ *arbitrary*

$\quad | \ \textit{Some } c \Rightarrow \textit{if ws-prog } G$
$\qquad\qquad \textit{then } f\ C\ c$
$\qquad\qquad\qquad (\textit{if } C = \textit{Object then } t$
$\qquad\qquad\qquad\qquad \textit{else class-rec } (G,\textit{super } c)\ t\ f)$
$\qquad\qquad \textit{else arbitrary})$
(**hints** *recdef-wf*: *wf-subcls1 intro*: *subcls1I*)
declare *class-rec.simps* [*simp del*]

lemma *class-rec*: $[\![\textit{class } G\ C = \textit{Some } c;\ \textit{ws-prog } G]\!] \Longrightarrow$
$\textit{class-rec } (G,C)\ t\ f =$
$f\ C\ c\ (\textit{if } C = \textit{Object then } t \textit{ else class-rec } (G,\textit{super } c)\ t\ f)$
apply (*rule class-rec.simps* [*THEN trans* [*THEN fun-cong* [*THEN fun-cong*]]])
apply *simp*
done

constdefs
$\textit{imethds}{::}\ \textit{prog} \Rightarrow \textit{qtname} \Rightarrow (\textit{sig},\textit{qtname} \times \textit{mhead})\ \textit{tables}$
— methods of an interface, with overriding and inheritance, cf. 9.2
imethds G I
$\equiv \textit{iface-rec } (G,I)$
$\qquad (\lambda I\ i\ ts.\ (\textit{Un-tables } ts) \oplus\oplus$
$\qquad\qquad (\textit{o2s} \circ \textit{table-of } (\textit{map } (\lambda(s,m).\ (s,I,m))\ (\textit{imethods } i))))$

end

Chapter 9

TypeRel

15 The relations between Java types

theory *TypeRel* **imports** *Decl* **begin**

simplifications:

- subinterface, subclass and widening relation includes identity

improvements over Java Specification 1.0:

- narrowing reference conversion also in cases where the return types of a pair of methods common to both types are in widening (rather identity) relation
- one could add similar constraints also for other cases

design issues:

- the type relations do not require *is-type* for their arguments
- the subint1 and subcls1 relations imply *is-iface*/*is-class* for their first arguments, which is required for their finiteness

consts

implmt1 :: *prog* ⇒ (*qtname* × *qtname*) *set* — direct implementation

syntax

-subint1 :: *prog* => [*qtname*, *qtname*] => *bool* (-|−-<:I1- [71,71,71] 70)
-subint :: *prog* => [*qtname*, *qtname*] => *bool* (-|−-<=:I -[71,71,71] 70)

@*implmt1* :: *prog* => [*qtname*, *qtname*] => *bool* (-|−-~>1- [71,71,71] 70)

syntax (*xsymbols*)

-subint1 :: *prog* ⇒ [*qtname*, *qtname*] ⇒ *bool* (-⊢-≺I1- [71,71,71] 70)
-subint :: *prog* ⇒ [*qtname*, *qtname*] ⇒ *bool* (-⊢-≼I - [71,71,71] 70)

-implmt1 :: *prog* ⇒ [*qtname*, *qtname*] ⇒ *bool* (-⊢-⇝1- [71,71,71] 70)

translations

$G{\vdash}I \prec I1\ J == (I,J) \in subint1\ G$
$G{\vdash}I \preceq I\ \ J == (I,J) \in (subint1\ G)\hat{}*$ — cf. 9.1.3

$G{\vdash}C \leadsto 1\ I == (C,I) \in implmt1\ G$

subclass and subinterface relations

lemmas *subcls-direct* = *subcls1I* [*THEN r-into-rtrancl*, *standard*]

lemma *subcls-direct1*:
$[\![class\ G\ C = Some\ c;\ C \neq Object; D=super\ c]\!] \Longrightarrow G{\vdash}C{\preceq_C}\ D$

apply (*auto dest*: *subcls-direct*)
done

lemma *subcls1I1*:
$[\![class\ G\ C = Some\ c;\ C \neq Object;D{=}super\ c]\!] \Longrightarrow G{\vdash}C{\prec}_{C1}\ D$
apply (*auto dest*: *subcls1I*)
done

lemma *subcls-direct2*:
$[\![class\ G\ C = Some\ c;\ C \neq Object;D{=}super\ c]\!] \Longrightarrow G{\vdash}C{\prec}_C\ D$
apply (*auto dest*: *subcls1I1*)
done

lemma *subclseq-trans*: $[\![G{\vdash}A \preceq_C B;\ G{\vdash}B \preceq_C C]\!] \Longrightarrow G{\vdash}A \preceq_C C$
by (*blast intro*: *rtrancl-trans*)

lemma *subcls-trans*: $[\![G{\vdash}A \prec_C B;\ G{\vdash}B \prec_C C]\!] \Longrightarrow G{\vdash}A \prec_C C$
by (*blast intro*: *trancl-trans*)

lemma *SXcpt-subcls-Throwable-lemma*:
$[\![class\ G\ (SXcpt\ xn) = Some\ xc;$
$super\ xc = (if\ xn = Throwable\ then\ Object\ else\ \ SXcpt\ Throwable)]\!]$
$\Longrightarrow G{\vdash}SXcpt\ xn{\preceq_C}\ SXcpt\ Throwable$
apply (*case-tac xn* = *Throwable*)
apply *simp-all*
apply (*drule subcls-direct*)
apply (*auto dest*: *sym*)
done

lemma *subcls-ObjectI*: $[\![is\text{-}class\ G\ C;\ ws\text{-}prog\ G]\!] \Longrightarrow G{\vdash}C{\preceq_C}\ Object$
apply (*erule ws-subcls1-induct*)
apply *clarsimp*
apply (*case-tac C* = *Object*)
apply (*fast intro*: *r-into-rtrancl* [*THEN rtrancl-trans*])+
done

lemma *subclseq-ObjectD* [*dest*!]: $G{\vdash}Object{\preceq_C}\ C \Longrightarrow C = Object$
apply (*erule rtrancl-induct*)
apply (*auto dest*: *subcls1D*)
done

lemma *subcls-ObjectD* [*dest*!]: $G{\vdash}Object{\prec_C}\ C \Longrightarrow False$
apply (*erule trancl-induct*)
apply (*auto dest*: *subcls1D*)
done

lemma *subcls-ObjectI1* [*intro*!]:
$[\![C \neq Object;is\text{-}class\ G\ C;ws\text{-}prog\ G]\!] \Longrightarrow G{\vdash}C \prec_C Object$
apply (*drule* (*1*) *subcls-ObjectI*)
apply (*auto intro*: *rtrancl-into-trancl3*)

done

lemma *subcls-is-class*: $(C,D) \in (subcls1\ G)\hat{}+ \Longrightarrow is\text{-}class\ G\ C$
apply (*erule trancl-trans-induct*)
apply (*auto dest*!: *subcls1D*)
done

lemma *subcls-is-class2* [*rule-format* (*no-asm*)]:
$G \vdash C \preceq_C D \Longrightarrow is\text{-}class\ G\ D \longrightarrow is\text{-}class\ G\ C$
apply (*erule rtrancl-induct*)
apply (*drule-tac* [*2*] *subcls1D*)
apply *auto*
done

lemma *single-inheritance*:
$[\![G \vdash A \prec_{C1} B;\ G \vdash A \prec_{C1} C]\!] \Longrightarrow B = C$
by (*auto simp add*: *subcls1-def*)

lemma *subcls-compareable*:
$[\![G \vdash A \preceq_C X;\ G \vdash A \preceq_C Y$
$]\!] \Longrightarrow G \vdash X \preceq_C Y \vee G \vdash Y \preceq_C X$
by (*rule triangle-lemma*) (*auto intro*: *single-inheritance*)

lemma *subcls1-irrefl*: $[\![G \vdash C \prec_{C1} D;\ ws\text{-}prog\ G\ ]\!]$
$\Longrightarrow C \neq D$
proof
  assume *ws*: *ws-prog G* **and**
    subcls1: $G \vdash C \prec_{C1} D$ **and**
     eq-C-D: $C=D$
  from *subcls1* **obtain** *c*
    where
      neq-C-Object: $C \neq Object$ **and**
              clsC: *class G C* = *Some c* **and**
          super-c: *super c* = *D*
    by (*auto simp add*: *subcls1-def*)
  with *super-c subcls1 eq-C-D*
  have *subcls-super-c-C*: $G \vdash super\ c \prec_C C$
    by *auto*
  from *ws clsC neq-C-Object*
  have $\neg\ G \vdash super\ c \prec_C C$
    by (*auto dest*: *ws-prog-cdeclD*)
  from *this subcls-super-c-C*
  show *False*
    by (*rule notE*)
qed

lemma *no-subcls-Object*: $G \vdash C \prec_C D \Longrightarrow C \neq Object$
by (*erule converse-trancl-induct*) (*auto dest*: *subcls1D*)

lemma *subcls-acyclic*: $[\![G \vdash C \prec_C D;\ ws\text{-}prog\ G]\!] \Longrightarrow \neg\ G \vdash D \prec_C C$
proof −
  assume *ws*: *ws-prog G*

assume *subcls-C-D*: $G \vdash C \prec_C D$
then show *?thesis*
proof (*induct rule*: *converse-trancl-induct*)
fix C
assume *subcls1-C-D*: $G \vdash C \prec_{C1} D$
then obtain c **where**
$C \neq Object$ **and**
$class\ G\ C = Some\ c$ **and**
$super\ c = D$
by (*auto simp add*: *subcls1-def*)
with *ws*
show $\neg\ G \vdash D \prec_C C$
by (*auto dest*: *ws-prog-cdeclD*)
next
fix $C\ Z$
assume *subcls1-C-Z*: $G \vdash C \prec_{C1} Z$ **and**
subcls-Z-D: $G \vdash Z \prec_C D$ **and**
nsubcls-D-Z: $\neg\ G \vdash D \prec_C Z$
show $\neg\ G \vdash D \prec_C C$
proof
assume *subcls-D-C*: $G \vdash D \prec_C C$
show *False*
proof −
from *subcls-D-C subcls1-C-Z*
have $G \vdash D \prec_C Z$
by (*auto dest*: *r-into-trancl trancl-trans*)
with *nsubcls-D-Z*
show *?thesis*
by (*rule notE*)
qed
qed
qed
qed

lemma *subclseq-cases* [*consumes 1*, *case-names Eq Subcls*]:
$[\![G \vdash C \preceq_C D;\ C = D \Longrightarrow P;\ G \vdash C \prec_C D \Longrightarrow P]\!] \Longrightarrow P$
by (*blast intro*: *rtrancl-cases*)

lemma *subclseq-acyclic*:
$[\![G \vdash C \preceq_C D;\ G \vdash D \preceq_C C;\ ws\text{-}prog\ G]\!] \Longrightarrow C = D$
by (*auto elim*: *subclseq-cases dest*: *subcls-acyclic*)

lemma *subcls-irrefl*: $[\![G \vdash C \prec_C D;\ ws\text{-}prog\ G]\!]$
$\Longrightarrow C \neq D$
proof −
assume *ws*: *ws-prog G*
assume *subcls*: $G \vdash C \prec_C D$
then show *?thesis*
proof (*induct rule*: *converse-trancl-induct*)
fix C
assume $G \vdash C \prec_{C1} D$
with *ws*
show $C \neq D$
by (*blast dest*: *subcls1-irrefl*)
next
fix $C\ Z$

assume *subcls1-C-Z*: $G\vdash C \prec_{C1}\ Z$ **and**
subcls-Z-D: $G\vdash Z \prec_C\ D$ **and**
neq-Z-D: $Z \neq D$
show $C{\neq}D$
proof
assume *eq-C-D*: $C{=}D$
show *False*
proof –
from *subcls1-C-Z eq-C-D*
have $G\vdash D \prec_C\ Z$
by (*auto*)
also
from *subcls-Z-D ws*
have $\neg\ G\vdash D \prec_C\ Z$
by (*rule subcls-acyclic*)
ultimately
show *?thesis*
by – (*rule notE*)
qed
qed
qed
qed

lemma *invert-subclseq*:
$[\![G\vdash C \preceq_C\ D;\ \textit{ws-prog}\ G]\!]$
$\Longrightarrow \neg\ G\vdash D \prec_C\ C$
proof –
assume *ws*: *ws-prog* G **and**
subclseq-C-D: $G\vdash C \preceq_C\ D$
show *?thesis*
proof (*cases* $D{=}C$)
case *True*
with *ws*
show *?thesis*
by (*auto dest*: *subcls-irrefl*)
next
case *False*
with *subclseq-C-D*
have $G\vdash C \prec_C\ D$
by (*blast intro*: *rtrancl-into-trancl3*)
with *ws*
show *?thesis*
by (*blast dest*: *subcls-acyclic*)
qed
qed

lemma *invert-subcls*:
$[\![G\vdash C \prec_C\ D;\ \textit{ws-prog}\ G]\!]$
$\Longrightarrow \neg\ G\vdash D \preceq_C\ C$
proof –
assume *ws*: *ws-prog* G **and**
subcls-C-D: $G\vdash C \prec_C\ D$
then
have *nsubcls-D-C*: $\neg\ G\vdash D \prec_C\ C$
by (*blast dest*: *subcls-acyclic*)
show *?thesis*
proof

assume $G\vdash D \preceq_C C$
then show *False*
proof (*cases rule*: *subclseq-cases*)
case *Eq*
with *ws subcls-C-D*
show *?thesis*
by (*auto dest*: *subcls-irrefl*)
next
case *Subcls*
with *nsubcls-D-C*
show *?thesis*
by *blast*
qed
qed
qed

lemma *subcls-superD*:
$[\![G\vdash C \prec_C D;\ class\ G\ C = Some\ c]\!] \Longrightarrow G\vdash(super\ c) \preceq_C D$
proof −
assume *clsC*: $class\ G\ C = Some\ c$
assume *subcls-C-C*: $G\vdash C \prec_C D$
then obtain S **where**
$G\vdash C \prec_{C1} S$ **and**
subclseq-S-D: $G\vdash S \preceq_C D$
by (*blast dest*: *tranclD*)
with *clsC*
have $S{=}super\ c$
by (*auto dest*: *subcls1D*)
with *subclseq-S-D* **show** *?thesis* **by** *simp*
qed

lemma *subclseq-superD*:
$[\![G\vdash C \preceq_C D;\ C{\neq}D;class\ G\ C = Some\ c]\!] \Longrightarrow G\vdash(super\ c) \preceq_C D$
proof −
assume *neq-C-D*: $C{\neq}D$
assume *clsC*: $class\ G\ C = Some\ c$
assume *subclseq-C-D*: $G\vdash C \preceq_C D$
then show *?thesis*
proof (*cases rule*: *subclseq-cases*)
case *Eq* **with** *neq-C-D* **show** *?thesis* **by** *contradiction*
next
case *Subcls*
with *clsC* **show** *?thesis* **by** (*blast dest*: *subcls-superD*)
qed
qed

implementation relation

defs
— direct implementation, cf. 8.1.3
implmt1-def:$implmt1\ G{\equiv}\{(C,I).\ C{\neq}Object \wedge (\exists\, c{\in}class\ G\ C{:}\ I{\in}set\ (superIfs\ c))\}$

lemma *implmt1D*: $G\vdash C{\leadsto}1I \Longrightarrow C{\neq}Object \wedge (\exists\, c{\in}class\ G\ C{:}\ I{\in}set\ (superIfs\ c))$
apply (*unfold implmt1-def*)
apply *auto*

done

inductive — implementation, cf. 8.1.4
implmt :: *prog* ⇒ *qtname* ⇒ *qtname* ⇒ *bool* (-⊢-$\leadsto$- [*71*,*71*,*71*] *70*)
for G :: *prog*
where
direct: $G \vdash C \leadsto 1J \Longrightarrow G \vdash C \leadsto J$
| *subint*: $[\![G \vdash C \leadsto 1I;\ G \vdash I \preceq I\ J]\!] \Longrightarrow G \vdash C \leadsto J$
| *subcls1*: $[\![G \vdash C \prec_{C1} D;\ G \vdash D \leadsto J\ ]\!] \Longrightarrow G \vdash C \leadsto J$

lemma *implmtD*: $G \vdash C \leadsto J \Longrightarrow (\exists I.\ G \vdash C \leadsto 1I \wedge G \vdash I \preceq I\ J) \vee (\exists D.\ G \vdash C \prec_{C1} D \wedge G \vdash D \leadsto J)$
apply (*erule implmt.induct*)
apply *fast+*
done

lemma *implmt-ObjectE* [*elim*!]: $G \vdash Object \leadsto I \Longrightarrow R$
by (*auto dest*!: *implmtD implmt1D subcls1D*)

lemma *subcls-implmt* [*rule-format* (*no-asm*)]: $G \vdash A \preceq_C B \Longrightarrow G \vdash B \leadsto K \longrightarrow G \vdash A \leadsto K$
apply (*erule rtrancl-induct*)
apply (*auto intro*: *implmt.subcls1*)
done

lemma *implmt-subint2*: $[\![\ G \vdash A \leadsto J;\ G \vdash J \preceq I\ K]\!] \Longrightarrow G \vdash A \leadsto K$
apply (*erule rev-mp*, *erule implmt.induct*)
apply (*auto dest*: *implmt.subint rtrancl-trans implmt.subcls1*)
done

lemma *implmt-is-class*: $G \vdash C \leadsto I \Longrightarrow$ *is-class G C*
apply (*erule implmt.induct*)
apply (*auto dest*: *implmt1D subcls1D*)
done

widening relation

inductive
— widening, viz. method invocation conversion, cf. 5.3 i.e. kind of syntactic subtyping
widen :: *prog* ⇒ *ty* ⇒ *ty* ⇒ *bool* (-⊢-$\preceq$- [*71*,*71*,*71*] *70*)
for G :: *prog*
where
refl: $G \vdash T \preceq T$ — identity conversion, cf. 5.1.1
| *subint*: $G \vdash I \preceq I\ J \Longrightarrow G \vdash Iface\ I \preceq Iface\ J$ — wid.ref.conv.,cf. 5.1.4
| *int-obj*: $G \vdash Iface\ I \preceq Class\ Object$
| *subcls*: $G \vdash C \preceq_C D \Longrightarrow G \vdash Class\ C \preceq Class\ D$
| *implmt*: $G \vdash C \leadsto I \Longrightarrow G \vdash Class\ C \preceq Iface\ I$
| *null*: $G \vdash NT \preceq RefT\ R$
| *arr-obj*: $G \vdash T.[] \preceq Class\ Object$
| *array*: $G \vdash RefT\ S \preceq RefT\ T \Longrightarrow G \vdash RefT\ S.[] \preceq RefT\ T.[]$

declare *widen.refl* [*intro*!]
declare *widen.intros* [*simp*]

lemma *widen-PrimT*: $G \vdash PrimT\ x \preceq T \Longrightarrow (\exists y.\ T = PrimT\ y)$
apply (*ind-cases* $G \vdash PrimT\ x \preceq T$)
by *auto*

lemma *widen-PrimT2*: $G \vdash S \preceq PrimT\ x \Longrightarrow \exists y.\ S = PrimT\ y$
apply (*ind-cases* $G \vdash S \preceq PrimT\ x$)
by *auto*

These widening lemmata hold in Bali but are to strong for ordinary Java. They would not work for real Java Integral Types, like short, long, int. These lemmata are just for documentation and are not used.

lemma *widen-PrimT-strong*: $G \vdash PrimT\ x \preceq T \Longrightarrow T = PrimT\ x$
by (*ind-cases* $G \vdash PrimT\ x \preceq T$) *simp-all*

lemma *widen-PrimT2-strong*: $G \vdash S \preceq PrimT\ x \Longrightarrow S = PrimT\ x$
by (*ind-cases* $G \vdash S \preceq PrimT\ x$) *simp-all*

Specialized versions for booleans also would work for real Java

lemma *widen-Boolean*: $G \vdash PrimT\ Boolean \preceq T \Longrightarrow T = PrimT\ Boolean$
by (*ind-cases* $G \vdash PrimT\ Boolean \preceq T$) *simp-all*

lemma *widen-Boolean2*: $G \vdash S \preceq PrimT\ Boolean \Longrightarrow S = PrimT\ Boolean$
by (*ind-cases* $G \vdash S \preceq PrimT\ Boolean$) *simp-all*

lemma *widen-RefT*: $G \vdash RefT\ R \preceq T \Longrightarrow \exists t.\ T = RefT\ t$
apply (*ind-cases* $G \vdash RefT\ R \preceq T$)
by *auto*

lemma *widen-RefT2*: $G \vdash S \preceq RefT\ R \Longrightarrow \exists t.\ S = RefT\ t$
apply (*ind-cases* $G \vdash S \preceq RefT\ R$)
by *auto*

lemma *widen-Iface*: $G \vdash Iface\ I \preceq T \Longrightarrow T = Class\ Object \lor (\exists J.\ T = Iface\ J)$
apply (*ind-cases* $G \vdash Iface\ I \preceq T$)
by *auto*

lemma *widen-Iface2*: $G \vdash S \preceq\ Iface\ J \Longrightarrow S = NT \lor (\exists I.\ S = Iface\ I) \lor (\exists D.\ S = Class\ D)$
apply (*ind-cases* $G \vdash S \preceq\ Iface\ J$)
by *auto*

lemma *widen-Iface-Iface*: $G \vdash Iface\ I \preceq\ Iface\ J \Longrightarrow G \vdash I \preceq I\ J$
apply (*ind-cases* $G \vdash Iface\ I \preceq\ Iface\ J$)
by *auto*

lemma *widen-Iface-Iface-eq* [*simp*]: $G \vdash Iface\ I \preceq\ Iface\ J = G \vdash I \preceq I\ J$
apply (*rule iffI*)
apply (*erule widen-Iface-Iface*)

apply (*erule widen.subint*)
done

lemma *widen-Class*: $G \vdash Class\ C \preceq T \Longrightarrow (\exists D.\ T = Class\ D) \lor (\exists I.\ T = Iface\ I)$
apply (*ind-cases* $G \vdash Class\ C \preceq T$)
by *auto*

lemma *widen-Class2*: $G \vdash S \preceq Class\ C \Longrightarrow C = Object \lor S = NT \lor (\exists D.\ S = Class\ D)$
apply (*ind-cases* $G \vdash S \preceq Class\ C$)
by *auto*

lemma *widen-Class-Class*: $G \vdash Class\ C \preceq Class\ cm \Longrightarrow G \vdash C \preceq_C cm$
apply (*ind-cases* $G \vdash Class\ C \preceq Class\ cm$)
by *auto*

lemma *widen-Class-Class-eq* [*simp*]: $G \vdash Class\ C \preceq Class\ cm = G \vdash C \preceq_C cm$
apply (*rule iffI*)
apply (*erule widen-Class-Class*)
apply (*erule widen.subcls*)
done

lemma *widen-Class-Iface*: $G \vdash Class\ C \preceq Iface\ I \Longrightarrow G \vdash C \leadsto I$
apply (*ind-cases* $G \vdash Class\ C \preceq Iface\ I$)
by *auto*

lemma *widen-Class-Iface-eq* [*simp*]: $G \vdash Class\ C \preceq Iface\ I = G \vdash C \leadsto I$
apply (*rule iffI*)
apply (*erule widen-Class-Iface*)
apply (*erule widen.implmt*)
done

lemma *widen-Array*: $G \vdash S.[] \preceq T \Longrightarrow T = Class\ Object \lor (\exists T'.\ T = T'.[] \land G \vdash S \preceq T')$
apply (*ind-cases* $G \vdash S.[] \preceq T$)
by *auto*

lemma *widen-Array2*: $G \vdash S \preceq T.[] \Longrightarrow S = NT \lor (\exists S'.\ S = S'.[] \land G \vdash S' \preceq T)$
apply (*ind-cases* $G \vdash S \preceq T.[]$)
by *auto*

lemma *widen-ArrayPrimT*: $G \vdash PrimT\ t.[] \preceq T \Longrightarrow T = Class\ Object \lor T = PrimT\ t.[]$
apply (*ind-cases* $G \vdash PrimT\ t.[] \preceq T$)
by *auto*

lemma *widen-ArrayRefT*:
$G \vdash RefT\ t.[] \preceq T \Longrightarrow T = Class\ Object \lor (\exists s.\ T = RefT\ s.[] \land G \vdash RefT\ t \preceq RefT\ s)$
apply (*ind-cases* $G \vdash RefT\ t.[] \preceq T$)
by *auto*

lemma *widen-ArrayRefT-ArrayRefT-eq* [*simp*]:
$G \vdash RefT\ T.[] \preceq RefT\ T'.[] = G \vdash RefT\ T \preceq RefT\ T'$
apply (*rule iffI*)
apply (*drule widen-ArrayRefT*)
apply *simp*
apply (*erule widen.array*)
done

lemma *widen-Array-Array*: $G \vdash T.[] \preceq T'.[] \Longrightarrow G \vdash T \preceq T'$
apply (*drule widen-Array*)
apply *auto*
done

lemma *widen-Array-Class*: $G \vdash S.[] \preceq Class\ C \Longrightarrow C = Object$
by (*auto dest*: *widen-Array*)

lemma *widen-NT2*: $G \vdash S \preceq NT \Longrightarrow S = NT$
apply (*ind-cases* $G \vdash S \preceq NT$)
by *auto*

lemma *widen-Object*: $[\![isrtype\ G\ T; ws\text{-}prog\ G]\!] \Longrightarrow G \vdash RefT\ T \preceq Class\ Object$
apply (*case-tac T*)
apply (*auto*)
apply (*subgoal-tac* $G \vdash qtname\text{-}ext\text{-}type \preceq_C Object$)
apply (*auto intro*: *subcls-ObjectI*)
done

lemma *widen-trans-lemma* [*rule-format* (*no-asm*)]:
$[\![G \vdash S \preceq U; \forall C.\ is\text{-}class\ G\ C \longrightarrow G \vdash C \preceq_C Object]\!] \Longrightarrow \forall T.\ G \vdash U \preceq T \longrightarrow G \vdash S \preceq T$
apply (*erule widen.induct*)
apply *safe*
prefer *5* **apply** (*drule widen-RefT*) **apply** *clarsimp*
apply (*frule-tac* [*1*] *widen-Iface*)
apply (*frule-tac* [*2*] *widen-Class*)
apply (*frule-tac* [*3*] *widen-Class*)
apply (*frule-tac* [*4*] *widen-Iface*)
apply (*frule-tac* [*5*] *widen-Class*)
apply (*frule-tac* [*6*] *widen-Array*)
apply *safe*
apply (*rule widen.int-obj*)
prefer *6* **apply** (*drule implmt-is-class*) **apply** *simp*
apply (*tactic ALLGOALS* (*etac thin-rl*))
prefer *6* **apply** *simp*
apply (*rule-tac* [*9*] *widen.arr-obj*)
apply (*rotate-tac* [*9*] *−1*)
apply (*frule-tac* [*9*] *widen-RefT*)
apply (*auto elim*!: *rtrancl-trans subcls-implmt implmt-subint2*)
done

lemma *ws-widen-trans*: $[\![G \vdash S \preceq U; G \vdash U \preceq T; ws\text{-}prog\ G]\!] \Longrightarrow G \vdash S \preceq T$
by (*auto intro*: *widen-trans-lemma subcls-ObjectI*)

lemma *widen-antisym-lemma* [*rule-format* (*no-asm*)]: $[\![G\vdash S\preceq T;$
$\forall I\ J.\ G\vdash I\preceq I\ J \wedge G\vdash J\preceq I\ I \longrightarrow I = J;$
$\forall C\ D.\ G\vdash C\preceq_C D \wedge G\vdash D\preceq_C C \longrightarrow C = D;$
$\forall I\ .\ G\vdash Object\leadsto I \longrightarrow False]\!] \Longrightarrow G\vdash T\preceq S \longrightarrow S = T$
apply (*erule widen.induct*)
apply (*auto dest*: *widen-Iface widen-NT2 widen-Class*)
done

lemmas *subint-antisym* =
subint1-acyclic [*THEN acyclic-impl-antisym-rtrancl*, *standard*]
lemmas *subcls-antisym* =
subcls1-acyclic [*THEN acyclic-impl-antisym-rtrancl*, *standard*]

lemma *widen-antisym*: $[\![G\vdash S\preceq T;\ G\vdash T\preceq S;\ ws\text{-}prog\ G]\!] \Longrightarrow S=T$
by (*fast elim*: *widen-antisym-lemma subint-antisym* [*THEN antisymD*]
subcls-antisym [*THEN antisymD*])

lemma *widen-ObjectD* [*dest*!]: $G\vdash Class\ Object\preceq T \Longrightarrow T=Class\ Object$
apply (*frule widen-Class*)
apply (*fast dest*: *widen-Class-Class widen-Class-Iface*)
done

constdefs
widens :: *prog* ⇒ [*ty list*, *ty list*] ⇒ *bool* (-⊢-[≼]- [*71*,*71*,*71*] *70*)
$G\vdash Ts[\preceq]Ts' \equiv list\text{-}all2\ (\lambda T\ T'.\ G\vdash T\preceq T')\ Ts\ Ts'$

lemma *widens-Nil* [*simp*]: $G\vdash[][\preceq][]$
apply (*unfold widens-def*)
apply *auto*
done

lemma *widens-Cons* [*simp*]: $G\vdash(S\#Ss)[\preceq](T\#Ts) = (G\vdash S\preceq T \wedge G\vdash Ss[\preceq]Ts)$
apply (*unfold widens-def*)
apply *auto*
done

narrowing relation

inductive — narrowing reference conversion, cf. 5.1.5
narrow :: *prog* ⇒ *ty* ⇒ *ty* ⇒ *bool* (-⊢-≻- [*71*,*71*,*71*] *70*)
for *G* :: *prog*
where
subcls: $G\vdash C\preceq_C D \Longrightarrow G\vdash Class\ D\succ Class\ C$
| *implmt*: $\neg G\vdash C\leadsto I \Longrightarrow G\vdash Class\ C\succ Iface\ I$
| *obj-arr*: $G\vdash Class\ Object\succ T.[]$
| *int-cls*: $G\vdash Iface\ I\succ Class\ C$
| *subint*: *imethds G I hidings imethds G J entails*
$(\lambda(md, mh)\ (md',mh').\ G\vdash mrt\ mh\preceq mrt\ mh') \Longrightarrow$
$\neg G\vdash I\preceq I\ J \Longrightarrow G\vdash Iface\ I\succ Iface\ J$
| *array*: $G\vdash RefT\ S\succ RefT\ T \Longrightarrow G\vdash RefT\ S.[]\succ RefT\ T.[]$

lemma *narrow-RefT*: $G\vdash RefT\ R\succ T \Longrightarrow \exists\, t.\ T=RefT\ t$
apply (*ind-cases* $G\vdash RefT\ R\succ T$)
by *auto*

lemma *narrow-RefT2*: $G\vdash S\succ RefT\ R \Longrightarrow \exists\, t.\ S=RefT\ t$
apply (*ind-cases* $G\vdash S\succ RefT\ R$)
by *auto*

lemma *narrow-PrimT*: $G\vdash PrimT\ pt\succ T \Longrightarrow \exists\, t.\ T=PrimT\ t$
by (*ind-cases* $G\vdash PrimT\ pt\succ T$)

lemma *narrow-PrimT2*: $G\vdash S\succ PrimT\ pt \Longrightarrow$
$\exists\, t.\ S=PrimT\ t \wedge G\vdash PrimT\ t\preceq PrimT\ pt$
by (*ind-cases* $G\vdash S\succ PrimT\ pt$)

These narrowing lemmata hold in Bali but are to strong for ordinary Java. They would not work for real Java Integral Types, like short, long, int. These lemmata are just for documentation and are not used.

lemma *narrow-PrimT-strong*: $G\vdash PrimT\ pt\succ T \Longrightarrow T=PrimT\ pt$
by (*ind-cases* $G\vdash PrimT\ pt\succ T$)

lemma *narrow-PrimT2-strong*: $G\vdash S\succ PrimT\ pt \Longrightarrow S=PrimT\ pt$
by (*ind-cases* $G\vdash S\succ PrimT\ pt$)

Specialized versions for booleans also would work for real Java

lemma *narrow-Boolean*: $G\vdash PrimT\ Boolean\succ T \Longrightarrow T=PrimT\ Boolean$
by (*ind-cases* $G\vdash PrimT\ Boolean\succ T$)

lemma *narrow-Boolean2*: $G\vdash S\succ PrimT\ Boolean \Longrightarrow S=PrimT\ Boolean$
by (*ind-cases* $G\vdash S\succ PrimT\ Boolean$)

casting relation

inductive — casting conversion, cf. 5.5
  cast :: $prog \Rightarrow ty \Rightarrow ty \Rightarrow bool$ (-$\vdash$-$\preceq$? - [71,71,71] 70)
  for G :: *prog*
where
  widen: $G\vdash S\preceq T \Longrightarrow G\vdash S\preceq?\ T$
| *narrow*: $G\vdash S\succ T \Longrightarrow G\vdash S\preceq?\ T$

lemma *cast-RefT*: $G\vdash RefT\ R\preceq?\ T \Longrightarrow \exists\, t.\ T=RefT\ t$
apply (*ind-cases* $G\vdash RefT\ R\preceq?\ T$)
by (*auto dest*: *widen-RefT narrow-RefT*)

lemma *cast-RefT2*: $G\vdash S\preceq?\ RefT\ R \Longrightarrow \exists\, t.\ S=RefT\ t$
apply (*ind-cases* $G\vdash S\preceq?\ RefT\ R$)
by (*auto dest*: *widen-RefT2 narrow-RefT2*)

lemma *cast-PrimT*: $G\vdash PrimT\ pt\preceq?\ T \Longrightarrow \exists\, t.\ T{=}PrimT\ t$
apply (*ind-cases* $G\vdash PrimT\ pt\preceq?\ T$)
by (*auto dest*: *widen-PrimT narrow-PrimT*)

lemma *cast-PrimT2*: $G\vdash S\preceq?\ PrimT\ pt \Longrightarrow \exists\, t.\ S{=}PrimT\ t \wedge G\vdash PrimT\ t\preceq PrimT\ pt$
apply (*ind-cases* $G\vdash S\preceq?\ PrimT\ pt$)
by (*auto dest*: *widen-PrimT2 narrow-PrimT2*)

lemma *cast-Boolean*:
  assumes *bool-cast*: $G\vdash PrimT\ Boolean\preceq?\ T$
  shows $T{=}PrimT\ Boolean$
using *bool-cast*
proof (*cases*)
  case *widen*
  hence $G\vdash PrimT\ Boolean\preceq\ T$
    by *simp*
  thus *?thesis* **by** (*rule widen-Boolean*)
next
  case *narrow*
  hence $G\vdash PrimT\ Boolean\succ T$
    by *simp*
  thus *?thesis* **by** (*rule narrow-Boolean*)
qed

lemma *cast-Boolean2*:
  assumes *bool-cast*: $G\vdash S\preceq?\ PrimT\ Boolean$
  shows $S = PrimT\ Boolean$
using *bool-cast*
proof (*cases*)
  case *widen*
  hence $G\vdash S\preceq\ PrimT\ Boolean$
    by *simp*
  thus *?thesis* **by** (*rule widen-Boolean2*)
next
  case *narrow*
  hence $G\vdash S\succ PrimT\ Boolean$
    by *simp*
  thus *?thesis* **by** (*rule narrow-Boolean2*)
qed

end

Chapter 10

DeclConcepts

16 Advanced concepts on Java declarations like overriding, inheritance, dynamic method lookup

theory *DeclConcepts* **imports** *TypeRel* **begin**

access control (cf. 6.6), overriding and hiding (cf. 8.4.6.1)

constdefs
is-public :: *prog* ⇒ *qtname* ⇒ *bool*
is-public G qn ≡ (*case class G qn of*
    None ⇒ (*case iface G qn of*
        None ⇒ *False*
        | *Some iface* ⇒ *access iface* = *Public*)
    | *Some class* ⇒ *access class* = *Public*)

17 accessibility of types (cf. 6.6.1)

Primitive types are always accessible, interfaces and classes are accessible in their package or if they are defined public, an array type is accessible if its element type is accessible

consts *accessible-in* :: *prog* ⇒ *ty* ⇒ *pname* ⇒ *bool*
    (- ⊢ - *accessible′-in* - [*61*,*61*,*61*] *60*)
  rt-accessible-in:: *prog* ⇒ *ref-ty* ⇒ *pname* ⇒ *bool*
    (- ⊢ - *accessible′-in′* - [*61*,*61*,*61*] *60*)
primrec
G⊢(*PrimT p*) *accessible-in pack* = *True*
accessible-in-RefT-simp:
G⊢(*RefT r*) *accessible-in pack* = *G*⊢*r accessible-in′ pack*

G⊢(*NullT*) *accessible-in′ pack* = *True*
G⊢(*IfaceT I*) *accessible-in′ pack* = ((*pid I* = *pack*) ∨ *is-public G I*)
G⊢(*ClassT C*) *accessible-in′ pack* = ((*pid C* = *pack*) ∨ *is-public G C*)
G⊢(*ArrayT ty*) *accessible-in′ pack* = *G*⊢*ty accessible-in pack*

declare *accessible-in-RefT-simp* [*simp del*]

constdefs
  is-acc-class :: *prog* ⇒ *pname* ⇒ *qtname* ⇒ *bool*
    is-acc-class G P C ≡ *is-class G C* ∧ *G*⊢(*Class C*) *accessible-in P*
  is-acc-iface :: *prog* ⇒ *pname* ⇒ *qtname* ⇒ *bool*
    is-acc-iface G P I ≡ *is-iface G I* ∧ *G*⊢(*Iface I*) *accessible-in P*
  is-acc-type :: *prog* ⇒ *pname* ⇒ *ty* ⇒ *bool*
    is-acc-type G P T ≡ *is-type G T* ∧ *G*⊢*T accessible-in P*
  is-acc-reftype :: *prog* ⇒ *pname* ⇒ *ref-ty* ⇒ *bool*
  is-acc-reftype G P T ≡ *isrtype G T* ∧ *G*⊢*T accessible-in′ P*

lemma *is-acc-classD*:
is-acc-class G P C ⟹ *is-class G C* ∧ *G*⊢(*Class C*) *accessible-in P*
by (*simp add*: *is-acc-class-def*)

lemma *is-acc-class-is-class*: *is-acc-class G P C* ⟹ *is-class G C*
by (*auto simp add*: *is-acc-class-def*)

lemma *is-acc-ifaceD*:
is-acc-iface G P I ⟹ *is-iface G I* ∧ *G*⊢(*Iface I*) *accessible-in P*
by (*simp add*: *is-acc-iface-def*)

lemma *is-acc-typeD*:
is-acc-type G P $T \Longrightarrow$ *is-type* G T $\wedge$ $G \vdash T$ *accessible-in* P
by (*simp add*: *is-acc-type-def*)

lemma *is-acc-reftypeD*:
is-acc-reftype G P $T \Longrightarrow$ *isrtype* G T $\wedge$ $G \vdash T$ *accessible-in'* P
by (*simp add*: *is-acc-reftype-def*)

18 accessibility of members

The accessibility of members is more involved as the accessibility of types. We have to distinguish several cases to model the different effects of accessibility during inheritance, overriding and ordinary member access

Various technical conversion and selection functions

overloaded selector *accmodi* to select the access modifier out of various HOL types

axclass *has-accmodi* < *type*
consts *accmodi*:: *'a*::*has-accmodi* $\Rightarrow$ *acc-modi*

instance *acc-modi*::*has-accmodi* **..**

defs (**overloaded**)
acc-modi-accmodi-def: *accmodi* (*a*::*acc-modi*) $\equiv$ *a*

lemma *acc-modi-accmodi-simp*[*simp*]: *accmodi* (*a*::*acc-modi*) = *a*
by (*simp add*: *acc-modi-accmodi-def*)

instance *decl-ext-type*:: (*type*) *has-accmodi* **..**

defs (**overloaded**)
decl-acc-modi-def: *accmodi* (*d*::(*'a*:: *type*) *decl-scheme*) $\equiv$ *access d*

lemma *decl-acc-modi-simp*[*simp*]: *accmodi* (*d*::(*'a*::*type*) *decl-scheme*) = *access d*
by (*simp add*: *decl-acc-modi-def*)

instance $*$ :: (*type*,*has-accmodi*) *has-accmodi* **..**

defs (**overloaded**)
pair-acc-modi-def: *accmodi p* $\equiv$ (*accmodi* (*snd p*))

lemma *pair-acc-modi-simp*[*simp*]: *accmodi* (*x*,*a*) = (*accmodi a*)
by (*simp add*: *pair-acc-modi-def*)

instance *memberdecl* :: *has-accmodi* **..**

defs (**overloaded**)
memberdecl-acc-modi-def: *accmodi m* $\equiv$ (*case m of*
fdecl f $\Rightarrow$ *accmodi f*
| *mdecl m* $\Rightarrow$ *accmodi m*)

lemma *memberdecl-fdecl-acc-modi-simp*[*simp*]:
accmodi (*fdecl m*) = *accmodi m*
by (*simp add*: *memberdecl-acc-modi-def*)

lemma *memberdecl-mdecl-acc-modi-simp*[*simp*]:
accmodi (*mdecl m*) = *accmodi m*
by (*simp add*: *memberdecl-acc-modi-def*)

overloaded selector *declclass* to select the declaring class out of various HOL types

axclass *has-declclass* < *type*
consts *declclass*:: *'a*::*has-declclass* ⇒ *qtname*

instance *qtname-ext-type*::(*type*) *has-declclass* **..**

defs (**overloaded**)
qtname-declclass-def: *declclass* (*q*::*qtname*) ≡ *q*

lemma *qtname-declclass-simp*[*simp*]: *declclass* (*q*::*qtname*) = *q*
by (*simp add*: *qtname-declclass-def*)

instance ∗ :: (*has-declclass*,*type*) *has-declclass* **..**

defs (**overloaded**)
pair-declclass-def: *declclass p* ≡ *declclass* (*fst p*)

lemma *pair-declclass-simp*[*simp*]: *declclass* (*c*,*x*) = *declclass c*
by (*simp add*: *pair-declclass-def*)

overloaded selector *is-static* to select the static modifier out of various HOL types

axclass *has-static* < *type*
consts *is-static* :: *'a*::*has-static* ⇒ *bool*

instance *decl-ext-type* :: (*has-static*) *has-static* **..**

defs (**overloaded**)
decl-is-static-def:
is-static (*m*::(*'a*::*has-static*) *decl-scheme*) ≡ *is-static* (*Decl.decl.more m*)

instance *member-ext-type* :: (*type*) *has-static* **..**

defs (**overloaded**)
static-field-type-is-static-def:
is-static (*m*::(*'b*::*type*) *member-ext-type*) ≡ *static-sel m*

lemma *member-is-static-simp*: *is-static* (*m*::*'a member-scheme*) = *static m*
apply (*cases m*)
apply (*simp add*: *static-field-type-is-static-def*
decl-is-static-def Decl.member.dest-convs)
done

instance ∗ :: (*type*,*has-static*) *has-static* **..**

defs (**overloaded**)
pair-is-static-def: *is-static p* ≡ *is-static* (*snd p*)

lemma *pair-is-static-simp* [*simp*]: *is-static* (*x*,*s*) = *is-static s*
by (*simp add*: *pair-is-static-def*)

lemma *pair-is-static-simp1*: *is-static p* = *is-static* (*snd p*)
by (*simp add*: *pair-is-static-def*)

instance *memberdecl*:: *has-static* **..**

defs (**overloaded**)
memberdecl-is-static-def:
is-static m ≡ (*case m of*
fdecl f ⇒ *is-static f*
| *mdecl m* ⇒ *is-static m*)

lemma *memberdecl-is-static-fdecl-simp*[*simp*]:
is-static (*fdecl f*) = *is-static f*
by (*simp add*: *memberdecl-is-static-def*)

lemma *memberdecl-is-static-mdecl-simp*[*simp*]:
is-static (*mdecl m*) = *is-static m*
by (*simp add*: *memberdecl-is-static-def*)

lemma *mhead-static-simp* [*simp*]: *is-static* (*mhead m*) = *is-static m*
by (*cases m*) (*simp add*: *mhead-def member-is-static-simp*)

constdefs — some mnemotic selectors for various pairs

decliface:: (*qtname* × (*'a*::*type*) *decl-scheme*) ⇒ *qtname*
decliface ≡ *fst* — get the interface component

mbr:: (*qtname* × *memberdecl*) ⇒ *memberdecl*
mbr ≡ *snd* — get the memberdecl component

mthd:: (*'b* × *'a*) ⇒ *'a*
— also used for mdecl, mhead
mthd ≡ *snd* — get the method component

fld:: (*'b* × (*'a*::*type*) *decl-scheme*) ⇒ (*'a*::*type*) *decl-scheme*
— also used for ((*vname* × *qtname*)× *field*)
fld ≡ *snd* — get the field component

constdefs — some mnemotic selectors for (*vname* × *qtname*)
fname:: (*vname* × *'a*) ⇒ *vname* — also used for fdecl
fname ≡ *fst*

declclassf:: (*vname* × *qtname*) ⇒ *qtname*
declclassf ≡ *snd*

lemma *decliface-simp*[*simp*]: *decliface* (*I*,*m*) = *I*

by (*simp add*: *decliface-def*)

lemma *mbr-simp*[*simp*]: *mbr* (*C*,*m*) = *m*
by (*simp add*: *mbr-def*)

lemma *access-mbr-simp* [*simp*]: (*accmodi* (*mbr m*)) = *accmodi m*
by (*cases m*) (*simp add*: *mbr-def*)

lemma *mthd-simp*[*simp*]: *mthd* (*C*,*m*) = *m*
by (*simp add*: *mthd-def*)

lemma *fld-simp*[*simp*]: *fld* (*C*,*f*) = *f*
by (*simp add*: *fld-def*)

lemma *accmodi-simp*[*simp*]: *accmodi* (*C*,*m*) = *access m*
by (*simp*)

lemma *access-mthd-simp* [*simp*]: (*access* (*mthd m*)) = *accmodi m*
by (*cases m*) (*simp add*: *mthd-def*)

lemma *access-fld-simp* [*simp*]: (*access* (*fld f*)) = *accmodi f*
by (*cases f*) (*simp add*: *fld-def*)

lemma *static-mthd-simp*[*simp*]: *static* (*mthd m*) = *is-static m*
by (*cases m*) (*simp add*: *mthd-def member-is-static-simp*)

lemma *mthd-is-static-simp* [*simp*]: *is-static* (*mthd m*) = *is-static m*
by (*cases m*) *simp*

lemma *static-fld-simp*[*simp*]: *static* (*fld f*) = *is-static f*
by (*cases f*) (*simp add*: *fld-def member-is-static-simp*)

lemma *ext-field-simp* [*simp*]: (*declclass f*,*fld f*) = *f*
by (*cases f*) (*simp add*: *fld-def*)

lemma *ext-method-simp* [*simp*]: (*declclass m*,*mthd m*) = *m*
by (*cases m*) (*simp add*: *mthd-def*)

lemma *ext-mbr-simp* [*simp*]: (*declclass m*,*mbr m*) = *m*
by (*cases m*) (*simp add*: *mbr-def*)

lemma *fname-simp*[*simp*]:*fname* (*n*,*c*) = *n*
by (*simp add*: *fname-def*)

lemma *declclassf-simp*[*simp*]:*declclassf* (*n*,*c*) = *c*
by (*simp add*: *declclassf-def*)

constdefs — some mnemotic selectors for (*vname* × *qtname*)
fldname :: (*vname* × *qtname*) ⇒ *vname*
fldname ≡ *fst*

fldclass :: (*vname* × *qtname*) ⇒ *qtname*
fldclass ≡ *snd*

lemma *fldname-simp*[*simp*]: *fldname* (*n*,*c*) = *n*
by (*simp add*: *fldname-def*)

lemma *fldclass-simp*[*simp*]: *fldclass* (*n*,*c*) = *c*
by (*simp add*: *fldclass-def*)

lemma *ext-fieldname-simp*[*simp*]: (*fldname f*,*fldclass f*) = *f*
by (*simp add*: *fldname-def fldclass-def*)

Convert a qualified method declaration (qualified with its declaring class) to a qualified member declaration: *methdMembr*

constdefs
methdMembr :: (*qtname* × *mdecl*) ⇒ (*qtname* × *memberdecl*)
methdMembr m ≡ (*fst m*,*mdecl* (*snd m*))

lemma *methdMembr-simp*[*simp*]: *methdMembr* (*c*,*m*) = (*c*,*mdecl m*)
by (*simp add*: *methdMembr-def*)

lemma *accmodi-methdMembr-simp*[*simp*]: *accmodi* (*methdMembr m*) = *accmodi m*
by (*cases m*) (*simp add*: *methdMembr-def*)

lemma *is-static-methdMembr-simp*[*simp*]: *is-static* (*methdMembr m*) = *is-static m*
by (*cases m*) (*simp add*: *methdMembr-def*)

lemma *declclass-methdMembr-simp*[*simp*]: *declclass* (*methdMembr m*) = *declclass m*
by (*cases m*) (*simp add*: *methdMembr-def*)

Convert a qualified method (qualified with its declaring class) to a qualified member declaration: *method*

constdefs
method :: *sig* ⇒ (*qtname* × *methd*) ⇒ (*qtname* × *memberdecl*)
method sig m ≡ (*declclass m*, *mdecl* (*sig*, *mthd m*))

lemma *method-simp*[*simp*]: *method sig* (*C*,*m*) = (*C*,*mdecl* (*sig*,*m*))
by (*simp add*: *method-def*)

lemma *accmodi-method-simp*[*simp*]: *accmodi* (*method sig m*) = *accmodi m*
by (*simp add*: *method-def*)

lemma *declclass-method-simp*[*simp*]: *declclass* (*method sig m*) = *declclass m*
by (*simp add*: *method-def*)

lemma *is-static-method-simp*[*simp*]: *is-static* (*method sig m*) = *is-static m*
by (*cases m*) (*simp add*: *method-def*)

lemma *mbr-method-simp*[*simp*]: *mbr* (*method sig m*) = *mdecl* (*sig*,*mthd m*)
by (*simp add*: *mbr-def method-def*)

lemma *memberid-method-simp*[*simp*]: *memberid* (*method sig m*) = *mid sig*
by (*simp add*: *method-def*)

constdefs
fieldm :: *vname* ⇒ (*qtname* × *field*) ⇒ (*qtname* × *memberdecl*)
fieldm n f ≡ (*declclass f*, *fdecl* (*n*, *fld f*))

lemma *fieldm-simp*[*simp*]: *fieldm n* (*C*,*f*) = (*C*,*fdecl* (*n*,*f*))
by (*simp add*: *fieldm-def*)

lemma *accmodi-fieldm-simp*[*simp*]: *accmodi* (*fieldm n f*) = *accmodi f*
by (*simp add*: *fieldm-def*)

lemma *declclass-fieldm-simp*[*simp*]: *declclass* (*fieldm n f*) = *declclass f*
by (*simp add*: *fieldm-def*)

lemma *is-static-fieldm-simp*[*simp*]: *is-static* (*fieldm n f*) = *is-static f*
by (*cases f*) (*simp add*: *fieldm-def*)

lemma *mbr-fieldm-simp*[*simp*]: *mbr* (*fieldm n f*) = *fdecl* (*n*,*fld f*)
by (*simp add*: *mbr-def fieldm-def*)

lemma *memberid-fieldm-simp*[*simp*]: *memberid* (*fieldm n f*) = *fid n*
by (*simp add*: *fieldm-def*)

Select the signature out of a qualified method declaration: *msig*

constdefs *msig*:: (*qtname* × *mdecl*) ⇒ *sig*
msig m ≡ *fst* (*snd m*)

lemma *msig-simp*[*simp*]: *msig* (*c*,(*s*,*m*)) = *s*
by (*simp add*: *msig-def*)

Convert a qualified method (qualified with its declaring class) to a qualified method declaration: *qmdecl*

constdefs *qmdecl* :: *sig* ⇒ (*qtname* × *methd*) ⇒ (*qtname* × *mdecl*)
qmdecl sig m ≡ (*declclass m*, (*sig*,*mthd m*))

lemma *qmdecl-simp*[*simp*]: *qmdecl sig* (*C*,*m*) = (*C*,(*sig*,*m*))
by (*simp add*: *qmdecl-def*)

lemma *declclass-qmdecl-simp*[*simp*]: *declclass* (*qmdecl sig m*) = *declclass m*
by (*simp add*: *qmdecl-def*)

lemma *accmodi-qmdecl-simp*[*simp*]: *accmodi* (*qmdecl sig m*) = *accmodi m*
by (*simp add*: *qmdecl-def*)

lemma *is-static-qmdecl-simp*[*simp*]: *is-static* (*qmdecl sig m*) = *is-static m*
by (*cases m*) (*simp add*: *qmdecl-def*)

lemma *msig-qmdecl-simp*[*simp*]: *msig* (*qmdecl sig m*) = *sig*
by (*simp add*: *qmdecl-def*)

lemma *mdecl-qmdecl-simp*[*simp*]:
mdecl (*mthd* (*qmdecl sig new*)) = *mdecl* (*sig*, *mthd new*)
by (*simp add*: *qmdecl-def*)

lemma *methdMembr-qmdecl-simp* [*simp*]:
methdMembr (*qmdecl sig old*) = *method sig old*
by (*simp add*: *methdMembr-def qmdecl-def method-def*)

overloaded selector *resTy* to select the result type out of various HOL types

axclass *has-resTy* < *type*
consts *resTy*:: *'a*::*has-resTy* ⇒ *ty*

instance *decl-ext-type* :: (*has-resTy*) *has-resTy* **..**

defs (**overloaded**)
decl-resTy-def:
resTy (*m*::(*'a*::*has-resTy*) *decl-scheme*) ≡ *resTy* (*Decl.decl.more m*)

instance *member-ext-type* :: (*has-resTy*) *has-resTy* **..**

defs (**overloaded**)
member-ext-type-resTy-def:
resTy (*m*::(*'b*::*has-resTy*) *member-ext-type*)
≡ *resTy* (*member.more-sel m*)

instance *mhead-ext-type* :: (*type*) *has-resTy* **..**

defs (**overloaded**)
mhead-ext-type-resTy-def:
resTy (*m*::(*'b mhead-ext-type*))
≡ *resT-sel m*

lemma *mhead-resTy-simp*: *resTy* (*m*::*'a mhead-scheme*) = *resT m*
apply (*cases m*)
apply (*simp add*: *decl-resTy-def member-ext-type-resTy-def*
mhead-ext-type-resTy-def
member.dest-convs mhead.dest-convs)

done

lemma *resTy-mhead* [*simp*]:*resTy* (*mhead m*) = *resTy m*
by (*simp add*: *mhead-def mhead-resTy-simp*)

instance ∗ :: (*type*,*has-resTy*) *has-resTy* **..**

defs (**overloaded**)
pair-resTy-def: *resTy p* ≡ *resTy* (*snd p*)

lemma *pair-resTy-simp*[*simp*]: *resTy* (*x*,*m*) = *resTy m*
by (*simp add*: *pair-resTy-def*)

lemma *qmdecl-resTy-simp* [*simp*]: *resTy* (*qmdecl sig m*) = *resTy m*
by (*cases m*) (*simp*)

lemma *resTy-mthd* [*simp*]:*resTy* (*mthd m*) = *resTy m*
by (*cases m*) (*simp add*: *mthd-def*)

inheritable-in

G⊢*m inheritable-in P*: m can be inherited by classes in package P if:

- the declaration class of m is accessible in P and
- the member m is declared with protected or public access or if it is declared with default (package) access, the package of the declaration class of m is also P. If the member m is declared with private access it is not accessible for inheritance at all.

constdefs
inheritable-in::
prog ⇒ (*qtname* × *memberdecl*) ⇒ *pname* ⇒ *bool*
(- ⊢ - *inheritable′-in* - [*61*,*61*,*61*] *60*)
G⊢*membr inheritable-in pack*
≡ (*case* (*accmodi membr*) *of*
Private ⇒ *False*
| *Package* ⇒ (*pid* (*declclass membr*)) = *pack*
| *Protected* ⇒ *True*
| *Public* ⇒ *True*)

syntax
Method-inheritable-in::
prog ⇒ (*qtname* × *mdecl*) ⇒ *pname* ⇒ *bool*
(- ⊢*Method* - *inheritable′-in* - [*61*,*61*,*61*] *60*)

translations
G⊢*Method m inheritable-in p* == *G*⊢*methdMembr m inheritable-in p*

syntax
Methd-inheritable-in::
prog ⇒ *sig* ⇒ (*qtname* × *methd*) ⇒ *pname* ⇒ *bool*
(- ⊢*Methd* - - *inheritable′-in* - [*61*,*61*,*61*,*61*] *60*)

translations
G⊢*Methd s m inheritable-in p* == *G*⊢(*method s m*) *inheritable-in p*

declared-in/undeclared-in

constdefs *cdeclaredmethd*:: *prog* ⇒ *qtname* ⇒ (*sig*,*methd*) *table*
cdeclaredmethd G C
  ≡ (*case class G C of*
      None ⇒ λ *sig*. *None*
    | *Some c* ⇒ *table-of* (*methods c*)
)

constdefs
cdeclaredfield:: *prog* ⇒ *qtname* ⇒ (*vname*,*field*) *table*
cdeclaredfield G C
  ≡ (*case class G C of*
      None ⇒ λ *sig*. *None*
    | *Some c* ⇒ *table-of* (*cfields c*)
)

constdefs
declared-in:: *prog* ⇒ *memberdecl* ⇒ *qtname* ⇒ *bool*
                (-⊢ - *declared′-in* - [*61*,*61*,*61*] *60*)
G⊢*m declared-in C* ≡ (*case m of*
              fdecl (*fn*,*f*) ⇒ *cdeclaredfield G C fn* = *Some f*
            | *mdecl* (*sig*,*m*) ⇒ *cdeclaredmethd G C sig* = *Some m*)

syntax
method-declared-in:: *prog* ⇒ (*qtname* × *mdecl*) ⇒ *qtname* ⇒ *bool*
                (-⊢*Method* - *declared′-in* - [*61*,*61*,*61*] *60*)
translations
G⊢*Method m declared-in C* == *G*⊢*mdecl* (*mthd m*) *declared-in C*

syntax
methd-declared-in:: *prog* ⇒ *sig* ⇒(*qtname* × *methd*) ⇒ *qtname* ⇒ *bool*
              (-⊢*Methd* - - *declared′-in* - [*61*,*61*,*61*,*61*] *60*)
translations
G⊢*Methd s m declared-in C* == *G*⊢*mdecl* (*s*,*mthd m*) *declared-in C*

lemma *declared-in-classD*:
 G⊢*m declared-in C* ⟹ *is-class G C*
by (*cases m*)
  (*auto simp add*: *declared-in-def cdeclaredmethd-def cdeclaredfield-def*)

constdefs
undeclared-in:: *prog* ⇒ *memberid* ⇒ *qtname* ⇒ *bool*
                (-⊢ - *undeclared′-in* - [*61*,*61*,*61*] *60*)

G⊢*m undeclared-in C* ≡ (*case m of*
              fid fn ⇒ *cdeclaredfield G C fn* = *None*
            | *mid sig* ⇒ *cdeclaredmethd G C sig* = *None*)

members

inductive
  members :: *prog* ⇒ (*qtname* × *memberdecl*) ⇒ *qtname* ⇒ *bool*
   (- ⊢ - *member′-of* - [*61*,*61*,*61*] *60*)
  for *G* :: *prog*
where

$\quad$ *Immediate*: $[\![G\vdash mbr\ m\ declared\text{-}in\ C; declclass\ m = C]\!] \Longrightarrow G\vdash m\ member\text{-}of\ C$
$|$ *Inherited*: $[\![G\vdash m\ inheritable\text{-}in\ (pid\ C);\ G\vdash memberid\ m\ undeclared\text{-}in\ C;$
$\qquad G\vdash C \prec_{C1} S;\ G\vdash(Class\ S)\ accessible\text{-}in\ (pid\ C); G\vdash m\ member\text{-}of\ S$
$\qquad]\!] \Longrightarrow G\vdash m\ member\text{-}of\ C$

Note that in the case of an inherited member only the members of the direct superclass are concerned. If a member of a superclass of the direct superclass isn't inherited in the direct superclass (not member of the direct superclass) than it can't be a member of the class. E.g. If a member of a class A is defined with package access it isn't member of a subclass S if S isn't in the same package as A. Any further subclasses of S will not inherit the member, regardless if they are in the same package as A or not.

syntax
method-member-of:: $prog \Rightarrow (qtname \times mdecl) \Rightarrow qtname \Rightarrow bool$
$\qquad (\text{-} \vdash Method\ \text{-}\ member'\text{-}of\ \text{-}\ [61,61,61]\ 60)$

translations
$G\vdash Method\ m\ member\text{-}of\ C \rightleftharpoons G\vdash(methdMembr\ m)\ member\text{-}of\ C$

syntax
methd-member-of:: $prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow qtname \Rightarrow bool$
$\qquad (\text{-} \vdash Methd\ \text{-}\ \text{-}\ member'\text{-}of\ \text{-}\ [61,61,61,61]\ 60)$

translations
$G\vdash Methd\ s\ m\ member\text{-}of\ C \rightleftharpoons G\vdash(method\ s\ m)\ member\text{-}of\ C$

syntax
fieldm-member-of:: $prog \Rightarrow vname \Rightarrow (qtname \times field) \Rightarrow qtname \Rightarrow bool$
$\qquad (\text{-} \vdash Field\ \text{-}\ \text{-}\ member'\text{-}of\ \text{-}\ [61,61,61]\ 60)$

translations
$G\vdash Field\ n\ f\ member\text{-}of\ C \rightleftharpoons G\vdash fieldm\ n\ f\ member\text{-}of\ C$

constdefs
inherits:: $prog \Rightarrow qtname \Rightarrow (qtname \times memberdecl) \Rightarrow bool$
$\qquad (\text{-} \vdash \text{-}\ inherits\ \text{-}\ [61,61,61]\ 60)$
$G\vdash C\ inherits\ m$
$\quad \equiv G\vdash m\ inheritable\text{-}in\ (pid\ C) \wedge G\vdash memberid\ m\ undeclared\text{-}in\ C \wedge$
$\quad (\exists\ S.\ G\vdash C \prec_{C1} S \wedge G\vdash(Class\ S)\ accessible\text{-}in\ (pid\ C) \wedge G\vdash m\ member\text{-}of\ S)$

lemma *inherits-member*: $G\vdash C\ inherits\ m \Longrightarrow G\vdash m\ member\text{-}of\ C$
by (*auto simp add*: *inherits-def intro*: *members.Inherited*)

constdefs *member-in*::$prog \Rightarrow (qtname \times memberdecl) \Rightarrow qtname \Rightarrow bool$
$\qquad (\text{-} \vdash \text{-}\ member'\text{-}in\ \text{-}\ [61,61,61]\ 60)$
$G\vdash m\ member\text{-}in\ C \equiv \exists\ provC.\ G\vdash C \preceq_C provC \wedge G \vdash m\ member\text{-}of\ provC$

A member is in a class if it is member of the class or a superclass. If a member is in a class we can select this member. This additional notion is necessary since not all members are inherited to subclasses. So such members are not member-of the subclass but member-in the subclass.

syntax
method-member-in:: $prog \Rightarrow (qtname \times mdecl) \Rightarrow qtname \Rightarrow bool$
$\qquad (\text{-} \vdash Method\ \text{-}\ member'\text{-}in\ \text{-}\ [61,61,61]\ 60)$

translations
$G\vdash Method\ m\ member\text{-}in\ C \rightleftharpoons G\vdash(methdMembr\ m)\ member\text{-}in\ C$

syntax
methd-member-in:: $prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow qtname \Rightarrow bool$
$(\text{-} \vdash Methd \text{ - - } member'\text{-}in \text{ - } [61,61,61,61]\ 60)$

translations
$G\vdash Methd\ s\ m\ member\text{-}in\ C \rightleftharpoons G\vdash(method\ s\ m)\ member\text{-}in\ C$

lemma *member-inD*: $G\vdash m\ member\text{-}in\ C$
$\Longrightarrow \exists\ provC.\ G\vdash\ C \preceq_C provC \land G \vdash m\ member\text{-}of\ provC$
by (*auto simp add*: *member-in-def*)

lemma *member-inI*: $[\![G \vdash m\ member\text{-}of\ provC;G\vdash\ C \preceq_C provC]\!] \Longrightarrow\ G\vdash m\ member\text{-}in\ C$
by (*auto simp add*: *member-in-def*)

lemma *member-of-to-member-in*: $G \vdash m\ member\text{-}of\ C \Longrightarrow G \vdash m\ member\text{-}in\ C$
by (*auto intro*: *member-inI*)

overriding

Unfortunately the static notion of overriding (used during the typecheck of the compiler) and the dynamic notion of overriding (used during execution in the JVM) are not exactly the same.

Static overriding (used during the typecheck of the compiler)

inductive
stat-overridesR :: $prog \Rightarrow (qtname \times mdecl) \Rightarrow (qtname \times mdecl) \Rightarrow bool$
$(\text{-} \vdash \text{ - } overrides_S \text{ - } [61,61,61]\ 60)$
for G :: *prog*
where

Direct: $[\![\neg\ is\text{-}static\ new;\ msig\ new = msig\ old;$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new);$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old);$
$G\vdash Method\ old\ inheritable\text{-}in\ pid\ (declclass\ new);$
$G\vdash(declclass\ new) \prec_{C1} superNew;$
$G \vdash Method\ old\ member\text{-}of\ superNew$
$]\!] \Longrightarrow G\vdash new\ overrides_S\ old$

| *Indirect*: $[\![G\vdash new\ overrides_S\ inter;\ G\vdash inter\ overrides_S\ old]\!]$
$\Longrightarrow G\vdash new\ overrides_S\ old$

Dynamic overriding (used during the typecheck of the compiler)

inductive
overridesR :: $prog \Rightarrow (qtname \times mdecl) \Rightarrow (qtname \times mdecl) \Rightarrow bool$
$(\text{-} \vdash \text{ - } overrides \text{ - } [61,61,61]\ 60)$
for G :: *prog*
where

Direct: $[\![\neg\ is\text{-}static\ new;\ \neg\ is\text{-}static\ old;\ accmodi\ new \neq Private;$
$msig\ new = msig\ old;$
$G\vdash(declclass\ new) \prec_C (declclass\ old);$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new);$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old);$
$G\vdash Method\ old\ inheritable\text{-}in\ pid\ (declclass\ new);$
$G\vdash resTy\ new \preceq resTy\ old$
$]\!] \Longrightarrow G\vdash new\ overrides\ old$

| *Indirect*: $[\![G\vdash new\ overrides\ inter;\ G\vdash inter\ overrides\ old]\!]$
$\Longrightarrow G\vdash new\ overrides\ old$

syntax
sig-stat-overrides::
$prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow (qtname \times methd) \Rightarrow bool$
$(\text{-,-}\vdash \text{ - } overrides_S \text{ - } [61,61,61,61]\ 60)$
translations
$G,s\vdash new\ overrides_S\ old \rightharpoonup G\vdash(qmdecl\ s\ new)\ overrides_S\ (qmdecl\ s\ old)$

syntax
sig-overrides:: $prog \Rightarrow sig \Rightarrow (qtname \times methd) \Rightarrow (qtname \times methd) \Rightarrow bool$
$(\text{-,-}\vdash \text{ - } overrides \text{ - } [61,61,61,61]\ 60)$
translations
$G,s\vdash new\ overrides\ old \rightharpoonup G\vdash(qmdecl\ s\ new)\ overrides\ (qmdecl\ s\ old)$

Hiding

constdefs *hides*::
$prog \Rightarrow (qtname \times mdecl) \Rightarrow (qtname \times mdecl) \Rightarrow bool$
$(\text{-}\vdash \text{ - } hides \text{ - } [61,61,61]\ 60)$
$G\vdash new\ hides\ old$
$\equiv is\text{-}static\ new \wedge msig\ new = msig\ old \wedge$
$G\vdash(declclass\ new) \prec_C (declclass\ old) \wedge$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old) \wedge$
$G\vdash Method\ old\ inheritable\text{-}in\ pid\ (declclass\ new)$

syntax
sig-hides:: $prog \Rightarrow sig \Rightarrow (qtname \times mdecl) \Rightarrow (qtname \times mdecl) \Rightarrow bool$
$(\text{-,-}\vdash \text{ - } hides \text{ - } [61,61,61,61]\ 60)$
translations
$G,s\vdash new\ hides\ old \rightharpoonup G\vdash(qmdecl\ s\ new)\ hides\ (qmdecl\ s\ old)$

lemma *hidesI*:
$[\![is\text{-}static\ new;\ msig\ new = msig\ old;$
$G\vdash(declclass\ new) \prec_C (declclass\ old);$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new);$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old);$
$G\vdash Method\ old\ inheritable\text{-}in\ pid\ (declclass\ new)$
$]\!] \Longrightarrow G\vdash new\ hides\ old$
by (*auto simp add*: *hides-def*)

lemma *hidesD*:
$[\![G\vdash new\ hides\ old]\!] \Longrightarrow$
$declclass\ new \neq Object \wedge is\text{-}static\ new \wedge msig\ new = msig\ old \wedge$
$G\vdash(declclass\ new) \prec_C (declclass\ old) \wedge$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old)$
by (*auto simp add*: *hides-def*)

lemma *overrides-commonD*:
$[\![G\vdash new\ overrides\ old]\!] \Longrightarrow$
$declclass\ new \neq Object \wedge \neg\ is\text{-}static\ new \wedge \neg\ is\text{-}static\ old \wedge$
$accmodi\ new \neq Private \wedge$

$msig\ new = msig\ old\ \wedge$
$G\vdash(declclass\ new) \prec_C (declclass\ old) \wedge$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old)$
by (*induct set*: *overridesR*) (*auto intro*: *trancl-trans*)

lemma *ws-overrides-commonD*:
$[\![G\vdash new\ overrides\ old; ws\text{-}prog\ G]\!] \Longrightarrow$
$declclass\ new \neq Object \wedge \neg\ is\text{-}static\ new \wedge \neg\ is\text{-}static\ old \wedge$
$accmodi\ new \neq Private \wedge G\vdash resTy\ new \preceq resTy\ old \wedge$
$msig\ new = msig\ old\ \wedge$
$G\vdash(declclass\ new) \prec_C (declclass\ old) \wedge$
$G\vdash Method\ new\ declared\text{-}in\ (declclass\ new) \wedge$
$G\vdash Method\ old\ declared\text{-}in\ (declclass\ old)$
by (*induct set*: *overridesR*) (*auto intro*: *trancl-trans ws-widen-trans*)

lemma *overrides-eq-sigD*:
$[\![G\vdash new\ overrides\ old]\!] \Longrightarrow msig\ old{=}msig\ new$
by (*auto dest*: *overrides-commonD*)

lemma *hides-eq-sigD*:
$[\![G\vdash new\ hides\ old]\!] \Longrightarrow msig\ old{=}msig\ new$
by (*auto simp add*: *hides-def*)

permits access

constdefs
permits-acc::
$prog \Rightarrow (qtname \times memberdecl) \Rightarrow qtname \Rightarrow qtname \Rightarrow bool$
(- ⊢ - *in* - *permits'-acc'-from* - [*61*,*61*,*61*,*61*] *60*)

$G\vdash membr\ in\ class\ permits\text{-}acc\text{-}from\ accclass$
$\equiv (case\ (accmodi\ membr)\ of$
$\quad Private \Rightarrow (declclass\ membr = accclass)$
$\quad |\ Package \Rightarrow (pid\ (declclass\ membr) = pid\ accclass)$
$\quad |\ Protected \Rightarrow (pid\ (declclass\ membr) = pid\ accclass)$
$\qquad \vee$
$\qquad (G\vdash accclass \prec_C declclass\ membr$
$\qquad \wedge (G\vdash class \preceq_C accclass \vee is\text{-}static\ membr))$
$\quad |\ Public \Rightarrow True)$

The subcondition of the *Protected* case: $G\vdash accclass \prec_C declclass\ membr$ could also be relaxed to: $G\vdash accclass \preceq_C declclass\ membr$ since in case both classes are the same the other condition $pid\ (declclass\ membr) = pid\ accclass$ holds anyway.

Like in case of overriding, the static and dynamic accessibility of members is not uniform.

- Statically the class/interface of the member must be accessible for the member to be accessible. During runtime this is not necessary. For Example, if a class is accessible and we are allowed to access a member of this class (statically) we expect that we can access this member in an arbitrary subclass (during runtime). It's not intended to restrict the access to accessible subclasses during runtime.

- Statically the member we want to access must be "member of" the class. Dynamically it must only be "member in" the class.

inductive
accessible-fromR :: *prog* ⇒ *qtname* ⇒ (*qtname* × *memberdecl*) ⇒ *qtname* ⇒ *bool*
and *accessible-from* :: *prog* ⇒ (*qtname* × *memberdecl*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢ - *of* - *accessible′-from* - [*61*,*61*,*61*,*61*] *60*)
and *method-accessible-from* :: *prog* ⇒ (*qtname* × *mdecl*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢*Method* - *of* - *accessible′-from* - [*61*,*61*,*61*,*61*] *60*)
for *G* :: *prog* **and** *accclass* :: *qtname*
where
G⊢*membr of cls accessible-from accclass* ≡ *accessible-fromR G accclass membr cls*

| *G*⊢*Method m of cls accessible-from accclass* ≡ *accessible-fromR G accclass* (*methdMembr m*) *cls*

| *Immediate*: ⟦*G*⊢*membr member-of class*;
G⊢(*Class class*) *accessible-in* (*pid accclass*);
G⊢*membr in class permits-acc-from accclass*
⟧ ⟹ *G*⊢*membr of class accessible-from accclass*

| *Overriding*: ⟦*G*⊢*membr member-of class*;
G⊢(*Class class*) *accessible-in* (*pid accclass*);
membr=(*C*,*mdecl new*);
G⊢(*C*,*new*) $overrides_S$ *old*;
G⊢*class* $\prec_C$ *supr*;
G⊢*Method old of supr accessible-from accclass*
⟧⟹ *G*⊢*membr of class accessible-from accclass*

syntax
methd-accessible-from::
prog ⇒ *sig* ⇒ (*qtname* × *methd*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢*Methd* - - *of* - *accessible′-from* - [*61*,*61*,*61*,*61*,*61*] *60*)

translations
G⊢*Methd s m of cls accessible-from accclass*
⇌ *G*⊢(*method s m*) *of cls accessible-from accclass*

syntax
field-accessible-from::
prog ⇒ *vname* ⇒ (*qtname* × *field*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢*Field* - - *of* - *accessible′-from* - [*61*,*61*,*61*,*61*,*61*] *60*)

translations
G⊢*Field fn f of C accessible-from accclass*
⇌ *G*⊢(*fieldm fn f*) *of C accessible-from accclass*

inductive
dyn-accessible-fromR :: *prog* ⇒ *qtname* ⇒ (*qtname* × *memberdecl*) ⇒ *qtname* ⇒ *bool*
and *dyn-accessible-from′* :: *prog* ⇒ (*qtname* × *memberdecl*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢ - *in* - *dyn′-accessible′-from* - [*61*,*61*,*61*,*61*] *60*)
and *method-dyn-accessible-from* :: *prog* ⇒ (*qtname* × *mdecl*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢*Method* - *in* - *dyn′-accessible′-from* - [*61*,*61*,*61*,*61*] *60*)
for *G* :: *prog* **and** *accclass* :: *qtname*
where
G⊢*membr in C dyn-accessible-from accC* ≡ *dyn-accessible-fromR G accC membr C*

| *G*⊢*Method m in C dyn-accessible-from accC* ≡ *dyn-accessible-fromR G accC* (*methdMembr m*) *C*

| *Immediate*: ⟦*G*⊢*membr member-in class*;
G⊢*membr in class permits-acc-from accclass*
⟧ ⟹ *G*⊢*membr in class dyn-accessible-from accclass*

| *Overriding*: $[\![G\vdash membr\ member\text{-}in\ class;$
$membr=(C,mdecl\ new);$
$G\vdash(C,new)\ overrides\ old;$
$G\vdash class \prec_C supr;$
$G\vdash Method\ old\ in\ supr\ dyn\text{-}accessible\text{-}from\ accclass$
$]\!] \Longrightarrow G\vdash membr\ in\ class\ dyn\text{-}accessible\text{-}from\ accclass$

syntax
methd-dyn-accessible-from::
prog ⇒ *sig* ⇒ (*qtname* × *methd*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢*Methd* - - *in* - *dyn′-accessible′-from* - [*61*,*61*,*61*,*61*,*61*] *60*)

translations
G⊢*Methd s m in C dyn-accessible-from accC*
⇌ *G*⊢(*method s m*) *in C dyn-accessible-from accC*

syntax
field-dyn-accessible-from::
prog ⇒ *vname* ⇒ (*qtname* × *field*) ⇒ *qtname* ⇒ *qtname* ⇒ *bool*
(- ⊢*Field* - - *in* - *dyn′-accessible′-from* - [*61*,*61*,*61*,*61*,*61*] *60*)

translations
G⊢*Field fn f in dynC dyn-accessible-from accC*
⇌ *G*⊢(*fieldm fn f*) *in dynC dyn-accessible-from accC*

lemma *accessible-from-commonD*: *G*⊢*m of C accessible-from S*
⟹ *G*⊢*m member-of C* ∧ *G*⊢(*Class C*) *accessible-in* (*pid S*)
by (*auto elim*: *accessible-fromR.induct*)

lemma *unique-declaration*:
⟦*G*⊢*m declared-in C*; *G*⊢*n declared-in C*; *memberid m* = *memberid n* ⟧
⟹ *m* = *n*
apply (*cases m*)
apply (*cases n*,
auto simp add: *declared-in-def cdeclaredmethd-def cdeclaredfield-def*)+
done

lemma *declared-not-undeclared*:
G⊢*m declared-in C* ⟹ ¬ *G*⊢ *memberid m undeclared-in C*
by (*cases m*) (*auto simp add*: *declared-in-def undeclared-in-def*)

lemma *undeclared-not-declared*:
G⊢ *memberid m undeclared-in C* ⟹ ¬ *G*⊢ *m declared-in C*
by (*cases m*) (*auto simp add*: *declared-in-def undeclared-in-def*)

lemma *not-undeclared-declared*:
¬ *G*⊢ *membr-id undeclared-in C* ⟹ (∃ *m*. *G*⊢*m declared-in C* ∧
membr-id = *memberid m*)
proof −
assume *not-undecl*:¬ *G*⊢ *membr-id undeclared-in C*
show *?thesis* (**is** *?P membr-id*)
proof (*cases membr-id*)
case (*fid vname*)

with *not-undecl*
obtain *fld* **where**
G⊢fdecl (vname,fld) declared-in C
by (*auto simp add*: *undeclared-in-def declared-in-def cdeclaredfield-def*)
with *fid* **show** *?thesis*
by *auto*
next
case (*mid sig*)
with *not-undecl*
obtain *mthd* **where**
G⊢mdecl (sig,mthd) declared-in C
by (*auto simp add*: *undeclared-in-def declared-in-def cdeclaredmethd-def*)
with *mid* **show** *?thesis*
by *auto*
qed
qed

lemma *unique-declared-in*:
⟦*G⊢m declared-in C*; *G⊢n declared-in C*; *memberid m = memberid n*⟧
⟹ *m = n*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def cdeclaredfield-def split*: *memberdecl.splits*)

lemma *unique-member-of*:
assumes *n*: *G⊢n member-of C* **and**
m: *G⊢m member-of C* **and**
eqid: *memberid n = memberid m*
shows *n=m*
proof −
from *n m eqid*
show *n=m*
proof (*induct*)
case (*Immediate n C*)
assume *member-n*: *G⊢ mbr n declared-in C declclass n = C*
assume *eqid*: *memberid n = memberid m*
assume *G ⊢ m member-of C*
then show *n=m*
proof (*cases*)
case (*Immediate m′ -*)
with *eqid*
have *m=m′*
memberid n = memberid m
G⊢ mbr m declared-in C
declclass m = C
by *auto*
with *member-n*
show *?thesis*
by (*cases n*, *cases m*)
(*auto simp add*: *declared-in-def cdeclaredmethd-def cdeclaredfield-def split*: *memberdecl.splits*)
next
case (*Inherited m′ - -*)
then have *G⊢ memberid m undeclared-in C*
by *simp*

with *eqid member-n*
show *?thesis*
by (*cases n*) (*auto dest*: *declared-not-undeclared*)
qed
next
case (*Inherited n C S*)
assume *undecl*: $G\vdash$ *memberid n undeclared-in C*
assume *super*: $G\vdash C \prec_{C1} S$
assume *hyp*: $[\![G \vdash m \text{ member-of } S; \text{memberid } n = \text{memberid } m]\!] \Longrightarrow n = m$
assume *eqid*: *memberid n* = *memberid m*
assume $G \vdash$ *m member-of C*
then show *n=m*
proof (*cases*)
case *Immediate*
then have $G\vdash$ *mbr m declared-in C* **by** *simp*
with *eqid undecl*
show *?thesis*
by (*cases m*) (*auto dest*: *declared-not-undeclared*)
next
case *Inherited*
with *super* **have** $G \vdash$ *m member-of S*
by (*auto dest*!: *subcls1D*)
with *eqid hyp*
show *?thesis*
by *blast*
qed
qed
qed

lemma *member-of-is-classD*: $G\vdash m$ *member-of C* $\Longrightarrow$ *is-class G C*
proof (*induct set*: *members*)
case (*Immediate m C*)
assume $G\vdash$ *mbr m declared-in C*
then show *is-class G C*
by (*cases mbr m*)
(*auto simp add*: *declared-in-def cdeclaredmethd-def cdeclaredfield-def*)
next
case (*Inherited m C S*)
assume $G\vdash C \prec_{C1} S$ **and** *is-class G S*
then show *is-class G C*
by − (*rule subcls-is-class2*,*auto*)
qed

lemma *member-of-declC*:
$G\vdash m$ *member-of C*
$\Longrightarrow$ $G\vdash$*mbr m declared-in* (*declclass m*)
by (*induct set*: *members*) *auto*

lemma *member-of-member-of-declC*:
$G\vdash m$ *member-of C*
$\Longrightarrow$ $G\vdash m$ *member-of* (*declclass m*)
by (*auto dest*: *member-of-declC intro*: *members.Immediate*)

lemma *member-of-class-relation*:
$G\vdash m$ *member-of C* $\Longrightarrow$ $G\vdash C \preceq_C$ *declclass m*

proof (*induct set*: *members*)
  case (*Immediate m C*)
  then show $G\vdash C \preceq_C$ *declclass m* **by** *simp*
next
  case (*Inherited m C S*)
  then show $G\vdash C \preceq_C$ *declclass m*
    by (*auto dest*: *r-into-rtrancl intro*: *rtrancl-trans*)
qed

lemma *member-in-class-relation*:
  $G\vdash m$ *member-in* $C \Longrightarrow G\vdash C \preceq_C$ *declclass m*
by (*auto dest*: *member-inD member-of-class-relation*
    intro: *rtrancl-trans*)

lemma *stat-override-declclasses-relation*:
$[\![G\vdash(\mathit{declclass\ new}) \prec_{C1} \mathit{superNew};\ G \vdash \mathit{Method\ old\ member\text{-}of\ superNew}\ ]\!]$
$\Longrightarrow G\vdash(\mathit{declclass\ new}) \prec_C (\mathit{declclass\ old})$
apply (*rule trancl-rtrancl-trancl*)
apply (*erule r-into-trancl*)
apply (*cases old*)
apply (*auto dest*: *member-of-class-relation*)
done

lemma *stat-overrides-commonD*:
$[\![G\vdash \mathit{new\ overrides}_S\ \mathit{old}]\!] \Longrightarrow$
  $\mathit{declclass\ new} \neq \mathit{Object} \wedge \neg\ \mathit{is\text{-}static\ new} \wedge \mathit{msig\ new} = \mathit{msig\ old} \wedge$
  $G\vdash(\mathit{declclass\ new}) \prec_C (\mathit{declclass\ old}) \wedge$
  $G\vdash\mathit{Method\ new\ declared\text{-}in}\ (\mathit{declclass\ new}) \wedge$
  $G\vdash\mathit{Method\ old\ declared\text{-}in}\ (\mathit{declclass\ old})$
apply (*induct set*: *stat-overridesR*)
apply (*frule* (*1*) *stat-override-declclasses-relation*)
apply (*auto intro*: *trancl-trans*)
done

lemma *member-of-Package*:
 $[\![G\vdash m\ \mathit{member\text{-}of}\ C;\ \mathit{accmodi}\ m = \mathit{Package}]\!]$
  $\Longrightarrow \mathit{pid}\ (\mathit{declclass}\ m) = \mathit{pid}\ C$
proof −
  assume *member*: $G\vdash m$ *member-of C*
  then show $\mathit{accmodi}\ m = \mathit{Package} \Longrightarrow\ ?\mathit{thesis}$ (**is** *PROP ?P m C*)
  proof (*induct rule*: *members.induct*)
    fix *C m*
    assume   *C*: $\mathit{declclass}\ m = C$
    then show $\mathit{pid}\ (\mathit{declclass}\ m) = \mathit{pid}\ C$
      by *simp*
  next
    fix *C S m*
    assume *inheritable*: $G \vdash m$ *inheritable-in pid C*
    assume   *hyp*: *PROP ?P m S* **and**
        package-acc: $\mathit{accmodi}\ m = \mathit{Package}$
    with *inheritable package-acc hyp*
    show $\mathit{pid}\ (\mathit{declclass}\ m) = \mathit{pid}\ C$
      by (*auto simp add*: *inheritable-in-def*)
  qed
qed

lemma *member-in-declC*: $G\vdash m$ *member-in* $C \Longrightarrow G\vdash m$ *member-in* (*declclass* m)
proof –
  assume *member-in-C*: $G\vdash m$ *member-in* C
  from *member-in-C*
  obtain *provC* **where**
    subclseq-C-provC: $G\vdash C \preceq_C$ *provC* **and**
    member-of-provC: $G \vdash m$ *member-of provC*
    by (*auto simp add*: *member-in-def*)
  from *member-of-provC*
  have $G \vdash m$ *member-of declclass* m
    by (*rule member-of-member-of-declC*)
  moreover
  from *member-in-C*
  have $G\vdash C \preceq_C$ *declclass* m
    by (*rule member-in-class-relation*)
  ultimately
  show *?thesis*
    by (*auto simp add*: *member-in-def*)
qed

lemma *dyn-accessible-from-commonD*: $G\vdash m$ *in* C *dyn-accessible-from* S
$\Longrightarrow G\vdash m$ *member-in* C
by (*auto elim*: *dyn-accessible-fromR.induct*)

lemma *no-Private-stat-override*:
$[\![G\vdash new\ overrides_S\ old]\!] \Longrightarrow accmodi\ old \neq Private$
by (*induct set*: *stat-overridesR*) (*auto simp add*: *inheritable-in-def*)

lemma *no-Private-override*: $[\![G\vdash new\ overrides\ old]\!] \Longrightarrow accmodi\ old \neq Private$
by (*induct set*: *overridesR*) (*auto simp add*: *inheritable-in-def*)

lemma *permits-acc-inheritance*:
$[\![G\vdash m$ *in statC permits-acc-from accC*; $G\vdash dynC \preceq_C statC$
$]\!] \Longrightarrow G\vdash m$ *in dynC permits-acc-from accC*
by (*cases accmodi* m)
  (*auto simp add*: *permits-acc-def*
      intro: *subclseq-trans*)

lemma *permits-acc-static-declC*:
$[\![G\vdash m$ *in* C *permits-acc-from accC*; $G\vdash m$ *member-in* C; *is-static* m
$]\!] \Longrightarrow G\vdash m$ *in* (*declclass* m) *permits-acc-from accC*
by (*cases accmodi* m) (*auto simp add*: *permits-acc-def*)

lemma *dyn-accessible-from-static-declC*:
  assumes *acc-C*: $G\vdash m$ *in* C *dyn-accessible-from accC* **and**
      static: *is-static* m
  shows $G\vdash m$ *in* (*declclass* m) *dyn-accessible-from accC*
proof –
  from *acc-C static*
  show $G\vdash m$ *in* (*declclass* m) *dyn-accessible-from accC*
  proof (*induct*)

case (*Immediate m C*)
then show *?case*
by (*auto intro*!: *dyn-accessible-fromR.Immediate*
dest: *member-in-declC permits-acc-static-declC*)
next
case (*Overriding m C declCNew new old sup*)
then have ¬ *is-static m*
by (*auto dest*: *overrides-commonD*)
moreover
assume *is-static m*
ultimately show *?case*
by *contradiction*
qed
qed

lemma *field-accessible-fromD*:
⟦*G*⊢*membr of C accessible-from accC*;*is-field membr*⟧
⟹ *G*⊢*membr member-of C* ∧
G⊢(*Class C*) *accessible-in* (*pid accC*) ∧
G⊢*membr in C permits-acc-from accC*
by (*cases set*: *accessible-fromR*)
(*auto simp add*: *is-field-def split*: *memberdecl.splits*)

lemma *field-accessible-from-permits-acc-inheritance*:
⟦*G*⊢*membr of statC accessible-from accC*; *is-field membr*; $G \vdash dynC \preceq_C statC$⟧
⟹ *G*⊢*membr in dynC permits-acc-from accC*
by (*auto dest*: *field-accessible-fromD intro*: *permits-acc-inheritance*)

lemma *accessible-fieldD*:
⟦*G*⊢*membr of C accessible-from accC*; *is-field membr*⟧
⟹ *G*⊢*membr member-of C* ∧
G⊢(*Class C*) *accessible-in* (*pid accC*) ∧
G⊢*membr in C permits-acc-from accC*
by (*induct rule*: *accessible-fromR.induct*) (*auto dest*: *is-fieldD*)

lemma *member-of-Private*:
⟦*G*⊢*m member-of C*; *accmodi m* = *Private*⟧ ⟹ *declclass m* = *C*
by (*induct set*: *members*) (*auto simp add*: *inheritable-in-def*)

lemma *member-of-subclseq-declC*:
G⊢*m member-of C* ⟹ $G \vdash C \preceq_C$ *declclass m*
by (*induct set*: *members*) (*auto dest*: *r-into-rtrancl intro*: *rtrancl-trans*)

lemma *member-of-inheritance*:
assumes *m*: *G*⊢*m member-of D* **and**
subclseq-D-C: $G \vdash D \preceq_C C$ **and**
subclseq-C-m: $G \vdash C \preceq_C$ *declclass m* **and**
ws: *ws-prog G*

shows $G \vdash m$ *member-of* C
proof –
from *m subclseq-D-C subclseq-C-m*
show *?thesis*
proof (*induct*)
case (*Immediate m D*)
assume *declclass* $m = D$ **and**
$G \vdash D \preceq_C C$ **and** $G \vdash C \preceq_C$ *declclass m*
with *ws* **have** $D = C$
by (*auto intro*: *subclseq-acyclic*)
with *Immediate*
show $G \vdash m$ *member-of* C
by (*auto intro*: *members.Immediate*)
next
case (*Inherited m D S*)
assume *member-of-D-props*:
$G \vdash m$ *inheritable-in pid D*
$G \vdash$ *memberid m undeclared-in D*
$G \vdash$ *Class S accessible-in pid D*
$G \vdash m$ *member-of S*
assume *super*: $G \vdash D \prec_{C1} S$
assume *hyp*: $[\![G \vdash S \preceq_C C;\ G \vdash C \preceq_C \textit{declclass } m]\!] \Longrightarrow G \vdash m \textit{ member-of } C$
assume *subclseq-C-m*: $G \vdash C \preceq_C$ *declclass m*
assume $G \vdash D \preceq_C C$
then show $G \vdash m$ *member-of* C
proof (*cases rule*: *subclseq-cases*)
case *Eq*
assume $D = C$
with *super member-of-D-props*
show *?thesis*
by (*auto intro*: *members.Inherited*)
next
case *Subcls*
assume $G \vdash D \prec_C C$
with *super*
have $G \vdash S \preceq_C C$
by (*auto dest*: *subcls1D subcls-superD*)
with *subclseq-C-m hyp* **show** *?thesis*
by *blast*
qed
qed
qed

lemma *member-of-subcls*:
assumes *old*: $G \vdash$ *old member-of C* **and**
new: $G \vdash$ *new member-of D* **and**
eqid: *memberid new* = *memberid old* **and**
subclseq-D-C: $G \vdash D \preceq_C C$ **and**
subcls-new-old: $G \vdash$ *declclass new* $\prec_C$ *declclass old* **and**
ws: *ws-prog G*
shows $G \vdash D \prec_C C$
proof –
from *old*
have *subclseq-C-old*: $G \vdash C \preceq_C$ *declclass old*
by (*auto dest*: *member-of-subclseq-declC*)
from *new*
have *subclseq-D-new*: $G \vdash D \preceq_C$ *declclass new*
by (*auto dest*: *member-of-subclseq-declC*)

from *subcls-new-old ws*
have *neq-new-old*: $new \neq old$
  by (*cases new*,*cases old*) (*auto dest*: *subcls-irrefl*)
from *subclseq-D-new subclseq-D-C*
have $G \vdash (declclass\ new) \preceq_C C \lor G \vdash C \preceq_C (declclass\ new)$
  by (*rule subcls-compareable*)
then have $G \vdash (declclass\ new) \preceq_C C$
proof
  assume $G \vdash declclass\ new \preceq_C C$ **then show** *?thesis* **.**
next
  assume $G \vdash C \preceq_C (declclass\ new)$
  with *new subclseq-D-C ws*
  have $G \vdash$*new member-of C*
    by (*blast intro*: *member-of-inheritance*)
  with *eqid old*
  have *new=old*
    by (*blast intro*: *unique-member-of*)
  with *neq-new-old*
  show *?thesis*
    by *contradiction*
qed
then show *?thesis*
proof (*cases rule*: *subclseq-cases*)
  case *Eq*
  assume *declclass new = C*
  with *new* **have** $G \vdash$*new member-of C*
    by (*auto dest*: *member-of-member-of-declC*)
  with *eqid old*
  have *new=old*
    by (*blast intro*: *unique-member-of*)
  with *neq-new-old*
  show *?thesis*
    by *contradiction*
next
  case *Subcls*
  assume $G \vdash declclass\ new \prec_C C$
  with *subclseq-D-new*
  show $G \vdash D \prec_C C$
    by (*rule rtrancl-trancl-trancl*)
qed
qed

corollary *member-of-overrides-subcls*:
$[\![G \vdash Methd\ sig\ old\ member\text{-}of\ C;\ G \vdash Methd\ sig\ new\ member\text{-}of\ D; G \vdash D \preceq_C C;$
$G,sig \vdash new\ overrides\ old;\ ws\text{-}prog\ G]\!]$
$\Longrightarrow G \vdash D \prec_C C$
by (*drule overrides-commonD*) (*auto intro*: *member-of-subcls*)

corollary *member-of-stat-overrides-subcls*:
$[\![G \vdash Methd\ sig\ old\ member\text{-}of\ C;\ G \vdash Methd\ sig\ new\ member\text{-}of\ D; G \vdash D \preceq_C C;$
$G,sig \vdash new\ overrides_S\ old;\ ws\text{-}prog\ G]\!]$
$\Longrightarrow G \vdash D \prec_C C$
by (*drule stat-overrides-commonD*) (*auto intro*: *member-of-subcls*)

lemma *inherited-field-access*:
  assumes *stat-acc*: $G \vdash$*membr of statC accessible-from accC* **and**

is-field: *is-field membr* **and**
subclseq: $G \vdash dynC \preceq_C statC$
shows $G \vdash$*membr in dynC dyn-accessible-from accC*
proof −
from *stat-acc is-field subclseq*
show *?thesis*
by (*auto dest*: *accessible-fieldD*
intro: *dyn-accessible-fromR.Immediate*
member-inI
permits-acc-inheritance)
qed

lemma *accessible-inheritance*:
assumes *stat-acc*: $G \vdash$*m of statC accessible-from accC* **and**
subclseq: $G \vdash dynC \preceq_C statC$ **and**
member-dynC: $G \vdash$*m member-of dynC* **and**
dynC-acc: $G \vdash$(*Class dynC*) *accessible-in* (*pid accC*)
shows $G \vdash$*m of dynC accessible-from accC*
proof −
from *stat-acc*
have *member-statC*: $G \vdash$*m member-of statC*
by (*auto dest*: *accessible-from-commonD*)
from *stat-acc*
show *?thesis*
proof (*cases*)
case *Immediate*
with *member-dynC member-statC subclseq dynC-acc*
show *?thesis*
by (*auto intro*: *accessible-fromR.Immediate permits-acc-inheritance*)
next
case *Overriding*
with *member-dynC subclseq dynC-acc*
show *?thesis*
by (*auto intro*: *accessible-fromR.Overriding rtrancl-trancl-trancl*)
qed
qed

fields and methods

types
fspec = *vname* × *qtname*

translations
fspec <= (*type*) *vname* × *qtname*

constdefs
imethds:: *prog* ⇒ *qtname* ⇒ (*sig*,*qtname* × *mhead*) *tables*
imethds G I
≡ *iface-rec* (*G*,*I*)
(λ*I i ts*. (*Un-tables ts*) ⊕⊕
(*o2s* ∘ *table-of* (*map* (λ(*s*,*m*). (*s*,*I*,*m*)) (*imethods i*))))

methods of an interface, with overriding and inheritance, cf. 9.2

constdefs
accimethds:: *prog* ⇒ *pname* ⇒ *qtname* ⇒ (*sig*,*qtname* × *mhead*) *tables*
accimethds G pack I
≡ *if* $G \vdash$*Iface I accessible-in pack*
then imethds G I

else λ *k.* {}

only returns imethds if the interface is accessible

constdefs
methd:: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ (*sig*,*qtname* $\times$ *methd*) *table*

methd G C
$\equiv$ *class-rec* (*G*,*C*) *empty*
 (λ*C c subcls-mthds.*
 filter-tab (λ*sig m. G*$\vdash$*C inherits method sig m*)
 subcls-mthds
 ++
 table-of (*map* (λ(*s*,*m*). (*s*,*C*,*m*)) (*methods c*)))

methd G C: methods of a class C (statically visible from C), with inheritance and hiding cf. 8.4.6; Overriding is captured by *dynmethd*. Every new method with the same signature coalesces the method of a superclass.

constdefs
accmethd:: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname* $\Rightarrow$ (*sig*,*qtname* $\times$ *methd*) *table*
accmethd G S C
$\equiv$ *filter-tab* (λ*sig m. G*$\vdash$*method sig m of C accessible-from S*)
 (*methd G C*)

accmethd G S C: only those methods of *methd G C*, accessible from S

Note the class component in the accessibility filter. The class where method *m* is declared (*declC*) isn't necessarily accessible from the current scope *S*. The method can be made accessible through inheritance, too. So we must test accessibility of method *m* of class *C* (not *declclass m*)

constdefs
dynmethd:: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname* $\Rightarrow$ (*sig*,*qtname* $\times$ *methd*) *table*
dynmethd G statC dynC
$\equiv$ λ *sig.*
 (*if G*$\vdash$*dynC* $\preceq_C$ *statC*
 then (*case methd G statC sig of*
 None $\Rightarrow$ *None*
 | *Some statM*
 $\Rightarrow$ (*class-rec* (*G*,*dynC*) *empty*
 (λ*C c subcls-mthds.*
 subcls-mthds
 ++
 (*filter-tab*
 (λ - *dynM. G*,*sig*$\vdash$*dynM overrides statM* $\vee$ *dynM*=*statM*)
 (*methd G C*)))
) *sig*
)
 else None)

dynmethd G statC dynC: dynamic method lookup of a reference with dynamic class *dynC* and static class *statC*

Note some kind of duality between *methd* and *dynmethd* in the *class-rec* arguments. Whereas *methd* filters the subclass methods (to get only the inherited ones), *dynmethd* filters the new methods (to get only those methods which actually override the methods of the static class)

constdefs
dynimethd:: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname* $\Rightarrow$ (*sig*,*qtname* $\times$ *methd*) *table*
dynimethd G I dynC
$\equiv$ λ *sig. if imethds G I sig* $\neq$ {}

then methd G dynC sig
else dynmethd G Object dynC sig

dynimethd G I dynC: dynamic method lookup of a reference with dynamic class dynC and static interface type I

When calling an interface method, we must distinguish if the method signature was defined in the interface or if it must be an Object method in the other case. If it was an interface method we search the class hierarchy starting at the dynamic class of the object up to Object to find the first matching method (*methd*). Since all interface methods have public access the method can't be coalesced due to some odd visibility effects like in case of dynmethd. The method will be inherited or overridden in all classes from the first class implementing the interface down to the actual dynamic class.

constdefs
dynlookup::*prog* $\Rightarrow$ *ref-ty* $\Rightarrow$ *qtname* $\Rightarrow$ (*sig*,*qtname* $\times$ *methd*) *table*
dynlookup G statT dynC
$\equiv$ (*case statT of*
NullT $\Rightarrow$ *empty*
| *IfaceT I* $\Rightarrow$ *dynimethd G I dynC*
| *ClassT statC* $\Rightarrow$ *dynmethd G statC dynC*
| *ArrayT ty* $\Rightarrow$ *dynmethd G Object dynC*)

dynlookup G statT dynC: dynamic lookup of a method within the static reference type statT and the dynamic class dynC. In a wellformd context statT will not be NullT and in case statT is an array type, dynC=Object

constdefs
fields:: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ ((*vname* $\times$ *qtname*) $\times$ *field*) *list*
fields G C
$\equiv$ *class-rec* (*G*,*C*) [] (λ*C c ts*. *map* (λ(*n*,*t*). ((*n*,*C*),*t*)) (*cfields c*) @ *ts*)

DeclConcepts.fields G C list of fields of a class, including all the fields of the superclasses (private, inherited and hidden ones) not only the accessible ones (an instance of a object allocates all these fields

constdefs
accfield:: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname* $\Rightarrow$ (*vname*, *qtname* $\times$ *field*) *table*
accfield G S C
$\equiv$ *let field-tab* = *table-of*((*map* (λ((*n*,*d*),*f*).(*n*,(*d*,*f*)))) (*fields G C*))
in filter-tab (λ*n* (*declC*,*f*). *G*$\vdash$ (*declC*,*fdecl* (*n*,*f*)) *of C accessible-from S*)
field-tab

accfield G C S: fields of a class *C* which are accessible from scope of class *S* with inheritance and hiding, cf. 8.3

note the class component in the accessibility filter (see also *methd*). The class declaring field *f* (*declC*) isn't necessarily accessible from scope *S*. The field can be made visible through inheritance, too. So we must test accessibility of field *f* of class *C* (not *declclass f*)

constdefs

is-methd :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *sig* $\Rightarrow$ *bool*
is-methd G $\equiv$ λ*C sig*. *is-class G C* $\wedge$ *methd G C sig* $\neq$ *None*

constdefs *efname*:: ((*vname* $\times$ *qtname*) $\times$ *field*) $\Rightarrow$ (*vname* $\times$ *qtname*)
efname $\equiv$ *fst*

lemma *efname-simp*[*simp*]:*efname* (*n*,*f*) = *n*
by (*simp add*: *efname-def*)

19 imethds

lemma *imethds-rec*: $\llbracket$*iface G I* = *Some i*; *ws-prog G*$\rrbracket \Longrightarrow$
  imethds G I = *Un-tables* ((λJ. *imethds G J*)‘*set* (*isuperIfs i*)) $\oplus\oplus$
    (*o2s* $\circ$ *table-of* (*map* (λ(*s*,*mh*). (*s*,*I*,*mh*)) (*imethods i*)))
apply (*unfold imethds-def*)
apply (*rule iface-rec* [*THEN trans*])
apply *auto*
done

lemma *imethds-norec*:
  $\llbracket$*iface G md* = *Some i*; *ws-prog G*; *table-of* (*imethods i*) *sig* = *Some mh*$\rrbracket \Longrightarrow$
  (*md*, *mh*) $\in$ *imethds G md sig*
apply (*subst imethds-rec*)
apply *assumption*+
apply (*rule iffD2*)
apply (*rule overrides-t-Some-iff*)
apply (*rule disjI1*)
apply (*auto elim*: *table-of-map-SomeI*)
done

lemma *imethds-declI*: $\llbracket$*m* $\in$ *imethds G I sig*; *ws-prog G*; *is-iface G I*$\rrbracket \Longrightarrow$
  ($\exists$ *i*. *iface G* (*decliface m*) = *Some i* $\wedge$
  table-of (*imethods i*) *sig* = *Some* (*mthd m*)) $\wedge$
  (*I*,*decliface m*) $\in$ (*subint1 G*)^$*$ $\wedge$ *m* $\in$ *imethds G* (*decliface m*) *sig*
apply (*erule rev-mp*)
apply (*rule ws-subint1-induct*, *assumption*, *assumption*)
apply (*subst imethds-rec*, *erule conjunct1*, *assumption*)
apply (*force elim*: *imethds-norec intro*: *rtrancl-into-rtrancl2*)
done

lemma *imethds-cases* [*consumes 3*, *case-names NewMethod InheritedMethod*]:
  assumes *im*: *im* $\in$ *imethds G I sig* **and**
    ifI: *iface G I* = *Some i* **and**
    ws: *ws-prog G* **and**
   hyp-new: *table-of* (*map* (λ(*s*, *mh*). (*s*, *I*, *mh*)) (*imethods i*)) *sig*
     = *Some im* $\Longrightarrow$ *P* **and**
   hyp-inh: $\bigwedge$ *J*. $\llbracket$*J* $\in$ *set* (*isuperIfs i*); *im* $\in$ *imethds G J sig*$\rrbracket \Longrightarrow$ *P*
  shows *P*
proof –
  from *ifI ws im hyp-new hyp-inh*
  show *P*
   by (*auto simp add*: *imethds-rec*)
qed

20 accimethd

lemma *accimethds-simp* [*simp*]:
G$\vdash$*Iface I accessible-in pack* $\Longrightarrow$ *accimethds G pack I* = *imethds G I*
by (*simp add*: *accimethds-def*)

lemma *accimethdsD*:
 im $\in$ *accimethds G pack I sig*

$\Longrightarrow$ *im* $\in$ *imethds G I sig* $\wedge$ *G*$\vdash$*Iface I accessible-in pack*
by (*auto simp add*: *accimethds-def*)

lemma *accimethdsI*:
$[\![$*im* $\in$ *imethds G I sig*;*G*$\vdash$*Iface I accessible-in pack*$]\!]$
$\Longrightarrow$ *im* $\in$ *accimethds G pack I sig*
by *simp*

21 methd

lemma *methd-rec*: $[\![$*class G C* = *Some c*; *ws-prog G*$]\!]$ $\Longrightarrow$
methd G C
= (*if C* = *Object*
then empty
else filter-tab (λ*sig m*. *G*$\vdash$*C inherits method sig m*)
(*methd G* (*super c*)))
++ *table-of* (*map* (λ(*s*,*m*). (*s*,*C*,*m*)) (*methods c*))
apply (*unfold methd-def*)
apply (*erule class-rec* [*THEN trans*], *assumption*)
apply (*simp*)
done

lemma *methd-norec*:
$[\![$*class G declC* = *Some c*; *ws-prog G*;*table-of* (*methods c*) *sig* = *Some m*$]\!]$
$\Longrightarrow$ *methd G declC sig* = *Some* (*declC*, *m*)
apply (*simp only*: *methd-rec*)
apply (*rule disjI1* [*THEN map-add-Some-iff* [*THEN iffD2*]])
apply (*auto elim*: *table-of-map-SomeI*)
done

lemma *methd-declC*:
$[\![$*methd G C sig* = *Some m*; *ws-prog G*;*is-class G C*$]\!]$ $\Longrightarrow$
($\exists$ *d*. *class G* (*declclass m*)=*Some d* $\wedge$ *table-of* (*methods d*) *sig*=*Some* (*mthd m*)) $\wedge$
G$\vdash$*C* $\preceq_C$ (*declclass m*) $\wedge$ *methd G* (*declclass m*) *sig* = *Some m*
apply (*erule rev-mp*)
apply (*rule ws-subcls1-induct*, *assumption*, *assumption*)
apply (*subst methd-rec*, *assumption*)
apply (*case-tac Ca*=*Object*)
apply (*force elim*: *methd-norec*)

apply *simp*
apply (*case-tac table-of* (*map* (λ(*s*, *m*). (*s*, *Ca*, *m*)) (*methods c*)) *sig*)
apply (*force intro*: *rtrancl-into-rtrancl2*)

apply (*auto intro*: *methd-norec*)
done

lemma *methd-inheritedD*:
$[\![$*class G C* = *Some c*; *ws-prog G*;*methd G C sig* = *Some m*$]\!]$
$\Longrightarrow$ (*declclass m* $\neq$ *C* $\longrightarrow$ *G* $\vdash$*C inherits method sig m*)
by (*auto simp add*: *methd-rec*)

lemma *methd-diff-cls*:
⟦*ws-prog G*; *is-class G C*; *is-class G D*;
methd G C sig = *m*; *methd G D sig* = *n*; *m*≠*n*
⟧ ⟹ *C*≠*D*
by (*auto simp add*: *methd-rec*)

lemma *method-declared-inI*:
⟦*table-of* (*methods c*) *sig* = *Some m*; *class G C* = *Some c*⟧
⟹ *G*⊢*mdecl* (*sig*,*m*) *declared-in C*
by (*auto simp add*: *cdeclaredmethd-def declared-in-def*)

lemma *methd-declared-in-declclass*:
⟦*methd G C sig* = *Some m*; *ws-prog G*;*is-class G C*⟧
⟹ *G*⊢*Methd sig m declared-in* (*declclass m*)
by (*auto dest*: *methd-declC method-declared-inI*)

lemma *member-methd*:
assumes *member-of*: *G*⊢*Methd sig m member-of C* **and**
ws: *ws-prog G*
shows *methd G C sig* = *Some m*
proof −
from *member-of*
have *iscls-C*: *is-class G C*
by (*rule member-of-is-classD*)
from *iscls-C ws member-of*
show *?thesis* (**is** *?Methd C*)
proof (*induct rule*: *ws-class-induct′*)
case (*Object co*)
assume *G* ⊢*Methd sig m member-of Object*
then have *G*⊢*Methd sig m declared-in Object* ∧ *declclass m* = *Object*
by (*cases set*: *members*) (*cases m*, *auto dest*: *subcls1D*)
with *ws Object*
show *?Methd Object*
by (*cases m*)
(*auto simp add*: *declared-in-def cdeclaredmethd-def methd-rec*
intro: *table-of-mapconst-SomeI*)
next
case (*Subcls C c*)
assume *clsC*: *class G C* = *Some c* **and**
neq-C-Obj: *C* ≠ *Object* **and**
hyp: *G* ⊢*Methd sig m member-of super c* ⟹ *?Methd* (*super c*) **and**
member-of: *G* ⊢*Methd sig m member-of C*
from *member-of*
show *?Methd C*
proof (*cases*)
case (*Immediate membr Ca*)
then have *Ca*=*C membr* = *method sig m* **and**
G⊢*Methd sig m declared-in C declclass m* = *C*
by (*cases m*,*auto*)
with *clsC*
have *table-of* (*map* (λ(*s*, *m*). (*s*, *C*, *m*)) (*methods c*)) *sig* = *Some m*
by (*cases m*)
(*auto simp add*: *declared-in-def cdeclaredmethd-def*
intro: *table-of-mapconst-SomeI*)
with *clsC neq-C-Obj ws*
show *?thesis*

    by (*simp add*: *methd-rec*)
  next
    case (*Inherited membr Ca S*)
    with *clsC*
    have *eq-Ca-C*: *Ca=C* **and**
      undecl: *G⊢mid sig undeclared-in C* **and**
      super: *G ⊢Methd sig m member-of* (*super c*)
    by (*auto dest*: *subcls1D*)
    from *eq-Ca-C clsC undecl*
    have *table-of* (*map* (λ(*s*, *m*). (*s*, *C*, *m*)) (*methods c*)) *sig* = *None*
      by (*auto simp add*: *undeclared-in-def cdeclaredmethd-def*
        intro: *table-of-mapconst-NoneI*)
    moreover
    from *Inherited* **have** *G* ⊢ *C inherits* (*method sig m*)
      by (*auto simp add*: *inherits-def*)
    moreover
    note *clsC neq-C-Obj ws super hyp*
    ultimately
    show *?thesis*
      by (*auto simp add*: *methd-rec intro*: *filter-tab-SomeI*)
  qed
 qed
qed

lemma *finite-methd*:*ws-prog G* ⟹ *finite* {*methd G C sig* |*sig C*. *is-class G C*}
apply (*rule finite-is-class* [*THEN finite-SetCompr2*])
apply (*intro strip*)
apply (*erule-tac ws-subcls1-induct*, *assumption*)
apply (*subst methd-rec*)
apply (*assumption*)
apply (*auto intro*!: *finite-range-map-of finite-range-filter-tab finite-range-map-of-map-add*)
done

lemma *finite-dom-methd*:
⟦*ws-prog G*; *is-class G C*⟧ ⟹ *finite* (*dom* (*methd G C*))
apply (*erule-tac ws-subcls1-induct*)
apply *assumption*
apply (*subst methd-rec*)
apply (*assumption*)
apply (*auto intro*!: *finite-dom-map-of finite-dom-filter-tab*)
done

22 accmethd

lemma *accmethd-SomeD*:
accmethd G S C sig = *Some m*
⟹ *methd G C sig* = *Some m* ∧ *G⊢method sig m of C accessible-from S*
by (*auto simp add*: *accmethd-def dest*: *filter-tab-SomeD*)

lemma *accmethd-SomeI*:
⟦*methd G C sig* = *Some m*; *G⊢method sig m of C accessible-from S*⟧
⟹ *accmethd G S C sig* = *Some m*
by (*auto simp add*: *accmethd-def intro*: *filter-tab-SomeI*)

lemma *accmethd-declC*:
$[\![accmethd\ G\ S\ C\ sig = Some\ m;\ ws\text{-}prog\ G;\ is\text{-}class\ G\ C]\!] \Longrightarrow$
$(\exists\, d.\ class\ G\ (declclass\ m){=}Some\ d \wedge$
$table\text{-}of\ (methods\ d)\ sig{=}Some\ (mthd\ m)) \wedge$
$G{\vdash}C \preceq_C (declclass\ m) \wedge methd\ G\ (declclass\ m)\ sig = Some\ m \wedge$
$G{\vdash}method\ sig\ m\ of\ C\ accessible\text{-}from\ S$
by (*auto dest*: *accmethd-SomeD methd-declC accmethd-SomeI*)

lemma *finite-dom-accmethd*:
$[\![ws\text{-}prog\ G;\ is\text{-}class\ G\ C]\!] \Longrightarrow finite\ (dom\ (accmethd\ G\ S\ C))$
by (*auto simp add*: *accmethd-def intro*: *finite-dom-filter-tab finite-dom-methd*)

23 dynmethd

lemma *dynmethd-rec*:
$[\![class\ G\ dynC = Some\ c;\ ws\text{-}prog\ G]\!] \Longrightarrow$
dynmethd G statC dynC sig
$\quad = (if\ G{\vdash}dynC \preceq_C statC$
$\qquad then\ (case\ methd\ G\ statC\ sig\ of$
$\qquad\qquad None \Rightarrow None$
$\qquad\qquad |\ Some\ statM$
$\qquad\qquad\quad \Rightarrow (case\ methd\ G\ dynC\ sig\ of$
$\qquad\qquad\qquad None \Rightarrow dynmethd\ G\ statC\ (super\ c)\ sig$
$\qquad\qquad\qquad |\ Some\ dynM \Rightarrow$
$\qquad\qquad\qquad\quad (if\ G{,}sig{\vdash}\ dynM\ overrides\ statM \vee dynM = statM$
$\qquad\qquad\qquad\qquad then\ Some\ dynM$
$\qquad\qquad\qquad\qquad else\ (dynmethd\ G\ statC\ (super\ c)\ sig)$
$\qquad\qquad\qquad)))$
$\qquad else\ None)$
$\quad$(**is** - $\Longrightarrow$ - $\Longrightarrow$ *?Dynmethd-def dynC sig* = *?Dynmethd-rec dynC c sig*)
proof −
  assume *clsDynC*: *class G dynC = Some c* **and**
        ws: *ws-prog G*
  then show *?Dynmethd-def dynC sig* = *?Dynmethd-rec dynC c sig*
  proof (*induct rule*: *ws-class-induct''*)
    case (*Object co*)
    show *?Dynmethd-def Object sig* = *?Dynmethd-rec Object co sig*
    proof (*cases* $G{\vdash}Object \preceq_C statC$)
      case *False*
      then show *?thesis* **by** (*simp add*: *dynmethd-def*)
    next
      case *True*
      then have *eq-statC-Obj*: *statC = Object* **..**
      show *?thesis*
      proof (*cases methd G statC sig*)
        case *None* **then show** *?thesis* **by** (*simp add*: *dynmethd-def*)
      next
        case *Some*
        with *True Object ws eq-statC-Obj*
        show *?thesis*
          by (*auto simp add*: *dynmethd-def class-rec*
                intro: *filter-tab-SomeI*)
      qed
    qed
  next
    case (*Subcls dynC c sc*)
    show *?Dynmethd-def dynC sig* = *?Dynmethd-rec dynC c sig*

```
proof (cases G⊢dynC ≼C statC)
  case False
  then show ?thesis by (simp add: dynmethd-def)
next
  case True
  note subclseq-dynC-statC = True
  show ?thesis
  proof (cases methd G statC sig)
    case None then show ?thesis by (simp add: dynmethd-def)
  next
    case (Some statM)
    note statM = Some
    let ?filter C =
          filter-tab
            (λ- dynM. G,sig ⊢ dynM overrides statM ∨ dynM = statM)
            (methd G C)
    let ?class-rec C =
          (class-rec (G, C) empty
                       (λC c subcls-mthds. subcls-mthds ++ (?filter C)))
    from statM Subcls ws subclseq-dynC-statC
    have dynmethd-dynC-def:
         ?Dynmethd-def dynC sig =
           ((?class-rec (super c))
            ++
           (?filter dynC)) sig
     by (simp (no-asm-simp) only: dynmethd-def class-rec)
        auto
   show ?thesis
   proof (cases dynC = statC)
     case True
     with subclseq-dynC-statC statM dynmethd-dynC-def
     have ?Dynmethd-def dynC sig = Some statM
       by (auto intro: map-add-find-right filter-tab-SomeI)
     with subclseq-dynC-statC True Some
     show ?thesis
       by auto
   next
     case False
     with subclseq-dynC-statC Subcls
     have subclseq-super-statC: G⊢(super c) ≼C statC
       by (blast dest: subclseq-superD)
     show ?thesis
     proof (cases methd G dynC sig)
       case None
       then have ?filter dynC sig = None
         by (rule filter-tab-None)
       then have ?Dynmethd-def dynC sig=?class-rec (super c) sig
         by (simp add: dynmethd-dynC-def)
       with  subclseq-super-statC statM None
       have ?Dynmethd-def dynC sig = ?Dynmethd-def (super c) sig
         by (auto simp add: empty-def dynmethd-def)
       with None subclseq-dynC-statC statM
       show ?thesis
         by simp
     next
       case (Some dynM)
       note dynM = Some
       let ?Termination = G ⊢ qmdecl sig dynM overrides qmdecl sig statM ∨
                          dynM = statM
```

show *?thesis*
proof (*cases ?filter dynC sig*)
case *None*
with *dynM*
have *no-termination*: ¬ *?Termination*
by (*simp add*: *filter-tab-def*)
from *None*
have *?Dynmethd-def dynC sig=?class-rec* (*super c*) *sig*
by (*simp add*: *dynmethd-dynC-def*)
with *subclseq-super-statC statM dynM no-termination*
show *?thesis*
by (*auto simp add*: *empty-def dynmethd-def*)
next
case *Some*
with *dynM*
have *termination*: *?Termination*
by (*auto*)
with *Some dynM*
have *?Dynmethd-def dynC sig=Some dynM*
by (*auto simp add*: *dynmethd-dynC-def*)
with *subclseq-super-statC statM dynM termination*
show *?thesis*
by (*auto simp add*: *dynmethd-def*)
qed
qed
qed
qed
qed
qed
qed

lemma *dynmethd-C-C*:⟦*is-class G C*; *ws-prog G*⟧
$\Longrightarrow$ *dynmethd G C C sig* = *methd G C sig*
apply (*auto simp add*: *dynmethd-rec*)
done

lemma *dynmethdSomeD*:
⟦*dynmethd G statC dynC sig* = *Some dynM*; *is-class G dynC*; *ws-prog G*⟧
$\Longrightarrow$ *G*⊢*dynC* $\preceq_C$ *statC* ∧ (∃ *statM*. *methd G statC sig* = *Some statM*)
by (*auto simp add*: *dynmethd-rec*)

lemma *dynmethd-Some-cases* [*consumes 3*, *case-names Static Overrides*]:
assumes *dynM*: *dynmethd G statC dynC sig* = *Some dynM* **and**
is-cls-dynC: *is-class G dynC* **and**
ws: *ws-prog G* **and**
hyp-static: *methd G statC sig* = *Some dynM* $\Longrightarrow$ *P* **and**
hyp-override: $\bigwedge$ *statM*. ⟦*methd G statC sig* = *Some statM*;*dynM*≠*statM*;
G,*sig*⊢*dynM overrides statM*⟧ $\Longrightarrow$ *P*
shows *P*
proof −
from *is-cls-dynC* **obtain** *dc* **where** *clsDynC*: *class G dynC* = *Some dc* **by** *blast*
from *clsDynC ws dynM hyp-static hyp-override*
show *P*
proof (*induct rule*: *ws-class-induct*)
case (*Object co*)
with *ws* **have** *statC* = *Object*

by (*auto simp add*: *dynmethd-rec*)
with *ws Object* **show** *?thesis* **by** (*auto simp add*: *dynmethd-C-C*)
next
case (*Subcls C c*)
with *ws* **show** *?thesis*
by (*auto simp add*: *dynmethd-rec*)
qed
qed

lemma *no-override-in-Object*:
assumes *dynM*: *dynmethd G statC dynC sig = Some dynM* **and**
is-cls-dynC: *is-class G dynC* **and**
ws: *ws-prog G* **and**
statM: *methd G statC sig = Some statM* **and**
neq-dynM-statM: $dynM \neq statM$
shows $dynC \neq Object$
proof −
from *is-cls-dynC* **obtain** *dc* **where** *clsDynC*: *class G dynC = Some dc* **by** *blast*
from *clsDynC ws dynM statM neq-dynM-statM*
show *?thesis* (**is** *?P dynC*)
proof (*induct rule*: *ws-class-induct*)
case (*Object co*)
with *ws* **have** *statC = Object*
by (*auto simp add*: *dynmethd-rec*)
with *ws Object* **show** *?P Object* **by** (*auto simp add*: *dynmethd-C-C*)
next
case (*Subcls dynC c*)
with *ws* **show** *?P dynC*
by (*auto simp add*: *dynmethd-rec*)
qed
qed

lemma *dynmethd-Some-rec-cases* [*consumes 3*,
case-names Static Override Recursion]:
assumes *dynM*: *dynmethd G statC dynC sig = Some dynM* **and**
clsDynC: *class G dynC = Some c* **and**
ws: *ws-prog G* **and**
hyp-static: *methd G statC sig = Some dynM* $\Longrightarrow$ *P* **and**
hyp-override: $\bigwedge$ *statM*. $[\![$*methd G statC sig = Some statM*;
methd G dynC sig = Some dynM; $statM \neq dynM$;
G,sig$\vdash$ *dynM overrides statM*$]\!]$ $\Longrightarrow$ *P* **and**

hyp-recursion: $[\![dynC \neq Object$;
dynmethd G statC (*super c*) *sig = Some dynM*$]\!]$ $\Longrightarrow$ *P*
shows *P*
proof −
from *clsDynC* **have** *is-class G dynC* **by** *simp*
note *no-override-in-Object′ = no-override-in-Object* [*OF dynM this ws*]
from *ws clsDynC dynM hyp-static hyp-override hyp-recursion*
show *?thesis*
by (*auto simp add*: *dynmethd-rec dest*: *no-override-in-Object′*)
qed

lemma *dynmethd-declC*:
$[\![$*dynmethd G statC dynC sig = Some m*;

is-class G statC;*ws-prog G*
⟧ $\Longrightarrow$
($\exists$ *d*. *class G* (*declclass m*)=*Some d* $\wedge$ *table-of* (*methods d*) *sig*=*Some* (*mthd m*)) $\wedge$
G$\vdash$*dynC* $\preceq_C$ (*declclass m*) $\wedge$ *methd G* (*declclass m*) *sig* = *Some m*
proof −
assume *is-cls-statC*: *is-class G statC*
assume *ws*: *ws-prog G*
assume *m*: *dynmethd G statC dynC sig* = *Some m*
from *m*
have *G*$\vdash$*dynC* $\preceq_C$ *statC* **by** (*auto simp add*: *dynmethd-def*)
from *this is-cls-statC*
have *is-cls-dynC*: *is-class G dynC* **by** (*rule subcls-is-class2*)
from *is-cls-dynC ws m*
show *?thesis* (**is** *?P dynC*)
proof (*induct rule*: *ws-class-induct'*)
case (*Object co*)
with *ws* **have** *statC*=*Object* **by** (*auto simp add*: *dynmethd-rec*)
with *ws Object*
show *?P Object*
by (*auto simp add*: *dynmethd-C-C dest*: *methd-declC*)
next
case (*Subcls dynC c*)
assume *hyp*: *dynmethd G statC* (*super c*) *sig* = *Some m* $\Longrightarrow$ *?P* (*super c*) **and**
clsDynC: *class G dynC* = *Some c* **and**
m': *dynmethd G statC dynC sig* = *Some m* **and**
neq-dynC-Obj: *dynC* $\neq$ *Object*
from *ws this* **obtain** *statM* **where**
subclseq-dynC-statC: *G*$\vdash$*dynC* $\preceq_C$ *statC* **and**
statM: *methd G statC sig* = *Some statM*
by (*blast dest*: *dynmethdSomeD*)
from *clsDynC neq-dynC-Obj*
have *subclseq-dynC-super*: *G*$\vdash$*dynC* $\preceq_C$ (*super c*)
by (*auto intro*: *subcls1I*)
from *m' clsDynC ws*
show *?P dynC*
proof (*cases rule*: *dynmethd-Some-rec-cases*)
case *Static*
with *is-cls-statC ws subclseq-dynC-statC*
show *?thesis*
by (*auto intro*: *rtrancl-trans dest*: *methd-declC*)
next
case *Override*
with *clsDynC ws*
show *?thesis*
by (*auto dest*: *methd-declC*)
next
case *Recursion*
with *hyp subclseq-dynC-super*
show *?thesis*
by (*auto intro*: *rtrancl-trans*)
qed
qed
qed

lemma *methd-Some-dynmethd-Some*:
assumes *statM*: *methd G statC sig* = *Some statM* **and**
subclseq: *G*$\vdash$*dynC* $\preceq_C$ *statC* **and**
is-cls-statC: *is-class G statC* **and**

```
            ws: ws-prog G
  shows ∃ dynM. dynmethd G statC dynC sig = Some dynM
    (is ?P dynC)
proof −
  from subclseq is-cls-statC
  have is-cls-dynC: is-class G dynC by (rule subcls-is-class2)
  then obtain dc where
    clsDynC: class G dynC = Some dc by blast
  from clsDynC ws subclseq
  show ?thesis
  proof (induct rule: ws-class-induct)
    case (Object co)
    with ws  have statC = Object
      by (auto)
    with ws Object statM
    show ?P Object
      by (auto simp add: dynmethd-C-C)
  next
    case (Subcls dynC dc)
    assume clsDynC': class G dynC = Some dc
    assume neq-dynC-Obj: dynC ≠ Object
    assume hyp: G⊢super dc≼C statC ⟹ ?P (super dc)
    assume subclseq': G⊢dynC≼C statC
    then
    show ?P dynC
    proof (cases rule: subclseq-cases)
      case Eq
      with ws statM clsDynC'
      show ?thesis
        by (auto simp add: dynmethd-rec)
    next
      case Subcls
      assume G⊢dynC≺C statC
      from this clsDynC'
      have G⊢super dc≼C statC by (rule subcls-superD)
      with hyp ws clsDynC' subclseq' statM
      show ?thesis
        by (auto simp add: dynmethd-rec)
    qed
  qed
qed


lemma dynmethd-cases [consumes 4, case-names Static Overrides]:
  assumes     statM: methd G statC sig = Some statM and
           subclseq: G⊢dynC ≼C statC and
        is-cls-statC: is-class G statC and
                 ws: ws-prog G and
         hyp-static: dynmethd G statC dynC sig = Some statM ⟹ P and
       hyp-override: ⋀ dynM. ⟦dynmethd G statC dynC sig = Some dynM;
                                 dynM≠statM;
                            G,sig⊢dynM overrides statM⟧ ⟹ P
  shows P
proof −
  from subclseq is-cls-statC
  have is-cls-dynC: is-class G dynC by (rule subcls-is-class2)
  then obtain dc where
    clsDynC: class G dynC = Some dc by blast
  from statM subclseq is-cls-statC ws
```

obtain *dynM*
where *dynM*: *dynmethd G statC dynC sig = Some dynM*
by (*blast dest*: *methd-Some-dynmethd-Some*)
from *dynM is-cls-dynC ws*
show *?thesis*
proof (*cases rule*: *dynmethd-Some-cases*)
case *Static*
with *hyp-static dynM statM* **show** *?thesis* **by** *simp*
next
case *Overrides*
with *hyp-override dynM statM* **show** *?thesis* **by** *simp*
qed
qed

lemma *ws-dynmethd*:
assumes *statM*: *methd G statC sig = Some statM* **and**
subclseq: $G \vdash dynC \preceq_C statC$ **and**
is-cls-statC: *is-class G statC* **and**
ws: *ws-prog G*
shows
$\exists$ *dynM*. *dynmethd G statC dynC sig = Some dynM* $\wedge$
is-static dynM = is-static statM $\wedge$ $G \vdash$*resTy dynM*$\preceq$*resTy statM*
proof −
from *statM subclseq is-cls-statC ws*
show *?thesis*
proof (*cases rule*: *dynmethd-cases*)
case *Static*
with *statM*
show *?thesis*
by *simp*
next
case *Overrides*
with *ws*
show *?thesis*
by (*auto dest*: *ws-overrides-commonD*)
qed
qed

24 dynlookup

lemma *dynlookup-cases* [*consumes 1*, *case-names NullT IfaceT ClassT ArrayT*]:
⟦*dynlookup G statT dynC sig = x*;
⟦*statT = NullT* ; *empty sig = x* ⟧ ⟹ *P*;
⋀ *I*. ⟦*statT = IfaceT I* ; *dynimethd G I dynC sig = x*⟧ ⟹ *P*;
⋀ *statC*.⟦*statT = ClassT statC*; *dynmethd G statC dynC sig = x*⟧ ⟹ *P*;
⋀ *ty*. ⟦*statT = ArrayT ty* ; *dynmethd G Object dynC sig = x*⟧ ⟹ *P*
⟧ ⟹ *P*
by (*cases statT*) (*auto simp add*: *dynlookup-def*)

25 fields

lemma *fields-rec*: ⟦*class G C = Some c*; *ws-prog G*⟧ ⟹
fields G C = map (λ(*fn*,*ft*). ((*fn*,*C*),*ft*)) (*cfields c*) @
(*if C = Object then* [] *else fields G* (*super c*))
apply (*simp only*: *fields-def*)
apply (*erule class-rec* [*THEN trans*])
apply *assumption*
apply *clarsimp*

done

lemma *fields-norec*:
$[\![$*class G fd* = *Some c*; *ws-prog G*; *table-of* (*cfields c*) *fn* = *Some f*$]\!]$
$\Longrightarrow$ *table-of* (*fields G fd*) (*fn*,*fd*) = *Some f*
apply (*subst fields-rec*)
apply *assumption+*
apply (*subst map-of-append*)
apply (*rule disjI1* [*THEN map-add-Some-iff* [*THEN iffD2*]])
apply (*auto elim*: *table-of-map2-SomeI*)
done

lemma *table-of-fieldsD*:
table-of (*map* (λ(*fn*,*ft*). ((*fn*,*C*),*ft*)) (*cfields c*)) *efn* = *Some f*
$\Longrightarrow$ (*declclassf efn*) = *C* $\wedge$ *table-of* (*cfields c*) (*fname efn*) = *Some f*
apply (*case-tac efn*)
by *auto*

lemma *fields-declC*:
$[\![$*table-of* (*fields G C*) *efn* = *Some f*; *ws-prog G*; *is-class G C*$]\!]$ $\Longrightarrow$
($\exists$ *d*. *class G* (*declclassf efn*) = *Some d* $\wedge$
table-of (*cfields d*) (*fname efn*)=*Some f*) $\wedge$
G$\vdash$*C* $\preceq_C$ (*declclassf efn*) $\wedge$ *table-of* (*fields G* (*declclassf efn*)) *efn* = *Some f*
apply (*erule rev-mp*)
apply (*rule ws-subcls1-induct*, *assumption*, *assumption*)
apply (*subst fields-rec*, *assumption*)
apply *clarify*
apply (*simp only*: *map-of-append*)
apply (*case-tac table-of* (*map* (*split* (λ*fn*. *Pair* (*fn*, *Ca*))) (*cfields c*)) *efn*)
apply (*force intro*:*rtrancl-into-rtrancl2 simp add*: *map-add-def*)

apply (*frule-tac fd*=*Ca* **in** *fields-norec*)
apply *assumption*
apply *blast*
apply (*frule table-of-fieldsD*)
apply (*frule-tac n*=*table-of* (*map* (*split* (λ*fn*. *Pair* (*fn*, *Ca*))) (*cfields c*))
and *m*=*table-of* (*if Ca* = *Object then* [] *else fields G* (*super c*))
in *map-add-find-right*)
apply (*case-tac efn*)
apply (*simp*)
done

lemma *fields-emptyI*: $\bigwedge$*y*. $[\![$*ws-prog G*; *class G C* = *Some c*;*cfields c* = [];
C $\neq$ *Object* $\longrightarrow$ *class G* (*super c*) = *Some y* $\wedge$ *fields G* (*super c*) = []$]\!]$ $\Longrightarrow$
fields G C = []
apply (*subst fields-rec*)
apply *assumption*
apply *auto*
done

lemma *fields-mono-lemma*:

$[\![x \in set\ (fields\ G\ C);\ G \vdash D \preceq_C C;\ ws\text{-}prog\ G]\!]$
$\Longrightarrow x \in set\ (fields\ G\ D)$
apply (*erule rev-mp*)
apply (*erule converse-rtrancl-induct*)
apply *fast*
apply (*drule subcls1D*)
apply *clarsimp*
apply (*subst fields-rec*)
apply *auto*
done

lemma *ws-unique-fields-lemma*:
$[\![(efn,fd) \in set\ (fields\ G\ (super\ c));\ fc \in set\ (cfields\ c);\ ws\text{-}prog\ G;$
$fname\ efn = fname\ fc;\ declclassf\ efn = C;$
$class\ G\ C = Some\ c;\ C \neq Object;\ class\ G\ (super\ c) = Some\ d]\!] \Longrightarrow R$
apply (*frule-tac ws-prog-cdeclD* [*THEN conjunct2*], *assumption*, *assumption*)
apply (*drule-tac weak-map-of-SomeI*)
apply (*frule-tac subcls1I* [*THEN subcls1-irrefl*], *assumption*, *assumption*)
apply (*auto dest*: *fields-declC* [*THEN conjunct2* [*THEN conjunct1*[*THEN rtranclD*]]])
done

lemma *ws-unique-fields*: $[\![is\text{-}class\ G\ C;\ ws\text{-}prog\ G;$
$\bigwedge C\ c.\ [\![class\ G\ C = Some\ c]\!] \Longrightarrow unique\ (cfields\ c)\ ]\!] \Longrightarrow$
$unique\ (fields\ G\ C)$
apply (*rule ws-subcls1-induct*, *assumption*, *assumption*)
apply (*subst fields-rec*, *assumption*)
apply (*auto intro*!: *unique-map-inj inj-onI*
elim!: *unique-append ws-unique-fields-lemma fields-norec*)
done

26 accfield

lemma *accfield-fields*:
$accfield\ G\ S\ C\ fn = Some\ f$
$\Longrightarrow table\text{-}of\ (fields\ G\ C)\ (fn,\ declclass\ f) = Some\ (fld\ f)$
apply (*simp only*: *accfield-def Let-def*)
apply (*rule table-of-remap-SomeD*)
apply (*auto dest*: *filter-tab-SomeD*)
done

lemma *accfield-declC-is-class*:
$[\![is\text{-}class\ G\ C;\ accfield\ G\ S\ C\ en = Some\ (fd,\ f);\ ws\text{-}prog\ G]\!] \Longrightarrow$
$is\text{-}class\ G\ fd$
apply (*drule accfield-fields*)
apply (*drule fields-declC* [*THEN conjunct1*], *assumption*)
apply *auto*
done

lemma *accfield-accessibleD*:
$accfield\ G\ S\ C\ fn = Some\ f \Longrightarrow G \vdash Field\ fn\ f\ of\ C\ accessible\text{-}from\ S$
by (*auto simp add*: *accfield-def Let-def*)

27 is methd

lemma *is-methdI*:
$[\![$*class G C = Some y; methd G C sig = Some b*$]\!] \Longrightarrow$ *is-methd G C sig*
apply (*unfold is-methd-def*)
apply *auto*
done

lemma *is-methdD*:
is-methd G C sig $\Longrightarrow$ *class G C* $\neq$ *None* $\wedge$ *methd G C sig* $\neq$ *None*
apply (*unfold is-methd-def*)
apply *auto*
done

lemma *finite-is-methd*:
ws-prog G $\Longrightarrow$ *finite* (*Collect* (*split* (*is-methd G*)))
apply (*unfold is-methd-def*)
apply (*subst SetCompr-Sigma-eq*)
apply (*rule finite-is-class* [*THEN finite-SigmaI*])
apply (*simp only*: *mem-Collect-eq*)
apply (*fold dom-def*)
apply (*erule finite-dom-methd*)
apply *assumption*
done

calculation of the superclasses of a class

constdefs
superclasses:: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *qtname set*
superclasses G C $\equiv$ *class-rec* (*G*,*C*) {}
(λ *C c superclss*. (*if C*=*Object*
then {}
else insert (*super c*) *superclss*))

lemma *superclasses-rec*: $[\![$*class G C = Some c*; *ws-prog G*$]\!] \Longrightarrow$
superclasses G C
= (*if* (*C*=*Object*)
then {}
else insert (*super c*) (*superclasses G* (*super c*)))
apply (*unfold superclasses-def*)
apply (*erule class-rec* [*THEN trans*], *assumption*)
apply (*simp*)
done

lemma *superclasses-mono*:
$[\![G \vdash C \prec_C D$;*ws-prog G*; *class G C = Some c*;
$\bigwedge$ *C c*. $[\![$*class G C = Some c*;*C*$\neq$*Object*$]\!] \Longrightarrow \exists$ *sc. class G* (*super c*) = *Some sc*;
x$\in$*superclasses G D*
$]\!] \Longrightarrow$ *x*$\in$*superclasses G C*
proof −

assume *ws*: *ws-prog G* **and**
cls-C: *class G C = Some c* **and**
wf: $\bigwedge$*C c*. $[\![$*class G C = Some c*; *C* $\neq$ *Object*$]\!]$
$\Longrightarrow \exists$ *sc. class G* (*super c*) = *Some sc*

```
  assume clsrel: G⊢C≺C D
  thus ⋀ c. ⟦class G C = Some c; x∈superclasses G D⟧⟹
       x∈superclasses G C (is PROP ?P C
                            is ⋀ c. ?CLS C c ⟹ ?SUP D ⟹ ?SUP C)
  proof (induct ?P C  rule: converse-trancl-induct)
    fix C c
    assume G⊢C≺C1D class G C = Some c x ∈ superclasses G D
    with wf ws show ?SUP C
      by (auto    intro: no-subcls1-Object
               simp add: superclasses-rec subcls1-def)
  next
    fix C S c
    assume clsrel': G⊢C ≺C1 S G⊢S ≺C D
       and    hyp : ⋀ s. ⟦class G S = Some s; x ∈ superclasses G D⟧
                       ⟹ x ∈ superclasses G S
       and  cls-C': class G C = Some c
       and       x: x ∈ superclasses G D
    moreover note wf ws
    moreover from calculation
    have ?SUP S
      by (force intro: no-subcls1-Object simp add: subcls1-def)
    moreover from calculation
    have super c = S
      by (auto intro: no-subcls1-Object simp add: subcls1-def)
    ultimately show ?SUP C
      by (auto intro: no-subcls1-Object simp add: superclasses-rec)
  qed
qed


lemma subclsEval:
⟦G⊢C ≺C D;ws-prog G; class G C = Some c;
  ⋀ C c. ⟦class G C = Some c;C≠Object⟧ ⟹ ∃ sc. class G (super c) = Some sc
 ⟧ ⟹ D∈superclasses G C
proof -
  note converse-trancl-induct
       = converse-trancl-induct [consumes 1,case-names Single Step]
  assume
            ws: ws-prog G           and
         cls-C: class G C = Some c and
            wf: ⋀C c. ⟦class G C = Some c; C ≠ Object⟧
                ⟹ ∃sc. class G (super c) = Some sc
  assume clsrel: G⊢C≺C D
  thus ⋀ c. class G C = Some c⟹ D∈superclasses G C
    (is PROP ?P C  is ⋀ c. ?CLS C c  ⟹ ?SUP C)
  proof (induct ?P C  rule: converse-trancl-induct)
    fix C c
    assume G⊢C ≺C1 D class G C = Some c
    with ws wf show ?SUP C
      by (auto intro: no-subcls1-Object simp add: superclasses-rec subcls1-def)
  next
    fix C S c
    assume G⊢C ≺C1 S G⊢S≺C D
           ⋀s. class G S = Some s ⟹ D ∈ superclasses G S
           class G C = Some c
    with ws wf show ?SUP C
      by - (rule superclasses-mono,
            auto dest: no-subcls1-Object simp add: subcls1-def )
  qed
```

qed

end

Chapter 11

WellType

28 Well-typedness of Java programs

theory *WellType* **imports** *DeclConcepts* **begin**

improvements over Java Specification 1.0:

- methods of Object can be called upon references of interface or array type

simplifications:

- the type rules include all static checks on statements and expressions, e.g. definedness of names (of parameters, locals, fields, methods)

design issues:

- unified type judgment for statements, variables, expressions, expression lists
- statements are typed like expressions with dummy type Void
- the typing rules take an extra argument that is capable of determining the dynamic type of objects. Therefore, they can be used for both checking static types and determining runtime types in transition semantics.

types *lenv*
  = (*lname*, *ty*) *table* — local variables, including This and Result

record *env* =
  prg:: *prog* — program
  cls:: *qtname* — current package and class name
  lcl:: *lenv* — local environment

translations
lenv <= (*type*) (*lname*, *ty*) *table*
lenv <= (*type*) *lname* ⇒ *ty option*
env <= (*type*) (|*prg*::*prog*,*cls*::*qtname*,*lcl*::*lenv*|)
env <= (*type*) (|*prg*::*prog*,*cls*::*qtname*,*lcl*::*lenv*,...::'*a*|)

syntax
pkg :: *env* ⇒ *pname* — select the current package from an environment
translations
pkg e == *pid* (*cls e*)

Static overloading: maximally specific methods

types
emhead = *ref-ty* × *mhead*

— Some mnemotic selectors for emhead
constdefs
declrefT :: *emhead* ⇒ *ref-ty*
declrefT ≡ *fst*

mhd :: *emhead* ⇒ *mhead*
mhd ≡ *snd*

lemma *declrefT-simp*[*simp*]:*declrefT* (*r*,*m*) = *r*

by (*simp add*: *declrefT-def*)

lemma *mhd-simp*[*simp*]:*mhd* (*r*,*m*) = *m*
by (*simp add*: *mhd-def*)

lemma *static-mhd-simp*[*simp*]: *static* (*mhd m*) = *is-static m*
by (*cases m*) (*simp add*: *member-is-static-simp mhd-def*)

lemma *mhd-resTy-simp* [*simp*]: *resTy* (*mhd m*) = *resTy m*
by (*cases m*) *simp*

lemma *mhd-is-static-simp* [*simp*]: *is-static* (*mhd m*) = *is-static m*
by (*cases m*) *simp*

lemma *mhd-accmodi-simp* [*simp*]: *accmodi* (*mhd m*) = *accmodi m*
by (*cases m*) *simp*

consts
cmheads :: *prog* ⇒ *qtname* ⇒ *qtname* ⇒ *sig* ⇒ *emhead set*
Objectmheads :: *prog* ⇒ *qtname* ⇒ *sig* ⇒ *emhead set*
accObjectmheads:: *prog* ⇒ *qtname* ⇒ *ref-ty* ⇒ *sig* ⇒ *emhead set*
mheads :: *prog* ⇒ *qtname* ⇒ *ref-ty* ⇒ *sig* ⇒ *emhead set*
defs
cmheads-def:
cmheads G S C
≡ λ*sig*. (λ(*Cls*,*mthd*). (*ClassT Cls*,(*mhead mthd*))) ‘ *o2s* (*accmethd G S C sig*)
Objectmheads-def:
Objectmheads G S
≡ λ*sig*. (λ(*Cls*,*mthd*). (*ClassT Cls*,(*mhead mthd*)))
‘ *o2s* (*filter-tab* (λ*sig m*. *accmodi m* ≠ *Private*) (*accmethd G S Object*) *sig*)
accObjectmheads-def:
accObjectmheads G S T
≡ *if G*⊢*RefT T accessible-in* (*pid S*)
then Objectmheads G S
else λ*sig*. {}
primrec
mheads G S NullT = (λ*sig*. {})
mheads G S (*IfaceT I*) = (λ*sig*. (λ(*I*,*h*).(*IfaceT I*,*h*))
‘ *accimethds G* (*pid S*) *I sig* ∪
accObjectmheads G S (*IfaceT I*) *sig*)
mheads G S (*ClassT C*) = *cmheads G S C*
mheads G S (*ArrayT T*) = *accObjectmheads G S* (*ArrayT T*)

constdefs
— applicable methods, cf. 15.11.2.1
appl-methds :: *prog* ⇒ *qtname* ⇒ *ref-ty* ⇒ *sig* ⇒ (*emhead* × *ty list*) *set*
appl-methds G S rt ≡ λ *sig*.
{(*mh*,*pTs*′) |*mh pTs*′. *mh* ∈ *mheads G S rt* (|*name*=*name sig*,*parTs*=*pTs*′|) ∧
G⊢(*parTs sig*)[⪯]*pTs*′}

— more specific methods, cf. 15.11.2.2
more-spec :: *prog* ⇒ *emhead* × *ty list* ⇒ *emhead* × *ty list* ⇒ *bool*
more-spec G ≡ λ(*mh*,*pTs*). λ(*mh*′,*pTs*′). *G*⊢*pTs*[⪯]*pTs*′

— maximally specific methods, cf. 15.11.2.2
$max\text{-}spec :: prog \Rightarrow qtname \Rightarrow ref\text{-}ty \Rightarrow sig \Rightarrow (emhead \times ty\ list)\ set$

$max\text{-}spec\ G\ S\ rt\ sig \equiv \{m.\ m \in appl\text{-}methds\ G\ S\ rt\ sig\ \land$
$(\forall m' \in appl\text{-}methds\ G\ S\ rt\ sig.\ more\text{-}spec\ G\ m'\ m \longrightarrow m'=m)\}$

lemma *max-spec2appl-meths*:
$x \in max\text{-}spec\ G\ S\ T\ sig \Longrightarrow x \in appl\text{-}methds\ G\ S\ T\ sig$
by (*auto simp*: *max-spec-def*)

lemma *appl-methsD*: $(mh,pTs') \in appl\text{-}methds\ G\ S\ T\ (|name=mn,parTs=pTs|) \Longrightarrow$
$mh \in mheads\ G\ S\ T\ (|name=mn,parTs=pTs'|) \land G \vdash pTs[\preceq]pTs'$
by (*auto simp*: *appl-methds-def*)

lemma *max-spec2mheads*:
$max\text{-}spec\ G\ S\ rt\ (|name=mn,parTs=pTs|) = insert\ (mh,\ pTs')\ A$
$\Longrightarrow mh \in mheads\ G\ S\ rt\ (|name=mn,parTs=pTs'|) \land G \vdash pTs[\preceq]pTs'$
apply (*auto dest*: *equalityD2 subsetD max-spec2appl-meths appl-methsD*)
done

constdefs
$empty\text{-}dt :: dyn\text{-}ty$
$empty\text{-}dt \equiv \lambda a.\ None$

$invmode :: ('a::type)member\text{-}scheme \Rightarrow expr \Rightarrow inv\text{-}mode$
$invmode\ m\ e \equiv if\ is\text{-}static\ m$
$then\ Static$
$else\ if\ e=Super\ then\ SuperM\ else\ IntVir$

lemma *invmode-nonstatic* [*simp*]:
$invmode\ (|access=a,static=False,\ldots=x|)\ (Acc\ (LVar\ e)) = IntVir$
apply (*unfold invmode-def*)
apply (*simp* (*no-asm*) *add*: *member-is-static-simp*)
done

lemma *invmode-Static-eq* [*simp*]: $(invmode\ m\ e = Static) = is\text{-}static\ m$
apply (*unfold invmode-def*)
apply (*simp* (*no-asm*))
done

lemma *invmode-IntVir-eq*: $(invmode\ m\ e = IntVir) = (\neg(is\text{-}static\ m) \land e \neq Super)$
apply (*unfold invmode-def*)
apply (*simp* (*no-asm*))
done

lemma *Null-staticD*:
$a'=Null \longrightarrow (is\text{-}static\ m) \Longrightarrow invmode\ m\ e = IntVir \longrightarrow a' \neq Null$

apply (*clarsimp simp add*: *invmode-IntVir-eq*)
done

Typing for unary operations

consts *unop-type* :: *unop* ⇒ *prim-ty*
primrec
unop-type UPlus = *Integer*
unop-type UMinus = *Integer*
unop-type UBitNot = *Integer*
unop-type UNot = *Boolean*

consts *wt-unop* :: *unop* ⇒ *ty* ⇒ *bool*
primrec
wt-unop UPlus t = (*t* = *PrimT Integer*)
wt-unop UMinus t = (*t* = *PrimT Integer*)
wt-unop UBitNot t = (*t* = *PrimT Integer*)
wt-unop UNot t = (*t* = *PrimT Boolean*)

Typing for binary operations

consts *binop-type* :: *binop* ⇒ *prim-ty*
primrec
binop-type Mul = *Integer*
binop-type Div = *Integer*
binop-type Mod = *Integer*
binop-type Plus = *Integer*
binop-type Minus = *Integer*
binop-type LShift = *Integer*
binop-type RShift = *Integer*
binop-type RShiftU = *Integer*
binop-type Less = *Boolean*
binop-type Le = *Boolean*
binop-type Greater = *Boolean*
binop-type Ge = *Boolean*
binop-type Eq = *Boolean*
binop-type Neq = *Boolean*
binop-type BitAnd = *Integer*
binop-type And = *Boolean*
binop-type BitXor = *Integer*
binop-type Xor = *Boolean*
binop-type BitOr = *Integer*
binop-type Or = *Boolean*
binop-type CondAnd = *Boolean*
binop-type CondOr = *Boolean*

consts *wt-binop* :: *prog* ⇒ *binop* ⇒ *ty* ⇒ *ty* ⇒ *bool*
primrec
wt-binop G Mul t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Div t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Mod t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Plus t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Minus t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G LShift t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G RShift t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G RShiftU t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Less t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Le t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Greater t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))

wt-binop G Ge t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Eq t1 t2 = ($G \vdash t1 \preceq t2$ ∨ $G \vdash t2 \preceq t1$)
wt-binop G Neq t1 t2 = ($G \vdash t1 \preceq t2$ ∨ $G \vdash t2 \preceq t1$)
wt-binop G BitAnd t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G And t1 t2 = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
wt-binop G BitXor t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Xor t1 t2 = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
wt-binop G BitOr t1 t2 = ((*t1* = *PrimT Integer*) ∧ (*t2* = *PrimT Integer*))
wt-binop G Or t1 t2 = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
wt-binop G CondAnd t1 t2 = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))
wt-binop G CondOr t1 t2 = ((*t1* = *PrimT Boolean*) ∧ (*t2* = *PrimT Boolean*))

Typing for terms

types *tys* = *ty* + *ty list*
translations
tys <= (*type*) *ty* + *ty list*

inductive
wt :: *env* ⇒ *dyn-ty* ⇒ [*term*,*tys*] ⇒ *bool* (-,-⊨-::- [51,51,51,51] 50)
and *wt-stmt* :: *env* ⇒ *dyn-ty* ⇒ *stmt* ⇒ *bool* (-,-⊨-::√ [51,51,51] 50)
and *ty-expr* :: *env* ⇒ *dyn-ty* ⇒ [*expr* ,*ty*] ⇒ *bool* (-,-⊨-::−- [51,51,51,51] 50)
and *ty-var* :: *env* ⇒ *dyn-ty* ⇒ [*var* ,*ty*] ⇒ *bool* (-,-⊨-::=- [51,51,51,51] 50)
and *ty-exprs* :: *env* ⇒ *dyn-ty* ⇒ [*expr list*, *ty list*] ⇒ *bool*
(-,-⊨-::≐- [51,51,51,51] 50)
where

E,*dt*⊨*s*::√ ≡ *E*,*dt*⊨*In1r s*::*Inl* (*PrimT Void*)
| *E*,*dt*⊨*e*::−*T* ≡ *E*,*dt*⊨*In1l e*::*Inl T*
| *E*,*dt*⊨*e*::=*T* ≡ *E*,*dt*⊨*In2 e*::*Inl T*
| *E*,*dt*⊨*e*::≐*T* ≡ *E*,*dt*⊨*In3 e*::*Inr T*

— well-typed statements

| *Skip*: *E*,*dt*⊨*Skip*::√

| *Expr*: ⟦*E*,*dt*⊨*e*::−*T*⟧ ⟹
E,*dt*⊨*Expr e*::√
— cf. 14.6
| *Lab*: *E*,*dt*⊨*c*::√ ⟹
E,*dt*⊨*l*• *c*::√

| *Comp*: ⟦*E*,*dt*⊨*c1*::√;
E,*dt*⊨*c2*::√⟧ ⟹
E,*dt*⊨*c1*;; *c2*::√

— cf. 14.8
| *If*: ⟦*E*,*dt*⊨*e*::−*PrimT Boolean*;
E,*dt*⊨*c1*::√;
E,*dt*⊨*c2*::√⟧ ⟹
E,*dt*⊨*If*(*e*) *c1 Else c2*::√

— cf. 14.10
| *Loop*: ⟦*E*,*dt*⊨*e*::−*PrimT Boolean*;
E,*dt*⊨*c*::√⟧ ⟹
E,*dt*⊨*l*• *While*(*e*) *c*::√
— cf. 14.13, 14.15, 14.16
| *Jmp*: *E*,*dt*⊨*Jmp jump*::√

— cf. 14.16
| *Throw*: $[\![E,dt\models e::-Class\ tn;$
$prg\ E\vdash tn\preceq_C\ SXcpt\ Throwable]\!] \Longrightarrow$
$E,dt\models Throw\ e::\surd$
— cf. 14.18
| *Try*: $[\![E,dt\models c1::\surd;\ prg\ E\vdash tn\preceq_C\ SXcpt\ Throwable;$
$lcl\ E\ (VName\ vn)=None;\ E\ (\!|lcl := lcl\ E(VName\ vn\mapsto Class\ tn)|\!),dt\models c2::\surd]\!]$
$\Longrightarrow$
$E,dt\models Try\ c1\ Catch(tn\ vn)\ c2::\surd$

— cf. 14.18
| *Fin*: $[\![E,dt\models c1::\surd;\ E,dt\models c2::\surd]\!] \Longrightarrow$
$E,dt\models c1\ Finally\ c2::\surd$

| *Init*: $[\![is\text{-}class\ (prg\ E)\ C]\!] \Longrightarrow$
$E,dt\models Init\ C::\surd$
— *Init* is created on the fly during evaluation (see Eval.thy). The class isn't necessarily accessible from the points *Init* is called. Therefor we only demand *is-class* and not *is-acc-class* here.

— well-typed expressions

— cf. 15.8
| *NewC*: $[\![is\text{-}acc\text{-}class\ (prg\ E)\ (pkg\ E)\ C]\!] \Longrightarrow$
$E,dt\models NewC\ C::-Class\ C$
— cf. 15.9
| *NewA*: $[\![is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ T;$
$E,dt\models i::-PrimT\ Integer]\!] \Longrightarrow$
$E,dt\models New\ T[i]::-T.[]$

— cf. 15.15
| *Cast*: $[\![E,dt\models e::-T;\ is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ T';$
$prg\ E\vdash T\preceq?\ T']\!] \Longrightarrow$
$E,dt\models Cast\ T'\ e::-T'$

— cf. 15.19.2
| *Inst*: $[\![E,dt\models e::-RefT\ T;\ is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ (RefT\ T');$
$prg\ E\vdash RefT\ T\preceq?\ RefT\ T']\!] \Longrightarrow$
$E,dt\models e\ InstOf\ T'::-PrimT\ Boolean$

— cf. 15.7.1
| *Lit*: $[\![typeof\ dt\ x = Some\ T]\!] \Longrightarrow$
$E,dt\models Lit\ x::-T$

| *UnOp*: $[\![E,dt\models e::-Te;\ wt\text{-}unop\ unop\ Te;\ T=PrimT\ (unop\text{-}type\ unop)]\!]$
$\Longrightarrow$
$E,dt\models UnOp\ unop\ e::-T$

| *BinOp*: $[\![E,dt\models e1::-T1;\ E,dt\models e2::-T2;\ wt\text{-}binop\ (prg\ E)\ binop\ T1\ T2;$
$T=PrimT\ (binop\text{-}type\ binop)]\!]$
$\Longrightarrow$
$E,dt\models BinOp\ binop\ e1\ e2::-T$

— cf. 15.10.2, 15.11.1
| *Super*: $[\![lcl\ E\ This = Some\ (Class\ C);\ C \neq Object;$
$class\ (prg\ E)\ C = Some\ c]\!] \Longrightarrow$
$E,dt\models Super::-Class\ (super\ c)$

— cf. 15.13.1, 15.10.1, 15.12

| *Acc*: $[\![E,dt \models va::=T]\!] \Longrightarrow$

$$E,dt \models Acc\ va::-T$$

— cf. 15.25, 15.25.1
| *Ass*: $[\![E,dt \models va::=T;\ va \neq LVar\ This;$
$E,dt \models v\ ::-T';$
$prg\ E \vdash T' \preceq T]\!] \Longrightarrow$

$$E,dt \models va:=v::-T'$$

— cf. 15.24
| *Cond*: $[\![E,dt \models e0::-PrimT\ Boolean;$
$E,dt \models e1::-T1;\ E,dt \models e2::-T2;$
$prg\ E \vdash T1 \preceq T2 \wedge T = T2 \ \vee\ prg\ E \vdash T2 \preceq T1 \wedge T = T1]\!] \Longrightarrow$

$$E,dt \models e0\ ?\ e1\ :\ e2::-T$$

— cf. 15.11.1, 15.11.2, 15.11.3
| *Call*: $[\![E,dt \models e::-RefT\ statT;$
$E,dt \models ps::\doteq pTs;$
$max\text{-}spec\ (prg\ E)\ (cls\ E)\ statT\ (\!|name=mn,parTs=pTs|\!)$
$= \{((statDeclT,m),pTs')\}$
$]\!] \Longrightarrow$

$$E,dt \models \{cls\ E,statT,invmode\ m\ e\}e\cdot mn(\{pTs'\}ps)::-(resTy\ m)$$

| *Methd*: $[\![is\text{-}class\ (prg\ E)\ C;$
$methd\ (prg\ E)\ C\ sig = Some\ m;$
$E,dt \models Body\ (declclass\ m)\ (stmt\ (mbody\ (mthd\ m)))::-T]\!] \Longrightarrow$

$$E,dt \models Methd\ C\ sig::-T$$

— The class C is the dynamic class of the method call (cf. Eval.thy). It hasn't got to be directly accessible from the current package *pkg E*. Only the static class must be accessible (enshured indirectly by *Call*). Note that l is just a dummy value. It is only used in the smallstep semantics. To proof typesafety directly for the smallstep semantics we would have to assume conformance of l here!

| *Body*: $[\![is\text{-}class\ (prg\ E)\ D;$
$E,dt \models blk::\surd;$
$(lcl\ E)\ Result = Some\ T;$
$is\text{-}type\ (prg\ E)\ T]\!] \Longrightarrow$

$$E,dt \models Body\ D\ blk::-T$$

— The class D implementing the method must not directly be accessible from the current package *pkg E*, but can also be indirectly accessible due to inheritance (enshured in *Call*) The result type hasn't got to be accessible in Java! (If it is not accessible you can only assign it to Object). For dummy value l see rule *Methd*.

— well-typed variables

— cf. 15.13.1
| *LVar*: $[\![lcl\ E\ vn = Some\ T;\ is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ T]\!] \Longrightarrow$

$$E,dt \models LVar\ vn::=T$$

— cf. 15.10.1
| *FVar*: $[\![E,dt \models e::-Class\ C;$
$accfield\ (prg\ E)\ (cls\ E)\ C\ fn = Some\ (statDeclC,f)]\!] \Longrightarrow$

$$E,dt \models \{cls\ E,statDeclC,is\text{-}static\ f\}e..fn::=(type\ f)$$

— cf. 15.12
| *AVar*: $[\![E,dt \models e::-T.[];$
$E,dt \models i::-PrimT\ Integer]\!] \Longrightarrow$

$$E,dt \models e.[i]::=T$$

— well-typed expression lists

— cf. 15.11.???
| *Nil*: $E,dt \models [] ::\doteq []$

— cf. 15.11.???
| *Cons*: $[\![E,dt \models e ::- T;$
$E,dt \models es ::\doteq Ts]\!] \Longrightarrow$
$E,dt \models e\#es ::\doteq T\#Ts$

syntax
-wt :: *env* ⇒ [*term*,*tys*] ⇒ *bool* (-|−-::- [*51*,*51*,*51*] *50*)
-wt-stmt :: *env* ⇒ *stmt* ⇒ *bool* (-|−-:<> [*51*,*51*] *50*)
-ty-expr :: *env* ⇒ [*expr* ,*ty*] ⇒ *bool* (-|−-:−- [*51*,*51*,*51*] *50*)
-ty-var :: *env* ⇒ [*var* ,*ty*] ⇒ *bool* (-|−-:=- [*51*,*51*,*51*] *50*)
-ty-exprs:: *env* ⇒ [*expr list*,
ty list] ⇒ *bool* (-|−-:#- [*51*,*51*,*51*] *50*)

syntax (*xsymbols*)
-wt :: *env* ⇒ [*term*,*tys*] ⇒ *bool* (-⊢-::- [*51*,*51*,*51*] *50*)
-wt-stmt :: *env* ⇒ *stmt* ⇒ *bool* (-⊢-::√ [*51*,*51*] *50*)
-ty-expr :: *env* ⇒ [*expr* ,*ty*] ⇒ *bool* (-⊢-::−- [*51*,*51*,*51*] *50*)
-ty-var :: *env* ⇒ [*var* ,*ty*] ⇒ *bool* (-⊢-::=- [*51*,*51*,*51*] *50*)
-ty-exprs :: *env* ⇒ [*expr list*,
ty list] ⇒ *bool* (-⊢-::≐- [*51*,*51*,*51*] *50*)

translations
$E \vdash t ::\ T == E, empty\text{-}dt \models t ::\ T$
$E \vdash s ::\surd\ == E \vdash In1r\ s :: Inl\ (PrimT\ Void)$
$E \vdash e ::- T == E \vdash In1l\ e :: Inl\ T$
$E \vdash e ::= T == E \vdash In2\ \ e :: Inl\ T$
$E \vdash e ::\doteq T == E \vdash In3\ \ e :: Inr\ T$

declare *not-None-eq* [*simp del*]
declare *split-if* [*split del*] *split-if-asm* [*split del*]
declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss delloop split-all-tac*)) ⟫

inductive-cases *wt-elim-cases* [*cases set*]:
$E,dt \models In2\ \ (LVar\ vn) :: T$
$E,dt \models In2\ \ (\{accC,statDeclC,s\}e..fn) :: T$
$E,dt \models In2\ \ (e.[i]) :: T$
$E,dt \models In1l\ (NewC\ C) :: T$
$E,dt \models In1l\ (New\ T'[i]) :: T$
$E,dt \models In1l\ (Cast\ T'\ e) :: T$
$E,dt \models In1l\ (e\ InstOf\ T') :: T$
$E,dt \models In1l\ (Lit\ x) :: T$
$E,dt \models In1l\ (UnOp\ unop\ e) :: T$
$E,dt \models In1l\ (BinOp\ binop\ e1\ e2) :: T$
$E,dt \models In1l\ (Super) :: T$
$E,dt \models In1l\ (Acc\ va) :: T$
$E,dt \models In1l\ (Ass\ va\ v) :: T$
$E,dt \models In1l\ (e0\ ?\ e1\ :\ e2) :: T$
$E,dt \models In1l\ (\{accC,statT,mode\}e\cdot mn(\{pT'\}p)) :: T$
$E,dt \models In1l\ (Methd\ C\ sig) :: T$
$E,dt \models In1l\ (Body\ D\ blk) :: T$
$E,dt \models In3\ \ ([]) :: Ts$
$E,dt \models In3\ \ (e\#es) :: Ts$
$E,dt \models In1r\ \ Skip :: x$

$E,dt \models In1r\ (Expr\ e)\ :: x$
$E,dt \models In1r\ (c1;;\ c2)\ :: x$
$E,dt \models In1r\ (l\bullet\ c)\ :: x$
$E,dt \models In1r\ (If(e)\ c1\ Else\ c2)\ :: x$
$E,dt \models In1r\ (l\bullet\ While(e)\ c)\ :: x$
$E,dt \models In1r\ (Jmp\ jump)\ :: x$
$E,dt \models In1r\ (Throw\ e)\ :: x$
$E,dt \models In1r\ (Try\ c1\ Catch(tn\ vn)\ c2) :: x$
$E,dt \models In1r\ (c1\ Finally\ c2)\ :: x$
$E,dt \models In1r\ (Init\ C)\ :: x$

declare *not-None-eq* [*simp*]
declare *split-if* [*split*] *split-if-asm* [*split*]
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss => ss addloop* (*split-all-tac*, *split-all-tac*))) ⟫

lemma *is-acc-class-is-accessible*:
is-acc-class G P C ⟹ *G*⊢(*Class C*) *accessible-in P*
by (*auto simp add*: *is-acc-class-def*)

lemma *is-acc-iface-is-iface*: *is-acc-iface G P I* ⟹ *is-iface G I*
by (*auto simp add*: *is-acc-iface-def*)

lemma *is-acc-iface-Iface-is-accessible*:
is-acc-iface G P I ⟹ *G*⊢(*Iface I*) *accessible-in P*
by (*auto simp add*: *is-acc-iface-def*)

lemma *is-acc-type-is-type*: *is-acc-type G P T* ⟹ *is-type G T*
by (*auto simp add*: *is-acc-type-def*)

lemma *is-acc-iface-is-accessible*:
is-acc-type G P T ⟹ *G*⊢*T accessible-in P*
by (*auto simp add*: *is-acc-type-def*)

lemma *wt-Methd-is-methd*:
E⊢*In1l* (*Methd C sig*)::*T* ⟹ *is-methd* (*prg E*) *C sig*
apply (*erule-tac wt-elim-cases*)
apply *clarsimp*
apply (*erule is-methdI*, *assumption*)
done

Special versions of some typing rules, better suited to pattern match the conclusion (no selectors in the conclusion)

lemma *wt-Call*:
$[\![E,dt \models e::-RefT\ statT;\ E,dt \models ps::\doteq pTs;$
$max\text{-}spec\ (prg\ E)\ (cls\ E)\ statT\ (\!|name=mn,parTs=pTs|\!)$
$= \{((statDeclC,m),pTs')\};rT=(resTy\ m);accC=cls\ E;$
$mode = invmode\ m\ e]\!] \Longrightarrow E,dt \models \{accC,statT,mode\}e\cdot mn(\{pTs'\}ps)::-rT$
by (*auto elim*: *wt.Call*)

lemma *invocationTypeExpr-noClassD*:
$[\![\ E \vdash e::-RefT\ statT]\!]$

$\Longrightarrow (\forall\ statC.\ statT \neq ClassT\ statC) \longrightarrow invmode\ m\ e \neq SuperM$
proof −
  assume *wt*: $E\vdash e::-RefT\ statT$
  show *?thesis*
  proof (*cases e=Super*)
    case *True*
    with *wt* **obtain** *C* **where** $statT = ClassT\ C$ **by** (*blast elim*: *wt-elim-cases*)
    then show *?thesis* **by** *blast*
  next
    case *False* **then show** *?thesis*
      by (*auto simp add*: *invmode-def split*: *split-if-asm*)
  qed
qed

lemma *wt-Super*:
$[\![lcl\ E\ This = Some\ (Class\ C);\ C \neq Object;\ class\ (prg\ E)\ C = Some\ c;\ D{=}super\ c]\!]$
$\Longrightarrow E,dt\models Super::-Class\ D$
by (*auto elim*: *wt.Super*)

lemma *wt-FVar*:
$[\![E,dt\models e::-Class\ C;\ accfield\ (prg\ E)\ (cls\ E)\ C\ fn = Some\ (statDeclC,f);$
  $sf{=}is\text{-}static\ f;\ fT{=}(type\ f);\ accC{=}cls\ E]\!]$
$\Longrightarrow E,dt\models\{accC,statDeclC,sf\}e..fn::=fT$
by (*auto dest*: *wt.FVar*)

lemma *wt-init* [*iff*]: $E,dt\models Init\ C::\surd = is\text{-}class\ (prg\ E)\ C$
by (*auto elim*: *wt-elim-cases intro*: *wt.Init*)

declare *wt.Skip* [*iff*]

lemma *wt-StatRef*:
  $is\text{-}acc\text{-}type\ (prg\ E)\ (pkg\ E)\ (RefT\ rt) \Longrightarrow E\vdash StatRef\ rt::-RefT\ rt$
apply (*rule wt.Cast*)
apply (*rule wt.Lit*)
apply (*simp* (*no-asm*))
apply (*simp* (*no-asm-simp*))
apply (*rule cast.widen*)
apply (*simp* (*no-asm*))
done

lemma *wt-Inj-elim*:
  $\bigwedge E.\ E,dt\models t::U \Longrightarrow case\ t\ of$
    $In1\ ec \Rightarrow (case\ ec\ of$
      $Inl\ e \Rightarrow \exists\ T.\ U{=}Inl\ T$
      $\mid Inr\ s \Rightarrow U{=}Inl\ (PrimT\ Void))$
    $\mid In2\ e \Rightarrow (\exists\ T.\ U{=}Inl\ T)$
    $\mid In3\ e \Rightarrow (\exists\ T.\ U{=}Inr\ T)$
apply (*erule wt.induct*)
apply *auto*
done

— In the special syntax to distinguish the typing judgements for expressions, statements, variables and expression lists the kind of term corresponds to the kind of type in the end e.g. An statement (injection

In3 into terms, always has type void (injection *Inl* into the generalised types. The following simplification procedures establish these kinds of correlation.

lemma *wt-expr-eq*: $E,dt \models In1l\ t::U = (\exists T.\ U=Inl\ T \wedge E,dt \models t::-T)$
by (*auto*, *frule wt-Inj-elim*, *auto*)

lemma *wt-var-eq*: $E,dt \models In2\ t::U = (\exists T.\ U=Inl\ T \wedge E,dt \models t::=T)$
by (*auto*, *frule wt-Inj-elim*, *auto*)

lemma *wt-exprs-eq*: $E,dt \models In3\ t::U = (\exists Ts.\ U=Inr\ Ts \wedge E,dt \models t::\doteq Ts)$
by (*auto*, *frule wt-Inj-elim*, *auto*)

lemma *wt-stmt-eq*: $E,dt \models In1r\ t::U = (U=Inl(PrimT\ Void) \wedge E,dt \models t::\surd)$
by (*auto*, *frule wt-Inj-elim*, *auto*, *frule wt-Inj-elim*, *auto*)

simproc-setup *wt-expr* $(E,dt \models In1l\ t::U) = \langle\langle$
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ (*Const* - \$ -)) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm wt-expr-eq*})) $\rangle\rangle$

simproc-setup *wt-var* $(E,dt \models In2\ t::U) = \langle\langle$
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ (*Const* - \$ -)) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm wt-var-eq*})) $\rangle\rangle$

simproc-setup *wt-exprs* $(E,dt \models In3\ t::U) = \langle\langle$
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ (*Const* - \$ -)) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm wt-exprs-eq*})) $\rangle\rangle$

simproc-setup *wt-stmt* $(E,dt \models In1r\ t::U) = \langle\langle$
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ (*Const* - \$ -)) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm wt-stmt-eq*})) $\rangle\rangle$

lemma *wt-elim-BinOp*:
$[\![E,dt \models In1l\ (BinOp\ binop\ e1\ e2)::T;$
$\bigwedge T1\ T2\ T3.$
$[\![E,dt \models e1::-T1;\ E,dt \models e2::-T2;\ wt\text{-}binop\ (prg\ E)\ binop\ T1\ T2;$
$E,dt \models (if\ b\ then\ In1l\ e2\ else\ In1r\ Skip)::T3;$
$T = Inl\ (PrimT\ (binop\text{-}type\ binop))]\!]$
$\Longrightarrow P]\!]$
$\Longrightarrow P$
apply (*erule wt-elim-cases*)
apply (*cases b*)
apply *auto*
done

lemma *Inj-eq-lemma* [*simp*]:

$(\forall\, T.\ (\exists\, T'.\ T = Inj\ T' \wedge P\ T') \longrightarrow Q\ T) = (\forall\, T'.\ P\ T' \longrightarrow Q\ (Inj\ T'))$
by *auto*

lemma *single-valued-tys-lemma* [*rule-format* (*no-asm*)]:
$\forall\, S\ T.\ G \vdash S \preceq T \longrightarrow G \vdash T \preceq S \longrightarrow S = T \Longrightarrow E,dt \models t::T \Longrightarrow$
$G = prg\ E \longrightarrow (\forall\, T'.\ E,dt \models t::T' \longrightarrow T = T')$
apply (*cases E*, *erule wt.induct*)
apply (*safe del*: *disjE*)
apply (*simp-all* (*no-asm-use*) *split del*: *split-if-asm*)
apply (*safe del*: *disjE*)

apply (*tactic* ⟪ *ALLGOALS* (*fn i* => *if i* = *11 then EVERY'*[*thin-tac ?E,dt*⊨*e0*::−*PrimT Boolean*, *thin-tac ?E,dt*⊨*e1*::−*?T1*, *thin-tac ?E,dt*⊨*e2*::−*?T2*] *i else thin-tac All ?P i*) ⟫)

apply (*tactic* ⟪*ALLGOALS* (*eresolve-tac* (*thms wt-elim-cases*))⟫)
apply (*simp-all* (*no-asm-use*) *split del*: *split-if-asm*)
apply (*erule-tac* [*12*] *V* = *All ?P* **in** *thin-rl*)
apply ((*blast del*: *equalityCE dest*: *sym* [*THEN trans*])+)
done

lemma *single-valued-tys*:
ws-prog (*prg E*) $\Longrightarrow$ *single-valued* $\{(t,T).\ E,dt \models t::T\}$
apply (*unfold single-valued-def*)
apply *clarsimp*
apply (*rule single-valued-tys-lemma*)
apply (*auto intro*!: *widen-antisym*)
done

lemma *typeof-empty-is-type* [*rule-format* (*no-asm*)]:
$typeof\ (\lambda a.\ None)\ v = Some\ T \longrightarrow is\text{-}type\ G\ T$
apply (*rule val.induct*)
apply *auto*
done

lemma *typeof-is-type* [*rule-format* (*no-asm*)]:
$(\forall\, a.\ v \neq Addr\ a) \longrightarrow (\exists\, T.\ typeof\ dt\ v = Some\ T \wedge is\text{-}type\ G\ T)$
apply (*rule val.induct*)
prefer *5*
apply *fast*
apply (*simp-all* (*no-asm*))
done

end

Chapter 12

DefiniteAssignment

29 Definite Assignment

theory *DefiniteAssignment* **imports** *WellType* **begin**

Definite Assignment Analysis (cf. 16)

The definite assignment analysis approximates the sets of local variables that will be assigned at a certain point of evaluation, and ensures that we will only read variables which previously were assigned. It should conform to the following idea: If the evaluation of a term completes normally (no abruption (exception, break, continue, return) appeared) , the set of local variables calculated by the analysis is a subset of the variables that were actually assigned during evaluation.

To get more precise information about the sets of assigned variables the analysis includes the following optimisations:

- Inside of a while loop we also take care of the variables assigned before break statements, since the break causes the while loop to continue normally.
- For conditional statements we take care of constant conditions to statically determine the path of evaluation.
- Inside a distinct path of a conditional statements we know to which boolean value the condition has evaluated to, and so can retrieve more information about the variables assigned during evaluation of the boolean condition.

Since in our model of Java the return values of methods are stored in a local variable we also ensure that every path of (normal) evaluation will assign the result variable, or in the sense of real Java every path ends up in and return instruction.

Not covered yet:

- analysis of definite unassigned
- special treatment of final fields

Correct nesting of jump statements

For definite assignment it becomes crucial, that jumps (break, continue, return) are nested correctly i.e. a continue jump is nested in a matching while statement, a break jump is nested in a proper label statement, a class initialiser does not terminate abruptly with a return. With this we can for example ensure that evaluation of an expression will never end up with a jump, since no breaks, continues or returns are allowed in an expression.

consts *jumpNestingOkS* :: *jump set* ⇒ *stmt* ⇒ *bool*
primrec
jumpNestingOkS jmps (*Skip*) = *True*
jumpNestingOkS jmps (*Expr e*) = *True*
jumpNestingOkS jmps (*j*· *s*) = *jumpNestingOkS* ({*j*} ∪ *jmps*) *s*
jumpNestingOkS jmps (*c1*;;*c2*) = (*jumpNestingOkS jmps c1* ∧
jumpNestingOkS jmps c2)
jumpNestingOkS jmps (*If*(*e*) *c1 Else c2*) = (*jumpNestingOkS jmps c1* ∧
jumpNestingOkS jmps c2)
jumpNestingOkS jmps (*l*· *While*(*e*) *c*) = *jumpNestingOkS* ({*Cont l*} ∪ *jmps*) *c*
— The label of the while loop only handles continue jumps. Breaks are only handled by *Lab*
jumpNestingOkS jmps (*Jmp j*) = (*j* ∈ *jmps*)
jumpNestingOkS jmps (*Throw e*) = *True*
jumpNestingOkS jmps (*Try c1 Catch*(*C vn*) *c2*) = (*jumpNestingOkS jmps c1* ∧
jumpNestingOkS jmps c2)
jumpNestingOkS jmps (*c1 Finally c2*) = (*jumpNestingOkS jmps c1* ∧

jumpNestingOkS jmps c2)
jumpNestingOkS jmps (*Init C*) = *True*
— wellformedness of the program must enshure that for all initializers jumpNestingOkS holds
— Dummy analysis for intermediate smallstep term *FinA*
jumpNestingOkS jmps (*FinA a c*) = *False*

constdefs *jumpNestingOk* :: *jump set* ⇒ *term* ⇒ *bool*
jumpNestingOk jmps t ≡ (*case t of*
  In1 se ⇒ (*case se of*
    Inl e ⇒ *True*
    | *Inr s* ⇒ *jumpNestingOkS jmps s*)
  | *In2 v* ⇒ *True*
  | *In3 es* ⇒ *True*)

lemma *jumpNestingOk-expr-simp* [*simp*]: *jumpNestingOk jmps* (*In1l e*) = *True*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-expr-simp1* [*simp*]: *jumpNestingOk jmps* ⟨*e*::*expr*⟩ = *True*
by (*simp add*: *inj-term-simps*)

lemma *jumpNestingOk-stmt-simp* [*simp*]:
jumpNestingOk jmps (*In1r s*) = *jumpNestingOkS jmps s*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-stmt-simp1* [*simp*]:
jumpNestingOk jmps ⟨*s*::*stmt*⟩ = *jumpNestingOkS jmps s*
by (*simp add*: *inj-term-simps*)

lemma *jumpNestingOk-var-simp* [*simp*]: *jumpNestingOk jmps* (*In2 v*) = *True*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-var-simp1* [*simp*]: *jumpNestingOk jmps* ⟨*v*::*var*⟩ = *True*
by (*simp add*: *inj-term-simps*)

lemma *jumpNestingOk-expr-list-simp* [*simp*]: *jumpNestingOk jmps* (*In3 es*) = *True*
by (*simp add*: *jumpNestingOk-def*)

lemma *jumpNestingOk-expr-list-simp1* [*simp*]:
jumpNestingOk jmps ⟨*es*::*expr list*⟩ = *True*
by (*simp add*: *inj-term-simps*)

Calculation of assigned variables for boolean expressions

30 Very restricted calculation fallback calculation

consts *the-LVar-name*:: *var* ⇒ *lname*
primrec
the-LVar-name (*LVar n*) = *n*

consts *assignsE* :: *expr* ⇒ *lname set*

$assignsV$ :: $var \Rightarrow lname\ set$
$assignsEs$:: $expr\ list \Rightarrow lname\ set$

primrec
$assignsE\ (NewC\ c) = \{\}$
$assignsE\ (NewA\ t\ e) = assignsE\ e$
$assignsE\ (Cast\ t\ e) = assignsE\ e$
$assignsE\ (e\ InstOf\ r) = assignsE\ e$
$assignsE\ (Lit\ val) = \{\}$
$assignsE\ (UnOp\ unop\ e) = assignsE\ e$
$assignsE\ (BinOp\ binop\ e1\ e2) = (if\ binop{=}CondAnd \lor binop{=}CondOr$
$then\ (assignsE\ e1)$
$else\ (assignsE\ e1) \cup (assignsE\ e2))$
$assignsE\ (Super) = \{\}$
$assignsE\ (Acc\ v) = assignsV\ v$
$assignsE\ (v{:=}e) = (assignsV\ v) \cup (assignsE\ e) \cup$
$(if\ \exists\ n.\ v{=}(LVar\ n)\ then\ \{the\text{-}LVar\text{-}name\ v\}$
$else\ \{\})$
$assignsE\ (b?\ e1 : e2) = (assignsE\ b) \cup ((assignsE\ e1) \cap (assignsE\ e2))$
$assignsE\ (\{accC,statT,mode\}objRef{\cdot}mn(\{pTs\}args))$
$= (assignsE\ objRef) \cup (assignsEs\ args)$
— Only dummy analysis for intermediate expressions *Methd*, *Body*, *InsInitE* and *Callee*
$assignsE\ (Methd\ C\ sig) = \{\}$
$assignsE\ (Body\ C\ s) = \{\}$
$assignsE\ (InsInitE\ s\ e) = \{\}$
$assignsE\ (Callee\ l\ e) = \{\}$

$assignsV\ (LVar\ n) = \{\}$
$assignsV\ (\{accC,statDeclC,stat\}objRef..fn) = assignsE\ objRef$
$assignsV\ (e1.[e2]) = assignsE\ e1 \cup assignsE\ e2$

$assignsEs\ [] = \{\}$
$assignsEs\ (e\#es) = assignsE\ e \cup assignsEs\ es$

constdefs $assigns$:: $term \Rightarrow lname\ set$
$assigns\ t \equiv (case\ t\ of$
$In1\ se \Rightarrow (case\ se\ of$
$Inl\ e \Rightarrow assignsE\ e$
$|\ Inr\ s \Rightarrow \{\})$
$|\ In2\ v \Rightarrow assignsV\ v$
$|\ In3\ es \Rightarrow assignsEs\ es)$

lemma *assigns-expr-simp* [*simp*]: $assigns\ (In1l\ e) = assignsE\ e$
by (*simp add*: *assigns-def*)

lemma *assigns-expr-simp1* [*simp*]: $assigns\ (\langle e\rangle) = assignsE\ e$
by (*simp add*: *inj-term-simps*)

lemma *assigns-stmt-simp* [*simp*]: $assigns\ (In1r\ s) = \{\}$
by (*simp add*: *assigns-def*)

lemma *assigns-stmt-simp1* [*simp*]: $assigns\ (\langle s{::}stmt\rangle) = \{\}$
by (*simp add*: *inj-term-simps*)

lemma *assigns-var-simp* [*simp*]: *assigns* (*In2 v*) = *assignsV v*
by (*simp add*: *assigns-def*)

lemma *assigns-var-simp1* [*simp*]: *assigns* ($\langle v \rangle$) = *assignsV v*
by (*simp add*: *inj-term-simps*)

lemma *assigns-expr-list-simp* [*simp*]: *assigns* (*In3 es*) = *assignsEs es*
by (*simp add*: *assigns-def*)

lemma *assigns-expr-list-simp1* [*simp*]: *assigns* ($\langle es \rangle$) = *assignsEs es*
by (*simp add*: *inj-term-simps*)

31 Analysis of constant expressions

consts *constVal* :: *expr* ⇒ *val option*
primrec

```
constVal (NewC c)       = None
constVal (NewA t e)     = None
constVal (Cast t e)     = None
constVal (Inst e r)     = None
constVal (Lit val)      = Some val
constVal (UnOp unop e) = (case (constVal e) of
                            None   ⇒ None
                          | Some v ⇒ Some (eval-unop unop v))
constVal (BinOp binop e1 e2) = (case (constVal e1) of
                                  None    ⇒ None
                                | Some v1 ⇒ (case (constVal e2) of
                                               None    ⇒ None
                                             | Some v2 ⇒ Some (eval-binop
                                                                binop v1 v2)))
constVal (Super)        = None
constVal (Acc v)        = None
constVal (Ass v e)      = None
constVal (Cond b e1 e2)  = (case (constVal b) of
                             None   ⇒ None
                           | Some bv⇒ (case the-Bool bv of
                                         True ⇒ (case (constVal e2) of
                                                   None   ⇒ None
                                                 | Some v ⇒ constVal e1)
                                       | False⇒ (case (constVal e1) of
                                                   None   ⇒ None
                                                 | Some v ⇒ constVal e2)))
```

— Note that *constVal* (*Cond b e1 e2*) is stricter as it could be. It requires that all tree expressions are constant even if we can decide which branch to choose, provided the constant value of *b*

```
constVal (Call accC statT mode objRef mn pTs args) = None
constVal (Methd C sig)   = None
constVal (Body  C s)     = None
constVal (InsInitE s e)  = None
constVal (Callee l e)    = None
```

lemma *constVal-Some-induct* [*consumes 1*, *case-names Lit UnOp BinOp CondL CondR*]:
  assumes *const*: *constVal e* = *Some v* **and**
    hyp-Lit: $\bigwedge$ *v*. *P* (*Lit v*) **and**
    hyp-UnOp: $\bigwedge$ *unop e'*. *P e'* ⟹ *P* (*UnOp unop e'*) **and**
    hyp-BinOp: $\bigwedge$ *binop e1 e2*. ⟦*P e1*; *P e2*⟧ ⟹ *P* (*BinOp binop e1 e2*) **and**

```
    hyp-CondL: ⋀ b bv e1 e2. ⟦constVal b = Some bv; the-Bool bv; P b; P e1⟧
                    ⟹ P (b? e1 : e2) and
    hyp-CondR: ⋀ b bv e1 e2. ⟦constVal b = Some bv; ¬the-Bool bv; P b; P e2⟧
                    ⟹ P (b? e1 : e2)
  shows P e
proof −
  have True and ⋀ v. constVal e = Some v ⟹ P e and True and True
  proof (induct x::var and e and s::stmt and es::expr list)
    case Lit
    show ?case by (rule hyp-Lit)
  next
    case UnOp
    thus ?case
      by (auto intro: hyp-UnOp)
  next
    case BinOp
    thus ?case
      by (auto intro: hyp-BinOp)
  next
    case (Cond b e1 e2)
    then obtain v where   v: constVal (b ? e1 : e2) = Some v
      by blast
    then obtain bv where bv: constVal b = Some bv
      by simp
    show ?case
    proof (cases the-Bool bv)
      case True
      with Cond show ?thesis using v bv
        by (auto intro: hyp-CondL)
    next
      case False
      with Cond show ?thesis using v bv
        by (auto intro: hyp-CondR)
    qed
  qed (simp-all)
  with const
  show ?thesis
    by blast
qed


lemma assignsE-const-simp: constVal e = Some v ⟹ assignsE e = {}
  by (induct rule: constVal-Some-induct) simp-all
```

32 Main analysis for boolean expressions

Assigned local variables after evaluating the expression if it evaluates to a specific boolean value. If the expression cannot evaluate to a *Boolean* value UNIV is returned. If we expect true/false the opposite constant false/true will also lead to UNIV.

```
consts assigns-if:: bool ⇒ expr ⇒ lname set
primrec
assigns-if b (NewC c)          = UNIV — can never evaluate to Boolean
assigns-if b (NewA t e)        = UNIV — can never evaluate to Boolean
assigns-if b (Cast t e)        = assigns-if b e
assigns-if b (Inst e r)        = assignsE e — Inst has type Boolean but e is a reference type
assigns-if b (Lit val)         = (if val=Bool b then {} else UNIV)
assigns-if b (UnOp unop e)     = (case constVal (UnOp unop e) of
                                    None   ⇒ (if unop = UNot
```

```
                                          then assigns-if (¬b) e
                                          else UNIV)
                              | Some v ⇒ (if v=Bool b
                                          then {}
                                          else UNIV))
assigns-if b (BinOp binop e1 e2)
  = (case constVal (BinOp binop e1 e2) of
       None ⇒ (if binop=CondAnd then
                 (case b of
                     True  ⇒ assigns-if True  e1 ∪ assigns-if True e2
                  |  False ⇒ assigns-if False e1 ∩
                              (assigns-if True e1 ∪ assigns-if False e2))
               else
              (if binop=CondOr then
                 (case b of
                     True  ⇒ assigns-if True e1 ∩
                              (assigns-if False e1 ∪ assigns-if True e2)
                  |  False ⇒ assigns-if False  e1 ∪ assigns-if False e2)
               else assignsE e1 ∪ assignsE e2))
     | Some v ⇒ (if v=Bool b then {} else UNIV))

assigns-if b (Super)      = UNIV — can never evaluate to Boolean
assigns-if b (Acc v)      = (assignsV v)
assigns-if b (v := e)     = (assignsE (Ass v e))
assigns-if b (c? e1 : e2) = (assignsE c) ∪
                              (case (constVal c) of
                                 None    ⇒ (assigns-if b e1) ∩
                                              (assigns-if b e2)
                              | Some bv ⇒ (case the-Bool bv of
                                                True  ⇒ assigns-if b e1
                                              | False ⇒ assigns-if b e2))
assigns-if b ({accC,statT,mode}objRef·mn({pTs}args))
          = assignsE ({accC,statT,mode}objRef·mn({pTs}args))
— Only dummy analysis for intermediate expressions Methd, Body, InsInitE and Callee
assigns-if b (Methd C sig)   = {}
assigns-if b (Body  C s)     = {}
assigns-if b (InsInitE s e)  = {}
assigns-if b (Callee l e)    = {}
```

lemma *assigns-if-const-b-simp*:
  assumes *boolConst*: *constVal e = Some (Bool b)* (**is** *?Const b e*)
  shows *assigns-if b e = {}* (**is** *?Ass b e*)
proof −
  have *True* **and** ⋀ *b. ?Const b e ⟹ ?Ass b e* **and** *True* **and** *True*
  proof (*induct* - **and** *e* **and** - **and** - *rule*: *var-expr-stmt.inducts*)
    case *Lit*
    thus *?case* **by** *simp*
  next
    case *UnOp*
    thus *?case* **by** *simp*
  next
    case (*BinOp binop*)
    thus *?case*
      by (*cases binop*) (*simp-all*)
  next
    case (*Cond c e1 e2 b*)
    note *hyp-c* = ‹⋀ *b. ?Const b c ⟹ ?Ass b c*›
    note *hyp-e1* = ‹⋀ *b. ?Const b e1 ⟹ ?Ass b e1*›

```
    note hyp-e2 = ‹⋀ b. ?Const b e2 ⟹ ?Ass b e2›
    note const = ‹constVal (c ? e1 : e2) = Some (Bool b)›
    then obtain bv where bv: constVal c = Some bv
      by simp
    hence emptyC: assignsE c = {} by (rule assignsE-const-simp)
    show ?case
    proof (cases the-Bool bv)
      case True
      with const bv
      have ?Const b e1 by simp
      hence ?Ass b e1 by (rule hyp-e1)
      with emptyC bv True
      show ?thesis
        by simp
    next
      case False
      with const bv
      have ?Const b e2 by simp
      hence ?Ass b e2 by (rule hyp-e2)
      with emptyC bv False
      show ?thesis
        by simp
    qed
  qed (simp-all)
  with boolConst
  show ?thesis
    by blast
qed


lemma assigns-if-const-not-b-simp:
  assumes boolConst: constVal e = Some (Bool b)        (is ?Const b e)
  shows assigns-if (¬b) e = UNIV                    (is ?Ass b e)
proof −
  have True and ⋀ b. ?Const b e ⟹ ?Ass b e and True and True
  proof (induct - and e and - and - rule: var-expr-stmt.inducts)
    case Lit
    thus ?case by simp
  next
    case UnOp
    thus ?case by simp
  next
    case (BinOp binop)
    thus ?case
      by (cases binop) (simp-all)
  next
    case (Cond c e1 e2 b)
    note hyp-c = ‹⋀ b. ?Const b c ⟹ ?Ass b c›
    note hyp-e1 = ‹⋀ b. ?Const b e1 ⟹ ?Ass b e1›
    note hyp-e2 = ‹⋀ b. ?Const b e2 ⟹ ?Ass b e2›
    note const = ‹constVal (c ? e1 : e2) = Some (Bool b)›
    then obtain bv where bv: constVal c = Some bv
      by simp
    show ?case
    proof (cases the-Bool bv)
      case True
      with const bv
      have ?Const b e1 by simp
      hence ?Ass b e1 by (rule hyp-e1)
```

```
      with bv True
      show ?thesis
        by simp
    next
      case False
      with const bv
      have ?Const b e2 by simp
      hence ?Ass b e2 by (rule hyp-e2)
      with bv False
      show ?thesis
        by simp
    qed
  qed (simp-all)
  with boolConst
  show ?thesis
    by blast
qed
```

33 Lifting set operations to range of tables (map to a set)

constdefs
union-ts:: $('a,'b)$ *tables* $\Rightarrow$ $('a,'b)$ *tables* $\Rightarrow$ $('a,'b)$ *tables*
(- $\Rightarrow\cup$ - [67,67] 65)
$A \Rightarrow\cup B \equiv \lambda\ k.\ A\ k \cup B\ k$

constdefs
intersect-ts:: $('a,'b)$ *tables* $\Rightarrow$ $('a,'b)$ *tables* $\Rightarrow$ $('a,'b)$ *tables*
(- $\Rightarrow\cap$ - [72,72] 71)
$A \Rightarrow\cap B \equiv \lambda\ k.\ A\ k \cap B\ k$

constdefs
all-union-ts:: $('a,'b)$ *tables* $\Rightarrow$ $'b$ *set* $\Rightarrow$ $('a,'b)$ *tables*
(**infixl** $\Rightarrow\cup_\forall$ 40)
$A \Rightarrow\cup_\forall B \equiv \lambda\ k.\ A\ k \cup B$

Binary union of tables

lemma *union-ts-iff* [*simp*]: $(c \in (A \Rightarrow\cup B)\ k) = (c \in A\ k \vee c \in B\ k)$
by (*unfold union-ts-def*) *blast*

lemma *union-tsI1* [*elim?*]: $c \in A\ k \Longrightarrow c \in (A \Rightarrow\cup B)\ k$
by *simp*

lemma *union-tsI2* [*elim?*]: $c \in B\ k \Longrightarrow c \in (A \Rightarrow\cup B)\ k$
by *simp*

lemma *union-tsCI* [*intro*!]: $(c \notin B\ k \Longrightarrow c \in A\ k) \Longrightarrow c \in (A \Rightarrow\cup B)\ k$
by *auto*

lemma *union-tsE* [*elim*!]:
$[\![c \in (A \Rightarrow\cup B)\ k; (c \in A\ k \Longrightarrow P); (c \in B\ k \Longrightarrow P)]\!] \Longrightarrow P$
by (*unfold union-ts-def*) *blast*

Binary intersection of tables

lemma *intersect-ts-iff* [*simp*]: $c \in (A \Rightarrow\cap\ B)\ k = (c \in A\ k \wedge c \in B\ k)$
  by (*unfold intersect-ts-def*) *blast*

lemma *intersect-tsI* [*intro*!]: $[\![c \in A\ k;\ c \in B\ k]\!] \Longrightarrow c \in\ (A \Rightarrow\cap\ B)\ k$
  by *simp*

lemma *intersect-tsD1*: $c \in (A \Rightarrow\cap\ B)\ k \Longrightarrow c \in A\ k$
  by *simp*

lemma *intersect-tsD2*: $c \in (A \Rightarrow\cap\ B)\ k \Longrightarrow c \in B\ k$
  by *simp*

lemma *intersect-tsE* [*elim*!]:
  $[\![c \in (A \Rightarrow\cap\ B)\ k;\ [\![c \in A\ k;\ c \in B\ k]\!] \Longrightarrow P]\!] \Longrightarrow P$
  by *simp*

All-Union of tables and set

lemma *all-union-ts-iff* [*simp*]: $(c \in (A \Rightarrow\cup_\forall\ B)\ k) = (c \in A\ k \vee\ c \in B)$
  by (*unfold all-union-ts-def*) *blast*

lemma *all-union-tsI1* [*elim?*]: $c \in A\ k \Longrightarrow c \in (A \Rightarrow\cup_\forall\ B)\ k$
  by *simp*

lemma *all-union-tsI2* [*elim?*]: $c \in B \Longrightarrow c \in (A \Rightarrow\cup_\forall\ B)\ k$
  by *simp*

lemma *all-union-tsCI* [*intro*!]: $(c \notin B \Longrightarrow c \in A\ k) \Longrightarrow c \in (A \Rightarrow\cup_\forall\ B)\ k$
  by *auto*

lemma *all-union-tsE* [*elim*!]:
 $[\![c \in (A \Rightarrow\cup_\forall\ B)\ k;\ (c \in A\ k \Longrightarrow P);\ (c \in B \Longrightarrow P)]\!] \Longrightarrow P$
  by (*unfold all-union-ts-def*) *blast*

The rules of definite assignment

types *breakass* = (*label*, *lname*) *tables*
— Mapping from a break label, to the set of variables that will be assigned if the evaluation terminates with this break

record *assigned* =
  nrm :: *lname set* — Definetly assigned variables for normal completion
  brk :: *breakass* — Definetly assigned variables for abrupt completion with a break

constdefs *rmlab* :: $'a \Rightarrow ('a,'b)\ tables \Rightarrow ('a,'b)\ tables$
rmlab $k\ A \equiv \lambda\ x.\ if\ x{=}k\ then\ UNIV\ else\ A\ x$

constdefs *range-inter-ts* :: $('a,'b)\ tables \Rightarrow\ 'b\ set\ (\Rightarrow\bigcap\text{-}\ 80)$

$\Rightarrow\bigcap A \equiv \{x\ |x.\ \forall\ k.\ x \in A\ k\}$

In $E\vdash B$ »t» A, B denotes the "assigned" variables before evaluating term t, whereas A denotes the "assigned" variables after evaluating term t. The environment E is only needed for the conditional *-? - : -*. The definite assignment rules refer to the typing rules here to distinguish boolean and other expressions.

inductive
da :: *env* ⇒ *lname set* ⇒ *term* ⇒ *assigned* ⇒ *bool* (-⊢ - »-» - [65,65,65,65] 71)
where
Skip: *Env*⊢ *B* »⟨*Skip*⟩» (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|)

| *Expr*: *Env*⊢ *B* »⟨*e*⟩» *A*
⟹
Env⊢ *B* »⟨*Expr e*⟩» *A*
| *Lab*: ⟦*Env*⊢ *B* »⟨*c*⟩» *C*; *nrm A* = *nrm C* ∩ (*brk C*) *l*; *brk A* = *rmlab l* (*brk C*)⟧
⟹
Env⊢ *B* »⟨*Break l*· *c*⟩» *A*

| *Comp*: ⟦*Env*⊢ *B* »⟨*c1*⟩» *C1*; *Env*⊢ *nrm C1* »⟨*c2*⟩» *C2*;
nrm A = *nrm C2*; *brk A* = (*brk C1*) ⇒∩ (*brk C2*)⟧
⟹
Env⊢ *B* »⟨*c1*;; *c2*⟩» *A*

| *If*: ⟦*Env*⊢ *B* »⟨*e*⟩» *E*;
Env⊢ (*B* ∪ *assigns-if True e*) »⟨*c1*⟩» *C1*;
Env⊢ (*B* ∪ *assigns-if False e*) »⟨*c2*⟩» *C2*;
nrm A = *nrm C1* ∩ *nrm C2*;
brk A = *brk C1* ⇒∩ *brk C2* ⟧
⟹
Env⊢ *B* »⟨*If*(*e*) *c1 Else c2*⟩» *A*

— Note that E is not further used, because we take the specialized sets that also consider if the expression evaluates to true or false. Inside of e there is no `break` or `finally`, so the break map of E will be the trivial one. So *Env*⊢ *B* »⟨*e*⟩» *E* is just used to ensure the definite assignment in expression e. Notice the implicit analysis of a constant boolean expression e in this rule. For example, if e is constantly *True* then *assigns-if False e* = *UNIV* and therefor *nrm C2* = *UNIV*. So finally *nrm A* = *nrm C1*. For the break maps this trick workd too, because the trival break map will map all labels to *UNIV*. In the example, if no break occurs in *c2* the break maps will trivially map to *UNIV* and if a break occurs it will map to *UNIV* too, because *assigns-if False e* = *UNIV*. So in the intersection of the break maps the path *c2* will have no contribution.

| *Loop*: ⟦*Env*⊢ *B* »⟨*e*⟩» *E*;
Env⊢ (*B* ∪ *assigns-if True e*) »⟨*c*⟩» *C*;
nrm A = *nrm C* ∩ (*B* ∪ *assigns-if False e*);
brk A = *brk C*⟧
⟹
Env⊢ *B* »⟨*l*· *While*(*e*) *c*⟩» *A*
— The *Loop* rule resembles some of the ideas of the *If* rule. For the *nrm A* the set *B* ∪ *assigns-if False e* will be *UNIV* if the condition is constantly true. To normally exit the while loop, we must consider the body c to be completed normally (*nrm C*) or with a break. But in this model, the label l of the loop only handles continue labels, not break labels. The break label will be handled by an enclosing *Lab* statement. So we don't have to handle the breaks specially.

| *Jmp*: ⟦*jump*=*Ret* ⟶ *Result* ∈ *B*;
nrm A = *UNIV*;
brk A = (*case jump of*
Break l ⇒ λ *k*. *if k*=*l then B else UNIV*
| *Cont l* ⇒ λ *k*. *UNIV*
| *Ret* ⇒ λ *k*. *UNIV*)⟧

$\Longrightarrow$
$Env\vdash\ B\ »\langle Jmp\ jump\rangle»\ A$

— In case of a break to label l the corresponding break set is all variables assigned before the break. The assigned variables for normal completion of the *Jmp* is *UNIV*, because the statement will never complete normally. For continue and return the break map is the trivial one. In case of a return we enshure that the result value is assigned.

| *Throw*: $[\![Env\vdash\ B\ »\langle e\rangle»\ E;\ nrm\ A = UNIV;\ brk\ A = (\lambda\ l.\ UNIV)]\!]$
$\Longrightarrow Env\vdash\ B\ »\langle Throw\ e\rangle»\ A$

| *Try*: $[\![Env\vdash\ B\ »\langle c1\rangle»\ C1;$
$Env(\!|lcl := lcl\ Env(VName\ vn\mapsto Class\ C)|\!)\vdash\ (B \cup \{VName\ vn\})\ »\langle c2\rangle»\ C2;$
$nrm\ A = nrm\ C1 \cap nrm\ C2;$
$brk\ A = brk\ C1 \Rightarrow\cap\ brk\ C2]\!]$
$\Longrightarrow Env\vdash\ B\ »\langle Try\ c1\ Catch(C\ vn)\ c2\rangle»\ A$

| *Fin*: $[\![Env\vdash\ B\ »\langle c1\rangle»\ C1;$
$Env\vdash\ B\ »\langle c2\rangle»\ C2;$
$nrm\ A = nrm\ C1 \cup nrm\ C2;$
$brk\ A = ((brk\ C1) \Rightarrow\cup_\forall\ (nrm\ C2)) \Rightarrow\cap\ (brk\ C2)]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle c1\ Finally\ c2\rangle»\ A$

— The set of assigned variables before execution *c2* are the same as before execution *c1*, because *c1* could throw an exception and so we can't guarantee that any variable will be assigned in *c1*. The *Finally* statement completes normally if both *c1* and *c2* complete normally. If *c1* completes abruptly with a break, then *c2* also will be executed and may terminate normally or with a break. The overall break map then is the intersection of the maps of both paths. If *c2* terminates normally we have to extend all break sets in *brk C1* with *nrm C2* ($\Rightarrow\cup_\forall$). If *c2* exits with a break this break will appear in the overall result state. We don't know if *c1* completed normally or abruptly (maybe with an exception not only a break) so *c1* has no contribution to the break map following this path.

— Evaluation of expressions and the break sets of definite assignment: Thinking of a Java expression we assume that we can never have a break statement inside of a expression. So for all expressions the break sets could be set to the trivial one: $\lambda l.\ UNIV$. But we can't trivially proof, that evaluating an expression will never result in a break, allthough Java expressions allready syntactically don't allow nested stetements in them. The reason are the nested class initialzation statements which are inserted by the evaluation rules. So to proof the absence of a break we need to ensure, that the initialization statements will never end up in a break. In a wellfromed initialization statement, of course, were breaks are nested correctly inside of *Lab* or *Loop* statements evaluation of the whole initialization statement will never result in a break, because this break will be handled inside of the statement. But for simplicity we haven't added the analysis of the correct nesting of breaks in the typing judgments right now. So we have decided to adjust the rules of definite assignment to fit to these circumstances. If an initialization is involved during evaluation of the expression (evaluation rules *FVar*, *NewC* and *NewA*

| *Init*: $Env\vdash\ B\ »\langle Init\ C\rangle»\ (\!|nrm{=}B,brk{=}\lambda\ l.\ UNIV|\!)$

— Wellformedness of a program will ensure, that every static initialiser is definetly assigned and the jumps are nested correctly. The case here for *Init* is just for convenience, to get a proper precondition for the induction hypothesis in various proofs, so that we don't have to expand the initialisation on every point where it is triggerred by the evaluation rules.

| *NewC*: $Env\vdash\ B\ »\langle NewC\ C\rangle»\ (\!|nrm{=}B,brk{=}\lambda\ l.\ UNIV|\!)$

| *NewA*: $Env\vdash\ B\ »\langle e\rangle»\ A$
$\Longrightarrow$
$Env\vdash\ B\ »\langle New\ T[e]\rangle»\ A$

| *Cast*: $Env\vdash\ B\ »\langle e\rangle»\ A$
$\Longrightarrow$
$Env\vdash\ B\ »\langle Cast\ T\ e\rangle»\ A$

| *Inst*: $Env\vdash B \gg\langle e\rangle\gg A$
$\Longrightarrow$
$Env\vdash B \gg\langle e\ InstOf\ T\rangle\gg A$

| *Lit*: $Env\vdash B \gg\langle Lit\ v\rangle\gg (|nrm{=}B,brk{=}\lambda\ l.\ UNIV|)$

| *UnOp*: $Env\vdash B \gg\langle e\rangle\gg A$
$\Longrightarrow$
$Env\vdash B \gg\langle UnOp\ unop\ e\rangle\gg A$

| *CondAnd*: $[\![Env\vdash B \gg\langle e1\rangle\gg E1;\ Env\vdash (B \cup assigns\text{-}if\ True\ e1) \gg\langle e2\rangle\gg E2;$
$nrm\ A = B \cup (assigns\text{-}if\ True\ (BinOp\ CondAnd\ e1\ e2) \cap$
$assigns\text{-}if\ False\ (BinOp\ CondAnd\ e1\ e2));$
$brk\ A = (\lambda\ l.\ UNIV)\ ]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle BinOp\ CondAnd\ e1\ e2\rangle\gg A$

| *CondOr*: $[\![Env\vdash B \gg\langle e1\rangle\gg E1;\ Env\vdash (B \cup assigns\text{-}if\ False\ e1) \gg\langle e2\rangle\gg E2;$
$nrm\ A = B \cup (assigns\text{-}if\ True\ (BinOp\ CondOr\ e1\ e2) \cap$
$assigns\text{-}if\ False\ (BinOp\ CondOr\ e1\ e2));$
$brk\ A = (\lambda\ l.\ UNIV)\ ]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle BinOp\ CondOr\ e1\ e2\rangle\gg A$

| *BinOp*: $[\![Env\vdash B \gg\langle e1\rangle\gg E1;\ Env\vdash nrm\ E1 \gg\langle e2\rangle\gg A;$
$binop \neq CondAnd;\ binop \neq CondOr]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle BinOp\ binop\ e1\ e2\rangle\gg A$

| *Super*: $This \in B$
$\Longrightarrow$
$Env\vdash B \gg\langle Super\rangle\gg (|nrm{=}B,brk{=}\lambda\ l.\ UNIV|)$

| *AccLVar*: $[\![vn \in B;$
$nrm\ A = B;\ brk\ A = (\lambda\ k.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle Acc\ (LVar\ vn)\rangle\gg A$

— To properly access a local variable we have to test the definite assignment here. The variable must occur in the set B

| *Acc*: $[\![\forall\ vn.\ v \neq LVar\ vn;$
$Env\vdash B \gg\langle v\rangle\gg A]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle Acc\ v\rangle\gg A$

| *AssLVar*: $[\![Env\vdash B \gg\langle e\rangle\gg E;\ nrm\ A = nrm\ E \cup \{vn\};\ brk\ A = brk\ E]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle (LVar\ vn) := e\rangle\gg A$

| *Ass*: $[\![\forall\ vn.\ v \neq LVar\ vn;\ Env\vdash B \gg\langle v\rangle\gg V;\ Env\vdash nrm\ V \gg\langle e\rangle\gg A]\!]$
$\Longrightarrow$
$Env\vdash B \gg\langle v := e\rangle\gg A$

| *CondBool*: $[\![Env\vdash(c\ ?\ e1 : e2)::-(PrimT\ Boolean);$
$Env\vdash B \gg\langle c\rangle\gg C;$
$Env\vdash (B \cup assigns\text{-}if\ True\ \ c) \gg\langle e1\rangle\gg E1;$
$Env\vdash (B \cup assigns\text{-}if\ False\ c) \gg\langle e2\rangle\gg E2;$
$nrm\ A = B \cup (assigns\text{-}if\ True\ \ (c\ ?\ e1 : e2) \cap$
$assigns\text{-}if\ False\ (c\ ?\ e1 : e2));$

$brk\ A = (\lambda\ l.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle c\ ?\ e1\ :\ e2\rangle»\ A$

| *Cond*: $[\![\neg\ Env\vdash(c\ ?\ e1\ :\ e2)::-(PrimT\ Boolean);$
$Env\vdash\ B\ »\langle c\rangle»\ C;$
$Env\vdash\ (B \cup assigns\text{-}if\ True\ \ c)\ »\langle e1\rangle»\ E1;$
$Env\vdash\ (B \cup assigns\text{-}if\ False\ c)\ »\langle e2\rangle»\ E2;$
$nrm\ A = nrm\ E1 \cap nrm\ E2;\ brk\ A = (\lambda\ l.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle c\ ?\ e1\ :\ e2\rangle»\ A$

| *Call*: $[\![Env\vdash\ B\ »\langle e\rangle»\ E;\ Env\vdash\ nrm\ E\ »\langle args\rangle»\ A]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle\{accC,statT,mode\}e\cdot mn(\{pTs\}args)\rangle»\ A$

— The interplay of *Call*, *Methd* and *Body*: Why rules for *Methd* and *Body* at all? Note that a Java source program will not include bare *Methd* or *Body* terms. These terms are just introduced during evaluation. So definite assignment of *Call* does not consider *Methd* or *Body* at all. So for definite assignment alone we could omit the rules for *Methd* and *Body*. But since evaluation of the method invocation is split up into three rules we must ensure that we have enough information about the call even in the *Body* term to make sure that we can proof type safety. Also we must be able transport this information from *Call* to *Methd* and then further to *Body* during evaluation to establish the definite assignment of *Methd* during evaluation of *Call*, and of *Body* during evaluation of *Methd*. This is necessary since definite assignment will be a precondition for each induction hypothesis coming out of the evaluation rules, and therefor we have to establish the definite assignment of the sub-evaluation during the type-safety proof. Note that well-typedness is also a precondition for type-safety and so we can omit some assertion that are already ensured by well-typedness.
| *Methd*: $[\![methd\ (prg\ Env)\ D\ sig = Some\ m;$
$Env\vdash\ B\ »\langle Body\ (declclass\ m)\ (stmt\ (mbody\ (mthd\ m)))\rangle»\ A$
$]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle Methd\ D\ sig\rangle»\ A$

| *Body*: $[\![Env\vdash\ B\ »\langle c\rangle»\ C;\ jumpNestingOkS\ \{Ret\}\ c;\ Result \in nrm\ C;$
$nrm\ A = B;\ brk\ A = (\lambda\ l.\ UNIV)]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle Body\ D\ c\rangle»\ A$
— Note that *A* is not correlated to *C*. If the body statement returns abruptly with return, evaluation of *Body* will absorb this return and complete normally. So we cannot trivially get the assigned variables of the body statement since it has not completed normally or with a break. If the body completes normally we guarantee that the result variable is set with this rule. But if the body completes abruptly with a return we can't guarantee that the result variable is set here, since definite assignment only talks about normal completion and breaks. So for a return the *Jump* rule ensures that the result variable is set and then this information must be carried over to the *Body* rule by the conformance predicate of the state.
| *LVar*: $Env\vdash\ B\ »\langle LVar\ vn\rangle»\ (\!|nrm{=}B,\ brk{=}\lambda\ l.\ UNIV|\!)$

| *FVar*: $Env\vdash\ B\ »\langle e\rangle»\ A$
$\Longrightarrow$
$Env\vdash\ B\ »\langle\{accC,statDeclC,stat\}e..fn\rangle»\ A$

| *AVar*: $[\![Env\vdash\ B\ »\langle e1\rangle»\ E1;\ Env\vdash\ nrm\ E1\ »\langle e2\rangle»\ A]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle e1.[e2]\rangle»\ A$

| *Nil*: $Env\vdash\ B\ »\langle[]::expr\ list\rangle»\ (\!|nrm{=}B,\ brk{=}\lambda\ l.\ UNIV|\!)$

| *Cons*: $[\![Env\vdash\ B\ »\langle e::expr\rangle»\ E;\ Env\vdash\ nrm\ E\ »\langle es\rangle»\ A]\!]$
$\Longrightarrow$
$Env\vdash\ B\ »\langle e\#es\rangle»\ A$

declare *inj-term-sym-simps* [*simp*]
declare *assigns-if.simps* [*simp del*]
declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss => ss delloop split-all-tac*)) ⟫

inductive-cases *da-elim-cases* [*cases set*]:
Env⊢ *B* »⟨*Skip*⟩» *A*
Env⊢ *B* »*In1r Skip*» *A*
Env⊢ *B* »⟨*Expr e*⟩» *A*
Env⊢ *B* »*In1r* (*Expr e*)» *A*
Env⊢ *B* »⟨*l*• *c*⟩» *A*
Env⊢ *B* »*In1r* (*l*• *c*)» *A*
Env⊢ *B* »⟨*c1*;; *c2*⟩» *A*
Env⊢ *B* »*In1r* (*c1*;; *c2*)» *A*
Env⊢ *B* »⟨*If*(*e*) *c1 Else c2*⟩» *A*
Env⊢ *B* »*In1r* (*If*(*e*) *c1 Else c2*)» *A*
Env⊢ *B* »⟨*l*• *While*(*e*) *c*⟩» *A*
Env⊢ *B* »*In1r* (*l*• *While*(*e*) *c*)» *A*
Env⊢ *B* »⟨*Jmp jump*⟩» *A*
Env⊢ *B* »*In1r* (*Jmp jump*)» *A*
Env⊢ *B* »⟨*Throw e*⟩» *A*
Env⊢ *B* »*In1r* (*Throw e*)» *A*
Env⊢ *B* »⟨*Try c1 Catch*(*C vn*) *c2*⟩» *A*
Env⊢ *B* »*In1r* (*Try c1 Catch*(*C vn*) *c2*)» *A*
Env⊢ *B* »⟨*c1 Finally c2*⟩» *A*
Env⊢ *B* »*In1r* (*c1 Finally c2*)» *A*
Env⊢ *B* »⟨*Init C*⟩» *A*
Env⊢ *B* »*In1r* (*Init C*)» *A*
Env⊢ *B* »⟨*NewC C*⟩» *A*
Env⊢ *B* »*In1l* (*NewC C*)» *A*
Env⊢ *B* »⟨*New T*[*e*]⟩» *A*
Env⊢ *B* »*In1l* (*New T*[*e*])» *A*
Env⊢ *B* »⟨*Cast T e*⟩» *A*
Env⊢ *B* »*In1l* (*Cast T e*)» *A*
Env⊢ *B* »⟨*e InstOf T*⟩» *A*
Env⊢ *B* »*In1l* (*e InstOf T*)» *A*
Env⊢ *B* »⟨*Lit v*⟩» *A*
Env⊢ *B* »*In1l* (*Lit v*)» *A*
Env⊢ *B* »⟨*UnOp unop e*⟩» *A*
Env⊢ *B* »*In1l* (*UnOp unop e*)» *A*
Env⊢ *B* »⟨*BinOp binop e1 e2*⟩» *A*
Env⊢ *B* »*In1l* (*BinOp binop e1 e2*)» *A*
Env⊢ *B* »⟨*Super*⟩» *A*
Env⊢ *B* »*In1l* (*Super*)» *A*
Env⊢ *B* »⟨*Acc v*⟩» *A*
Env⊢ *B* »*In1l* (*Acc v*)» *A*
Env⊢ *B* »⟨*v* := *e*⟩» *A*
Env⊢ *B* »*In1l* (*v* := *e*)» *A*
Env⊢ *B* »⟨*c* ? *e1* : *e2*⟩» *A*
Env⊢ *B* »*In1l* (*c* ? *e1* : *e2*)» *A*
Env⊢ *B* »⟨{*accC*,*statT*,*mode*}*e*·*mn*({*pTs*}*args*)⟩» *A*
Env⊢ *B* »*In1l* ({*accC*,*statT*,*mode*}*e*·*mn*({*pTs*}*args*))» *A*
Env⊢ *B* »⟨*Methd C sig*⟩» *A*
Env⊢ *B* »*In1l* (*Methd C sig*)» *A*
Env⊢ *B* »⟨*Body D c*⟩» *A*
Env⊢ *B* »*In1l* (*Body D c*)» *A*
Env⊢ *B* »⟨*LVar vn*⟩» *A*

Env⊢ *B* »*In2* (*LVar vn*)» *A*
Env⊢ *B* »⟨{*accC*,*statDeclC*,*stat*}*e*..*fn*⟩» *A*
Env⊢ *B* »*In2* ({*accC*,*statDeclC*,*stat*}*e*..*fn*)» *A*
Env⊢ *B* »⟨*e1*.[*e2*]⟩» *A*
Env⊢ *B* »*In2* (*e1*.[*e2*])» *A*
Env⊢ *B* »⟨[]::*expr list*⟩» *A*
Env⊢ *B* »*In3* ([]::*expr list*)» *A*
Env⊢ *B* »⟨*e*#*es*⟩» *A*
Env⊢ *B* »*In3* (*e*#*es*)» *A*
declare *inj-term-sym-simps* [*simp del*]
declare *assigns-if.simps* [*simp*]
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*))) ⟫

lemma *da-Skip*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*Skip*⟩» *A*
  by (*auto intro*: *da.Skip*)

lemma *da-NewC*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*NewC C*⟩» *A*
  by (*auto intro*: *da.NewC*)

lemma *da-Lit*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*Lit v*⟩» *A*
  by (*auto intro*: *da.Lit*)

lemma *da-Super*: ⟦*This* ∈ *B*;*A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|)⟧ ⟹ *Env*⊢ *B* »⟨*Super*⟩» *A*
  by (*auto intro*: *da.Super*)

lemma *da-Init*: *A* = (|*nrm*=*B*,*brk*=λ *l*. *UNIV*|) ⟹ *Env*⊢ *B* »⟨*Init C*⟩» *A*
  by (*auto intro*: *da.Init*)

lemma *assignsE-subseteq-assigns-ifs*:
 assumes *boolEx*: *E*⊢*e*::−*PrimT Boolean* (**is** *?Boolean e*)
   shows *assignsE e* ⊆ *assigns-if True e* ∩ *assigns-if False e* (**is** *?Incl e*)
proof −
  have *True* **and** *?Boolean e* ⟹ *?Incl e* **and** *True* **and** *True*
  proof (*induct* - **and** *e* **and** - **and** - *rule*: *var-expr-stmt.inducts*)
    case (*Cast T e*)
    have *E*⊢*e*::− (*PrimT Boolean*)
    proof −
      from ‹*E*⊢(*Cast T e*)::− (*PrimT Boolean*)›
      obtain *Te* **where** *E*⊢*e*::−*Te*
                          prg E⊢*Te*≼? *PrimT Boolean*
        by *cases simp*
      thus *?thesis*
        by − (*drule cast-Boolean2*,*simp*)
    qed
    with *Cast.hyps*
    show *?case*
      by *simp*

```
  next
    case (Lit val)
    thus ?case
      by − (erule wt-elim-cases, cases val, auto simp add: empty-dt-def)
  next
    case (UnOp unop e)
    thus ?case
      by − (erule wt-elim-cases,cases unop,
            (fastsimp simp add: assignsE-const-simp)+)
  next
    case (BinOp binop e1 e2)
    from BinOp.prems obtain e1T e2T
      where E⊢e1::−e1T and E⊢e2::−e2T and wt-binop (prg E) binop e1T e2T
            and (binop-type binop) = Boolean
      by (elim wt-elim-cases) simp
    with BinOp.hyps
    show ?case
      by − (cases binop, auto simp add: assignsE-const-simp)
  next
    case (Cond c e1 e2)
    note hyp-c = ‹?Boolean c ⟹ ?Incl c›
    note hyp-e1 = ‹?Boolean e1 ⟹ ?Incl e1›
    note hyp-e2 = ‹?Boolean e2 ⟹ ?Incl e2›
    note wt = ‹E⊢(c ? e1 : e2)::−PrimT Boolean›
    then obtain
      boolean-c:  E⊢c::−PrimT Boolean and
      boolean-e1: E⊢e1::−PrimT Boolean and
      boolean-e2: E⊢e2::−PrimT Boolean
      by (elim wt-elim-cases) (auto dest: widen-Boolean2)
    show ?case
    proof (cases constVal c)
      case None
      with boolean-e1 boolean-e2
      show ?thesis
        using hyp-e1 hyp-e2
        by (auto)
    next
      case (Some bv)
      show ?thesis
      proof (cases the-Bool bv)
        case True
        with Some show ?thesis using hyp-e1 boolean-e1 by auto
      next
        case False
        with Some show ?thesis using hyp-e2 boolean-e2 by auto
      qed
    qed
  qed simp-all
  with boolEx
  show ?thesis
    by blast
qed
```

```
lemma rmlab-same-label [simp]: (rmlab l A) l = UNIV
  by (simp add: rmlab-def)
```

lemma *rmlab-same-label1* [*simp*]: $l{=}l' \Longrightarrow (rmlab\ l\ A)\ l' = UNIV$
  by (*simp add*: *rmlab-def*)

lemma *rmlab-other-label* [*simp*]: $l{\neq}l' \Longrightarrow (rmlab\ l\ A)\ l' = A\ l'$
  by (*auto simp add*: *rmlab-def*)

lemma *range-inter-ts-subseteq* [*intro*]: $\forall\ k.\ A\ k\ \subseteq B\ k \Longrightarrow\ \Rightarrow\bigcap A \subseteq \Rightarrow\bigcap B$
  by (*auto simp add*: *range-inter-ts-def*)

lemma *range-inter-ts-subseteq'*:
  $[\![\forall\ k.\ A\ k\ \subseteq B\ k;\ x \in \Rightarrow\bigcap A]\!] \Longrightarrow x \in \Rightarrow\bigcap B$
  by (*auto simp add*: *range-inter-ts-def*)

lemma *da-monotone*:
  assumes *da*: $Env\vdash\ B\ \ \text{»}t\text{»}\ A$ **and**
    $B \subseteq B'$ **and**
    da': $Env\vdash\ B'\ \text{»}t\text{»}\ A'$
  shows $(nrm\ A \subseteq nrm\ A') \wedge (\forall\ \ l.\ (brk\ A\ l \subseteq brk\ A'\ l))$
proof −
  from *da*
  show $\bigwedge\ B'\ A'.\ [\![Env\vdash\ B'\ \text{»}t\text{»}\ A';\ B \subseteq B']\!]$
      $\Longrightarrow (nrm\ A \subseteq nrm\ A') \wedge (\forall\ \ l.\ (brk\ A\ l \subseteq brk\ A'\ l))$
    (**is** *PROP ?Hyp Env B t A*)
  proof (*induct*)
    case *Skip*
    from *Skip.prems Skip.hyps*
    show *?case* **by** *cases simp*
  next
    case *Expr*
    from *Expr.prems Expr.hyps*
    show *?case* **by** *cases simp*
  next
    case (*Lab Env B c C A l B' A'*)
    note $A = ‹nrm\ A = nrm\ C \cap brk\ C\ l›\ ‹brk\ A = rmlab\ l\ (brk\ C)›$
    note $‹PROP\ ?Hyp\ Env\ B\ \langle c\rangle\ C›$
    moreover
    note $‹B \subseteq B'›$
    moreover
    obtain C'
      where $Env\vdash\ B'\ \text{»}\langle c\rangle\text{»}\ C'$
        and A': $nrm\ A' = nrm\ C' \cap brk\ C'\ l\ brk\ A' = rmlab\ l\ (brk\ C')$
      using *Lab.prems*
      by − (*erule da-elim-cases*,*simp*)
    ultimately
    have $nrm\ C \subseteq nrm\ C'$ **and** *hyp-brk*: $(\forall l.\ brk\ C\ l \subseteq brk\ C'\ l)$ **by** *auto*
    then
    have $nrm\ C \cap brk\ C\ l \subseteq nrm\ C' \cap brk\ C'\ l$ **by** *auto*
    moreover
    {
      fix l'
      from *hyp-brk*
      have $rmlab\ l\ (brk\ C)\ l'\ \subseteq rmlab\ l\ (brk\ C')\ l'$
        by (*cases* $l{=}l'$) *simp-all*

```
   }
   moreover note A A'
   ultimately show ?case
     by simp
 next
   case (Comp Env B c1 C1 c2 C2 A B' A')
   note A = ‹nrm A = nrm C2› ‹brk A = brk C1 ⇒∩  brk C2›
   from ‹Env⊢ B' »⟨c1;; c2⟩» A'›
   obtain  C1' C2'
     where da-c1: Env⊢ B' »⟨c1⟩» C1' and
           da-c2: Env⊢ nrm C1' »⟨c2⟩» C2'  and
           A': nrm A' = nrm C2' brk A' = brk C1' ⇒∩  brk C2'
     by (rule da-elim-cases) auto
   note ‹PROP ?Hyp Env B ⟨c1⟩ C1›
   moreover note ‹B ⊆ B'›
   moreover note da-c1
   ultimately have C1': nrm C1 ⊆ nrm C1' (∀ l. brk C1 l ⊆ brk C1' l)
     by auto
   note ‹PROP ?Hyp Env (nrm C1) ⟨c2⟩ C2›
   with da-c2 C1'
   have C2': nrm C2 ⊆ nrm C2' (∀ l. brk C2 l ⊆ brk C2' l)
     by auto
   with A A' C1'
   show ?case
     by auto
 next
   case (If Env B e E c1 C1 c2 C2 A B' A')
   note A = ‹nrm A = nrm C1 ∩ nrm C2› ‹brk A = brk C1 ⇒∩  brk C2›
   from ‹Env⊢ B' »⟨If(e) c1 Else c2⟩» A'›
   obtain C1' C2'
     where da-c1: Env⊢ B' ∪ assigns-if True e »⟨c1⟩» C1' and
           da-c2: Env⊢ B' ∪ assigns-if False e »⟨c2⟩» C2' and
              A': nrm A' = nrm C1' ∩ nrm C2' brk A' = brk C1' ⇒∩  brk C2'
     by (rule da-elim-cases) auto
   note ‹PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c1⟩ C1›
   moreover note B' = ‹B ⊆ B'›
   moreover note da-c1
   ultimately obtain C1': nrm C1 ⊆ nrm C1' (∀ l. brk C1 l ⊆ brk C1' l)
     by blast
   note ‹PROP ?Hyp Env (B ∪ assigns-if False e) ⟨c2⟩ C2›
   with da-c2 B'
   obtain C2': nrm C2 ⊆ nrm C2' (∀ l. brk C2 l ⊆ brk C2' l)
     by blast
   with A A' C1'
   show ?case
     by auto
 next
   case (Loop Env B e E c C A l B' A')
   note A = ‹nrm A = nrm C ∩ (B ∪ assigns-if False e)› ‹brk A = brk C›
   from ‹Env⊢ B' »⟨l• While(e) c⟩» A'›
   obtain C'
     where
      da-c': Env⊢ B' ∪ assigns-if True e »⟨c⟩» C' and
         A': nrm A' = nrm C' ∩ (B' ∪ assigns-if False e)
             brk A' = brk C'
     by (rule da-elim-cases) auto
   note ‹PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c⟩ C›
   moreover note B' = ‹B ⊆ B'›
   moreover note da-c'
```

```
    ultimately obtain C': nrm C ⊆ nrm C' (∀ l. brk C l ⊆ brk C' l)
      by blast
    with A A' B'
    have nrm A ⊆ nrm A'
      by blast
    moreover
    { fix l'
      have  brk A l' ⊆ brk A' l'
      proof (cases constVal e)
        case None
        with A A' C'
        show ?thesis
           by (cases l=l') auto
      next
        case (Some bv)
        with A A' C'
        show ?thesis
          by (cases the-Bool bv, cases l=l') auto
      qed
    }
    ultimately show ?case
      by auto
  next
    case (Jmp jump B A Env B' A')
    thus ?case by (elim da-elim-cases) (auto split: jump.splits)
  next
    case Throw thus ?case by −  (erule da-elim-cases, auto)
  next
    case (Try Env B c1 C1 vn C c2 C2 A B' A')
    note A = ‹nrm A = nrm C1 ∩ nrm C2› ‹brk A = brk C1 ⇒∩  brk C2›
    from ‹Env⊢ B' »⟨Try c1 Catch(C vn) c2⟩» A'›
    obtain C1' C2'
      where da-c1': Env⊢ B' »⟨c1⟩» C1' and
            da-c2': Env(|lcl := lcl Env(VName vn↦Class C)|)⊢ B' ∪ {VName vn}
                     »⟨c2⟩» C2' and
            A': nrm A' = nrm C1' ∩ nrm C2'
                brk A' = brk C1' ⇒∩  brk C2'
      by (rule da-elim-cases) auto
    note ‹PROP ?Hyp Env B ⟨c1⟩ C1›
    moreover note B' = ‹B ⊆ B'›
    moreover note da-c1'
    ultimately obtain C1': nrm C1 ⊆ nrm C1' (∀ l. brk C1 l ⊆ brk C1' l)
      by blast
    note ‹PROP ?Hyp (Env(|lcl := lcl Env(VName vn↦Class C)|))
                 (B ∪ {VName vn}) ⟨c2⟩ C2›
    with B' da-c2'
    obtain nrm C2 ⊆ nrm C2' (∀ l. brk C2 l ⊆ brk C2' l)
      by blast
    with C1' A A'
    show ?case
      by auto
  next
    case (Fin Env B c1 C1 c2 C2 A B' A')
    note A = ‹nrm A = nrm C1 ∪ nrm C2›
      ‹brk A = (brk C1 ⇒∪∀  nrm C2) ⇒∩ (brk C2)›
    from ‹Env⊢ B' »⟨c1 Finally c2⟩» A'›
    obtain C1' C2'
      where  da-c1': Env⊢ B' »⟨c1⟩» C1' and
             da-c2': Env⊢ B' »⟨c2⟩» C2' and
```

```
          A': nrm A' = nrm C1' ∪ nrm C2'
              brk A' = (brk C1' ⇒∪∀ nrm C2') ⇒∩ (brk C2')
    by (rule da-elim-cases) auto
  note ‹PROP ?Hyp Env B ⟨c1⟩ C1›
  moreover note B' = ‹B ⊆ B'›
  moreover note da-c1'
  ultimately obtain C1': nrm C1 ⊆ nrm C1' (∀ l. brk C1 l ⊆ brk C1' l)
    by blast
  note hyp-c2 = ‹PROP ?Hyp Env B ⟨c2⟩ C2›
  from da-c2' B'
   obtain nrm C2 ⊆ nrm C2' (∀ l. brk C2 l ⊆ brk C2' l)
     by − (drule hyp-c2,auto)
   with A A' C1'
   show ?case
     by auto
 next
   case Init thus ?case by − (erule da-elim-cases, auto)
 next
   case NewC thus ?case by − (erule da-elim-cases, auto)
 next
   case NewA thus ?case by − (erule da-elim-cases, auto)
 next
   case Cast thus ?case by − (erule da-elim-cases, auto)
 next
   case Inst thus ?case by − (erule da-elim-cases, auto)
 next
   case Lit thus ?case by − (erule da-elim-cases, auto)
 next
   case UnOp thus ?case by − (erule da-elim-cases, auto)
 next
   case (CondAnd Env B e1 E1 e2 E2 A B' A')
   note A = ‹nrm A = B ∪
                  assigns-if True (BinOp CondAnd e1 e2) ∩
                  assigns-if False (BinOp CondAnd e1 e2)›
          ‹brk A = (λl. UNIV)›
   from ‹Env⊢ B' »⟨BinOp CondAnd e1 e2⟩» A'›
   obtain A': nrm A' = B' ∪
                        assigns-if True (BinOp CondAnd e1 e2) ∩
                        assigns-if False (BinOp CondAnd e1 e2)
                 brk A' = (λl. UNIV)
     by (rule da-elim-cases) auto
   note B' = ‹B ⊆ B'›
   with A A' show ?case
     by auto
 next
   case CondOr thus ?case by − (erule da-elim-cases, auto)
 next
   case BinOp thus ?case by − (erule da-elim-cases, auto)
 next
   case Super thus ?case by − (erule da-elim-cases, auto)
 next
   case AccLVar thus ?case by − (erule da-elim-cases, auto)
 next
   case Acc thus ?case by − (erule da-elim-cases, auto)
 next
   case AssLVar thus ?case by − (erule da-elim-cases, auto)
 next
   case Ass thus ?case by − (erule da-elim-cases, auto)
 next
```

```
  case (CondBool Env c e1 e2 B C E1 E2 A B' A')
  note A = ‹nrm A = B ∪
                  assigns-if True (c ? e1 : e2) ∩
                  assigns-if False (c ? e1 : e2)›
        ‹brk A = (λl. UNIV)›
  note ‹Env⊢ (c ? e1 : e2)::− (PrimT Boolean)›
  moreover
  note ‹Env⊢ B' »⟨c ? e1 : e2⟩» A'›
  ultimately
  obtain A': nrm A' = B' ∪
                        assigns-if True (c ? e1 : e2) ∩
                        assigns-if False (c ? e1 : e2)
             brk A' = (λl. UNIV)
    by − (erule da-elim-cases,auto simp add: inj-term-simps)

  note B' = ‹B ⊆ B'›
  with A A' show ?case
    by auto
 next
  case (Cond Env c e1 e2 B C E1 E2 A B' A')
  note A = ‹nrm A = nrm E1 ∩ nrm E2› ‹brk A = (λl. UNIV)›
  note not-bool = ‹¬ Env⊢ (c ? e1 : e2)::− (PrimT Boolean)›
  from ‹Env⊢ B' »⟨c ? e1 : e2⟩» A'›
  obtain E1' E2'
    where da-e1': Env⊢ B' ∪ assigns-if True c »⟨e1⟩» E1' and
          da-e2': Env⊢ B' ∪ assigns-if False c »⟨e2⟩» E2' and
            A': nrm A' = nrm E1' ∩ nrm E2'
              brk A' = (λl. UNIV)
    using not-bool
    by  − (erule da-elim-cases, auto simp add: inj-term-simps)

  note ‹PROP ?Hyp Env (B ∪ assigns-if True c) ⟨e1⟩ E1›
  moreover note B' = ‹B ⊆ B'›
  moreover note da-e1'
  ultimately obtain E1': nrm E1 ⊆ nrm E1' (∀ l. brk E1 l ⊆ brk E1' l)
    by blast
  note ‹PROP ?Hyp Env (B ∪ assigns-if False c) ⟨e2⟩ E2›
  with B' da-e2'
  obtain nrm E2 ⊆ nrm E2' (∀ l. brk E2 l ⊆ brk E2' l)
    by blast
 with E1' A A'
 show ?case
   by auto
next
 case Call
 from Call.prems and Call.hyps
 show ?case by cases auto
next
 case Methd thus ?case by −  (erule da-elim-cases, auto)
next
 case Body thus ?case by −  (erule da-elim-cases, auto)
next
 case LVar thus ?case by −  (erule da-elim-cases, auto)
next
 case FVar thus ?case by −  (erule da-elim-cases, auto)
next
 case AVar thus ?case by −  (erule da-elim-cases, auto)
next
 case Nil thus ?case by −  (erule da-elim-cases, auto)
```

```
  next
    case Cons thus ?case by -  (erule da-elim-cases, auto)
  qed
qed (rule da', rule ‹B ⊆ B'›)


lemma da-weaken:
  assumes da: Env⊢ B »t» A and B ⊆ B'
  shows ∃ A'. Env ⊢ B' »t» A'
proof -
  note assigned.select-convs [Pure.intro]
  from da
  show ⋀ B'. B ⊆ B' ⟹ ∃ A'. Env⊢ B' »t» A' (is PROP ?Hyp Env B t)
  proof (induct)
    case Skip thus ?case by (iprover intro: da.Skip)
  next
    case Expr thus ?case by (iprover intro: da.Expr)
  next
    case (Lab Env B c C A l B')
    note ‹PROP ?Hyp Env B ⟨c⟩›
    moreover
    note B' = ‹B ⊆ B'›
    ultimately obtain C' where Env⊢ B' »⟨c⟩» C'
      by iprover
    then obtain A' where Env⊢ B' »⟨Break l· c⟩» A'
      by (iprover intro: da.Lab)
    thus ?case ..
  next
    case (Comp Env B c1 C1 c2 C2 A B')
    note da-c1 = ‹Env⊢ B »⟨c1⟩» C1›
    note ‹PROP ?Hyp Env B ⟨c1⟩›
    moreover
    note B' = ‹B ⊆ B'›
    ultimately obtain C1' where da-c1': Env⊢ B' »⟨c1⟩» C1'
      by iprover
    with da-c1 B'
    have
      nrm C1 ⊆ nrm C1'
      by (rule da-monotone [elim-format]) simp
    moreover
    note ‹PROP ?Hyp Env (nrm C1) ⟨c2⟩›
    ultimately obtain C2' where Env⊢ nrm C1' »⟨c2⟩» C2'
      by iprover
    with da-c1' obtain A' where Env⊢ B' »⟨c1;; c2⟩» A'
      by (iprover intro: da.Comp)
    thus ?case ..
  next
    case (If Env B e E c1 C1 c2 C2 A B')
    note B' = ‹B ⊆ B'›
    obtain  E' where Env⊢ B' »⟨e⟩» E'
    proof -
      have PROP ?Hyp Env B ⟨e⟩ by (rule If.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain C1' where Env⊢ (B' ∪ assigns-if True e) »⟨c1⟩» C1'
    proof -
      from B'
```

```
      have (B ∪ assigns-if True e) ⊆ (B' ∪ assigns-if True e)
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c1⟩ by (rule If.hyps)
      ultimately
      show ?thesis using that by iprover
    qed
    moreover
    obtain C2' where Env⊢ (B' ∪ assigns-if False e) »⟨c2⟩» C2'
    proof −
      from B' have (B ∪ assigns-if False e) ⊆ (B' ∪ assigns-if False e)
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if False e) ⟨c2⟩ by (rule If.hyps)
      ultimately
      show ?thesis using that by iprover
    qed
    ultimately
    obtain A' where Env⊢ B' »⟨If(e) c1 Else c2⟩» A'
      by (iprover intro: da.If)
    thus ?case ..
  next
    case (Loop Env B e E c C A l B')
    note B' = ‹B ⊆ B'›
    obtain E' where Env⊢ B' »⟨e⟩» E'
    proof −
      have PROP ?Hyp Env B ⟨e⟩ by (rule Loop.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain C' where Env⊢ (B' ∪ assigns-if True e) »⟨c⟩» C'
    proof −
      from B'
      have (B ∪ assigns-if True e) ⊆ (B' ∪ assigns-if True e)
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if True e) ⟨c⟩ by (rule Loop.hyps)
      ultimately
      show ?thesis using that by iprover
    qed
    ultimately
    obtain A' where Env⊢ B' »⟨l• While(e) c⟩» A'
      by (iprover intro: da.Loop )
    thus ?case ..
  next
    case (Jmp jump B A Env B')
    note B' = ‹B ⊆ B'›
    with Jmp.hyps have jump = Ret ⟶ Result ∈ B'
      by auto
    moreover
    obtain A'::assigned
              where  nrm A' = UNIV
                     brk A' = (case jump of
                                 Break l ⇒ λk. if k = l then B' else UNIV
                               | Cont l ⇒ λk. UNIV
                               | Ret ⇒ λk. UNIV)

      by  iprover
```

```
    ultimately have Env⊢ B' »⟨Jmp jump⟩» A'
      by (rule da.Jmp)
    thus ?case ..
  next
    case Throw thus ?case by (iprover intro: da.Throw )
  next
    case (Try Env B c1 C1 vn C c2 C2 A B')
    note B' = ‹B ⊆ B'›
    obtain C1' where Env⊢ B' »⟨c1⟩» C1'
    proof -
      have PROP ?Hyp Env B ⟨c1⟩ by (rule Try.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain C2' where
      Env(|lcl := lcl Env(VName vn↦Class C)|)⊢ B' ∪ {VName vn} »⟨c2⟩» C2'
    proof -
      from B' have B ∪ {VName vn} ⊆ B' ∪ {VName vn} by blast
      moreover
      have PROP ?Hyp (Env(|lcl := lcl Env(VName vn↦Class C)|))
                (B ∪ {VName vn}) ⟨c2⟩
        by (rule Try.hyps)
      ultimately
      show ?thesis using that by iprover
    qed
    ultimately
    obtain A' where Env⊢ B' »⟨Try c1 Catch(C vn) c2⟩» A'
      by (iprover intro: da.Try )
    thus ?case ..
  next
    case (Fin Env B c1 C1 c2 C2 A B')
    note B' = ‹B ⊆ B'›
    obtain C1' where C1': Env⊢ B' »⟨c1⟩» C1'
    proof -
      have PROP ?Hyp Env B ⟨c1⟩ by (rule Fin.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain C2' where Env⊢ B' »⟨c2⟩» C2'
    proof -
      have PROP ?Hyp Env B ⟨c2⟩ by (rule Fin.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    ultimately
    obtain A' where Env⊢ B' »⟨c1 Finally c2⟩» A'
      by (iprover intro: da.Fin )
    thus ?case ..
  next
    case Init thus ?case by (iprover intro: da.Init)
  next
    case NewC thus ?case by (iprover intro: da.NewC)
  next
    case NewA thus ?case by (iprover intro: da.NewA)
  next
    case Cast thus ?case by (iprover intro: da.Cast)
  next
```

```
    case Inst thus ?case by (iprover intro: da.Inst)
  next
    case Lit thus ?case by (iprover intro: da.Lit)
  next
    case UnOp thus ?case by (iprover intro: da.UnOp)
  next
    case (CondAnd Env B e1 E1 e2 E2 A B')
    note B' = ‹B ⊆ B'›
    obtain E1' where Env⊢ B' »⟨e1⟩» E1'
    proof -
      have PROP ?Hyp Env B ⟨e1⟩ by (rule CondAnd.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain E2' where Env⊢ B' ∪  assigns-if True e1 »⟨e2⟩» E2'
    proof -
      from B' have B ∪ assigns-if True e1 ⊆ B' ∪  assigns-if True e1
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if True e1) ⟨e2⟩ by (rule CondAnd.hyps)
      ultimately show ?thesis using that by iprover
    qed
    ultimately
    obtain A' where Env⊢ B' »⟨BinOp CondAnd e1 e2⟩» A'
      by (iprover intro: da.CondAnd)
    thus ?case ..
  next
    case (CondOr Env B e1 E1 e2 E2 A B')
    note B' = ‹B ⊆ B'›
    obtain E1' where Env⊢ B' »⟨e1⟩» E1'
    proof -
      have PROP ?Hyp Env B ⟨e1⟩ by (rule CondOr.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
    obtain E2' where Env⊢ B' ∪ assigns-if False e1 »⟨e2⟩» E2'
    proof -
      from B' have B ∪ assigns-if False e1 ⊆ B' ∪  assigns-if False e1
        by blast
      moreover
      have PROP ?Hyp Env (B ∪ assigns-if False e1) ⟨e2⟩ by (rule CondOr.hyps)
      ultimately show ?thesis using that by iprover
    qed
    ultimately
    obtain A' where Env⊢ B' »⟨BinOp CondOr e1 e2⟩» A'
      by (iprover intro: da.CondOr)
    thus ?case ..
  next
    case (BinOp Env B e1 E1 e2 A binop B')
    note B' = ‹B ⊆ B'›
    obtain E1' where E1': Env⊢ B' »⟨e1⟩» E1'
    proof -
      have PROP ?Hyp Env B ⟨e1⟩ by (rule BinOp.hyps)
      with B'
      show ?thesis using that by iprover
    qed
    moreover
```

```
  obtain A' where Env⊢ nrm E1' »⟨e2⟩» A'
  proof -
    have Env⊢ B »⟨e1⟩» E1 by (rule BinOp.hyps)
    from this B' E1'
    have nrm E1 ⊆ nrm E1'
      by (rule da-monotone [THEN conjE])
    moreover
    have PROP ?Hyp Env (nrm E1) ⟨e2⟩ by (rule BinOp.hyps)
    ultimately show ?thesis using that by iprover
  qed
  ultimately
  have Env⊢ B' »⟨BinOp binop e1 e2⟩» A'
    using BinOp.hyps by (iprover intro: da.BinOp)
  thus ?case ..
next
  case (Super B Env B')
  note B' = ‹B ⊆ B'›
  with Super.hyps have This ∈ B'
    by auto
  thus ?case by (iprover intro: da.Super)
next
  case (AccLVar vn B A Env B')
  note ‹vn ∈ B›
  moreover
  note ‹B ⊆ B'›
  ultimately have vn ∈ B' by auto
  thus ?case by (iprover intro: da.AccLVar)
next
  case Acc thus ?case by (iprover intro: da.Acc)
next
  case (AssLVar Env B e E A vn B')
  note B' = ‹B ⊆ B'›
  then obtain E' where Env⊢ B' »⟨e⟩» E'
    by (rule AssLVar.hyps [elim-format]) iprover
  then obtain A' where
    Env⊢ B' »⟨LVar vn:=e⟩» A'
    by (iprover intro: da.AssLVar)
  thus ?case ..
next
  case (Ass v Env B V e A B')
  note B' = ‹B ⊆ B'›
  note ‹∀ vn. v ≠ LVar vn›
  moreover
  obtain V' where V': Env⊢ B' »⟨v⟩» V'
  proof -
    have PROP ?Hyp Env B ⟨v⟩ by (rule Ass.hyps)
    with B'
    show ?thesis using that by iprover
  qed
  moreover
  obtain A' where Env⊢ nrm V' »⟨e⟩» A'
  proof -
    have Env⊢ B »⟨v⟩» V by (rule Ass.hyps)
    from this B' V'
    have nrm V ⊆ nrm V'
      by (rule da-monotone [THEN conjE])
    moreover
    have PROP ?Hyp Env (nrm V) ⟨e⟩ by (rule Ass.hyps)
    ultimately show ?thesis using that by iprover
```

qed
ultimately
have $Env\vdash\ B'\ \text{»}\langle v := e\rangle\text{»}\ A'$
by (*iprover intro*: *da.Ass*)
thus *?case* **..**
next
case (*CondBool Env c e1 e2 B C E1 E2 A B'*)
note $B' = \langle B \subseteq B'\rangle$
note $\langle Env\vdash(c\ ?\ e1 : e2)::-(PrimT\ Boolean)\rangle$
moreover obtain C' **where** C': $Env\vdash\ B'\ \text{»}\langle c\rangle\text{»}\ C'$
proof −
have *PROP ?Hyp Env B* $\langle c\rangle$ **by** (*rule CondBool.hyps*)
with B'
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain $E1'$ **where** $Env\vdash\ B' \cup$ *assigns-if True c* $\text{»}\langle e1\rangle\text{»}\ E1'$
proof −
from B'
have $(B \cup$ *assigns-if True c*$) \subseteq (B' \cup$ *assigns-if True c*$)$
by *blast*
moreover
have *PROP ?Hyp Env* $(B \cup$ *assigns-if True c*$)$ $\langle e1\rangle$ **by** (*rule CondBool.hyps*)
ultimately
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain $E2'$ **where** $Env\vdash\ B' \cup$ *assigns-if False c* $\text{»}\langle e2\rangle\text{»}\ E2'$
proof −
from B'
have $(B \cup$ *assigns-if False c*$) \subseteq (B' \cup$ *assigns-if False c*$)$
by *blast*
moreover
have *PROP ?Hyp Env* $(B \cup$ *assigns-if False c*$)$ $\langle e2\rangle$ **by**(*rule CondBool.hyps*)
ultimately
show *?thesis* **using** *that* **by** *iprover*
qed
ultimately
obtain A' **where** $Env\vdash\ B'\ \text{»}\langle c\ ?\ e1 : e2\rangle\text{»}\ A'$
by (*iprover intro*: *da.CondBool*)
thus *?case* **..**
next
case (*Cond Env c e1 e2 B C E1 E2 A B'*)
note $B' = \langle B \subseteq B'\rangle$
note $\langle\neg\ Env\vdash(c\ ?\ e1 : e2)::-(PrimT\ Boolean)\rangle$
moreover obtain C' **where** C': $Env\vdash\ B'\ \text{»}\langle c\rangle\text{»}\ C'$
proof −
have *PROP ?Hyp Env B* $\langle c\rangle$ **by** (*rule Cond.hyps*)
with B'
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain $E1'$ **where** $Env\vdash\ B' \cup$ *assigns-if True c* $\text{»}\langle e1\rangle\text{»}\ E1'$
proof −
from B'
have $(B \cup$ *assigns-if True c*$) \subseteq (B' \cup$ *assigns-if True c*$)$
by *blast*
moreover
have *PROP ?Hyp Env* $(B \cup$ *assigns-if True c*$)$ $\langle e1\rangle$ **by** (*rule Cond.hyps*)

ultimately
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain $E2'$ **where** $Env\vdash B' \cup$ *assigns-if False c* »⟨*e2*⟩» $E2'$
proof –
from B'
have $(B \cup$ *assigns-if False c*$) \subseteq (B' \cup$ *assigns-if False c*$)$
by *blast*
moreover
have *PROP ?Hyp Env* $(B \cup$ *assigns-if False c*$)$ ⟨*e2*⟩ **by** (*rule Cond.hyps*)
ultimately
show *?thesis* **using** *that* **by** *iprover*
qed
ultimately
obtain A' **where** $Env\vdash B'$ »⟨*c ? e1 : e2*⟩» A'
by (*iprover intro*: *da.Cond*)
thus *?case* **..**
next
case (*Call Env B e E args A accC statT mode mn pTs* B')
note $B' = \langle B \subseteq B' \rangle$
obtain E' **where** E': $Env\vdash B'$ »⟨*e*⟩» E'
proof –
have *PROP ?Hyp Env B* ⟨*e*⟩ **by** (*rule Call.hyps*)
with B'
show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain A' **where** $Env\vdash$ *nrm* E' »⟨*args*⟩» A'
proof –
have $Env\vdash B$ »⟨*e*⟩» E **by** (*rule Call.hyps*)
from *this* B' E'
have *nrm* $E \subseteq$ *nrm* E'
by (*rule da-monotone* [*THEN conjE*])
moreover
have *PROP ?Hyp Env* (*nrm E*) ⟨*args*⟩ **by** (*rule Call.hyps*)
ultimately show *?thesis* **using** *that* **by** *iprover*
qed
ultimately
have $Env\vdash B'$ »⟨{*accC*,*statT*,*mode*}*e*·*mn*({*pTs*}*args*)⟩» A'
by (*iprover intro*: *da.Call*)
thus *?case* **..**
next
case *Methd* **thus** *?case* **by** (*iprover intro*: *da.Methd*)
next
case (*Body Env B c C A D* B')
note $B' = \langle B \subseteq B' \rangle$
obtain C' **where** C': $Env\vdash B'$ »⟨*c*⟩» C' **and** *nrm-*C': *nrm* $C \subseteq$ *nrm* C'
proof –
have $Env\vdash B$ »⟨*c*⟩» C **by** (*rule Body.hyps*)
moreover note B'
moreover
from B' **obtain** C' **where** *da-c*: $Env\vdash B'$ »⟨*c*⟩» C'
by (*rule Body.hyps* [*elim-format*]) *blast*
ultimately
have *nrm* $C \subseteq$ *nrm* C'
by (*rule da-monotone* [*THEN conjE*])
with *da-c that* **show** *?thesis* **by** *iprover*
qed

moreover
note ‹*Result* ∈ *nrm C*›
with *nrm-C′* **have** *Result* ∈ *nrm C′*
  by *blast*
moreover note ‹*jumpNestingOkS* {*Ret*} *c*›
ultimately obtain *A′* **where**
  Env⊢ *B′* »⟨*Body D c*⟩» *A′*
  by (*iprover intro*: *da.Body*)
thus *?case* **..**
next
case *LVar* **thus** *?case* **by** (*iprover intro*: *da.LVar*)
next
case *FVar* **thus** *?case* **by** (*iprover intro*: *da.FVar*)
next
case (*AVar Env B e1 E1 e2 A B′*)
note *B′* = ‹*B* ⊆ *B′*›
obtain *E1′* **where** *E1′*: *Env*⊢ *B′* »⟨*e1*⟩» *E1′*
proof −
  have *PROP ?Hyp Env B* ⟨*e1*⟩ **by** (*rule AVar.hyps*)
  with *B′*
  show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain *A′* **where** *Env*⊢ *nrm E1′* »⟨*e2*⟩» *A′*
proof −
  have *Env*⊢ *B* »⟨*e1*⟩» *E1* **by** (*rule AVar.hyps*)
  from *this B′ E1′*
  have *nrm E1* ⊆ *nrm E1′*
    by (*rule da-monotone* [*THEN conjE*])
  moreover
  have *PROP ?Hyp Env* (*nrm E1*) ⟨*e2*⟩ **by** (*rule AVar.hyps*)
  ultimately show *?thesis* **using** *that* **by** *iprover*
qed
ultimately
have *Env*⊢ *B′* »⟨*e1*.[*e2*]⟩» *A′*
  by (*iprover intro*: *da.AVar*)
thus *?case* **..**
next
case *Nil* **thus** *?case* **by** (*iprover intro*: *da.Nil*)
next
case (*Cons Env B e E es A B′*)
note *B′* = ‹*B* ⊆ *B′*›
obtain *E′* **where** *E′*: *Env*⊢ *B′* »⟨*e*⟩» *E′*
proof −
  have *PROP ?Hyp Env B* ⟨*e*⟩ **by** (*rule Cons.hyps*)
  with *B′*
  show *?thesis* **using** *that* **by** *iprover*
qed
moreover
obtain *A′* **where** *Env*⊢ *nrm E′* »⟨*es*⟩» *A′*
proof −
  have *Env*⊢ *B* »⟨*e*⟩» *E* **by** (*rule Cons.hyps*)
  from *this B′ E′*
  have *nrm E* ⊆ *nrm E′*
    by (*rule da-monotone* [*THEN conjE*])
  moreover
  have *PROP ?Hyp Env* (*nrm E*) ⟨*es*⟩ **by** (*rule Cons.hyps*)
  ultimately show *?thesis* **using** *that* **by** *iprover*
qed

ultimately
have $Env \vdash B' \gg\langle e \ \# \ es\rangle\gg A'$
by (*iprover intro*: *da.Cons*)
thus *?case* **..**
qed
qed (*rule* ‹$B \subseteq B'$›)

corollary *da-weakenE* [*consumes 2*]:
assumes *da*: $Env \vdash B \ \gg t\gg \ A$ **and**
B': $B \subseteq B'$ **and**
ex-mono: $\bigwedge A'. \ [\![Env \vdash B' \gg t\gg A'; \ nrm \ A \subseteq nrm \ A';$
$\bigwedge l. \ brk \ A \ l \subseteq brk \ A' \ l]\!] \Longrightarrow P$
shows P
proof −
from *da* B'
obtain A' **where** A': $Env \vdash B' \gg t\gg A'$
by (*rule da-weaken* [*elim-format*]) *iprover*
with *da* B'
have $nrm \ A \subseteq nrm \ A' \wedge (\forall \ l. \ brk \ A \ l \subseteq brk \ A' \ l)$
by (*rule da-monotone*)
with A' *ex-mono*
show *?thesis*
by *iprover*
qed

end

Chapter 13

WellForm

34 Well-formedness of Java programs

theory *WellForm* **imports** *DefiniteAssignment* **begin**

For static checks on expressions and statements, see WellType.thy
improvements over Java Specification 1.0 (cf. 8.4.6.3, 8.4.6.4, 9.4.1):

- a method implementing or overwriting another method may have a result type that widens to the result type of the other method (instead of identical type)
- if a method hides another method (both methods have to be static!) there are no restrictions to the result type since the methods have to be static and there is no dynamic binding of static methods
- if an interface inherits more than one method with the same signature, the methods need not have identical return types

simplifications:

- Object and standard exceptions are assumed to be declared like normal classes

well-formed field declarations

well-formed field declaration (common part for classes and interfaces), cf. 8.3 and (9.3)

constdefs
wf-fdecl :: *prog* ⇒ *pname* ⇒ *fdecl* ⇒ *bool*
wf-fdecl G P ≡ λ(*fn*,*f*). *is-acc-type G P* (*type f*)

lemma *wf-fdecl-def2*: $\bigwedge$*fd*. *wf-fdecl G P fd* = *is-acc-type G P* (*type* (*snd fd*))
apply (*unfold wf-fdecl-def*)
apply *simp*
done

well-formed method declarations

A method head is wellformed if:

- the signature and the method head agree in the number of parameters
- all types of the parameters are visible
- the result type is visible
- the parameter names are unique

constdefs
wf-mhead :: *prog* ⇒ *pname* ⇒ *sig* ⇒ *mhead* ⇒ *bool*
wf-mhead G P ≡ λ *sig mh*. *length* (*parTs sig*) = *length* (*pars mh*) ∧
(∀ *T*∈*set* (*parTs sig*). *is-acc-type G P T*) ∧
is-acc-type G P (*resTy mh*) ∧
distinct (*pars mh*)

A method declaration is wellformed if:

- the method head is wellformed
- the names of the local variables are unique

- the types of the local variables must be accessible
- the local variables don't shadow the parameters
- the class of the method is defined
- the body statement is welltyped with respect to the modified environment of local names, were the local variables, the parameters the special result variable (Res) and This are assoziated with there types.

constdefs *callee-lcl*:: *qtname* ⇒ *sig* ⇒ *methd* ⇒ *lenv*
callee-lcl C sig m
 ≡ λ *k*. (*case k of*
  EName e
  ⇒ (*case e of*
   VNam v
   ⇒(*table-of* (*lcls* (*mbody m*))((*pars m*)[↦](*parTs sig*))) *v*
  | *Res* ⇒ *Some* (*resTy m*))
 | *This* ⇒ *if is-static m then None else Some* (*Class C*))

constdefs *parameters* :: *methd* ⇒ *lname set*
parameters m ≡ *set* (*map* (*EName* ∘ *VNam*) (*pars m*))
  ∪ (*if* (*static m*) *then* {} *else* {*This*})

constdefs
 wf-mdecl :: *prog* ⇒ *qtname* ⇒ *mdecl* ⇒ *bool*
 wf-mdecl G C ≡
  λ(*sig*,*m*).
   wf-mhead G (*pid C*) *sig* (*mhead m*) ∧
   unique (*lcls* (*mbody m*)) ∧
   (∀(*vn*,*T*)∈*set* (*lcls* (*mbody m*)). *is-acc-type G* (*pid C*) *T*) ∧
   (∀*pn*∈*set* (*pars m*). *table-of* (*lcls* (*mbody m*)) *pn* = *None*) ∧
   jumpNestingOkS {*Ret*} (*stmt* (*mbody m*)) ∧
   is-class G C ∧
   (|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|)⊢(*stmt* (*mbody m*))::√ ∧
   (∃ *A*. (|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|)
    ⊢ *parameters m* »⟨*stmt* (*mbody m*)⟩» *A*
   ∧ *Result* ∈ *nrm A*)

lemma *callee-lcl-VNam-simp* [*simp*]:
callee-lcl C sig m (*EName* (*VNam v*))
 = (*table-of* (*lcls* (*mbody m*))((*pars m*)[↦](*parTs sig*))) *v*
by (*simp add*: *callee-lcl-def*)

lemma *callee-lcl-Res-simp* [*simp*]:
callee-lcl C sig m (*EName Res*) = *Some* (*resTy m*)
by (*simp add*: *callee-lcl-def*)

lemma *callee-lcl-This-simp* [*simp*]:
callee-lcl C sig m (*This*) = (*if is-static m then None else Some* (*Class C*))
by (*simp add*: *callee-lcl-def*)

lemma *callee-lcl-This-static-simp*:
is-static m ⟹ *callee-lcl C sig m* (*This*) = *None*
by *simp*

lemma *callee-lcl-This-not-static-simp*:
$\neg$ *is-static m* $\Longrightarrow$ *callee-lcl C sig m* (*This*) = *Some* (*Class C*)
by *simp*

lemma *wf-mheadI*:
$[\![$*length* (*parTs sig*) = *length* (*pars m*); $\forall$ *T*$\in$*set* (*parTs sig*). *is-acc-type G P T*;
is-acc-type G P (*resTy m*); *distinct* (*pars m*)$]\!]$ $\Longrightarrow$
wf-mhead G P sig m
apply (*unfold wf-mhead-def*)
apply (*simp* (*no-asm-simp*))
done

lemma *wf-mdeclI*: $[\![$
wf-mhead G (*pid C*) *sig* (*mhead m*); *unique* (*lcls* (*mbody m*));
($\forall$ *pn*$\in$*set* (*pars m*). *table-of* (*lcls* (*mbody m*)) *pn* = *None*);
$\forall$ (*vn*,*T*)$\in$*set* (*lcls* (*mbody m*)). *is-acc-type G* (*pid C*) *T*;
jumpNestingOkS {*Ret*} (*stmt* (*mbody m*));
is-class G C;
(|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|)$\vdash$(*stmt* (*mbody m*))::$\surd$;
($\exists$ *A*. (|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|) $\vdash$ *parameters m* »$\langle$*stmt* (*mbody m*)$\rangle$» *A*
$\wedge$ *Result* $\in$ *nrm A*)
$]\!]$ $\Longrightarrow$
wf-mdecl G C (*sig*,*m*)
apply (*unfold wf-mdecl-def*)
apply *simp*
done

lemma *wf-mdeclE* [*consumes 1*]:
$[\![$*wf-mdecl G C* (*sig*,*m*);
$[\![$*wf-mhead G* (*pid C*) *sig* (*mhead m*); *unique* (*lcls* (*mbody m*));
$\forall$ *pn*$\in$*set* (*pars m*). *table-of* (*lcls* (*mbody m*)) *pn* = *None*;
$\forall$ (*vn*,*T*)$\in$*set* (*lcls* (*mbody m*)). *is-acc-type G* (*pid C*) *T*;
jumpNestingOkS {*Ret*} (*stmt* (*mbody m*));
is-class G C;
(|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|)$\vdash$(*stmt* (*mbody m*))::$\surd$;
($\exists$ *A*. (|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|)$\vdash$ *parameters m* »$\langle$*stmt* (*mbody m*)$\rangle$» *A*
$\wedge$ *Result* $\in$ *nrm A*)
$]\!]$ $\Longrightarrow$ *P*
$]\!]$ $\Longrightarrow$ *P*
by (*unfold wf-mdecl-def*) *simp*

lemma *wf-mdeclD1*:
wf-mdecl G C (*sig*,*m*) $\Longrightarrow$
wf-mhead G (*pid C*) *sig* (*mhead m*) $\wedge$ *unique* (*lcls* (*mbody m*)) $\wedge$
($\forall$ *pn*$\in$*set* (*pars m*). *table-of* (*lcls* (*mbody m*)) *pn* = *None*) $\wedge$
($\forall$ (*vn*,*T*)$\in$*set* (*lcls* (*mbody m*)). *is-acc-type G* (*pid C*) *T*)
apply (*unfold wf-mdecl-def*)
apply *simp*
done

lemma *wf-mdecl-bodyD*:

wf-mdecl G C (sig,m) $\Longrightarrow$
($\exists$ *T*. (|*prg*=*G*,*cls*=*C*,*lcl*=*callee-lcl C sig m*|)$\vdash$*Body C* (*stmt* (*mbody m*))::$-$*T* $\wedge$
G$\vdash$*T*$\preceq$(*resTy m*))
apply (*unfold wf-mdecl-def*)
apply *clarify*
apply (*rule-tac x*=(*resTy m*) **in** *exI*)
apply (*unfold wf-mhead-def*)
apply (*auto simp add*: *wf-mhead-def is-acc-type-def intro*: *wt.Body*)
done

lemma *rT-is-acc-type*:
wf-mhead G P sig m $\Longrightarrow$ *is-acc-type G P* (*resTy m*)
apply (*unfold wf-mhead-def*)
apply *auto*
done

well-formed interface declarations

A interface declaration is wellformed if:

- the interface hierarchy is wellstructured
- there is no class with the same name
- the method heads are wellformed and not static and have Public access
- the methods are uniquely named
- all superinterfaces are accessible
- the result type of a method overriding a method of Object widens to the result type of the overridden method. Shadowing static methods is forbidden.
- the result type of a method overriding a set of methods defined in the superinterfaces widens to each of the corresponding result types

constdefs
wf-idecl :: *prog* $\Rightarrow$ *idecl* $\Rightarrow$ *bool*
wf-idecl G $\equiv$
λ(*I*,*i*).
ws-idecl G I (*isuperIfs i*) $\wedge$
$\neg$*is-class G I* $\wedge$
($\forall$(*sig*,*mh*)$\in$*set* (*imethods i*). *wf-mhead G* (*pid I*) *sig mh* $\wedge$
$\neg$*is-static mh* $\wedge$
accmodi mh = *Public*) $\wedge$
unique (*imethods i*) $\wedge$
($\forall$ *J*$\in$*set* (*isuperIfs i*). *is-acc-iface G* (*pid I*) *J*) $\wedge$
(*table-of* (*imethods i*)
hiding (*methd G Object*)
under (λ *new old*. *accmodi old* $\neq$ *Private*)
entails (λ*new old*. *G*$\vdash$*resTy new*$\preceq$*resTy old* $\wedge$
is-static new = *is-static old*)) $\wedge$
(*o2s* $\circ$ *table-of* (*imethods i*)
hidings Un-tables((λ*J*.(*imethds G J*))‘*set* (*isuperIfs i*))
entails (λ*new old*. *G*$\vdash$*resTy new*$\preceq$*resTy old*))

lemma *wf-idecl-mhead*: $[\![$*wf-idecl G* (I,i); $(sig,mh) \in set$ $(imethods\ i)]\!] \Longrightarrow$
wf-mhead G (*pid I*) *sig mh* $\wedge$ $\neg$*is-static mh* $\wedge$ *accmodi mh* = *Public*
apply (*unfold wf-idecl-def*)
apply *auto*
done

lemma *wf-idecl-hidings*:
wf-idecl G $(I,\ i) \Longrightarrow$
($\lambda s.$ *o2s* (*table-of* (*imethods i*) *s*))
hidings Un-tables (($\lambda J.$ *imethds G J*) ‘ *set* (*isuperIfs i*))
entails λ*new old*. $G \vdash$*resTy new*$\preceq$*resTy old*
apply (*unfold wf-idecl-def o-def*)
apply *simp*
done

lemma *wf-idecl-hiding*:
wf-idecl G $(I,\ i) \Longrightarrow$
(*table-of* (*imethods i*)
hiding (*methd G Object*)
under (λ *new old*. *accmodi old* $\neq$ *Private*)
entails (λ*new old*. $G \vdash$*resTy new*$\preceq$*resTy old* $\wedge$
is-static new = *is-static old*))
apply (*unfold wf-idecl-def*)
apply *simp*
done

lemma *wf-idecl-supD*:
$[\![$*wf-idecl G* (I,i); $J \in$ *set* (*isuperIfs i*)$]\!]$
$\Longrightarrow$ *is-acc-iface G* (*pid I*) *J* $\wedge$ $(J,\ I) \notin$ (*subint1 G*) ^+
apply (*unfold wf-idecl-def ws-idecl-def*)
apply *auto*
done

well-formed class declarations

A class declaration is wellformed if:

- there is no interface with the same name
- all superinterfaces are accessible and for all methods implementing an interface method the result type widens to the result type of the interface method, the method is not static and offers at least as much access (this actually means that the method has Public access, since all interface methods have public access)
- all field declarations are wellformed and the field names are unique
- all method declarations are wellformed and the method names are unique
- the initialization statement is welltyped
- the classhierarchy is wellstructured
- Unless the class is Object:
  - the superclass is accessible

- for all methods overriding another method (of a superclass)the result type widens to the result type of the overridden method, the access modifier of the new method provides at least as much access as the overwritten one.
- for all methods hiding a method (of a superclass) the hidden method must be static and offer at least as much access rights. Remark: In contrast to the Java Language Specification we don't restrict the result types of the method (as in case of overriding), because there seems to be no reason, since there is no dynamic binding of static methods. (cf. 8.4.6.3 vs. 15.12.1). Stricly speaking the restrictions on the access rights aren't necessary to, since the static type and the access rights together determine which method is to be called statically. But if a class gains more then one static method with the same signature due to inheritance, it is confusing when the method selection depends on the access rights only: e.g. Class C declares static public method foo(). Class D is subclass of C and declares static method foo() with default package access. D.foo() ? if this call is in the same package as D then foo of class D is called, otherwise foo of class C.

constdefs *entails*:: $('a,'b)$ *table* $\Rightarrow$ $('b \Rightarrow$ *bool*$)$ $\Rightarrow$ *bool*
(- *entails* - *20*)
t entails P $\equiv \forall k.\ \forall\ x \in t\ k\colon P\ x$

lemma *entailsD*:
$[\![t\ \mathit{entails}\ P;\ t\ k = \mathit{Some}\ x]\!] \Longrightarrow P\ x$
by (*simp add*: *entails-def*)

lemma *empty-entails*[*simp*]: *empty entails P*
by (*simp add*: *entails-def*)

constdefs
wf-cdecl :: *prog* $\Rightarrow$ *cdecl* $\Rightarrow$ *bool*
wf-cdecl G $\equiv$
$\lambda(C,c)$.
$\neg$*is-iface G C* $\wedge$
$(\forall I{\in}set\ (superIfs\ c).\ is\text{-}acc\text{-}iface\ G\ (pid\ C)\ I\ \wedge$
$(\forall s.\ \forall\ im \in imethds\ G\ I\ s.$
$(\exists\ cm \in methd\ \ G\ C\ s\colon G{\vdash}resTy\ cm{\preceq}resTy\ im\ \wedge$
$\neg\ is\text{-}static\ cm\ \wedge$
$accmodi\ im \le accmodi\ cm))) \wedge$
$(\forall f{\in}set\ (cfields\ c).\ wf\text{-}fdecl\ G\ (pid\ C)\ f) \wedge unique\ (cfields\ c) \wedge$
$(\forall m{\in}set\ (methods\ c).\ wf\text{-}mdecl\ G\ C\ m) \wedge unique\ (methods\ c) \wedge$
$jumpNestingOkS\ \{\}\ (init\ c) \wedge$
$(\exists\ A.\ (\!|prg{=}G,cls{=}C,lcl{=}empty|\!){\vdash}\ \{\}\ \gg\langle init\ c\rangle\gg\ A) \wedge$
$(\!|prg{=}G,cls{=}C,lcl{=}empty|\!){\vdash}(init\ c){::}\surd \wedge ws\text{-}cdecl\ G\ C\ (super\ c) \wedge$
$(C \neq Object \longrightarrow$
$(is\text{-}acc\text{-}class\ G\ (pid\ C)\ (super\ c) \wedge$
$(table\text{-}of\ (map\ (\lambda\ (s,m).\ (s,C,m))\ (methods\ c))$
$entails\ (\lambda\ new.\ \forall\ \ old\ sig.$
$(G,sig{\vdash}new\ overrides_S\ old$
$\longrightarrow (G{\vdash}resTy\ new{\preceq}resTy\ old\ \wedge$
$accmodi\ old \le accmodi\ new\ \wedge$
$\neg is\text{-}static\ old)) \wedge$
$(G,sig{\vdash}new\ hides\ old$
$\longrightarrow (accmodi\ old \le accmodi\ new\ \wedge$
$is\text{-}static\ old))))$
$))$

lemma *wf-cdeclE* [*consumes 1*]:
$\llbracket$*wf-cdecl G* (*C*,*c*);
$\llbracket\neg$*is-iface G C*;
($\forall$ *I*$\in$*set* (*superIfs c*). *is-acc-iface G* (*pid C*) *I* $\wedge$
($\forall$ *s*. $\forall$ *im* $\in$ *imethds G I s*.
($\exists$ *cm* $\in$ *methd G C s*: *G*$\vdash$*resTy cm*$\preceq$*resTy im* $\wedge$
$\neg$ *is-static cm* $\wedge$
accmodi im $\leq$ *accmodi cm*)));
$\forall$ *f*$\in$*set* (*cfields c*). *wf-fdecl G* (*pid C*) *f*; *unique* (*cfields c*);
$\forall$ *m*$\in$*set* (*methods c*). *wf-mdecl G C m*; *unique* (*methods c*);
jumpNestingOkS {} (*init c*);
$\exists$ *A*. (|*prg*=*G*,*cls*=*C*,*lcl*=*empty*|)$\vdash$ {} »$\langle$*init c*$\rangle$» *A*;
(|*prg*=*G*,*cls*=*C*,*lcl*=*empty*|)$\vdash$(*init c*)::$\surd$;
ws-cdecl G C (*super c*);
(*C* $\neq$ *Object* $\longrightarrow$
(*is-acc-class G* (*pid C*) (*super c*) $\wedge$
(*table-of* (*map* (λ (*s*,*m*). (*s*,*C*,*m*)) (*methods c*))
entails (λ *new*. $\forall$ *old sig*.
(*G*,*sig*$\vdash$*new overrides*$_S$ *old*
$\longrightarrow$ (*G*$\vdash$*resTy new*$\preceq$*resTy old* $\wedge$
accmodi old $\leq$ *accmodi new* $\wedge$
$\neg$*is-static old*)) $\wedge$
(*G*,*sig*$\vdash$*new hides old*
$\longrightarrow$ (*accmodi old* $\leq$ *accmodi new* $\wedge$
is-static old))))
))$\rrbracket \Longrightarrow$ *P*
$\rrbracket \Longrightarrow$ *P*
by (*unfold wf-cdecl-def*) *simp*

lemma *wf-cdecl-unique*:
wf-cdecl G (*C*,*c*) $\Longrightarrow$ *unique* (*cfields c*) $\wedge$ *unique* (*methods c*)
apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-fdecl*:
$\llbracket$*wf-cdecl G* (*C*,*c*); *f*$\in$*set* (*cfields c*)$\rrbracket \Longrightarrow$ *wf-fdecl G* (*pid C*) *f*
apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-mdecl*:
$\llbracket$*wf-cdecl G* (*C*,*c*); *m*$\in$*set* (*methods c*)$\rrbracket \Longrightarrow$ *wf-mdecl G C m*
apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-impD*:
$\llbracket$*wf-cdecl G* (*C*,*c*); *I*$\in$*set* (*superIfs c*)$\rrbracket$
$\Longrightarrow$ *is-acc-iface G* (*pid C*) *I* $\wedge$
($\forall$ *s*. $\forall$ *im* $\in$ *imethds G I s*.
($\exists$ *cm* $\in$ *methd G C s*: *G*$\vdash$*resTy cm*$\preceq$*resTy im* $\wedge$ $\neg$*is-static cm* $\wedge$
accmodi im $\leq$ *accmodi cm*))

apply (*unfold wf-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-supD*:
$[\![$*wf-cdecl G* (*C*,*c*); *C* $\neq$ *Object*$]\!] \Longrightarrow$
is-acc-class G (*pid C*) (*super c*) $\wedge$ (*super c*,*C*) $\notin$ (*subcls1 G*)^+ $\wedge$
(*table-of* (*map* (λ (*s*,*m*). (*s*,*C*,*m*)) (*methods c*))
entails (λ *new*. $\forall$ *old sig*.
(*G*,*sig*$\vdash$*new overrides*$_S$ *old*
$\longrightarrow$ (*G*$\vdash$*resTy new*$\preceq$*resTy old* $\wedge$
accmodi old $\leq$ *accmodi new* $\wedge$
$\neg$*is-static old*)) $\wedge$
(*G*,*sig*$\vdash$*new hides old*
$\longrightarrow$ (*accmodi old* $\leq$ *accmodi new* $\wedge$
is-static old))))
apply (*unfold wf-cdecl-def ws-cdecl-def*)
apply *auto*
done

lemma *wf-cdecl-overrides-SomeD*:
$[\![$*wf-cdecl G* (*C*,*c*); *C* $\neq$ *Object*; *table-of* (*methods c*) *sig* = *Some newM*;
G,*sig*$\vdash$(*C*,*newM*) *overrides*$_S$ *old*
$]\!] \Longrightarrow$ *G*$\vdash$*resTy newM*$\preceq$*resTy old* $\wedge$
accmodi old $\leq$ *accmodi newM* $\wedge$
$\neg$ *is-static old*
apply (*drule* (*1*) *wf-cdecl-supD*)
apply (*clarify*)
apply (*drule entailsD*)
apply (*blast intro*: *table-of-map-SomeI*)
apply (*drule-tac x*=*old* **in** *spec*)
apply (*auto dest*: *overrides-eq-sigD simp add*: *msig-def*)
done

lemma *wf-cdecl-hides-SomeD*:
$[\![$*wf-cdecl G* (*C*,*c*); *C* $\neq$ *Object*; *table-of* (*methods c*) *sig* = *Some newM*;
G,*sig*$\vdash$(*C*,*newM*) *hides old*
$]\!] \Longrightarrow$ *accmodi old* $\leq$ *access newM* $\wedge$
is-static old
apply (*drule* (*1*) *wf-cdecl-supD*)
apply (*clarify*)
apply (*drule entailsD*)
apply (*blast intro*: *table-of-map-SomeI*)
apply (*drule-tac x*=*old* **in** *spec*)
apply (*auto dest*: *hides-eq-sigD simp add*: *msig-def*)
done

lemma *wf-cdecl-wt-init*:
wf-cdecl G (*C*, *c*) $\Longrightarrow$ (|*prg*=*G*,*cls*=*C*,*lcl*=*empty*|)$\vdash$*init c*::$\surd$
apply (*unfold wf-cdecl-def*)
apply *auto*
done

well-formed programs

A program declaration is wellformed if:

- the class ObjectC of Object is defined
- every method of Object has an access modifier distinct from Package. This is necessary since every interface automatically inherits from Object. We must know, that every time a Object method is "overriden" by an interface method this is also overriden by the class implementing the the interface (see *implement-dynmethd and class-mheadsD*)
- all standard Exceptions are defined
- all defined interfaces are wellformed
- all defined classes are wellformed

constdefs
wf-prog :: *prog* ⇒ *bool*
wf-prog G ≡ *let is* = *ifaces G*; *cs* = *classes G in*
ObjectC ∈ *set cs* ∧
(∀ *m*∈*set Object-mdecls*. *accmodi m* ≠ *Package*) ∧
(∀ *xn*. *SXcptC xn* ∈ *set cs*) ∧
(∀ *i*∈*set is*. *wf-idecl G i*) ∧ *unique is* ∧
(∀ *c*∈*set cs*. *wf-cdecl G c*) ∧ *unique cs*

lemma *wf-prog-idecl*: ⟦*iface G I* = *Some i*; *wf-prog G*⟧ ⟹ *wf-idecl G* (*I*,*i*)
apply (*unfold wf-prog-def Let-def*)
apply *simp*
apply (*fast dest*: *map-of-SomeD*)
done

lemma *wf-prog-cdecl*: ⟦*class G C* = *Some c*; *wf-prog G*⟧ ⟹ *wf-cdecl G* (*C*,*c*)
apply (*unfold wf-prog-def Let-def*)
apply *simp*
apply (*fast dest*: *map-of-SomeD*)
done

lemma *wf-prog-Object-mdecls*:
wf-prog G ⟹ (∀ *m*∈*set Object-mdecls*. *accmodi m* ≠ *Package*)
apply (*unfold wf-prog-def Let-def*)
apply *simp*
done

lemma *wf-prog-acc-superD*:
⟦*wf-prog G*; *class G C* = *Some c*; *C* ≠ *Object* ⟧
⟹ *is-acc-class G* (*pid C*) (*super c*)
by (*auto dest*: *wf-prog-cdecl wf-cdecl-supD*)

lemma *wf-ws-prog* [*elim!*,*simp*]: *wf-prog G* ⟹ *ws-prog G*
apply (*unfold wf-prog-def Let-def*)
apply (*rule ws-progI*)
apply (*simp-all* (*no-asm*))
apply (*auto simp add*: *is-acc-class-def is-acc-iface-def*

dest!: *wf-idecl-supD wf-cdecl-supD*)+
done

lemma *class-Object* [*simp*]:
wf-prog G $\Longrightarrow$
class G Object = *Some* (|*access*=*Public*,*cfields*=[],*methods*=*Object-mdecls*,
init=*Skip*,*super*=*arbitrary*,*superIfs*=[]|)
apply (*unfold wf-prog-def Let-def ObjectC-def*)
apply (*fast dest*!: *map-of-SomeI*)
done

lemma *methd-Object*[*simp*]: *wf-prog G* $\Longrightarrow$ *methd G Object* =
table-of (*map* (λ(*s*,*m*). (*s*, *Object*, *m*)) *Object-mdecls*)
apply (*subst methd-rec*)
apply (*auto simp add*: *Let-def*)
done

lemma *wf-prog-Object-methd*:
[[*wf-prog G*; *methd G Object sig* = *Some m*]] $\Longrightarrow$ *accmodi m* $\neq$ *Package*
by (*auto dest*!: *wf-prog-Object-mdecls*) (*auto dest*!: *map-of-SomeD*)

lemma *wf-prog-Object-is-public*[*intro*]:
wf-prog G $\Longrightarrow$ *is-public G Object*
by (*auto simp add*: *is-public-def dest*: *class-Object*)

lemma *class-SXcpt* [*simp*]:
wf-prog G $\Longrightarrow$
class G (*SXcpt xn*) = *Some* (|*access*=*Public*,*cfields*=[],*methods*=*SXcpt-mdecls*,
init=*Skip*,
super=*if xn* = *Throwable then Object*
else SXcpt Throwable,
superIfs=[]|)
apply (*unfold wf-prog-def Let-def SXcptC-def*)
apply (*fast dest*!: *map-of-SomeI*)
done

lemma *wf-ObjectC* [*simp*]:
wf-cdecl G ObjectC = ($\neg$*is-iface G Object* $\wedge$ *Ball* (*set Object-mdecls*)
(*wf-mdecl G Object*) $\wedge$ *unique Object-mdecls*)
apply (*unfold wf-cdecl-def ws-cdecl-def ObjectC-def*)
apply (*auto intro*: *da.Skip*)
done

lemma *Object-is-class* [*simp*,*elim*!]: *wf-prog G* $\Longrightarrow$ *is-class G Object*
apply (*simp* (*no-asm-simp*))
done

lemma *Object-is-acc-class* [*simp*,*elim*!]: *wf-prog G* $\Longrightarrow$ *is-acc-class G S Object*
apply (*simp* (*no-asm-simp*) *add*: *is-acc-class-def is-public-def*
accessible-in-RefT-simp)
done

lemma *SXcpt-is-class* [*simp*,*elim*!]: *wf-prog G* $\Longrightarrow$ *is-class G* (*SXcpt xn*)
apply (*simp* (*no-asm-simp*))
done

lemma *SXcpt-is-acc-class* [*simp*,*elim*!]:
wf-prog G $\Longrightarrow$ *is-acc-class G S* (*SXcpt xn*)
apply (*simp* (*no-asm-simp*) *add*: *is-acc-class-def is-public-def accessible-in-RefT-simp*)
done

lemma *fields-Object* [*simp*]: *wf-prog G* $\Longrightarrow$ *DeclConcepts.fields G Object* = []
by (*force intro*: *fields-emptyI*)

lemma *accfield-Object* [*simp*]:
wf-prog G $\Longrightarrow$ *accfield G S Object* = *empty*
apply (*unfold accfield-def*)
apply (*simp* (*no-asm-simp*) *add*: *Let-def*)
done

lemma *fields-Throwable* [*simp*]:
wf-prog G $\Longrightarrow$ *DeclConcepts.fields G* (*SXcpt Throwable*) = []
by (*force intro*: *fields-emptyI*)

lemma *fields-SXcpt* [*simp*]: *wf-prog G* $\Longrightarrow$ *DeclConcepts.fields G* (*SXcpt xn*) = []
apply (*case-tac xn* = *Throwable*)
apply (*simp* (*no-asm-simp*))
by (*force intro*: *fields-emptyI*)

lemmas *widen-trans* = *ws-widen-trans* [*OF* - - *wf-ws-prog*, *elim*]

lemma *widen-trans2* [*elim*]: $[\![G\vdash U\preceq T;\ G\vdash S\preceq U;\ \textit{wf-prog}\ G]\!] \Longrightarrow G\vdash S\preceq T$
apply (*erule* (*2*) *widen-trans*)
done

lemma *Xcpt-subcls-Throwable* [*simp*]:
wf-prog G $\Longrightarrow$ $G\vdash$*SXcpt xn*$\preceq_C$ *SXcpt Throwable*
apply (*rule SXcpt-subcls-Throwable-lemma*)
apply *auto*
done

lemma *unique-fields*:
$[\![$*is-class G C*; *wf-prog G*$]\!]$ $\Longrightarrow$ *unique* (*DeclConcepts.fields G C*)
apply (*erule ws-unique-fields*)
apply (*erule wf-ws-prog*)
apply (*erule* (*1*) *wf-prog-cdecl* [*THEN wf-cdecl-unique* [*THEN conjunct1*]])
done

lemma *fields-mono*:
$[\![$*table-of* (*DeclConcepts.fields G C*) *fn* = *Some f*; $G\vdash D\preceq_C$ *C*;

is-class G D; *wf-prog G*⟧
⟹ *table-of* (*DeclConcepts.fields G D*) *fn* = *Some f*
apply (*rule map-of-SomeI*)
apply (*erule* (*1*) *unique-fields*)
apply (*erule* (*1*) *map-of-SomeD* [*THEN fields-mono-lemma*])
apply (*erule wf-ws-prog*)
done

lemma *fields-is-type* [*elim*]:
⟦*table-of* (*DeclConcepts.fields G C*) *m* = *Some f*; *wf-prog G*; *is-class G C*⟧ ⟹
is-type G (*type f*)
apply (*frule wf-ws-prog*)
apply (*force dest*: *fields-declC* [*THEN conjunct1*]
wf-prog-cdecl [*THEN wf-cdecl-fdecl*]
simp add: *wf-fdecl-def2 is-acc-type-def*)
done

lemma *imethds-wf-mhead* [*rule-format* (*no-asm*)]:
⟦*m* ∈ *imethds G I sig*; *wf-prog G*; *is-iface G I*⟧ ⟹
wf-mhead G (*pid* (*decliface m*)) *sig* (*mthd m*) ∧
¬ *is-static m* ∧ *accmodi m* = *Public*
apply (*frule wf-ws-prog*)
apply (*drule* (*2*) *imethds-declI* [*THEN conjunct1*])
apply *clarify*
apply (*frule-tac I*=(*decliface m*) **in** *wf-prog-idecl*,*assumption*)
apply (*drule wf-idecl-mhead*)
apply (*erule map-of-SomeD*)
apply (*cases m*, *simp*)
done

lemma *methd-wf-mdecl*:
⟦*methd G C sig* = *Some m*; *wf-prog G*; *class G C* = *Some y*⟧ ⟹
G⊢*C*$\preceq_C$ (*declclass m*) ∧ *is-class G* (*declclass m*) ∧
wf-mdecl G (*declclass m*) (*sig*,(*mthd m*))
apply (*frule wf-ws-prog*)
apply (*drule* (*1*) *methd-declC*)
apply *fast*
apply *clarsimp*
apply (*frule* (*1*) *wf-prog-cdecl*, *erule wf-cdecl-mdecl*, *erule map-of-SomeD*)
done

lemma *methd-rT-is-type*:
⟦*wf-prog G*;*methd G C sig* = *Some m*;
class G C = *Some y*⟧
⟹ *is-type G* (*resTy m*)
apply (*drule* (*2*) *methd-wf-mdecl*)
apply *clarify*
apply (*drule wf-mdeclD1*)
apply *clarify*
apply (*drule rT-is-acc-type*)
apply (*cases m*, *simp add*: *is-acc-type-def*)

```
done


lemma accmethd-rT-is-type:
⟦wf-prog G;accmethd G S C sig = Some m;
  class G C = Some y⟧
⟹ is-type G (resTy m)
by (auto simp add: accmethd-def
       intro: methd-rT-is-type)


lemma methd-Object-SomeD:
⟦wf-prog G;methd G Object sig = Some m⟧
 ⟹ declclass m = Object
by (auto dest: class-Object simp add: methd-rec )


lemma wf-imethdsD:
 ⟦im ∈ imethds G I sig;wf-prog G; is-iface G I⟧
 ⟹ ¬is-static im ∧ accmodi im = Public
proof −
  assume asm: wf-prog G is-iface G I im ∈ imethds G I sig
  have wf-prog G ⟶
       (∀ i im. iface G I = Some i ⟶ im ∈ imethds G I sig
                ⟶ ¬is-static im ∧ accmodi im = Public) (is ?P G I)
  proof (rule iface-rec.induct,intro allI impI)
    fix G I i im
    assume hyp: ∀ J i. J ∈ set (isuperIfs i) ∧ ws-prog G ∧ iface G I = Some i
                ⟶ ?P G J
    assume wf: wf-prog G and if-I: iface G I = Some i and
           im: im ∈ imethds G I sig
    show ¬is-static im ∧ accmodi im = Public
    proof −
      let ?inherited = Un-tables (imethds G ‘ set (isuperIfs i))
      let ?new = (o2s ∘ table-of (map (λ(s, mh). (s, I, mh)) (imethods i)))
      from if-I wf im have imethds:im ∈ (?inherited ⊕⊕ ?new) sig
        by (simp add: imethds-rec)
      from wf if-I have
        wf-supI: ∀ J. J ∈ set (isuperIfs i) ⟶ (∃ j. iface G J = Some j)
        by (blast dest: wf-prog-idecl wf-idecl-supD is-acc-ifaceD)
      from wf if-I have
        ∀ im ∈ set (imethods i). ¬ is-static im ∧ accmodi im = Public
        by (auto dest!: wf-prog-idecl wf-idecl-mhead)
      then have new-ok: ∀ im. table-of (imethods i) sig = Some im
                        ⟶ ¬ is-static im ∧ accmodi im = Public
        by (auto dest!: table-of-Some-in-set)
      show ?thesis
        proof (cases ?new sig = {})
          case True
          from True wf wf-supI if-I imethds hyp
          show ?thesis by (auto simp del: split-paired-All)
        next
          case False
          from False wf wf-supI if-I imethds new-ok hyp
          show ?thesis by (auto dest: wf-idecl-hidings hidings-entailsD)
        qed
      qed
    qed
  with asm show ?thesis by (auto simp del: split-paired-All)
```

qed

lemma *wf-prog-hidesD*:
  assumes *hides*: $G \vdash$*new hides old* **and** *wf*: *wf-prog G*
  shows
   accmodi old $\leq$ *accmodi new* $\wedge$
    is-static old
proof −
  from *hides*
  obtain *c* **where**
    clsNew: *class G* (*declclass new*) = *Some c* **and**
    neqObj: *declclass new* $\neq$ *Object*
    by (*auto dest*: *hidesD declared-in-classD*)
  with *hides* **obtain** *newM oldM* **where**
    newM: *table-of* (*methods c*) (*msig new*) = *Some newM* **and**
     new: *new* = (*declclass new*,(*msig new*),*newM*) **and**
     old: *old* = (*declclass old*,(*msig old*),*oldM*) **and**
         msig new = *msig old*
    by (*cases new*,*cases old*)
       (*auto dest*: *hidesD*
        simp add: *cdeclaredmethd-def declared-in-def*)
  with *hides*
  have *hides′*:
        G,(*msig new*)$\vdash$(*declclass new*,*newM*) *hides* (*declclass old*,*oldM*)
    by *auto*
  from *clsNew wf*
  have *wf-cdecl G* (*declclass new*,*c*) **by** (*blast intro*: *wf-prog-cdecl*)
  note *wf-cdecl-hides-SomeD* [*OF this neqObj newM hides′*]
  with *new old*
  show *?thesis*
    by (*cases new*, *cases old*) *auto*
qed

Compare this lemma about static overriding $G \vdash$ *new overrides*$_S$ *old* with the definition of dynamic overriding $G \vdash$ *new overrides old*. Conforming result types and restrictions on the access modifiers of the old and the new method are not part of the predicate for static overriding. But they are enshured in a wellfromed program. Dynamic overriding has no restrictions on the access modifiers but enforces confrom result types as precondition. But with some efford we can guarantee the access modifier restriction for dynamic overriding, too. See lemma *wf-prog-dyn-override-prop*.

lemma *wf-prog-stat-overridesD*:
  assumes *stat-override*: $G \vdash$*new overrides*$_S$ *old* **and** *wf*: *wf-prog G*
  shows
   $G\vdash$*resTy new*$\preceq$*resTy old* $\wedge$
    accmodi old $\leq$ *accmodi new* $\wedge$
    $\neg$ *is-static old*
proof −
  from *stat-override*
  obtain *c* **where**
    clsNew: *class G* (*declclass new*) = *Some c* **and**
    neqObj: *declclass new* $\neq$ *Object*
    by (*auto dest*: *stat-overrides-commonD declared-in-classD*)
  with *stat-override* **obtain** *newM oldM* **where**
    newM: *table-of* (*methods c*) (*msig new*) = *Some newM* **and**
     new: *new* = (*declclass new*,(*msig new*),*newM*) **and**
     old: *old* = (*declclass old*,(*msig old*),*oldM*) **and**
         msig new = *msig old*
    by (*cases new*,*cases old*)

(*auto dest*: *stat-overrides-commonD*
simp add: *cdeclaredmethd-def declared-in-def*)
with *stat-override*
have *stat-override'*:
G,(*msig new*)⊢(*declclass new*,*newM*) *overrides*$_S$ (*declclass old*,*oldM*)
by *auto*
from *clsNew wf*
have *wf-cdecl G* (*declclass new*,*c*) **by** (*blast intro*: *wf-prog-cdecl*)
note *wf-cdecl-overrides-SomeD* [*OF this neqObj newM stat-override'*]
with *new old*
show *?thesis*
by (*cases new*, *cases old*) *auto*
qed

lemma *static-to-dynamic-overriding*:
assumes *stat-override*: *G*⊢*new overrides*$_S$ *old* **and** *wf* : *wf-prog G*
shows *G*⊢*new overrides old*
proof −
from *stat-override*
show *?thesis* (**is** *?Overrides new old*)
proof (*induct*)
case (*Direct new old superNew*)
then have *stat-override*:*G*⊢*new overrides*$_S$ *old*
by (*rule stat-overridesR.Direct*)
from *stat-override wf*
have *resTy-widen*: *G*⊢*resTy new*$\preceq$*resTy old* **and**
not-static-old: ¬ *is-static old*
by (*auto dest*: *wf-prog-stat-overridesD*)
have *not-private-new*: *accmodi new* ≠ *Private*
proof −
from *stat-override*
have *accmodi old* ≠ *Private*
by (*rule no-Private-stat-override*)
moreover
from *stat-override wf*
have *accmodi old* ≤ *accmodi new*
by (*auto dest*: *wf-prog-stat-overridesD*)
ultimately
show *?thesis*
by (*auto dest*: *acc-modi-bottom*)
qed
with *Direct resTy-widen not-static-old*
show *?Overrides new old*
by (*auto intro*: *overridesR.Direct stat-override-declclasses-relation*)
next
case (*Indirect new inter old*)
then show *?Overrides new old*
by (*blast intro*: *overridesR.Indirect*)
qed
qed

lemma *non-Package-instance-method-inheritance*:
assumes *old-inheritable*: *G*⊢*Method old inheritable-in* (*pid C*) **and**
accmodi-old: *accmodi old* ≠ *Package* **and**
instance-method: ¬ *is-static old* **and**
subcls: *G*⊢*C* $\prec_C$ *declclass old* **and**
old-declared: *G*⊢*Method old declared-in* (*declclass old*) **and**

```
                    wf: wf-prog G
  shows G⊢Method old member-of C ∨
   (∃ new. G⊢ new overrides_S old ∧ G⊢Method new member-of C)
proof −
  from wf have ws: ws-prog G by auto
  from old-declared have iscls-declC-old: is-class G (declclass old)
    by (auto simp add: declared-in-def cdeclaredmethd-def)
  from subcls have iscls-C: is-class G C
    by (blast dest: subcls-is-class)
  from iscls-C ws old-inheritable subcls
  show ?thesis (is ?P C old)
  proof (induct rule: ws-class-induct')
    case Object
    assume G⊢Object≺_C declclass old
    then show ?P Object old
      by blast
  next
    case (Subcls C c)
    assume cls-C: class G C = Some c and
       neq-C-Obj: C ≠ Object and
             hyp: ⟦G ⊢Method old inheritable-in pid (super c);
                   G⊢super c≺_C declclass old⟧ ⟹ ?P (super c) old and
     inheritable: G ⊢Method old inheritable-in pid C and
         subclsC: G⊢C≺_C declclass old
    from cls-C neq-C-Obj
    have super: G⊢C ≺_C1 super c
      by (rule subcls1I)
    from wf cls-C neq-C-Obj
    have accessible-super: G⊢(Class (super c)) accessible-in (pid C)
      by (auto dest: wf-prog-cdecl wf-cdecl-supD is-acc-classD)
    {
      fix old
      assume   member-super: G⊢Method old member-of (super c)
      assume    inheritable: G ⊢Method old inheritable-in pid C
      assume instance-method: ¬ is-static old
      from member-super
      have old-declared: G⊢Method old declared-in (declclass old)
       by (cases old) (auto dest: member-of-declC)
      have ?P C old
      proof (cases G⊢mid (msig old) undeclared-in C)
        case True
        with inheritable super accessible-super member-super
        have G⊢Method old member-of C
          by (cases old) (auto intro: members.Inherited)
        then show ?thesis
          by auto
      next
        case False
        then obtain new-member where
             G⊢new-member declared-in C and
             mid (msig old) = memberid new-member
          by (auto dest: not-undeclared-declared)
        then obtain new where
                new: G⊢Method new declared-in C and
             eq-sig: msig old = msig new and
          declC-new: declclass new = C
          by (cases new-member) auto
        then have member-new: G⊢Method new member-of C
          by (cases new) (auto intro: members.Immediate)
```

```
      from declC-new super member-super
      have subcls-new-old: G⊢declclass new ≺C declclass old
        by (auto dest!: member-of-subclseq-declC
                 dest: r-into-trancl intro: trancl-rtrancl-trancl)
      show ?thesis
      proof (cases is-static new)
        case False
        with eq-sig declC-new new old-declared inheritable
             super member-super subcls-new-old
        have G⊢new overridesS old
          by (auto intro!: stat-overridesR.Direct)
        with member-new show ?thesis
          by blast
      next
        case True
        with eq-sig declC-new subcls-new-old new old-declared inheritable
        have G⊢new hides old
          by (auto intro: hidesI)
        with wf
        have is-static old
          by (blast dest: wf-prog-hidesD)
        with instance-method
        show ?thesis
          by (contradiction)
      qed
    qed
  } note hyp-member-super = this
  from subclsC cls-C
  have G⊢(super c)≼C declclass old
    by (rule subcls-superD)
  then
  show ?P C old
  proof (cases rule: subclseq-cases)
    case Eq
    assume super c = declclass old
    with old-declared
    have G⊢Method old member-of (super c)
      by (cases old) (auto intro: members.Immediate)
    with inheritable instance-method
    show ?thesis
      by (blast dest: hyp-member-super)
  next
    case Subcls
    assume G⊢super c≺C declclass old
    moreover
    from inheritable accmodi-old
    have G ⊢Method old inheritable-in pid (super c)
      by (cases accmodi old) (auto simp add: inheritable-in-def)
    ultimately
    have ?P (super c) old
      by (blast dest: hyp)
    then show ?thesis
    proof
      assume G ⊢Method old member-of super c
      with inheritable instance-method
      show ?thesis
        by (blast dest: hyp-member-super)
    next
      assume ∃new. G ⊢ new overridesS old ∧ G ⊢Method new member-of super c
```

then obtain *super-new* **where**
super-new-override: $G \vdash$ *super-new overrides*$_S$ *old* **and**
super-new-member: $G \vdash$*Method super-new member-of super c*
by *blast*
from *super-new-override wf*
have *accmodi old* $\leq$ *accmodi super-new*
by (*auto dest*: *wf-prog-stat-overridesD*)
with *inheritable accmodi-old*
have $G \vdash$*Method super-new inheritable-in pid C*
by (*auto simp add*: *inheritable-in-def*
split: *acc-modi.splits*
dest: *acc-modi-le-Dests*)
moreover
from *super-new-override*
have $\neg$ *is-static super-new*
by (*auto dest*: *stat-overrides-commonD*)
moreover
note *super-new-member*
ultimately have *?P C super-new*
by (*auto dest*: *hyp-member-super*)
then show *?thesis*
proof
assume $G \vdash$*Method super-new member-of C*
with *super-new-override*
show *?thesis*
by *blast*
next
assume $\exists$ *new*. $G \vdash$ *new overrides*$_S$ *super-new* $\wedge$
$G \vdash$*Method new member-of C*
with *super-new-override* **show** *?thesis*
by (*blast intro*: *stat-overridesR.Indirect*)
qed
qed
qed
qed
qed

lemma *non-Package-instance-method-inheritance-cases* [*consumes 6*,
case-names Inheritance Overriding]:
assumes *old-inheritable*: $G\vdash$*Method old inheritable-in* (*pid C*) **and**
accmodi-old: *accmodi old* $\neq$ *Package* **and**
instance-method: $\neg$ *is-static old* **and**
subcls: $G\vdash C \prec_C$ *declclass old* **and**
old-declared: $G\vdash$*Method old declared-in* (*declclass old*) **and**
wf: *wf-prog G* **and**
inheritance: $G\vdash$*Method old member-of C* $\Longrightarrow$ *P* **and**
overriding: $\bigwedge$ *new*.
$[\![G\vdash$ *new overrides*$_S$ *old*;$G\vdash$*Method new member-of C*$]\!]$
$\Longrightarrow$ *P*
shows *P*
proof −
from *old-inheritable accmodi-old instance-method subcls old-declared wf*
inheritance overriding
show *?thesis*
by (*auto dest*: *non-Package-instance-method-inheritance*)
qed

lemma *dynamic-to-static-overriding*:
  assumes *dyn-override*: $G\vdash$ *new overrides old* **and**
      accmodi-old: *accmodi old* $\neq$ *Package* **and**
        wf: *wf-prog G*
  shows $G\vdash$ *new* $overrides_S$ *old*
proof –
  from *dyn-override accmodi-old*
  show *?thesis* (**is** *?Overrides new old*)
  proof (*induct rule*: *overridesR.induct*)
    case (*Direct new old*)
    assume *new-declared*: $G\vdash$*Method new declared-in declclass new*
    assume *eq-sig-new-old*: *msig new* = *msig old*
    assume *subcls-new-old*: $G\vdash$*declclass new* $\prec_C$ *declclass old*
    assume $G \vdash$*Method old inheritable-in pid* (*declclass new*) **and**
      accmodi old $\neq$ *Package* **and**
      $\neg$ *is-static old* **and**
      $G\vdash$*declclass new*$\prec_C$ *declclass old* **and**
      $G\vdash$*Method old declared-in declclass old*
    from *this wf*
    show *?Overrides new old*
    proof (*cases rule*: *non-Package-instance-method-inheritance-cases*)
      case *Inheritance*
      assume $G \vdash$*Method old member-of declclass new*
      then have $G\vdash$*mid* (*msig old*) *undeclared-in declclass new*
      proof *cases*
        case *Immediate*
        with *subcls-new-old wf* **show** *?thesis*
          by (*auto dest*: *subcls-irrefl*)
      next
        case *Inherited*
        then show *?thesis*
          by (*cases old*) *auto*
      qed
      with *eq-sig-new-old new-declared*
      show *?thesis*
        by (*cases old*,*cases new*) (*auto dest*!: *declared-not-undeclared*)
    next
      case (*Overriding new'*)
      assume *stat-override-new'*: $G \vdash$ *new'* $overrides_S$ *old*
      then have *msig new'* = *msig old*
        by (*auto dest*: *stat-overrides-commonD*)
      with *eq-sig-new-old* **have** *eq-sig-new-new'*: *msig new*=*msig new'*
        by *simp*
      assume $G \vdash$*Method new' member-of declclass new*
      then show *?thesis*
      proof (*cases*)
        case *Immediate*
        then have *declC-new*: *declclass new'* = *declclass new*
          by *auto*
        from *Immediate*
        have $G\vdash$*Method new' declared-in declclass new*
          by (*cases new'*) *auto*
        with *new-declared eq-sig-new-new' declC-new*
        have *new*=*new'*
          by (*cases new*, *cases new'*) (*auto dest*: *unique-declared-in*)
        with *stat-override-new'*
        show *?thesis*
          by *simp*
      next

```
      case Inherited
      then have G⊢mid (msig new') undeclared-in declclass new
        by (cases new') (auto)
      with eq-sig-new-new' new-declared
      show ?thesis
        by (cases new,cases new') (auto dest!: declared-not-undeclared)
    qed
  qed
next
  case (Indirect new inter old)
  assume accmodi-old: accmodi old ≠ Package
  assume accmodi old ≠ Package ⟹ G ⊢ inter overrides_S old
  with accmodi-old
  have stat-override-inter-old: G ⊢ inter overrides_S old
    by blast
  moreover
  assume hyp-inter: accmodi inter ≠ Package ⟹ G ⊢ new overrides_S inter
  moreover
  have accmodi inter ≠ Package
  proof −
    from stat-override-inter-old wf
    have accmodi old ≤ accmodi inter
      by (auto dest: wf-prog-stat-overridesD)
    with stat-override-inter-old accmodi-old
    show ?thesis
      by (auto dest!: no-Private-stat-override
               split: acc-modi.splits
               dest: acc-modi-le-Dests)
  qed
  ultimately show ?Overrides new old
    by (blast intro: stat-overridesR.Indirect)
qed
qed


lemma wf-prog-dyn-override-prop:
 assumes dyn-override: G ⊢ new overrides old and
                  wf: wf-prog G
 shows accmodi old ≤ accmodi new
proof (cases accmodi old = Package)
 case True
 note old-Package = this
 show ?thesis
 proof (cases accmodi old ≤ accmodi new)
   case True then show ?thesis .
 next
   case False
   with old-Package
   have accmodi new = Private
     by (cases accmodi new) (auto simp add: le-acc-def less-acc-def)
   with dyn-override
   show ?thesis
     by (auto dest: overrides-commonD)
 qed
next
 case False
 with dyn-override wf
 have G ⊢ new overrides_S old
   by (blast intro: dynamic-to-static-overriding)
```

with *wf*
show *?thesis*
by (*blast dest*: *wf-prog-stat-overridesD*)
qed

lemma *overrides-Package-old*:
assumes *dyn-override*: $G \vdash$ *new overrides old* **and**
accmodi-new: *accmodi new* = *Package* **and**
wf: *wf-prog G*
shows *accmodi old* = *Package*
proof (*cases accmodi old*)
case *Private*
with *dyn-override* **show** *?thesis*
by (*simp add*: *no-Private-override*)
next
case *Package*
then show *?thesis* **.**
next
case *Protected*
with *dyn-override wf*
have $G \vdash$ *new overrides*$_S$ *old*
by (*auto intro*: *dynamic-to-static-overriding*)
with *wf*
have *accmodi old* $\leq$ *accmodi new*
by (*auto dest*: *wf-prog-stat-overridesD*)
with *Protected accmodi-new*
show *?thesis*
by (*simp add*: *less-acc-def le-acc-def*)
next
case *Public*
with *dyn-override wf*
have $G \vdash$ *new overrides*$_S$ *old*
by (*auto intro*: *dynamic-to-static-overriding*)
with *wf*
have *accmodi old* $\leq$ *accmodi new*
by (*auto dest*: *wf-prog-stat-overridesD*)
with *Public accmodi-new*
show *?thesis*
by (*simp add*: *less-acc-def le-acc-def*)
qed

lemma *dyn-override-Package*:
assumes *dyn-override*: $G \vdash$ *new overrides old* **and**
accmodi-old: *accmodi old* = *Package* **and**
accmodi-new: *accmodi new* = *Package* **and**
wf: *wf-prog G*
shows *pid* (*declclass old*) = *pid* (*declclass new*)
proof −
from *dyn-override accmodi-old accmodi-new*
show *?thesis* (**is** *?EqPid old new*)
proof (*induct rule*: *overridesR.induct*)
case (*Direct new old*)
assume *accmodi old* = *Package*
$G \vdash$*Method old inheritable-in pid* (*declclass new*)
then show *pid* (*declclass old*) = *pid* (*declclass new*)
by (*auto simp add*: *inheritable-in-def*)
next

case (*Indirect new inter old*)
assume *accmodi-old*: *accmodi old* = *Package* **and**
accmodi-new: *accmodi new* = *Package*
assume $G \vdash$ *new overrides inter*
with *accmodi-new wf*
have *accmodi inter* = *Package*
by (*auto intro*: *overrides-Package-old*)
with *Indirect*
show *pid* (*declclass old*) = *pid* (*declclass new*)
by *auto*
qed
qed

lemma *dyn-override-Package-escape*:
assumes *dyn-override*: $G \vdash$ *new overrides old* **and**
accmodi-old: *accmodi old* = *Package* **and**
outside-pack: *pid* (*declclass old*) $\neq$ *pid* (*declclass new*) **and**
wf: *wf-prog G*
shows $\exists$ *inter*. $G \vdash$ *new overrides inter* $\wedge$ $G \vdash$ *inter overrides old* $\wedge$
pid (*declclass old*) = *pid* (*declclass inter*) $\wedge$
Protected $\leq$ *accmodi inter*
proof −
from *dyn-override accmodi-old outside-pack*
show *?thesis* (**is** *?P new old*)
proof (*induct rule*: *overridesR.induct*)
case (*Direct new old*)
assume *accmodi-old*: *accmodi old* = *Package*
assume *outside-pack*: *pid* (*declclass old*) $\neq$ *pid* (*declclass new*)
assume $G \vdash$*Method old inheritable-in pid* (*declclass new*)
with *accmodi-old*
have *pid* (*declclass old*) = *pid* (*declclass new*)
by (*simp add*: *inheritable-in-def*)
with *outside-pack*
show *?P new old*
by (*contradiction*)
next
case (*Indirect new inter old*)
assume *accmodi-old*: *accmodi old* = *Package*
assume *outside-pack*: *pid* (*declclass old*) $\neq$ *pid* (*declclass new*)
assume *override-new-inter*: $G \vdash$ *new overrides inter*
assume *override-inter-old*: $G \vdash$ *inter overrides old*
assume *hyp-new-inter*: $[\![$*accmodi inter* = *Package*;
pid (*declclass inter*) $\neq$ *pid* (*declclass new*)$]\!]$
$\Longrightarrow$ *?P new inter*
assume *hyp-inter-old*: $[\![$*accmodi old* = *Package*;
pid (*declclass old*) $\neq$ *pid* (*declclass inter*)$]\!]$
$\Longrightarrow$ *?P inter old*
show *?P new old*
proof (*cases pid* (*declclass old*) = *pid* (*declclass inter*))
case *True*
note *same-pack-old-inter* = *this*
show *?thesis*
proof (*cases pid* (*declclass inter*) = *pid* (*declclass new*))
case *True*
with *same-pack-old-inter outside-pack*
show *?thesis*
by *auto*
next

case *False*
note *diff-pack-inter-new* = *this*
show *?thesis*
proof (*cases accmodi inter* = *Package*)
case *True*
with *diff-pack-inter-new hyp-new-inter*
obtain *newinter* **where**
over-new-newinter: $G \vdash$ *new overrides newinter* **and**
over-newinter-inter: $G \vdash$ *newinter overrides inter* **and**
eq-pid: *pid* (*declclass inter*) = *pid* (*declclass newinter*) **and**
accmodi-newinter: *Protected* $\leq$ *accmodi newinter*
by *auto*
from *over-newinter-inter override-inter-old*
have $G \vdash$*newinter overrides old*
by (*rule overridesR.Indirect*)
moreover
from *eq-pid same-pack-old-inter*
have *pid* (*declclass old*) = *pid* (*declclass newinter*)
by *simp*
moreover
note *over-new-newinter accmodi-newinter*
ultimately show *?thesis*
by *blast*
next
case *False*
with *override-new-inter*
have *Protected* $\leq$ *accmodi inter*
by (*cases accmodi inter*) (*auto dest*: *no-Private-override*)
with *override-new-inter override-inter-old same-pack-old-inter*
show *?thesis*
by *blast*
qed
qed
next
case *False*
with *accmodi-old hyp-inter-old*
obtain *newinter* **where**
over-inter-newinter: $G \vdash$ *inter overrides newinter* **and**
over-newinter-old: $G \vdash$ *newinter overrides old* **and**
eq-pid: *pid* (*declclass old*) = *pid* (*declclass newinter*) **and**
accmodi-newinter: *Protected* $\leq$ *accmodi newinter*
by *auto*
from *override-new-inter over-inter-newinter*
have $G \vdash$ *new overrides newinter*
by (*rule overridesR.Indirect*)
with *eq-pid over-newinter-old accmodi-newinter*
show *?thesis*
by *blast*
qed
qed
qed

lemma *declclass-widen*[*rule-format*]:
wf-prog G
$\longrightarrow$ ($\forall$ *c m. class G C* = *Some c* $\longrightarrow$ *methd G C sig* = *Some m*
$\longrightarrow$ $G \vdash C \preceq_C$ *declclass m*) (**is** *?P G C*)
proof (*rule class-rec.induct*,*intro allI impI*)
fix *G C c m*

```
  assume Hyp: ∀ c. C ≠ Object ∧ ws-prog G ∧ class G C = Some c
              ⟶ ?P G (super c)
  assume wf: wf-prog G and cls-C: class G C = Some c and
         m:  methd G C sig = Some m
  show G⊢C≼C declclass m
  proof (cases C=Object)
    case True
    with wf m show ?thesis by (simp add: methd-Object-SomeD)
  next
    let ?filter=filter-tab (λsig m. G⊢C inherits method sig m)
    let ?table = table-of (map (λ(s, m). (s, C, m)) (methods c))
    case False
    with cls-C wf m
    have methd-C: (?filter (methd G (super c)) ++ ?table) sig = Some m
      by (simp add: methd-rec)
    show ?thesis
    proof (cases ?table sig)
      case None
      from this methd-C have ?filter (methd G (super c)) sig = Some m
        by simp
      moreover
      from wf cls-C False obtain sup where class G (super c) = Some sup
        by (blast dest: wf-prog-cdecl wf-cdecl-supD is-acc-class-is-class)
      moreover note wf False cls-C
      ultimately have G⊢super c ≼C declclass m
        by (auto intro: Hyp [rule-format])
      moreover from cls-C False have  G⊢C ≺C1 super c by (rule subcls1I)
      ultimately show ?thesis by − (rule rtrancl-into-rtrancl2)
    next
      case Some
      from this wf False cls-C methd-C show ?thesis by auto
    qed
  qed
qed
```

```
lemma declclass-methd-Object:
 ⟦wf-prog G; methd G Object sig = Some m⟧ ⟹ declclass m = Object
by auto
```

```
lemma methd-declaredD:
 ⟦wf-prog G; is-class G C;methd G C sig = Some m⟧
  ⟹ G⊢(mdecl (sig,mthd m)) declared-in (declclass m)
proof −
  assume    wf: wf-prog G
  then have ws: ws-prog G ..
  assume  clsC: is-class G C
  from clsC ws
  show methd G C sig = Some m
        ⟹ G⊢(mdecl (sig,mthd m)) declared-in (declclass m)
    (is PROP ?P C)
  proof (induct ?P C rule: ws-class-induct')
    case Object
    assume methd G Object sig = Some m
    with wf show ?thesis
      by − (rule method-declared-inI, auto)
  next
    case Subcls
```

```
    fix C c
    assume clsC: class G C = Some c
    and       m: methd G C sig = Some m
    and     hyp: methd G (super c) sig = Some m ⟹ ?thesis
    let ?newMethods = table-of (map (λ(s, m). (s, C, m)) (methods c))
    show ?thesis
    proof (cases ?newMethods sig)
      case None
      from None ws clsC m hyp
      show ?thesis by (auto intro: method-declared-inI simp add: methd-rec)
    next
      case Some
      from Some ws clsC m
      show ?thesis by (auto intro: method-declared-inI simp add: methd-rec)
    qed
  qed
qed


lemma methd-rec-Some-cases [consumes 4, case-names NewMethod InheritedMethod]:
  assumes methd-C: methd G C sig = Some m and
                ws: ws-prog G and
              clsC: class G C = Some c and
         neq-C-Obj: C≠Object
  shows
⟦table-of (map (λ(s, m). (s, C, m)) (methods c)) sig = Some m ⟹ P;
  ⟦G⊢C inherits (method sig m); methd G (super c) sig = Some m⟧ ⟹ P
 ⟧ ⟹ P
proof −
  let ?inherited   = filter-tab (λsig m. G⊢C inherits method sig m)
                              (methd G (super c))
  let ?new = table-of (map (λ(s, m). (s, C, m)) (methods c))
  from ws clsC neq-C-Obj methd-C
  have methd-unfold: (?inherited ++ ?new) sig = Some m
    by (simp add: methd-rec)
  assume NewMethod: ?new sig = Some m ⟹ P
  assume InheritedMethod: ⟦G⊢C inherits (method sig m);
                          methd G (super c) sig = Some m⟧ ⟹ P
  show P
  proof (cases ?new sig)
    case None
    with methd-unfold have ?inherited sig = Some m
      by (auto)
    with InheritedMethod show P by blast
  next
    case Some
    with methd-unfold have ?new sig = Some m
      by auto
    with NewMethod show P by blast
  qed
qed


lemma methd-member-of:
  assumes wf: wf-prog G
  shows
    ⟦is-class G C; methd G C sig = Some m⟧ ⟹ G⊢Methd sig m member-of C
  (is ?Class C ⟹ ?Method C ⟹ ?MemberOf C)
```

```
proof -
  from wf have ws: ws-prog G ..
  assume defC: is-class G C
  from defC ws
  show ?Class C ⟹ ?Method C ⟹ ?MemberOf C
  proof (induct rule: ws-class-induct')
    case Object
    with wf have declC: Object = declclass m
      by (simp add: declclass-methd-Object)
    from Object wf have G⊢Methd sig m declared-in Object
      by (auto intro: methd-declaredD simp add: declC)
    with declC
    show ?MemberOf Object
      by (auto intro!: members.Immediate
                simp del: methd-Object)
  next
    case (Subcls C c)
    assume clsC: class G C = Some c and
      neq-C-Obj: C ≠ Object
    assume methd: ?Method C
    from methd ws clsC neq-C-Obj
    show ?MemberOf C
    proof (cases rule: methd-rec-Some-cases)
      case NewMethod
      with clsC show ?thesis
        by (auto dest: method-declared-inI intro!: members.Immediate)
    next
      case InheritedMethod
      then show ?thesis
        by (blast dest: inherits-member)
    qed
  qed
qed
```

```
lemma current-methd:
      ⟦table-of (methods c) sig = Some new;
        ws-prog G; class G C = Some c; C ≠ Object;
        methd G (super c) sig = Some old⟧
    ⟹ methd G C sig = Some (C,new)
by (auto simp add: methd-rec
          intro: filter-tab-SomeI map-add-find-right table-of-map-SomeI)
```

```
lemma wf-prog-staticD:
  assumes     wf: wf-prog G and
            clsC: class G C = Some c and
       neq-C-Obj: C ≠ Object and
             old: methd G (super c) sig = Some old and
     accmodi-old: Protected ≤ accmodi old and
             new: table-of (methods c) sig = Some new
  shows is-static new = is-static old
proof -
  from clsC wf
  have wf-cdecl: wf-cdecl G (C,c) by (rule wf-prog-cdecl)
  from wf clsC neq-C-Obj
  have is-cls-super: is-class G (super c)
    by (blast dest: wf-prog-acc-superD is-acc-classD)
  from wf is-cls-super old
```

have *old-member-of*: $G\vdash$*Methd sig old member-of* (*super c*)
  by (*rule methd-member-of*)
from *old wf is-cls-super*
have *old-declared*: $G\vdash$*Methd sig old declared-in* (*declclass old*)
  by (*auto dest*: *methd-declared-in-declclass*)
from *new clsC*
have *new-declared*: $G\vdash$*Methd sig* (*C*,*new*) *declared-in C*
  by (*auto intro*: *method-declared-inI*)
note *trancl-rtrancl-tranc* = *trancl-rtrancl-trancl* [*trans*]
from *clsC neq-C-Obj*
have *subcls1-C-super*: $G\vdash C \prec_{C1}$ *super c*
  by (*rule subcls1I*)
then have $G\vdash C \prec_C$ *super c* **..**
also from *old wf is-cls-super*
have $G\vdash$*super c* $\preceq_C$ (*declclass old*) **by** (*auto dest*: *methd-declC*)
finally have *subcls-C-old*: $G\vdash C \prec_C$ (*declclass old*) **.**
from *accmodi-old*
have *inheritable*: $G\vdash$*Methd sig old inheritable-in pid C*
  by (*auto simp add*: *inheritable-in-def*
      dest: *acc-modi-le-Dests*)
show *?thesis*
proof (*cases is-static new*)
  case *True*
  with *subcls-C-old new-declared old-declared inheritable*
  have G,*sig*$\vdash$(*C*,*new*) *hides old*
    by (*auto intro*: *hidesI*)
  with *True wf-cdecl neq-C-Obj new*
  show *?thesis*
    by (*auto dest*: *wf-cdecl-hides-SomeD*)
next
  case *False*
  with *subcls-C-old new-declared old-declared inheritable subcls1-C-super*
    old-member-of
  have G,*sig*$\vdash$(*C*,*new*) *overrides*$_S$ *old*
    by (*auto intro*: *stat-overridesR.Direct*)
  with *False wf-cdecl neq-C-Obj new*
  show *?thesis*
    by (*auto dest*: *wf-cdecl-overrides-SomeD*)
qed
qed

lemma *inheritable-instance-methd*:
  assumes *subclseq-C-D*: $G\vdash C \preceq_C D$ **and**
    is-cls-D: *is-class G D* **and**
    wf: *wf-prog G* **and**
    old: *methd G D sig* = *Some old* **and**
    accmodi-old: *Protected* $\leq$ *accmodi old* **and**
    not-static-old: $\neg$ *is-static old*
  shows
  $\exists$ *new*. *methd G C sig* = *Some new* $\wedge$
    (*new* = *old* $\vee$ G,*sig*$\vdash$*new overrides*$_S$ *old*)
 (**is** ($\exists$ *new*. (*?Constraint C new old*)))
proof −
  from *subclseq-C-D is-cls-D*
  have *is-cls-C*: *is-class G C* **by** (*rule subcls-is-class2*)
  from *wf*
  have *ws*: *ws-prog G* **..**
  from *is-cls-C ws subclseq-C-D*

```
show ∃ new. ?Constraint C new old
proof (induct rule: ws-class-induct')
  case (Object co)
  then have eq-D-Obj: D=Object by auto
  with old
  have ?Constraint Object old old
    by auto
  with eq-D-Obj
  show ∃ new. ?Constraint Object new old by auto
next
  case (Subcls C c)
  assume hyp: G⊢super c≼C D ⟹ ∃ new. ?Constraint (super c) new old
  assume clsC: class G C = Some c
  assume neq-C-Obj: C≠Object
  from clsC wf
  have wf-cdecl: wf-cdecl G (C,c)
    by (rule wf-prog-cdecl)
  from ws clsC neq-C-Obj
  have is-cls-super: is-class G (super c)
    by (auto dest: ws-prog-cdeclD)
  from clsC wf neq-C-Obj
  have superAccessible: G⊢(Class (super c)) accessible-in (pid C) and
       subcls1-C-super: G⊢C ≺C1 super c
    by (auto dest: wf-prog-cdecl wf-cdecl-supD is-acc-classD
            intro: subcls1I)
  show ∃ new. ?Constraint C new old
  proof (cases G⊢super c≼C D)
    case False
    from False Subcls
    have eq-C-D: C=D
      by (auto dest: subclseq-superD)
    with old
    have ?Constraint C old old
      by auto
    with eq-C-D
    show ∃ new. ?Constraint C new old by auto
  next
    case True
    with hyp obtain super-method
      where super: ?Constraint (super c) super-method old by blast
    from super not-static-old
    have not-static-super: ¬ is-static super-method
      by (auto dest!: stat-overrides-commonD)
    from super old wf accmodi-old
    have accmodi-super-method: Protected ≤ accmodi super-method
      by (auto dest!: wf-prog-stat-overridesD)
    from super accmodi-old wf
    have inheritable: G⊢Methd sig super-method inheritable-in (pid C)
      by (auto dest!: wf-prog-stat-overridesD
                      acc-modi-le-Dests
            simp add: inheritable-in-def)
    from super wf is-cls-super
    have member: G⊢Methd sig super-method member-of (super c)
      by (auto intro: methd-member-of)
    from member
    have decl-super-method:
      G⊢Methd sig super-method declared-in (declclass super-method)
      by (auto dest: member-of-declC)
    from super subcls1-C-super ws is-cls-super
```

have *subcls-C-super*: $G\vdash C \prec_C$ (*declclass super-method*)
  by (*auto intro*: *rtrancl-into-trancl2 dest*: *methd-declC*)
show $\exists$ *new*. *?Constraint C new old*
proof (*cases methd G C sig*)
  case *None*
  have *methd G* (*super c*) *sig* = *None*
  proof −
    from *clsC ws None*
    have *no-new*: *table-of* (*methods c*) *sig* = *None*
      by (*auto simp add*: *methd-rec*)
    with *clsC*
    have *undeclared*: $G\vdash$*mid sig undeclared-in C*
      by (*auto simp add*: *undeclared-in-def cdeclaredmethd-def*)
    with *inheritable member superAccessible subcls1-C-super*
    have *inherits*: $G\vdash C$ *inherits* (*method sig super-method*)
      by (*auto simp add*: *inherits-def*)
    with *clsC ws no-new super neq-C-Obj*
    have *methd G C sig* = *Some super-method*
      by (*auto simp add*: *methd-rec map-add-def intro*: *filter-tab-SomeI*)
    with *None* **show** *?thesis*
      by *simp*
  qed
  with *super* **show** *?thesis* **by** *auto*
next
  case (*Some new*)
  from *this ws clsC neq-C-Obj*
  show *?thesis*
  proof (*cases rule*: *methd-rec-Some-cases*)
    case *InheritedMethod*
    with *super Some* **show** *?thesis*
      by *auto*
  next
    case *NewMethod*
    assume *new*: *table-of* (*map* (λ(*s*, *m*). (*s*, *C*, *m*)) (*methods c*)) *sig*
          = *Some new*
    from *new*
    have *declcls-new*: *declclass new* = *C*
      by *auto*
    from *wf clsC neq-C-Obj super new not-static-super accmodi-super-method*
    have *not-static-new*: $\neg$ *is-static new*
      by (*auto dest*: *wf-prog-staticD*)
    from *clsC new*
    have *decl-new*: $G\vdash$*Methd sig new declared-in C*
      by (*auto simp add*: *declared-in-def cdeclaredmethd-def*)
    from *not-static-new decl-new decl-super-method*
        member subcls1-C-super inheritable declcls-new subcls-C-super
    have *G*,*sig*$\vdash$ *new overrides*$_S$ *super-method*
      by (*auto intro*: *stat-overridesR.Direct*)
    with *super Some*
    show *?thesis*
      by (*auto intro*: *stat-overridesR.Indirect*)
  qed
qed
qed
qed
qed

lemma *inheritable-instance-methd-cases* [*consumes 6*

, *case-names Inheritance Overriding*]:
assumes *subclseq-C-D*: $G\vdash C \preceq_C D$ **and**
is-cls-D: *is-class G D* **and**
wf: *wf-prog G* **and**
old: *methd G D sig = Some old* **and**
accmodi-old: *Protected* $\leq$ *accmodi old* **and**
not-static-old: $\neg$ *is-static old* **and**
inheritance: *methd G C sig = Some old* $\Longrightarrow$ *P* **and**
overriding: $\bigwedge$ *new*. ⟦*methd G C sig = Some new*;
G,sig$\vdash$*new overrides*$_S$ *old*⟧ $\Longrightarrow$ *P*

shows *P*
proof –
from *subclseq-C-D is-cls-D wf old accmodi-old not-static-old*
show *?thesis*
by (*auto dest*: *inheritable-instance-methd intro*: *inheritance overriding*)
qed

lemma *inheritable-instance-methd-props*:
assumes *subclseq-C-D*: $G\vdash C \preceq_C D$ **and**
is-cls-D: *is-class G D* **and**
wf: *wf-prog G* **and**
old: *methd G D sig = Some old* **and**
accmodi-old: *Protected* $\leq$ *accmodi old* **and**
not-static-old: $\neg$ *is-static old*
shows
$\exists$ *new*. *methd G C sig = Some new* $\wedge$
$\neg$ *is-static new* $\wedge$ *G*$\vdash$*resTy new*$\preceq$*resTy old* $\wedge$ *accmodi old* $\leq$*accmodi new*
(**is** ($\exists$ *new*. (*?Constraint C new old*)))
proof –
from *subclseq-C-D is-cls-D wf old accmodi-old not-static-old*
show *?thesis*
proof (*cases rule*: *inheritable-instance-methd-cases*)
case *Inheritance*
with *not-static-old accmodi-old* **show** *?thesis* **by** *auto*
next
case (*Overriding new*)
then have $\neg$ *is-static new* **by** (*auto dest*: *stat-overrides-commonD*)
with *Overriding not-static-old accmodi-old wf*
show *?thesis*
by (*auto dest!*: *wf-prog-stat-overridesD*)
qed
qed

lemma *bexI'*: $x \in A \Longrightarrow P\ x \Longrightarrow \exists x{\in}A.\ P\ x$ **by** *blast*

lemma *ballE'*: $\forall x{\in}A.\ P\ x \Longrightarrow (x \notin A \Longrightarrow Q) \Longrightarrow (P\ x \Longrightarrow Q) \Longrightarrow Q$ **by** *blast*

lemma *subint-widen-imethds*:
⟦*G*$\vdash$*I*$\preceq$*I J*; *wf-prog G*; *is-iface G J*; *jm* $\in$ *imethds G J sig*⟧ $\Longrightarrow$
$\exists$ *im* $\in$ *imethds G I sig*. *is-static im = is-static jm* $\wedge$
accmodi im = accmodi jm $\wedge$
G$\vdash$*resTy im*$\preceq$*resTy jm*
proof –
assume *irel*: *G*$\vdash$*I*$\preceq$*I J* **and**

```
        wf: wf-prog G and
    is-iface: is-iface G J
from irel show jm ∈ imethds G J sig ⟹ ?thesis
            (is PROP ?P I is PROP ?Prem J ⟹ ?Concl I)
proof (induct ?P I rule: converse-rtrancl-induct)
  case Id
  assume jm ∈ imethds G J sig
  then show ?Concl J by (blast elim: bexI')
next
  case Step
  fix I SI
  assume subint1-I-SI: G⊢I ≺I1 SI and
         subint-SI-J: G⊢SI ≼I J and
                hyp: PROP ?P SI and
                 jm: jm ∈ imethds G J sig
  from subint1-I-SI
  obtain i where
    ifI: iface G I = Some i and
     SI: SI ∈ set (isuperIfs i)
    by (blast dest: subint1D)

  let ?newMethods
      = (o2s ∘ table-of (map (λ(sig, mh). (sig, I, mh)) (imethods i)))
  show ?Concl I
  proof (cases ?newMethods sig = {})
    case True
    with ifI SI hyp wf jm
    show ?thesis
      by (auto simp add: imethds-rec)
  next
    case False
    from ifI wf False
    have imethds: imethds G I sig = ?newMethods sig
      by (simp add: imethds-rec)
    from False
    obtain im where
      imdef: im ∈ ?newMethods sig
      by (blast)
    with imethds
    have im: im ∈ imethds G I sig
      by (blast)
    with im wf ifI
    obtain
       imStatic: ¬ is-static im and
       imPublic: accmodi im = Public
      by (auto dest!: imethds-wf-mhead)
    from ifI wf
    have wf-I: wf-idecl G (I,i)
      by (rule wf-prog-idecl)
    with SI wf
    obtain si where
       ifSI: iface G SI = Some si and
      wf-SI: wf-idecl G (SI,si)
      by (auto dest!: wf-idecl-supD is-acc-ifaceD
                dest: wf-prog-idecl)
    from jm hyp
    obtain sim::qtname × mhead where
                    sim: sim ∈ imethds G SI sig and
       eq-static-sim-jm: is-static sim = is-static jm and
```

eq-access-sim-jm: *accmodi sim* = *accmodi jm* **and**
resTy-widen-sim-jm: *G*⊢*resTy sim*≼*resTy jm*
by *blast*
with *wf-I SI imdef sim*
have *G*⊢*resTy im*≼*resTy sim*
by (*auto dest*!: *wf-idecl-hidings hidings-entailsD*)
with *wf resTy-widen-sim-jm*
have *resTy-widen-im-jm*: *G*⊢*resTy im*≼*resTy jm*
by (*blast intro*: *widen-trans*)
from *sim wf ifSI*
obtain
simStatic: ¬ *is-static sim* **and**
simPublic: *accmodi sim* = *Public*
by (*auto dest*!: *imethds-wf-mhead*)
from *im*
imStatic simStatic eq-static-sim-jm
imPublic simPublic eq-access-sim-jm
resTy-widen-im-jm
show *?thesis*
by *auto*
qed
qed
qed

lemma *implmt1-methd*:
⋀*sig*. ⟦*G*⊢*C*↝*1I*; *wf-prog G*; *im* ∈ *imethds G I sig*⟧ ⟹
∃ *cm* ∈*methd G C sig*: ¬ *is-static cm* ∧ ¬ *is-static im* ∧
G⊢*resTy cm*≼*resTy im* ∧
accmodi im = *Public* ∧ *accmodi cm* = *Public*
apply (*drule implmt1D*)
apply *clarify*
apply (*drule* (*2*) *wf-prog-cdecl* [*THEN wf-cdecl-impD*])
apply (*frule* (*1*) *imethds-wf-mhead*)
apply (*simp add*: *is-acc-iface-def*)
apply (*force*)
done

lemma *implmt-methd* [*rule-format* (*no-asm*)]:
⟦*wf-prog G*; *G*⊢*C*↝*I*⟧ ⟹ *is-iface G I* ⟶
(∀ *im* ∈*imethds G I* *sig*.
∃ *cm*∈*methd G C sig*: ¬*is-static cm* ∧ ¬ *is-static im* ∧
G⊢*resTy cm*≼*resTy im* ∧
accmodi im = *Public* ∧ *accmodi cm* = *Public*)
apply (*frule implmt-is-class*)
apply (*erule implmt.induct*)
apply *safe*
apply (*drule* (*2*) *implmt1-methd*)
apply *fast*
apply (*drule* (*1*) *subint-widen-imethds*)
apply *simp*

apply *assumption*
apply *clarify*
apply (*drule* (*2*) *implmt1-methd*)
apply (*force*)
apply (*frule subcls1D*)
apply (*drule* (*1*) *bspec*)
apply *clarify*
apply (*drule* (*3*) *r-into-rtrancl* [*THEN inheritable-instance-methd-props*,
OF - implmt-is-class])
apply *auto*
done

lemma *mheadsD* [*rule-format* (*no-asm*)]:
$emh \in mheads\ G\ S\ t\ sig \longrightarrow wf\text{-}prog\ G \longrightarrow$
$(\exists\, C\ D\ m.\ t = ClassT\ C \wedge declrefT\ emh = ClassT\ D \wedge$
$accmethd\ G\ S\ C\ sig = Some\ m \wedge$
$(declclass\ m = D) \wedge mhead\ (mthd\ m) = (mhd\ emh)) \vee$
$(\exists\, I.\ t = IfaceT\ I \wedge ((\exists\, im.\ im \in accimethds\ G\ (pid\ S)\ I\ sig \wedge$
$mthd\ im = mhd\ emh) \vee$
$(\exists\, m.\ G \vdash Iface\ I\ accessible\text{-}in\ (pid\ S) \wedge accmethd\ G\ S\ Object\ sig = Some\ m \wedge$
$accmodi\ m \neq Private \wedge$
$declrefT\ emh = ClassT\ Object \wedge mhead\ (mthd\ m) = mhd\ emh))) \vee$
$(\exists\, T\ m.\ t = ArrayT\ T \wedge G \vdash Array\ T\ accessible\text{-}in\ (pid\ S) \wedge$
$accmethd\ G\ S\ Object\ sig = Some\ m \wedge accmodi\ m \neq Private \wedge$
$declrefT\ emh = ClassT\ Object \wedge mhead\ (mthd\ m) = mhd\ emh)$
apply (*rule-tac ref-ty1=t* **in** *ref-ty-ty.induct* [*THEN conjunct1*])
apply *auto*
apply (*auto simp add*: *cmheads-def accObjectmheads-def Objectmheads-def*)
apply (*auto dest*!: *accmethd-SomeD*)
done

lemma *mheads-cases* [*consumes 2*, *case-names Class-methd*
Iface-methd Iface-Object-methd Array-Object-methd]:
$[\![emh \in mheads\ G\ S\ t\ sig;\ wf\text{-}prog\ G;$
$\bigwedge C\ D\ m.\ [\![t = ClassT\ C; declrefT\ emh = ClassT\ D;\ accmethd\ G\ S\ C\ sig = Some\ m;$
$(declclass\ m = D);\ mhead\ (mthd\ m) = (mhd\ emh)]\!] \Longrightarrow P\ emh;$
$\bigwedge I\ im.\ [\![t = IfaceT\ I;\ im \in accimethds\ G\ (pid\ S)\ I\ sig;\ mthd\ im = mhd\ emh]\!]$
$\Longrightarrow P\ emh;$
$\bigwedge I\ m.\ [\![t = IfaceT\ I;\ G \vdash Iface\ I\ accessible\text{-}in\ (pid\ S);$
$accmethd\ G\ S\ Object\ sig = Some\ m;\ accmodi\ m \neq Private;$
$declrefT\ emh = ClassT\ Object;\ mhead\ (mthd\ m) = mhd\ emh]\!] \Longrightarrow P\ emh;$
$\bigwedge T\ m.\ [\![t = ArrayT\ T; G \vdash Array\ T\ accessible\text{-}in\ (pid\ S);$
$accmethd\ G\ S\ Object\ sig = Some\ m;\ accmodi\ m \neq Private;$
$declrefT\ emh = ClassT\ Object;\ mhead\ (mthd\ m) = mhd\ emh]\!] \Longrightarrow P\ emh$
$]\!] \Longrightarrow P\ emh$
by (*blast dest*!: *mheadsD*)

lemma *declclassD*[*rule-format*]:
$[\![wf\text{-}prog\ G; class\ G\ C = Some\ c;\ methd\ G\ C\ sig = Some\ m;$
$class\ G\ (declclass\ m) = Some\ d]\!]$
$\Longrightarrow table\text{-}of\ (methods\ d)\ sig = Some\ (mthd\ m)$
proof −
assume *wf*: *wf-prog G*
then have *ws*: *ws-prog G* **..**
assume *clsC*: *class G C = Some c*
from *clsC ws*

```
  show ⋀ m d. ⟦methd G C sig = Some m; class G (declclass m) = Some d⟧
      ⟹ table-of (methods d) sig = Some (mthd m)
       (is PROP ?P C)
  proof (induct ?P C rule: ws-class-induct)
    case Object
    fix m d
    assume methd G Object sig = Some m
           class G (declclass m) = Some d
    with wf show ?thesis m d by auto
  next
    case Subcls
    fix C c m d
    assume hyp: PROP ?P (super c)
    and       m: methd G C sig = Some m
    and  declC: class G (declclass m) = Some d
    and   clsC: class G C = Some c
    and   nObj: C ≠ Object
    let ?newMethods = table-of (map (λ(s, m). (s, C, m)) (methods c)) sig
    show ?thesis m d
    proof (cases ?newMethods)
      case None
      from None clsC nObj ws m declC
      show ?thesis by (auto simp add: methd-rec) (rule hyp)
    next
      case Some
      from Some clsC nObj ws m declC
      show ?thesis
        by (auto simp add: methd-rec
                     dest: wf-prog-cdecl wf-cdecl-supD is-acc-class-is-class)
    qed
  qed
qed
```

```
lemma dynmethd-Object:
  assumes statM: methd G Object sig = Some statM and
        private: accmodi statM = Private and
       is-cls-C: is-class G C and
             wf: wf-prog G
  shows dynmethd G Object C sig = Some statM
proof -
  from is-cls-C wf
  have subclseq: G⊢C ≼C Object
    by (auto intro: subcls-ObjectI)
  from wf have ws: ws-prog G
    by simp
  from wf
  have is-cls-Obj: is-class G Object
    by simp
  from statM subclseq is-cls-Obj ws private
  show ?thesis
  proof (cases rule: dynmethd-cases)
    case Static then show ?thesis .
  next
    case Overrides
    with private show ?thesis
```

    by (*auto dest*: *no-Private-override*)
  qed
qed

lemma *wf-imethds-hiding-objmethdsD*:
  assumes *old*: *methd G Object sig = Some old* **and**
    is-if-I: *is-iface G I* **and**
    wf: *wf-prog G* **and**
    not-private: *accmodi old* $\neq$ *Private* **and**
    new: *new* $\in$ *imethds G I sig*
  shows $G\vdash$*resTy new*$\preceq$*resTy old* $\wedge$ *is-static new* = *is-static old* (**is** *?P new*)
proof −
  from *wf* **have** *ws*: *ws-prog G* **by** *simp*
  {
    fix *I i new*
    assume *ifI*: *iface G I = Some i*
    assume *new*: *table-of* (*imethods i*) *sig = Some new*
    from *ifI new not-private wf old*
    have *?P* (*I*,*new*)
      by (*auto dest*!: *wf-prog-idecl wf-idecl-hiding cond-hiding-entailsD*
        simp del: *methd-Object*)
  } **note** *hyp-newmethod* = *this*
  from *is-if-I ws new*
  show *?thesis*
  proof (*induct rule*: *ws-interface-induct*)
    case (*Step I i*)
    assume *ifI*: *iface G I = Some i*
    assume *new*: *new* $\in$ *imethds G I sig*
    from *Step*
    have *hyp*: $\forall$ *J* $\in$ *set* (*isuperIfs i*). (*new* $\in$ *imethds G J sig* $\longrightarrow$ *?P new*)
      by *auto*
    from *new ifI ws*
    show *?P new*
    proof (*cases rule*: *imethds-cases*)
      case *NewMethod*
      with *ifI hyp-newmethod*
      show *?thesis*
        by *auto*
    next
      case (*InheritedMethod J*)
      assume *J* $\in$ *set* (*isuperIfs i*)
        new $\in$ *imethds G J sig*
      with *hyp*
      show *?thesis*
        by *auto*
    qed
  qed
qed

Which dynamic classes are valid to look up a member of a distinct static type? We have to distinct class members (named static members in Java) from instance members. Class members are global to all Objects of a class, instance members are local to a single Object instance. If a member is equipped with the static modifier it is a class member, else it is an instance member. The following table gives an overview of the current framework. We assume to have a reference with static type statT and a dynamic class dynC. Between both of these types the widening relation holds $G\vdash$*Class dynC*$\preceq$*statT*. Unfortunately this ordinary widening relation isn't enough to describe the valid lookup classes, since we must cope the special cases of arrays and interfaces,too. If we statically expect an array or inteface we may lookup a field or a method in Object which isn't covered in the widening relation.

statT field instance method static (class) method ———————————————————— ——— NullT / / / Iface / dynC Object Class dynC dynC dynC Array / Object Object

In most cases we con lookup the member in the dynamic class. But as an interface can't declare new static methods, nor an array can define new methods at all, we have to lookup methods in the base class Object.

The limitation to classes in the field column is artificial and comes out of the typing rule for the field access (see rule *FVar* in the welltyping relation *wt* in theory WellType). I stems out of the fact, that Object indeed has no non private fields. So interfaces and arrays can actually have no fields at all and a field access would be senseless. (In Java interfaces are allowed to declare new fields but in current Bali not!). So there is no principal reason why we should not allow Objects to declare non private fields. Then we would get the following column:

statT field ————— NullT / Iface Object Class dynC Array Object

consts *valid-lookup-cls*:: *prog* ⇒ *ref-ty* ⇒ *qtname* ⇒ *bool* ⇒ *bool*
(-,- ⊢ - *valid′-lookup′-cls′-for* - [*61*,*61*,*61*,*61*] *60*)
primrec
G,*NullT* ⊢ *dynC valid-lookup-cls-for static-membr* = *False*
G,*IfaceT I* ⊢ *dynC valid-lookup-cls-for static-membr*
= (*if static-membr*
then dynC=*Object*
else G⊢*Class dynC*⪯ *Iface I*)
G,*ClassT C* ⊢ *dynC valid-lookup-cls-for static-membr* = *G*⊢*Class dynC*⪯ *Class C*
G,*ArrayT T* ⊢ *dynC valid-lookup-cls-for static-membr* = (*dynC*=*Object*)

lemma *valid-lookup-cls-is-class*:
assumes *dynC*: *G*,*statT* ⊢ *dynC valid-lookup-cls-for static-membr* **and**
ty-statT: *isrtype G statT* **and**
wf: *wf-prog G*
shows *is-class G dynC*
proof (*cases statT*)
case *NullT*
with *dynC ty-statT* **show** *?thesis*
by (*auto dest*: *widen-NT2*)
next
case (*IfaceT I*)
with *dynC wf* **show** *?thesis*
by (*auto dest*: *implmt-is-class*)
next
case (*ClassT C*)
with *dynC ty-statT* **show** *?thesis*
by (*auto dest*: *subcls-is-class2*)
next
case (*ArrayT T*)
with *dynC wf* **show** *?thesis*
by (*auto*)
qed

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss delloop split-all-tac*)) ⟫
declaration ⟪ *K* (*Classical.map-cs* (*fn cs* => *cs delSWrapper split-all-tac*)) ⟫

lemma *dynamic-mheadsD*:
⟦*emh* ∈ *mheads G S statT sig*;

```
  G,statT ⊢ dynC valid-lookup-cls-for (is-static emh);
  isrtype G statT; wf-prog G
 ⟧ ⟹ ∃ m ∈ dynlookup G statT dynC sig:
        is-static m=is-static emh ∧ G⊢resTy m≼resTy emh
proof −
  assume      emh: emh ∈ mheads G S statT sig
  and          wf: wf-prog G
  and   dynC-Prop: G,statT ⊢ dynC valid-lookup-cls-for (is-static emh)
  and      istype: isrtype G statT
  from dynC-Prop istype wf
  obtain y where
    dynC: class G dynC = Some y
    by (auto dest: valid-lookup-cls-is-class)
  from emh wf show ?thesis
  proof (cases rule: mheads-cases)
    case Class-methd
    fix statC statDeclC sm
    assume     statC: statT = ClassT statC
    assume            accmethd G S statC sig = Some sm
    then have     sm: methd G statC sig = Some sm
      by (blast dest: accmethd-SomeD)
    assume eq-mheads: mhead (mthd sm) = mhd emh
    from statC
    have dynlookup: dynlookup G statT dynC sig = dynmethd G statC dynC sig
      by (simp add: dynlookup-def)
    from wf statC istype dynC-Prop sm
    obtain dm where
      dynmethd G statC dynC sig = Some dm
      is-static dm = is-static sm
      G⊢resTy dm≼resTy sm
      by (force dest!: ws-dynmethd accmethd-SomeD)
    with dynlookup eq-mheads
    show ?thesis
      by (cases emh type: *) (auto)
  next
    case Iface-methd
    fix I im
    assume    statI: statT = IfaceT I and
         eq-mheads: mthd im = mhd emh and
                    im ∈ accimethds G (pid S) I sig
    then have im: im ∈ imethds G I sig
      by (blast dest: accimethdsD)
    with istype statI eq-mheads wf
    have not-static-emh: ¬ is-static emh
      by (cases emh) (auto dest: wf-prog-idecl imethds-wf-mhead)
    from statI im
    have dynlookup: dynlookup G statT dynC sig = methd G dynC sig
      by (auto simp add: dynlookup-def dynimethd-def)
    from wf dynC-Prop statI istype im not-static-emh
    obtain dm where
      methd G dynC sig = Some dm
      is-static dm = is-static im
      G⊢resTy (mthd dm)≼resTy (mthd im)
      by (force dest: implmt-methd)
    with dynlookup eq-mheads
    show ?thesis
      by (cases emh type: *) (auto)
  next
    case Iface-Object-methd
```

```
  fix I sm
  assume   statI: statT = IfaceT I and
              sm: accmethd G S Object sig = Some sm and
      eq-mheads: mhead (mthd sm) = mhd emh and
           nPriv: accmodi sm ≠ Private
   show ?thesis
   proof (cases imethds G I sig = {})
     case True
     with statI
     have dynlookup: dynlookup G statT dynC sig = dynmethd G Object dynC sig
       by (simp add: dynlookup-def dynimethd-def)
     from wf dynC
     have subclsObj: G⊢dynC ≼C Object
       by (auto intro: subcls-ObjectI)
     from wf dynC dynC-Prop istype sm subclsObj
     obtain dm where
       dynmethd G Object dynC sig = Some dm
       is-static dm = is-static sm
       G⊢resTy (mthd dm)≼resTy (mthd sm)
       by (auto dest!: ws-dynmethd accmethd-SomeD
                intro: class-Object [OF wf] intro: that)
     with dynlookup eq-mheads
     show ?thesis
       by (cases emh type: ∗) (auto)
   next
     case False
     with statI
     have dynlookup: dynlookup G statT dynC sig = methd G dynC sig
       by (simp add: dynlookup-def dynimethd-def)
     from istype statI
     have is-iface G I
       by auto
     with wf sm nPriv False
     obtain im where
            im: im ∈ imethds G I sig and
       eq-stat: is-static im = is-static sm and
       resProp: G⊢resTy (mthd im)≼resTy (mthd sm)
       by (auto dest: wf-imethds-hiding-objmethdsD accmethd-SomeD)
     from im wf statI istype eq-stat eq-mheads
     have not-static-sm: ¬ is-static emh
       by (cases emh) (auto dest: wf-prog-idecl imethds-wf-mhead)
     from im wf dynC-Prop dynC istype statI not-static-sm
     obtain dm where
       methd G dynC sig = Some dm
       is-static dm = is-static im
       G⊢resTy (mthd dm)≼resTy (mthd im)
       by (auto dest: implmt-methd)
     with wf eq-stat resProp dynlookup eq-mheads
     show ?thesis
       by (cases emh type: ∗) (auto intro: widen-trans)
   qed
next
  case Array-Object-methd
  fix T sm
  assume statArr: statT = ArrayT T and
              sm: accmethd G S Object sig = Some sm and
      eq-mheads: mhead (mthd sm) = mhd emh
  from statArr dynC-Prop wf
  have dynlookup: dynlookup G statT dynC sig = methd G Object sig
```

by (*auto simp add*: *dynlookup-def dynmethd-C-C*)
with *sm eq-mheads sm*
show *?thesis*
by (*cases emh type*: ∗) (*auto dest*: *accmethd-SomeD*)
qed
qed
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declaration ⟪ *K* (*Classical.map-cs* (*fn cs* => *cs addSbefore* (*split-all-tac*, *split-all-tac*))) ⟫
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*))) ⟫

lemma *methd-declclass*:
⟦*class G C* = *Some c*; *wf-prog G*; *methd G C sig* = *Some m*⟧
⟹ *methd G* (*declclass m*) *sig* = *Some m*
proof −
assume *asm*: *class G C* = *Some c wf-prog G methd G C sig* = *Some m*
have *wf-prog G* ⟶
(∀ *c m*. *class G C* = *Some c* ⟶ *methd G C sig* = *Some m*
⟶ *methd G* (*declclass m*) *sig* = *Some m*) (**is** *?P G C*)
proof (*rule class-rec.induct*,*intro allI impI*)
fix *G C c m*
assume *hyp*: ∀ *c*. *C* ≠ *Object* ∧ *ws-prog G* ∧ *class G C* = *Some c* ⟶
?P G (*super c*)
assume *wf*: *wf-prog G* **and** *cls-C*: *class G C* = *Some c* **and**
m: *methd G C sig* = *Some m*
show *methd G* (*declclass m*) *sig* = *Some m*
proof (*cases C*=*Object*)
case *True*
with *wf m* **show** *?thesis* **by** (*auto intro*: *table-of-map-SomeI*)
next
let *?filter*=*filter-tab* (λ*sig m*. *G*⊢*C inherits method sig m*)
let *?table* = *table-of* (*map* (λ(*s*, *m*). (*s*, *C*, *m*)) (*methods c*))
case *False*
with *cls-C wf m*
have *methd-C*: (*?filter* (*methd G* (*super c*)) ++ *?table*) *sig* = *Some m*
by (*simp add*: *methd-rec*)
show *?thesis*
proof (*cases ?table sig*)
case *None*
from *this methd-C* **have** *?filter* (*methd G* (*super c*)) *sig* = *Some m*
by *simp*
moreover
from *wf cls-C False* **obtain** *sup* **where** *class G* (*super c*) = *Some sup*
by (*blast dest*: *wf-prog-cdecl wf-cdecl-supD is-acc-class-is-class*)
moreover note *wf False cls-C*
ultimately show *?thesis* **by** (*auto intro*: *hyp* [*rule-format*])
next
case *Some*
from *this methd-C m* **show** *?thesis* **by** *auto*
qed
qed
qed
with *asm* **show** *?thesis* **by** *auto*
qed

lemma *dynmethd-declclass*:
⟦*dynmethd G statC dynC sig* = *Some m*;
wf-prog G; *is-class G statC*
⟧ ⟹ *methd G* (*declclass m*) *sig* = *Some m*
by (*auto dest*: *dynmethd-declC*)

lemma *dynlookup-declC*:
⟦*dynlookup G statT dynC sig* = *Some m*; *wf-prog G*;
is-class G dynC;*isrtype G statT*
⟧ ⟹ $G \vdash dynC \preceq_C (declclass\ m) \wedge is\text{-}class\ G\ (declclass\ m)$
by (*cases statT*)
(*auto simp add*: *dynlookup-def dynimethd-def*
dest: *methd-declC dynmethd-declC*)

lemma *dynlookup-Array-declclassD* [*simp*]:
⟦*dynlookup G* (*ArrayT T*) *Object sig* = *Some dm*;*wf-prog G*⟧
⟹ *declclass dm* = *Object*
proof −
assume *dynL*: *dynlookup G* (*ArrayT T*) *Object sig* = *Some dm*
assume *wf*: *wf-prog G*
from *wf* **have** *ws*: *ws-prog G* **by** *auto*
from *wf* **have** *is-cls-Obj*: *is-class G Object* **by** *auto*
from *dynL wf*
show *?thesis*
by (*auto simp add*: *dynlookup-def dynmethd-C-C* [*OF is-cls-Obj ws*]
dest: *methd-Object-SomeD*)
qed

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss delloop split-all-tac*)) ⟫
declaration ⟪ *K* (*Classical.map-cs* (*fn cs* => *cs delSWrapper split-all-tac*)) ⟫

lemma *wt-is-type*: $E,dt \models v{::}T$ ⟹ *wf-prog* (*prg E*) ⟶
dt=*empty-dt* ⟶ (*case T of*
Inl T ⇒ *is-type* (*prg E*) *T*
| *Inr Ts* ⇒ *Ball* (*set Ts*) (*is-type* (*prg E*)))
apply (*unfold empty-dt-def*)
apply (*erule wt.induct*)
apply (*auto split del*: *split-if-asm simp del*: *snd-conv*
simp add: *is-acc-class-def is-acc-type-def*)
apply (*erule typeof-empty-is-type*)
apply (*frule* (*1*) *wf-prog-cdecl* [*THEN wf-cdecl-supD*],
force simp del: *snd-conv*, *clarsimp simp add*: *is-acc-class-def*)
apply (*drule* (*1*) *max-spec2mheads* [*THEN conjunct1*, *THEN mheadsD*])
apply (*drule-tac* [*2*] *accfield-fields*)
apply (*frule class-Object*)
apply (*auto dest*: *accmethd-rT-is-type*
imethds-wf-mhead [*THEN conjunct1*, *THEN rT-is-acc-type*]
dest!:*accimethdsD*
simp del: *class-Object*
simp add: *is-acc-type-def*
)
done
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]

declaration ⟪ *K* (*Classical.map-cs* (*fn cs* => *cs addSbefore* (*split-all-tac*, *split-all-tac*))) ⟫
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*))) ⟫

lemma *ty-expr-is-type*:
$[\![E\vdash e{::}{-}T;\ \mathit{wf\text{-}prog}\ (\mathit{prg}\ E)]\!] \Longrightarrow \mathit{is\text{-}type}\ (\mathit{prg}\ E)\ T$
by (*auto dest*!: *wt-is-type*)

lemma *ty-var-is-type*:
$[\![E\vdash v{::}{=}T;\ \mathit{wf\text{-}prog}\ (\mathit{prg}\ E)]\!] \Longrightarrow \mathit{is\text{-}type}\ (\mathit{prg}\ E)\ T$
by (*auto dest*!: *wt-is-type*)

lemma *ty-exprs-is-type*:
$[\![E\vdash es{::}{\doteq}Ts;\ \mathit{wf\text{-}prog}\ (\mathit{prg}\ E)]\!] \Longrightarrow \mathit{Ball}\ (\mathit{set}\ Ts)\ (\mathit{is\text{-}type}\ (\mathit{prg}\ E))$
by (*auto dest*!: *wt-is-type*)

lemma *static-mheadsD*:
$[\![\ emh \in \mathit{mheads}\ G\ S\ t\ \mathit{sig};\ \mathit{wf\text{-}prog}\ G;\ E\vdash e{::}{-}\mathit{RefT}\ t;\ \mathit{prg}\ E{=}G\ ;$
$\mathit{invmode}\ (\mathit{mhd}\ emh)\ e \neq \mathit{IntVir}$
$]\!] \Longrightarrow \exists\, m.\ (\ \ (\exists\ C.\ t = \mathit{ClassT}\ C \wedge \mathit{accmethd}\ G\ S\ C\ \mathit{sig} = \mathit{Some}\ m)$
$\vee\ (\forall\ C.\ t \neq \mathit{ClassT}\ C \wedge \mathit{accmethd}\ G\ S\ \mathit{Object}\ \mathit{sig} = \mathit{Some}\ m\ ))\ \wedge$
$\mathit{declrefT}\ emh = \mathit{ClassT}\ (\mathit{declclass}\ m) \wedge\ \ \mathit{mhead}\ (\mathit{mthd}\ m) = (\mathit{mhd}\ emh)$
apply (*subgoal-tac is-static emh* ∨ *e* = *Super*)
defer apply (*force simp add*: *invmode-def*)
apply (*frule ty-expr-is-type*)
apply *simp*
apply (*case-tac is-static emh*)
apply (*frule* (*1*) *mheadsD*)
apply *clarsimp*
apply *safe*
apply *blast*
apply (*auto dest*!: *imethds-wf-mhead*
accmethd-SomeD
accimethdsD
simp add: *accObjectmheads-def Objectmheads-def*)

apply (*erule wt-elim-cases*)
apply (*force simp add*: *cmheads-def*)
done

lemma *wt-MethdI*:
$[\![\mathit{methd}\ G\ C\ \mathit{sig} = \mathit{Some}\ m;\ \mathit{wf\text{-}prog}\ G;$
$\mathit{class}\ G\ C = \mathit{Some}\ c]\!] \Longrightarrow$
$\exists\, T.\ (\!|\mathit{prg}{=}G,\mathit{cls}{=}(\mathit{declclass}\ m),$
$\mathit{lcl}{=}\mathit{callee\text{-}lcl}\ (\mathit{declclass}\ m)\ \mathit{sig}\ (\mathit{mthd}\ m)|\!)\vdash\ \mathit{Methd}\ C\ \mathit{sig}{::}{-}T\ \wedge\ G\vdash T\preceq \mathit{resTy}\ m$
apply (*frule* (*2*) *methd-wf-mdecl*, *clarify*)
apply (*force dest*!: *wf-mdecl-bodyD intro*!: *wt.Methd*)
done

35 accessibility concerns

lemma *mheads-type-accessible*:
$[\![emh \in \mathit{mheads}\ G\ S\ T\ \mathit{sig};\ \mathit{wf\text{-}prog}\ G]\!]$
$\Longrightarrow G\vdash \mathit{RefT}\ T\ \mathit{accessible\text{-}in}\ (\mathit{pid}\ S)$
by (*erule mheads-cases*)
(*auto dest*: *accmethd-SomeD accessible-from-commonD accimethdsD*)

lemma *static-to-dynamic-accessible-from-aux*:
⟦*G*⊢*m of C accessible-from accC*;*wf-prog G*⟧
⟹ *G*⊢*m in C dyn-accessible-from accC*
proof (*induct rule*: *accessible-fromR.induct*)
qed (*auto intro*: *dyn-accessible-fromR.intros*
    member-of-to-member-in
    static-to-dynamic-overriding)

lemma *static-to-dynamic-accessible-from*:
  assumes *stat-acc*: *G*⊢*m of statC accessible-from accC* **and**
    subclseq: *G*⊢*dynC* $\preceq_C$ *statC* **and**
      wf: *wf-prog G*
  shows *G*⊢*m in dynC dyn-accessible-from accC*
proof −
  from *stat-acc subclseq*
  show *?thesis* (**is** *?Dyn-accessible m*)
  proof (*induct rule*: *accessible-fromR.induct*)
    case (*Immediate m statC*)
    then show *?Dyn-accessible m*
      by (*blast intro*: *dyn-accessible-fromR.Immediate*
        member-inI
        permits-acc-inheritance)
  next
    case (*Overriding m - -*)
    with *wf* **show** *?Dyn-accessible m*
      by (*blast intro*: *dyn-accessible-fromR.Overriding*
        member-inI
        static-to-dynamic-overriding
        rtrancl-trancl-trancl
        static-to-dynamic-accessible-from-aux)
  qed
qed

lemma *static-to-dynamic-accessible-from-static*:
  assumes *stat-acc*: *G*⊢*m of statC accessible-from accC* **and**
    static: *is-static m* **and**
      wf: *wf-prog G*
  shows *G*⊢*m in* (*declclass m*) *dyn-accessible-from accC*
proof −
  from *stat-acc wf*
  have *G*⊢*m in statC dyn-accessible-from accC*
    by (*auto intro*: *static-to-dynamic-accessible-from*)
  from *this static*
  show *?thesis*
    by (*rule dyn-accessible-from-static-declC*)
qed

lemma *dynmethd-member-in*:
  assumes *m*: *dynmethd G statC dynC sig* = *Some m* **and**
  iscls-statC: *is-class G statC* **and**
    wf: *wf-prog G*
  shows *G*⊢*Methd sig m member-in dynC*
proof −
  from *m*

have *subclseq*: $G\vdash dynC \preceq_C statC$
by (*auto simp add*: *dynmethd-def*)
from *subclseq iscls-statC*
have *iscls-dynC*: *is-class G dynC*
by (*rule subcls-is-class2*)
from *iscls-dynC iscls-statC wf m*
have $G\vdash dynC \preceq_C (declclass\ m) \wedge is\text{-}class\ G\ (declclass\ m) \wedge$
$methd\ G\ (declclass\ m)\ sig = Some\ m$
by − (*drule dynmethd-declC*, *auto*)
with *wf*
show *?thesis*
by (*auto intro*: *member-inI dest*: *methd-member-of*)
qed

lemma *dynmethd-access-prop*:
assumes *statM*: *methd G statC sig = Some statM* **and**
stat-acc: *G⊢Methd sig statM of statC accessible-from accC* **and**
dynM: *dynmethd G statC dynC sig = Some dynM* **and**
wf: *wf-prog G*
shows *G⊢Methd sig dynM in dynC dyn-accessible-from accC*
proof −
from *wf* **have** *ws*: *ws-prog G* **..**
from *dynM*
have *subclseq*: $G\vdash dynC \preceq_C statC$
by (*auto simp add*: *dynmethd-def*)
from *stat-acc*
have *is-cls-statC*: *is-class G statC*
by (*auto dest*: *accessible-from-commonD member-of-is-classD*)
with *subclseq*
have *is-cls-dynC*: *is-class G dynC*
by (*rule subcls-is-class2*)
from *is-cls-statC statM wf*
have *member-statC*: *G⊢Methd sig statM member-of statC*
by (*auto intro*: *methd-member-of*)
from *stat-acc*
have *statC-acc*: *G⊢Class statC accessible-in* (*pid accC*)
by (*auto dest*: *accessible-from-commonD*)
from *statM subclseq is-cls-statC ws*
show *?thesis*
proof (*cases rule*: *dynmethd-cases*)
case *Static*
assume *dynmethd*: *dynmethd G statC dynC sig = Some statM*
with *dynM* **have** *eq-dynM-statM*: *dynM=statM*
by *simp*
with *stat-acc subclseq wf*
show *?thesis*
by (*auto intro*: *static-to-dynamic-accessible-from*)
next
case (*Overrides newM*)
assume *dynmethd*: *dynmethd G statC dynC sig = Some newM*
assume *override*: *G,sig⊢newM overrides statM*
assume *neq*: *newM≠statM*
from *dynmethd dynM*
have *eq-dynM-newM*: *dynM=newM*
by *simp*
from *dynmethd eq-dynM-newM wf is-cls-statC*
have *G⊢Methd sig dynM member-in dynC*
by (*auto intro*: *dynmethd-member-in*)

moreover
from *subclseq*
have $G \vdash dynC \prec_C statC$
proof (*cases rule*: *subclseq-cases*)
case *Eq*
assume *dynC=statC*
moreover
from *is-cls-statC* **obtain** *c*
where *class G statC = Some c*
by *auto*
moreover
note *statM ws dynmethd*
ultimately
have *newM=statM*
by (*auto simp add*: *dynmethd-C-C*)
with *neq* **show** *?thesis*
by (*contradiction*)
next
case *Subcls* **then show** *?thesis* **.**
qed
moreover
from *stat-acc wf*
have $G \vdash$*Methd sig statM in statC dyn-accessible-from accC*
by (*blast intro*: *static-to-dynamic-accessible-from*)
moreover
note *override eq-dynM-newM*
ultimately show *?thesis*
by (*cases dynM*,*cases statM*) (*auto intro*: *dyn-accessible-fromR.Overriding*)
qed
qed

lemma *implmt-methd-access*:
fixes *accC*::*qtname*
assumes *iface-methd*: *imethds G I sig* $\neq \{\}$ **and**
implements: $G \vdash dynC \rightsquigarrow I$ **and**
isif-I: *is-iface G I* **and**
wf: *wf-prog G*
shows $\exists$ *dynM*. *methd G dynC sig = Some dynM* $\wedge$
$G \vdash$*Methd sig dynM in dynC dyn-accessible-from accC*
proof −
from *implements*
have *iscls-dynC*: *is-class G dynC* **by** (*rule implmt-is-class*)
from *iface-methd*
obtain *im*
where *im* $\in$ *imethds G I sig*
by *auto*
with *wf implements isif-I*
obtain *dynM*
where *dynM*: *methd G dynC sig = Some dynM* **and**
pub: *accmodi dynM = Public*
by (*blast dest*: *implmt-methd*)
with *iscls-dynC wf*
have $G \vdash$*Methd sig dynM in dynC dyn-accessible-from accC*
by (*auto intro*!: *dyn-accessible-fromR.Immediate*
intro: *methd-member-of member-of-to-member-in*
simp add: *permits-acc-def*)
with *dynM*
show *?thesis*

by *blast*
qed

corollary *implmt-dynimethd-access*:
fixes *accC*::*qtname*
assumes *iface-methd*: *imethds G I sig* $\neq$ {} **and**
implements: *G*⊢*dynC*⇝*I* **and**
isif-I: *is-iface G I* **and**
wf: *wf-prog G*
shows ∃ *dynM*. *dynimethd G I dynC sig* = *Some dynM* ∧
G⊢*Methd sig dynM in dynC dyn-accessible-from accC*
proof −
from *iface-methd*
have *dynimethd G I dynC sig* = *methd G dynC sig*
by (*simp add*: *dynimethd-def*)
with *iface-methd implements isif-I wf*
show *?thesis*
by (*simp only*:)
(*blast intro*: *implmt-methd-access*)
qed

lemma *dynlookup-access-prop*:
assumes *emh*: *emh* ∈ *mheads G accC statT sig* **and**
dynM: *dynlookup G statT dynC sig* = *Some dynM* **and**
dynC-prop: *G*,*statT* ⊢ *dynC valid-lookup-cls-for is-static emh* **and**
isT-statT: *isrtype G statT* **and**
wf: *wf-prog G*
shows *G* ⊢*Methd sig dynM in dynC dyn-accessible-from accC*
proof −
from *emh wf*
have *statT-acc*: *G*⊢*RefT statT accessible-in* (*pid accC*)
by (*rule mheads-type-accessible*)
from *dynC-prop isT-statT wf*
have *iscls-dynC*: *is-class G dynC*
by (*rule valid-lookup-cls-is-class*)
from *emh dynC-prop isT-statT wf dynM*
have *eq-static*: *is-static emh* = *is-static dynM*
by (*auto dest*: *dynamic-mheadsD*)
from *emh wf* **show** *?thesis*
proof (*cases rule*: *mheads-cases*)
case (*Class-methd statC - statM*)
assume *statT*: *statT* = *ClassT statC*
assume *accmethd G accC statC sig* = *Some statM*
then have *statM*: *methd G statC sig* = *Some statM* **and**
stat-acc: *G*⊢*Methd sig statM of statC accessible-from accC*
by (*auto dest*: *accmethd-SomeD*)
from *dynM statT*
have *dynM'*: *dynmethd G statC dynC sig* = *Some dynM*
by (*simp add*: *dynlookup-def*)
from *statM stat-acc wf dynM'*
show *?thesis*
by (*auto dest!*: *dynmethd-access-prop*)
next
case (*Iface-methd I im*)
then have *iface-methd*: *imethds G I sig* $\neq$ {} **and**
statT-acc: *G*⊢*RefT statT accessible-in* (*pid accC*)
by (*auto dest*: *accimethdsD*)
assume *statT*: *statT* = *IfaceT I*

assume *im*: *im* ∈ *accimethds G* (*pid accC*) *I sig*
assume *eq-mhds*: *mthd im* = *mhd emh*
from *dynM statT*
have *dynM'*: *dynimethd G I dynC sig* = *Some dynM*
  by (*simp add*: *dynlookup-def*)
from *isT-statT statT*
have *isif-I*: *is-iface G I*
  by *simp*
show *?thesis*
proof (*cases is-static emh*)
  case *False*
  with *statT dynC-prop*
  have *widen-dynC*: *G⊢Class dynC* ≼ *RefT statT*
    by *simp*
  from *statT widen-dynC*
  have *implmnt*: *G⊢dynC⇝I*
    by *auto*
  from *eq-static False*
  have *not-static-dynM*: ¬ *is-static dynM*
    by *simp*
  from *iface-methd implmnt isif-I wf dynM'*
  show *?thesis*
    by − (*drule implmt-dynimethd-access*, *auto*)
next
  case *True*
  assume *is-static emh*
  moreover
  from *wf isT-statT statT im*
  have ¬ *is-static im*
    by (*auto dest*: *accimethdsD wf-prog-idecl imethds-wf-mhead*)
  moreover note *eq-mhds*
  ultimately show *?thesis*
    by (*cases emh*) *auto*
qed
next
case (*Iface-Object-methd I statM*)
assume *statT*: *statT* = *IfaceT I*
assume *accmethd G accC Object sig* = *Some statM*
then have *statM*: *methd G Object sig* = *Some statM* **and**
    stat-acc: *G⊢Methd sig statM of Object accessible-from accC*
  by (*auto dest*: *accmethd-SomeD*)
assume *not-Private-statM*: *accmodi statM* ≠ *Private*
assume *eq-mhds*: *mhead* (*mthd statM*) = *mhd emh*
from *iscls-dynC wf*
have *widen-dynC-Obj*: *G⊢dynC* $\preceq_C$ *Object*
  by (*auto intro*: *subcls-ObjectI*)
show *?thesis*
proof (*cases imethds G I sig* = {})
  case *True*
  from *dynM statT True*
  have *dynM'*: *dynmethd G Object dynC sig* = *Some dynM*
    by (*simp add*: *dynlookup-def dynimethd-def*)
  from *statT*
  have *G⊢RefT statT* ≼*Class Object*
    by *auto*
  with *statM statT-acc stat-acc widen-dynC-Obj statT isT-statT*
    wf dynM' eq-static dynC-prop
  show *?thesis*
    by − (*drule dynmethd-access-prop*,*force*+)

next
case *False*
then obtain *im* **where**
im: $im \in$ *imethds G I sig*
by *auto*
have *not-static-emh*: $\neg$ *is-static emh*
proof −
from *im statM statT isT-statT wf not-Private-statM*
have *is-static im* = *is-static statM*
by (*fastsimp dest*: *wf-imethds-hiding-objmethdsD*)
with *wf isT-statT statT im*
have $\neg$ *is-static statM*
by (*auto dest*: *wf-prog-idecl imethds-wf-mhead*)
with *eq-mhds*
show *?thesis*
by (*cases emh*) *auto*
qed
with *statT dynC-prop*
have *implmnt*: $G \vdash dynC \leadsto I$
by *simp*
with *isT-statT statT*
have *isif-I*: *is-iface G I*
by *simp*
from *dynM statT*
have *dynM'*: *dynimethd G I dynC sig* = *Some dynM*
by (*simp add*: *dynlookup-def*)
from *False implmnt isif-I wf dynM'*
show *?thesis*
by − (*drule implmt-dynimethd-access*, *auto*)
qed
next
case (*Array-Object-methd T statM*)
assume *statT*: *statT* = *ArrayT T*
assume *accmethd G accC Object sig* = *Some statM*
then have *statM*: *methd G Object sig* = *Some statM* **and**
stat-acc: $G \vdash$ *Methd sig statM of Object accessible-from accC*
by (*auto dest*: *accmethd-SomeD*)
from *statT dynC-prop*
have *dynC-Obj*: *dynC* = *Object*
by *simp*
then
have *widen-dynC-Obj*: $G \vdash$ *Class dynC* $\preceq$ *Class Object*
by *simp*
from *dynM statT*
have *dynM'*: *dynmethd G Object dynC sig* = *Some dynM*
by (*simp add*: *dynlookup-def*)
from *statM statT-acc stat-acc dynM' wf widen-dynC-Obj*
statT isT-statT
show *?thesis*
by − (*drule dynmethd-access-prop*, *simp*+)
qed
qed

lemma *dynlookup-access*:
assumes *emh*: $emh \in$ *mheads G accC statT sig* **and**
dynC-prop: *G*,*statT* $\vdash$ *dynC valid-lookup-cls-for* (*is-static emh*) **and**
isT-statT: *isrtype G statT* **and**
wf: *wf-prog G*

shows $\exists$ *dynM*. *dynlookup G statT dynC sig* = *Some dynM* $\wedge$
G$\vdash$*Methd sig dynM in dynC dyn-accessible-from accC*
proof −
from *dynC-prop isT-statT wf*
have *is-cls-dynC*: *is-class G dynC*
by (*auto dest*: *valid-lookup-cls-is-class*)
with *emh wf dynC-prop isT-statT*
obtain *dynM* **where**
dynlookup G statT dynC sig = *Some dynM*
by − (*drule dynamic-mheadsD*,*auto*)
with *emh dynC-prop isT-statT wf*
show *?thesis*
by (*blast intro*: *dynlookup-access-prop*)
qed

lemma *stat-overrides-Package-old*:
assumes *stat-override*: *G* $\vdash$ *new overrides*$_S$ *old* **and**
accmodi-new: *accmodi new* = *Package* **and**
wf: *wf-prog G*
shows *accmodi old* = *Package*
proof −
from *stat-override wf*
have *accmodi old* $\leq$ *accmodi new*
by (*auto dest*: *wf-prog-stat-overridesD*)
with *stat-override accmodi-new* **show** *?thesis*
by (*cases accmodi old*) (*auto dest*: *no-Private-stat-override*
dest: *acc-modi-le-Dests*)
qed

Properties of dynamic accessibility

lemma *dyn-accessible-Private*:
assumes *dyn-acc*: *G* $\vdash$ *m in C dyn-accessible-from accC* **and**
priv: *accmodi m* = *Private*
shows *accC* = *declclass m*
proof −
from *dyn-acc priv*
show *?thesis*
proof (*induct*)
case (*Immediate m C*)
from ‹*G* $\vdash$ *m in C permits-acc-from accC*› **and** ‹*accmodi m* = *Private*›
show *?case*
by (*simp add*: *permits-acc-def*)
next
case *Overriding*
then show *?case*
by (*auto dest*!: *overrides-commonD*)
qed
qed

dyn-accessible-Package only works with the *wf-prog* assumption. Without it. it is easy to leaf the Package!

lemma *dyn-accessible-Package*:
⟦*G* $\vdash$ *m in C dyn-accessible-from accC*; *accmodi m* = *Package*;
wf-prog G⟧
$\Longrightarrow$ *pid accC* = *pid* (*declclass m*)
proof −
assume *wf*: *wf-prog G*

assume *accessible*: $G \vdash m$ *in* C *dyn-accessible-from accC*
then show *accmodi m* = *Package*
$\Longrightarrow$ *pid accC* = *pid* (*declclass m*)
(**is** *?Pack m* $\Longrightarrow$ *?P m*)
proof (*induct rule*: *dyn-accessible-fromR.induct*)
case (*Immediate m C*)
assume $G \vdash m$ *member-in* C
$G \vdash m$ *in* C *permits-acc-from accC*
accmodi m = *Package*
then show *?P m*
by (*auto simp add*: *permits-acc-def*)
next
case (*Overriding new C declC newm old Sup*)
assume *member-new*: $G \vdash$ *new member-in* C **and**
new: *new* = (*declC*, *mdecl newm*) **and**
override: $G \vdash$ (*declC*, *newm*) *overrides old* **and**
subcls-C-Sup: $G \vdash C \prec_C$ *Sup* **and**
acc-old: $G \vdash$ *methdMembr old in Sup dyn-accessible-from accC* **and**
hyp: *?Pack* (*methdMembr old*) $\Longrightarrow$ *?P* (*methdMembr old*) **and**
accmodi-new: *accmodi new* = *Package*
from *override accmodi-new new wf*
have *accmodi-old*: *accmodi old* = *Package*
by (*auto dest*: *overrides-Package-old*)
with *hyp*
have *P-sup*: *?P* (*methdMembr old*)
by (*simp*)
from *wf override new accmodi-old accmodi-new*
have *eq-pid-new-old*: *pid* (*declclass new*) = *pid* (*declclass old*)
by (*auto dest*: *dyn-override-Package*)
with *eq-pid-new-old P-sup* **show** *?P new*
by *auto*
qed
qed

For fields we don't need the wellformedness of the program, since there is no overriding

lemma *dyn-accessible-field-Package*:
assumes *dyn-acc*: $G \vdash f$ *in* C *dyn-accessible-from accC* **and**
pack: *accmodi f* = *Package* **and**
field: *is-field f*
shows *pid accC* = *pid* (*declclass f*)
proof −
from *dyn-acc pack field*
show *?thesis*
proof (*induct*)
case (*Immediate f C*)
from ‹$G \vdash f$ *in* C *permits-acc-from accC*› **and** ‹*accmodi f* = *Package*›
show *?case*
by (*simp add*: *permits-acc-def*)
next
case *Overriding*
then show *?case* **by** (*simp add*: *is-field-def*)
qed
qed

dyn-accessible-instance-field-Protected only works for fields since methods can break the package bounds due to overriding

lemma *dyn-accessible-instance-field-Protected*:
assumes *dyn-acc*: $G \vdash f$ *in* C *dyn-accessible-from accC* **and**

prot: *accmodi f* = *Protected* **and**
field: *is-field f* **and**
instance-field: ¬ *is-static f* **and**
outside: *pid* (*declclass f*) ≠ *pid accC*
shows $G\vdash\ C \preceq_C\ accC$
proof –
from *dyn-acc prot field instance-field outside*
show *?thesis*
proof (*induct*)
case (*Immediate f C*)
note ⟨*G* ⊢ *f in C permits-acc-from accC*⟩
moreover
assume *accmodi f* = *Protected* **and** *is-field f* **and** ¬ *is-static f* **and**
pid (*declclass f*) ≠ *pid accC*
ultimately
show $G\vdash\ C \preceq_C\ accC$
by (*auto simp add*: *permits-acc-def*)
next
case *Overriding*
then show *?case* **by** (*simp add*: *is-field-def*)
qed
qed

lemma *dyn-accessible-static-field-Protected*:
assumes *dyn-acc*: *G* ⊢ *f in C dyn-accessible-from accC* **and**
prot: *accmodi f* = *Protected* **and**
field: *is-field f* **and**
static-field: *is-static f* **and**
outside: *pid* (*declclass f*) ≠ *pid accC*
shows $G\vdash\ accC \preceq_C\ declclass\ f\ \wedge\ G\vdash C \preceq_C\ declclass\ f$
proof –
from *dyn-acc prot field static-field outside*
show *?thesis*
proof (*induct*)
case (*Immediate f C*)
assume *accmodi f* = *Protected* **and** *is-field f* **and** *is-static f* **and**
pid (*declclass f*) ≠ *pid accC*
moreover
note ⟨*G* ⊢ *f in C permits-acc-from accC*⟩
ultimately
have $G\vdash\ accC \preceq_C\ declclass\ f$
by (*auto simp add*: *permits-acc-def*)
moreover
from ⟨*G* ⊢ *f member-in C*⟩
have $G\vdash C \preceq_C\ declclass\ f$
by (*rule member-in-class-relation*)
ultimately show *?case*
by *blast*
next
case *Overriding*
then show *?case* **by** (*simp add*: *is-field-def*)
qed
qed

end

Chapter 14

State

36 State for evaluation of Java expressions and statements

theory *State* **imports** *DeclConcepts* **begin**

design issues:

- all kinds of objects (class instances, arrays, and class objects) are handeled via a general object abstraction
- the heap and the map for class objects are combined into a single table (*recall* (*loc*, *obj*) *table* $\times$ (*qtname*, *obj*) *table* ~= (*loc* + *qtname*, *obj*) *table*)

objects

datatype *obj-tag* = — tag for generic object
  CInst qtname — class instance
  | *Arr ty int* — array with component type and length
— — CStat qtname the tag is irrelevant for a class object, i.e. the static fields of a class, since its type is given already by the reference to it (see below)

types *vn* = *fspec* + *int* — variable name
record *obj* =
  tag :: *obj-tag* — generalized object
  values :: (*vn*, *val*) *table*

translations
  fspec <= (*type*) *vname* $\times$ *qtname*
  vn <= (*type*) *fspec* + *int*
  obj <= (*type*) (|*tag*::*obj-tag*, *values*::*vn* $\Rightarrow$ *val option*|)
  obj <= (*type*) (|*tag*::*obj-tag*, *values*::*vn* $\Rightarrow$ *val option*,...::'*a*|)

constdefs

  the-Arr :: *obj option* $\Rightarrow$ *ty* $\times$ *int* $\times$ (*vn*, *val*) *table*
  the-Arr obj $\equiv$ *SOME* (*T*,*k*,*t*). *obj* = *Some* (|*tag*=*Arr T k*,*values*=*t*|)

lemma *the-Arr-Arr* [*simp*]: *the-Arr* (*Some* (|*tag*=*Arr T k*,*values*=*cs*|)) = (*T*,*k*,*cs*)
apply (*auto simp*: *the-Arr-def*)
done

lemma *the-Arr-Arr1* [*simp*,*intro*,*dest*]:
  $[\![$*tag obj* = *Arr T k*$]\!]$ $\Longrightarrow$ *the-Arr* (*Some obj*) = (*T*,*k*,*values obj*)
apply (*auto simp add*: *the-Arr-def*)
done

constdefs

  upd-obj :: *vn* $\Rightarrow$ *val* $\Rightarrow$ *obj* $\Rightarrow$ *obj*
  upd-obj n v $\equiv$ λ *obj* . *obj* (|*values*:=(*values obj*)(*n*$\mapsto$*v*)|)

lemma *upd-obj-def2* [*simp*]:
  upd-obj n v obj = *obj* (|*values*:=(*values obj*)(*n*$\mapsto$*v*)|)
apply (*auto simp*: *upd-obj-def*)
done

constdefs
$obj\text{-}ty \quad :: obj \Rightarrow ty$
$obj\text{-}ty\ obj \quad \equiv case\ tag\ obj\ of$
$\quad CInst\ C \Rightarrow Class\ C$
$\quad |\ Arr\ T\ k \Rightarrow T.[]$

lemma $obj\text{-}ty\text{-}eq$ [$intro!$]: $obj\text{-}ty\ (|tag{=}oi,values{=}x|) = obj\text{-}ty\ (|tag{=}oi,values{=}y|)$
by ($simp\ add$: $obj\text{-}ty\text{-}def$)

lemma $obj\text{-}ty\text{-}eq1$ [$intro!,dest$]:
$tag\ obj = tag\ obj' \Longrightarrow obj\text{-}ty\ obj = obj\text{-}ty\ obj'$
by ($simp\ add$: $obj\text{-}ty\text{-}def$)

lemma $obj\text{-}ty\text{-}cong$ [$simp$]:
$obj\text{-}ty\ (obj\ (|values{:=}vs|)) = obj\text{-}ty\ obj$
by $auto$

lemma $obj\text{-}ty\text{-}CInst$ [$simp$]:
$obj\text{-}ty\ (|tag{=}CInst\ C,values{=}vs|) = Class\ C$
by ($simp\ add$: $obj\text{-}ty\text{-}def$)

lemma $obj\text{-}ty\text{-}CInst1$ [$simp,intro!,dest$]:
$[\![tag\ obj = CInst\ C]\!] \Longrightarrow obj\text{-}ty\ obj = Class\ C$
by ($simp\ add$: $obj\text{-}ty\text{-}def$)

lemma $obj\text{-}ty\text{-}Arr$ [$simp$]:
$obj\text{-}ty\ (|tag{=}Arr\ T\ i,values{=}vs|) = T.[]$
by ($simp\ add$: $obj\text{-}ty\text{-}def$)

lemma $obj\text{-}ty\text{-}Arr1$ [$simp,intro!,dest$]:
$[\![tag\ obj = Arr\ T\ i]\!] \Longrightarrow obj\text{-}ty\ obj = T.[]$
by ($simp\ add$: $obj\text{-}ty\text{-}def$)

lemma $obj\text{-}ty\text{-}widenD$:
$G{\vdash}obj\text{-}ty\ obj{\preceq}RefT\ t \Longrightarrow (\exists\, C.\ tag\ obj = CInst\ C) \lor (\exists\, T\ k.\ tag\ obj = Arr\ T\ k)$
apply ($unfold\ obj\text{-}ty\text{-}def$)
apply ($auto\ split\ add$: $obj\text{-}tag.split\text{-}asm$)
done

constdefs

$obj\text{-}class :: obj \Rightarrow qtname$
$obj\text{-}class\ obj \equiv case\ tag\ obj\ of$
$\quad CInst\ C \Rightarrow C$
$\quad |\ Arr\ T\ k \Rightarrow Object$

lemma $obj\text{-}class\text{-}CInst$ [$simp$]: $obj\text{-}class\ (|tag{=}CInst\ C,values{=}vs|) = C$
by ($auto\ simp$: $obj\text{-}class\text{-}def$)

lemma *obj-class-CInst1* [*simp*,*intro*!,*dest*]:
tag obj = *CInst C* $\Longrightarrow$ *obj-class obj* = *C*
by (*auto simp*: *obj-class-def*)

lemma *obj-class-Arr* [*simp*]: *obj-class* (|*tag*=*Arr T k*,*values*=*vs*|) = *Object*
by (*auto simp*: *obj-class-def*)

lemma *obj-class-Arr1* [*simp*,*intro*!,*dest*]:
tag obj = *Arr T k* $\Longrightarrow$ *obj-class obj* = *Object*
by (*auto simp*: *obj-class-def*)

lemma *obj-ty-obj-class*: $G \vdash$*obj-ty obj*$\preceq$ *Class statC* = $G \vdash$*obj-class obj* $\preceq_C$ *statC*
apply (*case-tac tag obj*)
apply (*auto simp add*: *obj-ty-def obj-class-def*)
apply (*case-tac statC* = *Object*)
apply (*auto dest*: *widen-Array-Class*)
done

object references

types *oref* = *loc* + *qtname* — generalized object reference
syntax
Heap :: *loc* $\Rightarrow$ *oref*
Stat :: *qtname* $\Rightarrow$ *oref*

translations
Heap => *Inl*
Stat => *Inr*
oref <= (*type*) *loc* + *qtname*

constdefs
fields-table::
prog $\Rightarrow$ *qtname* $\Rightarrow$ (*fspec* $\Rightarrow$ *field* $\Rightarrow$ *bool*) $\Rightarrow$ (*fspec*, *ty*) *table*
fields-table G C P
$\equiv$ *option-map type* $\circ$ *table-of* (*filter* (*split P*) (*DeclConcepts.fields G C*))

lemma *fields-table-SomeI*:
[[*table-of* (*DeclConcepts.fields G C*) *n* = *Some f*; *P n f*]]
$\Longrightarrow$ *fields-table G C P n* = *Some* (*type f*)
apply (*unfold fields-table-def*)
apply *clarsimp*
apply (*rule exI*)
apply (*rule conjI*)
apply (*erule map-of-filter-in*)
apply *assumption*
apply *simp*
done

lemma *fields-table-SomeD'*: *fields-table G C P fn* = *Some T* $\Longrightarrow$
$\exists f$. (*fn*,*f*)$\in$*set*(*DeclConcepts.fields G C*) $\wedge$ *type f* = *T*
apply (*unfold fields-table-def*)

apply *clarsimp*
apply (*drule map-of-SomeD*)
apply *auto*
done

lemma *fields-table-SomeD*:
$[\![$*fields-table G C P fn* = *Some T*; *unique* (*DeclConcepts.fields G C*)$]\!] \Longrightarrow$
$\exists f$. *table-of* (*DeclConcepts.fields G C*) *fn* = *Some f* $\wedge$ *type f* = *T*
apply (*unfold fields-table-def*)
apply *clarsimp*
apply (*rule exI*)
apply (*rule conjI*)
apply (*erule table-of-filter-unique-SomeD*)
apply *assumption*
apply *simp*
done

constdefs
in-bounds :: *int* $\Rightarrow$ *int* $\Rightarrow$ *bool* ((-/ *in'-bounds* -) [50, 51] 50)
i in-bounds k $\equiv 0 \leq i \wedge i < k$

arr-comps :: *'a* $\Rightarrow$ *int* $\Rightarrow$ *int* $\Rightarrow$ *'a option*
arr-comps T k $\equiv \lambda i$. *if i in-bounds k then Some T else None*

var-tys :: *prog* $\Rightarrow$ *obj-tag* $\Rightarrow$ *oref* $\Rightarrow$ (*vn*, *ty*) *table*
var-tys G oi r
$\equiv$ *case r of*
Heap a $\Rightarrow$ (*case oi of*
CInst C $\Rightarrow$ *fields-table G C* ($\lambda n\, f$. $\neg$*static f*) (+) *empty*
| *Arr T k* $\Rightarrow$ *empty* (+) *arr-comps T k*)
| *Stat C* $\Rightarrow$ *fields-table G C* ($\lambda fn\, f$. *declclassf fn* = *C* $\wedge$ *static f*)
(+) *empty*

lemma *var-tys-Some-eq*:
var-tys G oi r n = *Some T*
= (*case r of*
Inl a $\Rightarrow$ (*case oi of*
CInst C $\Rightarrow$ ($\exists$ *nt*. *n* = *Inl nt* $\wedge$ *fields-table G C* ($\lambda n\, f$.
$\neg$*static f*) *nt* = *Some T*)
| *Arr t k* $\Rightarrow$ ($\exists$ *i*. *n* = *Inr i* $\wedge$ *i in-bounds k* $\wedge$ *t* = *T*))
| *Inr C* $\Rightarrow$ ($\exists$ *nt*. *n* = *Inl nt* $\wedge$
fields-table G C ($\lambda fn\, f$. *declclassf fn* = *C* $\wedge$ *static f*) *nt*
= *Some T*))
apply (*unfold var-tys-def arr-comps-def*)
apply (*force split add*: *sum.split-asm sum.split obj-tag.split*)
done

stores

types *globs* — global variables: heap and static variables
= (*oref* , *obj*) *table*
heap
= (*loc* , *obj*) *table*

translations
globs <= (*type*) (*oref* , *obj*) *table*

heap <= (*type*) (*loc* , *obj*) *table*

datatype *st* =
st globs locals

37 access

constdefs

globs :: *st* ⇒ *globs*
globs ≡ *st-case* ($\lambda g\ l.\ g$)

locals :: *st* ⇒ *locals*
locals ≡ *st-case* ($\lambda g\ l.\ l$)

heap :: *st* ⇒ *heap*
heap s ≡ *globs s* ∘ *Heap*

lemma *globs-def2* [*simp*]: *globs* (*st g l*) = *g*
by (*simp add*: *globs-def*)

lemma *locals-def2* [*simp*]: *locals* (*st g l*) = *l*
by (*simp add*: *locals-def*)

lemma *heap-def2* [*simp*]: *heap s a*=*globs s* (*Heap a*)
by (*simp add*: *heap-def*)

syntax
val-this :: *st* ⇒ *val*
lookup-obj :: *st* ⇒ *val* ⇒ *obj*

translations
val-this s == *the* (*locals s This*)
lookup-obj s a′ == *the* (*heap s* (*the-Addr a′*))

38 memory allocation

constdefs
new-Addr :: *heap* ⇒ *loc option*
new-Addr h ≡ *if* ($\forall a.\ h\ a \neq None$) *then None else Some* (*SOME a. h a* = *None*)

lemma *new-AddrD*: *new-Addr h* = *Some a* ⟹ *h a* = *None*
apply (*auto simp add*: *new-Addr-def*)
apply (*erule someI*)
done

lemma *new-AddrD2*: *new-Addr h* = *Some a* ⟹ $\forall b.\ h\ b \neq None \longrightarrow b \neq a$
apply (*drule new-AddrD*)
apply *auto*
done

lemma *new-Addr-SomeI*: $h\ a = None \Longrightarrow \exists\, b.\ new\text{-}Addr\ h = Some\ b \wedge h\ b = None$
apply (*simp add*: *new-Addr-def*)
apply (*fast intro*: *someI2*)
done

39 initialization

syntax

init-vals :: $('a,\ ty)\ table \Rightarrow ('a,\ val)\ table$

translations
init-vals vs == *option-map default-val* $\circ$ *vs*

lemma *init-arr-comps-base* [*simp*]: *init-vals* (*arr-comps T 0*) = *empty*
apply (*unfold arr-comps-def in-bounds-def*)
apply (*rule ext*)
apply *auto*
done

lemma *init-arr-comps-step* [*simp*]:
$0 < j \Longrightarrow$ *init-vals* (*arr-comps T j*) =
init-vals (*arr-comps T* $(j - 1)$)$(j - 1 \mapsto$ *default-val T*)
apply (*unfold arr-comps-def in-bounds-def*)
apply (*rule ext*)
apply *auto*
done

40 update

constdefs
gupd :: *oref* $\Rightarrow$ *obj* $\Rightarrow$ *st* $\Rightarrow$ *st* (*gupd*′(-$\mapsto$-′)[*10*,*10*]*1000*)
gupd r obj $\equiv$ *st-case* ($\lambda g\ l.$ *st* ($g(r\mapsto obj)$) l)

lupd :: *lname* $\Rightarrow$ *val* $\Rightarrow$ *st* $\Rightarrow$ *st* (*lupd*′(-$\mapsto$-′)[*10*,*10*]*1000*)
lupd vn v $\equiv$ *st-case* ($\lambda g\ l.$ *st g* ($l(vn\mapsto v)$))

upd-gobj :: *oref* $\Rightarrow$ *vn* $\Rightarrow$ *val* $\Rightarrow$ *st* $\Rightarrow$ *st*
upd-gobj r n v $\equiv$ *st-case* ($\lambda g\ l.$ *st* (*chg-map* (*upd-obj n v*) *r g*) *l*)

set-locals :: *locals* $\Rightarrow$ *st* $\Rightarrow$ *st*
set-locals l $\equiv$ *st-case* ($\lambda g\ l'.$ *st g l*)

init-obj :: *prog* $\Rightarrow$ *obj-tag* $\Rightarrow$ *oref* $\Rightarrow$ *st* $\Rightarrow$ *st*
init-obj G oi r $\equiv$ *gupd*($r\mapsto$(|*tag*=*oi*, *values*=*init-vals* (*var-tys G oi r*)|))

syntax
init-class-obj :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *st* $\Rightarrow$ *st*

translations
init-class-obj G C == *init-obj G arbitrary* (*Inr C*)

lemma *gupd-def2* [*simp*]: *gupd*($r\mapsto obj$) (*st g l*) = *st* ($g(r\mapsto obj)$) *l*
apply (*unfold gupd-def*)
apply (*simp* (*no-asm*))

done

lemma *lupd-def2* [*simp*]: $lupd(vn \mapsto v)$ $(st\ g\ l) = st\ g\ (l(vn \mapsto v))$
apply (*unfold lupd-def*)
apply (*simp* (*no-asm*))
done

lemma *globs-gupd* [*simp*]: $globs\ \ (gupd(r \mapsto obj)\ s) = globs\ s(r \mapsto obj)$
apply (*induct s*)
by (*simp add*: *gupd-def*)

lemma *globs-lupd* [*simp*]: $globs\ \ (lupd(vn \mapsto v\ )\ s) = globs\ \ s$
apply (*induct s*)
by (*simp add*: *lupd-def*)

lemma *locals-gupd* [*simp*]: $locals\ (gupd(r \mapsto obj)\ s) = locals\ s$
apply (*induct s*)
by (*simp add*: *gupd-def*)

lemma *locals-lupd* [*simp*]: $locals\ (lupd(vn \mapsto v\ )\ s) = locals\ s(vn \mapsto v\ )$
apply (*induct s*)
by (*simp add*: *lupd-def*)

lemma *globs-upd-gobj-new* [*rule-format* (*no-asm*), *simp*]:
  $globs\ s\ r = None \longrightarrow globs\ (upd\text{-}gobj\ r\ n\ v\ s) = globs\ s$
apply (*unfold upd-gobj-def*)
apply (*induct s*)
apply *auto*
done

lemma *globs-upd-gobj-upd* [*rule-format* (*no-asm*), *simp*]:
$globs\ s\ r{=}Some\ obj \longrightarrow globs\ (upd\text{-}gobj\ r\ n\ v\ s) = globs\ s(r \mapsto upd\text{-}obj\ n\ v\ obj)$
apply (*unfold upd-gobj-def*)
apply (*induct s*)
apply *auto*
done

lemma *locals-upd-gobj* [*simp*]: $locals\ (upd\text{-}gobj\ r\ n\ v\ s) = locals\ s$
apply (*induct s*)
by (*simp add*: *upd-gobj-def*)

lemma *globs-init-obj* [*simp*]: $globs\ (init\text{-}obj\ G\ oi\ r\ s)\ t =$
  $(if\ t{=}r\ then\ Some\ (\!|tag{=}oi,values{=}init\text{-}vals\ (var\text{-}tys\ G\ oi\ r)|\!)\ else\ globs\ s\ t)$
apply (*unfold init-obj-def*)
apply (*simp* (*no-asm*))
done

lemma *locals-init-obj* [*simp*]: $locals\ (init\text{-}obj\ G\ oi\ r\ s) = locals\ s$

by (*simp add*: *init-obj-def*)

lemma *surjective-st* [*simp*]: *st* (*globs s*) (*locals s*) = *s*
apply (*induct s*)
by *auto*

lemma *surjective-st-init-obj*:
st (*globs* (*init-obj G oi r s*)) (*locals s*) = *init-obj G oi r s*
apply (*subst locals-init-obj* [*THEN sym*])
apply (*rule surjective-st*)
done

lemma *heap-heap-upd* [*simp*]:
heap (*st* (*g*(*Inl a*↦*obj*)) *l*) = *heap* (*st g l*)(*a*↦*obj*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-stat-upd* [*simp*]: *heap* (*st* (*g*(*Inr C*↦*obj*)) *l*) = *heap* (*st g l*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-local-upd* [*simp*]: *heap* (*st g* (*l*(*vn*↦*v*))) = *heap* (*st g l*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-gupd-Heap* [*simp*]: *heap* (*gupd*(*Heap a*↦*obj*) *s*) = *heap s*(*a*↦*obj*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-gupd-Stat* [*simp*]: *heap* (*gupd*(*Stat C*↦*obj*) *s*) = *heap s*
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-lupd* [*simp*]: *heap* (*lupd*(*vn*↦*v*) *s*) = *heap s*
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *heap-upd-gobj-Stat* [*simp*]: *heap* (*upd-gobj* (*Stat C*) *n v s*) = *heap s*
apply (*rule ext*)
apply (*simp* (*no-asm*))
apply (*case-tac globs s* (*Stat C*))
apply *auto*
done

lemma *set-locals-def2* [*simp*]: *set-locals l* (*st g l′*) = *st g l*
apply (*unfold set-locals-def*)
apply (*simp* (*no-asm*))

done

lemma *set-locals-id* [*simp*]: *set-locals* (*locals s*) *s* = *s*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *set-set-locals* [*simp*]: *set-locals l* (*set-locals l′ s*) = *set-locals l s*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *locals-set-locals* [*simp*]: *locals* (*set-locals l s*) = *l*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *globs-set-locals* [*simp*]: *globs* (*set-locals l s*) = *globs s*
apply (*unfold set-locals-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *heap-set-locals* [*simp*]: *heap* (*set-locals l s*) = *heap s*
apply (*unfold heap-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

abrupt completion

consts

the-Xcpt :: *abrupt* ⇒ *xcpt*
the-Jump :: *abrupt* => *jump*
the-Loc :: *xcpt* ⇒ *loc*
the-Std :: *xcpt* ⇒ *xname*

primrec *the-Xcpt* (*Xcpt x*) = *x*
primrec *the-Jump* (*Jump j*) = *j*
primrec *the-Loc* (*Loc a*) = *a*
primrec *the-Std* (*Std x*) = *x*

constdefs
abrupt-if :: *bool* ⇒ *abopt* ⇒ *abopt* ⇒ *abopt*
abrupt-if c x′ x ≡ *if c* ∧ (*x* = *None*) *then x′ else x*

lemma *abrupt-if-True-None* [*simp*]: *abrupt-if True x None* = *x*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-True-not-None* [*simp*]: $x \neq$ *None* $\Longrightarrow$ *abrupt-if True x y* $\neq$ *None*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-False* [*simp*]: *abrupt-if False x y* = *y*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-Some* [*simp*]: *abrupt-if c x* (*Some y*) = *Some y*
by (*simp add*: *abrupt-if-def*)

lemma *abrupt-if-not-None* [*simp*]: $y \neq$ *None* $\Longrightarrow$ *abrupt-if c x y* = *y*
apply (*simp add*: *abrupt-if-def*)
by *auto*

lemma *split-abrupt-if*:
P (*abrupt-if c x′ x*) =
$\quad$((*c* ∧ *x* = *None* ⟶ *P x′*) ∧ (¬ (*c* ∧ *x* = *None*) ⟶ *P x*))
apply (*unfold abrupt-if-def*)
apply (*split split-if*)
apply *auto*
done

syntax

raise-if :: *bool* ⇒ *xname* ⇒ *abopt* ⇒ *abopt*
np :: *val* ⇒ *abopt* ⇒ *abopt*
check-neg:: *val* ⇒ *abopt* ⇒ *abopt*
error-if :: *bool* ⇒ *error* ⇒ *abopt* ⇒ *abopt*

translations

raise-if c xn == *abrupt-if c* (*Some* (*Xcpt* (*Std xn*)))
np v == *raise-if* (*v* = *Null*) *NullPointer*
check-neg i′ == *raise-if* (*the-Intg i′*<*0*) *NegArrSize*
error-if c e == *abrupt-if c* (*Some* (*Error e*))

lemma *raise-if-None* [*simp*]: (*raise-if c x y* = *None*) = (¬*c* ∧ *y* = *None*)
apply (*simp add*: *abrupt-if-def*)
by *auto*
declare *raise-if-None* [*THEN iffD1*, *dest*!]

lemma *if-raise-if-None* [*simp*]:
((*if b then y else raise-if c x y*) = *None*) = ((*c* ⟶ *b*) ∧ *y* = *None*)
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

lemma *raise-if-SomeD* [*dest*!]:

raise-if c x y = Some z $\Longrightarrow$ *c* $\wedge$ *z*=(*Xcpt* (*Std x*)) $\wedge$ *y*=*None* $\vee$ (*y*=*Some z*)
apply (*case-tac y*)
apply (*case-tac c*)
apply (*simp add*: *abrupt-if-def*)
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

lemma *error-if-None* [*simp*]: (*error-if c e y* = *None*) = ($\neg$*c* $\wedge$ *y* = *None*)
apply (*simp add*: *abrupt-if-def*)
by *auto*
declare *error-if-None* [*THEN iffD1*, *dest*!]

lemma *if-error-if-None* [*simp*]:
((*if b then y else error-if c e y*) = *None*) = ((*c* $\longrightarrow$ *b*) $\wedge$ *y* = *None*)
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

lemma *error-if-SomeD* [*dest*!]:
error-if c e y = Some z $\Longrightarrow$ *c* $\wedge$ *z*=(*Error e*) $\wedge$ *y*=*None* $\vee$ (*y*=*Some z*)
apply (*case-tac y*)
apply (*case-tac c*)
apply (*simp add*: *abrupt-if-def*)
apply (*simp add*: *abrupt-if-def*)
apply *auto*
done

constdefs
absorb :: *jump* $\Rightarrow$ *abopt* $\Rightarrow$ *abopt*
absorb j a $\equiv$ *if a*=*Some* (*Jump j*) *then None else a*

lemma *absorb-SomeD* [*dest*!]: *absorb j a* = *Some x* $\Longrightarrow$ *a* = *Some x*
by (*auto simp add*: *absorb-def*)

lemma *absorb-same* [*simp*]: *absorb j* (*Some* (*Jump j*)) = *None*
by (*auto simp add*: *absorb-def*)

lemma *absorb-other* [*simp*]: *a* $\neq$ *Some* (*Jump j*) $\Longrightarrow$ *absorb j a* = *a*
by (*auto simp add*: *absorb-def*)

lemma *absorb-Some-NoneD*: *absorb j* (*Some abr*) = *None* $\Longrightarrow$ *abr* = *Jump j*
by (*simp add*: *absorb-def*)

lemma *absorb-Some-JumpD*: *absorb j s* = *Some* (*Jump j'*) $\Longrightarrow$ *j'*$\neq$*j*
by (*simp add*: *absorb-def*)

full program state

types
state = *abopt* $\times$ *st* — state including abruption information

syntax
  Norm :: *st* ⇒ *state*
  abrupt :: *state* ⇒ *abopt*
  store :: *state* ⇒ *st*

translations

  Norm s == (*None*,*s*)
  abrupt => *fst*
  store => *snd*
  abopt <= (*type*) *State.abrupt option*
  abopt <= (*type*) *abrupt option*
  state <= (*type*) *abopt* × *State.st*
  state <= (*type*) *abopt* × *st*

lemma *single-stateE*: ∀ *Z*. *Z* = (*s*::*state*) ⟹ *False*
apply (*erule-tac x* = (*Some k*,*y*) **in** *all-dupE*)
apply (*erule-tac x* = (*None*,*y*) **in** *allE*)
apply *clarify*
done

lemma *state-not-single*: *All* (*op* = (*x*::*state*)) ⟹ *R*
apply (*drule-tac x* = (*if abrupt x* = *None then Some ?x else None*,*?y*) **in** *spec*)
apply *clarsimp*
done

constdefs

  normal :: *state* ⇒ *bool*
 normal ≡ λ*s*. *abrupt s* = *None*

lemma *normal-def2* [*simp*]: *normal s* = (*abrupt s* = *None*)
apply (*unfold normal-def*)
apply (*simp* (*no-asm*))
done

constdefs
  heap-free :: *nat* ⇒ *state* ⇒ *bool*
 heap-free n ≡ λ*s*. *atleast-free* (*heap* (*store s*)) *n*

lemma *heap-free-def2* [*simp*]: *heap-free n s* = *atleast-free* (*heap* (*store s*)) *n*
apply (*unfold heap-free-def*)
apply *simp*
done

41 update

constdefs

  abupd :: (*abopt* ⇒ *abopt*) ⇒ *state* ⇒ *state*
 abupd f ≡ *prod-fun f id*

supd :: $(st \Rightarrow st) \Rightarrow state \Rightarrow state$
supd $\equiv$ *prod-fun id*

lemma *abupd-def2* [*simp*]: *abupd f* (*x*,*s*) = (*f x*,*s*)
by (*simp add*: *abupd-def*)

lemma *abupd-abrupt-if-False* [*simp*]: $\bigwedge$ *s*. *abupd* (*abrupt-if False xo*) *s* = *s*
by *simp*

lemma *supd-def2* [*simp*]: *supd f* (*x*,*s*) = (*x*,*f s*)
by (*simp add*: *supd-def*)

lemma *supd-lupd* [*simp*]:
$\bigwedge$ *s*. *supd* (*lupd vn v*) *s* = (*abrupt s*,*lupd vn v* (*store s*))
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

lemma *supd-gupd* [*simp*]:
$\bigwedge$ *s*. *supd* (*gupd r obj*) *s* = (*abrupt s*,*gupd r obj* (*store s*))
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

lemma *supd-init-obj* [*simp*]:
supd (*init-obj G oi r*) *s* = (*abrupt s*,*init-obj G oi r* (*store s*))
apply (*unfold init-obj-def*)
apply (*simp* (*no-asm*))
done

lemma *abupd-store-invariant* [*simp*]: *store* (*abupd f s*) = *store s*
by (*cases s*) *simp*

lemma *supd-abrupt-invariant* [*simp*]: *abrupt* (*supd f s*) = *abrupt s*
by (*cases s*) *simp*

syntax

set-lvars :: $locals \Rightarrow state \Rightarrow state$
restore-lvars :: $state \Rightarrow state \Rightarrow state$

translations

set-lvars l == *supd* (*set-locals l*)
restore-lvars s' *s* == *set-lvars* (*locals* (*store* s')) *s*

lemma *set-set-lvars* [*simp*]: $\bigwedge$ *s*. *set-lvars l* (*set-lvars* l' *s*) = *set-lvars l s*
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))

done

lemma *set-lvars-id* [*simp*]: $\bigwedge s.\ set\text{-}lvars\ (locals\ (store\ s))\ s = s$
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

initialisation test

constdefs

$inited\ \ :: qtname \Rightarrow globs \Rightarrow bool$
$inited\ C\ g \equiv g\ (Stat\ C) \neq None$

$initd\ \ :: qtname \Rightarrow state \Rightarrow bool$
$initd\ C \equiv inited\ C \circ globs \circ store$

lemma *not-inited-empty* [*simp*]: $\neg inited\ C\ empty$
apply (*unfold inited-def*)
apply (*simp* (*no-asm*))
done

lemma *inited-gupdate* [*simp*]: $inited\ C\ (g(r \mapsto obj)) = (inited\ C\ g \vee r = Stat\ C)$
apply (*unfold inited-def*)
apply (*auto split add*: *st.split*)
done

lemma *inited-init-class-obj* [*intro!*]: $inited\ C\ (globs\ (init\text{-}class\text{-}obj\ G\ C\ s))$
apply (*unfold inited-def*)
apply (*simp* (*no-asm*))
done

lemma *not-initedD*: $\neg\ inited\ C\ g \Longrightarrow g\ (Stat\ C) = None$
apply (*unfold inited-def*)
apply (*erule notnotD*)
done

lemma *initedD*: $inited\ C\ g \Longrightarrow \exists\ obj.\ g\ (Stat\ C) = Some\ obj$
apply (*unfold inited-def*)
apply *auto*
done

lemma *initd-def2* [*simp*]: $initd\ C\ s = inited\ C\ (globs\ (store\ s))$
apply (*unfold initd-def*)
apply (*simp* (*no-asm*))
done

error-free

constdefs *error-free*:: $state \Rightarrow bool$
$error\text{-}free\ s \equiv \neg\ (\exists\ err.\ abrupt\ s = Some\ (Error\ err))$

lemma *error-free-Norm* [*simp*,*intro*]: *error-free* (*Norm s*)
by (*simp add*: *error-free-def*)

lemma *error-free-normal* [*simp*,*intro*]: *normal s* $\Longrightarrow$ *error-free s*
by (*simp add*: *error-free-def*)

lemma *error-free-Xcpt* [*simp*]: *error-free* (*Some* (*Xcpt x*),*s*)
by (*simp add*: *error-free-def*)

lemma *error-free-Jump* [*simp*,*intro*]: *error-free* (*Some* (*Jump j*),*s*)
by (*simp add*: *error-free-def*)

lemma *error-free-Error* [*simp*]: *error-free* (*Some* (*Error e*),*s*) = *False*
by (*simp add*: *error-free-def*)

lemma *error-free-Some* [*simp*,*intro*]:
$\neg$ ($\exists$ *err*. *x*=*Error err*) $\Longrightarrow$ *error-free* ((*Some x*),*s*)
by (*auto simp add*: *error-free-def*)

lemma *error-free-abupd-absorb* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* (*abupd* (*absorb j*) *s*)
by (*cases s*)
(*auto simp add*: *error-free-def absorb-def*
split: *split-if-asm*)

lemma *error-free-absorb* [*simp*,*intro*]:
error-free (*a*,*s*) $\Longrightarrow$ *error-free* (*absorb j a*, *s*)
by (*auto simp add*: *error-free-def absorb-def*
split: *split-if-asm*)

lemma *error-free-abrupt-if* [*simp*,*intro*]:
[[*error-free s*; $\neg$ ($\exists$ *err*. *x*=*Error err*)]]
$\Longrightarrow$ *error-free* (*abupd* (*abrupt-if p* (*Some x*)) *s*)
by (*cases s*)
(*auto simp add*: *abrupt-if-def*
split: *split-if*)

lemma *error-free-abrupt-if1* [*simp*,*intro*]:
[[*error-free* (*a*,*s*); $\neg$ ($\exists$ *err*. *x*=*Error err*)]]
$\Longrightarrow$ *error-free* (*abrupt-if p* (*Some x*) *a*, *s*)
by (*auto simp add*: *abrupt-if-def*
split: *split-if*)

lemma *error-free-abrupt-if-Xcpt* [*simp*,*intro*]:
error-free s
$\Longrightarrow$ *error-free* (*abupd* (*abrupt-if p* (*Some* (*Xcpt x*))) *s*)
by *simp*

lemma *error-free-abrupt-if-Xcpt1* [*simp*,*intro*]:
error-free (*a*,*s*)
$\Longrightarrow$ *error-free* (*abrupt-if p* (*Some* (*Xcpt x*)) *a*, *s*)
by *simp*

lemma *error-free-abrupt-if-Jump* [*simp*,*intro*]:
error-free s
$\Longrightarrow$ *error-free* (*abupd* (*abrupt-if p* (*Some* (*Jump j*))) *s*)
by *simp*

lemma *error-free-abrupt-if-Jump1* [*simp*,*intro*]:
error-free (*a*,*s*)
$\Longrightarrow$ *error-free* (*abrupt-if p* (*Some* (*Jump j*)) *a*, *s*)
by *simp*

lemma *error-free-raise-if* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* (*abupd* (*raise-if p x*) *s*)
by *simp*

lemma *error-free-raise-if1* [*simp*,*intro*]:
error-free (*a*,*s*) $\Longrightarrow$ *error-free* ((*raise-if p x a*), *s*)
by *simp*

lemma *error-free-supd* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* (*supd f s*)
by (*cases s*) (*simp add*: *error-free-def*)

lemma *error-free-supd1* [*simp*,*intro*]:
error-free (*a*,*s*) $\Longrightarrow$ *error-free* (*a*,*f s*)
by (*simp add*: *error-free-def*)

lemma *error-free-set-lvars* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* ((*set-lvars l*) *s*)
by (*cases s*) *simp*

lemma *error-free-set-locals* [*simp*,*intro*]:
error-free (*x*, *s*)
$\Longrightarrow$ *error-free* (*x*, *set-locals l s'*)
by (*simp add*: *error-free-def*)

end

Chapter 15

Eval

42 Operational evaluation (big-step) semantics of Java expressions and statements

theory *Eval* **imports** *State DeclConcepts* **begin**

improvements over Java Specification 1.0:

- dynamic method lookup does not need to consider the return type (cf.15.11.4.4)
- throw raises a NullPointer exception if a null reference is given, and each throw of a standard exception yield a fresh exception object (was not specified)
- if there is not enough memory even to allocate an OutOfMemory exception, evaluation/execution fails, i.e. simply stops (was not specified)
- array assignment checks lhs (and may throw exceptions) before evaluating rhs
- fixed exact positions of class initializations (immediate at first active use)

design issues:

- evaluation vs. (single-step) transition semantics evaluation semantics chosen, because:
  - ++ less verbose and therefore easier to read (and to handle in proofs)
  - \+ more abstract
  - \+ intermediate values (appearing in recursive rules) need not be stored explicitly, e.g. no call body construct or stack of invocation frames containing local variables and return addresses for method calls needed
  - \+ convenient rule induction for subject reduction theorem
  - \- no interleaving (for parallelism) can be described
  - \- stating a property of infinite executions requires the meta-level argument that this property holds for any finite prefixes of it (e.g. stopped using a counter that is decremented to zero and then throwing an exception)
- unified evaluation for variables, expressions, expression lists, statements
- the value entry in statement rules is redundant
- the value entry in rules is irrelevant in case of exceptions, but its full inclusion helps to make the rule structure independent of exception occurence.
- as irrelevant value entries are ignored, it does not matter if they are unique. For simplicity, (fixed) arbitrary values are preferred over "free" values.
- the rule format is such that the start state may contain an exception.
  - ++ faciliates exception handling
  - \+ symmetry
- the rules are defined carefully in order to be applicable even in not type-correct situations (yielding undefined values), e.g. *the-Addr* (*Val* (*Bool* *b*)) = *arbitrary*.
  - ++ fewer rules
  - \- less readable because of auxiliary functions like *the-Addr*

  Alternative: "defensive" evaluation throwing some InternalError exception in case of (impossible, for correct programs) type mismatches

- there is exactly one rule per syntactic construct

  + no redundancy in case distinctions

- halloc fails iff there is no free heap address. When there is only one free heap address left, it returns an OutOfMemory exception. In this way it is guaranteed that when an OutOfMemory exception is thrown for the first time, there is a free location on the heap to allocate it.

- the allocation of objects that represent standard exceptions is deferred until execution of any enclosing catch clause, which is transparent to the program.

  - requires an auxiliary execution relation

  ++ avoids copies of allocation code and awkward case distinctions (whether there is enough memory to allocate the exception) in evaluation rules

- unfortunately *new-Addr* is not directly executable because of Hilbert operator.

simplifications:

- local variables are initialized with default values (no definite assignment)
- garbage collection not considered, therefore also no finalizers
- stack overflow and memory overflow during class initialization not modelled
- exceptions in initializations not replaced by ExceptionInInitializerError

types *vvar* = *val* × (*val* ⇒ *state* ⇒ *state*)
vals = (*val*, *vvar*, *val list*) *sum3*
translations
vvar <= (*type*) *val* × (*val* ⇒ *state* ⇒ *state*)
vals <= (*type*)(*val*, *vvar*, *val list*) *sum3*

To avoid redundancy and to reduce the number of rules, there is only one evaluation rule for each syntactic term. This is also true for variables (e.g. see the rules below for *LVar*, *FVar* and *AVar*). So evaluation of a variable must capture both possible further uses: read (rule *Acc*) or write (rule *Ass*) to the variable. Therefor a variable evaluates to a special value *vvar*, which is a pair, consisting of the current value (for later read access) and an update function (for later write access). Because during assignment to an array variable an exception may occur if the types don't match, the update function is very generic: it transforms the full state. This generic update function causes some technical trouble during some proofs (e.g. type safety, correctness of definite assignment). There we need to prove some additional invariant on this update function to prove the assignment correct, since the update function could potentially alter the whole state in an arbitrary manner. This invariant must be carried around through the whole induction. So for future approaches it may be better not to take such a generic update function, but only to store the address and the kind of variable (array (+ element type), local variable or field) for later assignment.

syntax (*xsymbols*)
dummy-res :: *vals* (◇)
translations
◇ == *In1 Unit*

syntax
val-inj-vals:: *expr* ⇒ *term* ($\lfloor$-$\rfloor_e$ *1000*)
var-inj-vals:: *var* ⇒ *term* ($\lfloor$-$\rfloor_v$ *1000*)
lst-inj-vals:: *expr list* ⇒ *term* ($\lfloor$-$\rfloor_l$ *1000*)

translations

$\lfloor e \rfloor_e \rightharpoonup$ *In1 e*
$\lfloor v \rfloor_v \rightharpoonup$ *In2 v*
$\lfloor es \rfloor_l \rightharpoonup$ *In3 es*

constdefs
arbitrary3 :: ('*al* + '*ar*, '*b*, '*c*) *sum3* ⇒ *vals*
arbitrary3 ≡ *sum3-case* (*In1* ∘ *sum-case* (λ*x*. *arbitrary*) (λ*x*. *Unit*))
(λ*x*. *In2 arbitrary*) (λ*x*. *In3 arbitrary*)

lemma [*simp*]: *arbitrary3* (*In1l x*) = *In1 arbitrary*
by (*simp add*: *arbitrary3-def*)

lemma [*simp*]: *arbitrary3* (*In1r x*) = ♢
by (*simp add*: *arbitrary3-def*)

lemma [*simp*]: *arbitrary3* (*In2 x*) = *In2 arbitrary*
by (*simp add*: *arbitrary3-def*)

lemma [*simp*]: *arbitrary3* (*In3 x*) = *In3 arbitrary*
by (*simp add*: *arbitrary3-def*)

exception throwing and catching

constdefs
throw :: *val* ⇒ *abopt* ⇒ *abopt*
throw a' x ≡ *abrupt-if True* (*Some* (*Xcpt* (*Loc* (*the-Addr a'*)))) (*np a' x*)

lemma *throw-def2*:
throw a' x = *abrupt-if True* (*Some* (*Xcpt* (*Loc* (*the-Addr a'*)))) (*np a' x*)
apply (*unfold throw-def*)
apply (*simp* (*no-asm*))
done

constdefs
fits :: *prog* ⇒ *st* ⇒ *val* ⇒ *ty* ⇒ *bool* (-,-⊢- *fits* -[*61*,*61*,*61*,*61*]*60*)
G,*s*⊢*a'* *fits T* ≡ (∃ *rt*. *T*=*RefT rt*) ⟶ *a'*=*Null* ∨ *G*⊢*obj-ty*(*lookup-obj s a'*)≼*T*

lemma *fits-Null* [*simp*]: *G*,*s*⊢*Null fits T*
by (*simp add*: *fits-def*)

lemma *fits-Addr-RefT* [*simp*]:
G,*s*⊢*Addr a fits RefT t* = *G*⊢*obj-ty* (*the* (*heap s a*))≼*RefT t*
by (*simp add*: *fits-def*)

lemma *fitsD*: ⋀*X*. *G*,*s*⊢*a'* *fits T* ⟹ (∃ *pt*. *T* = *PrimT pt*) ∨
(∃ *t*. *T* = *RefT t*) ∧ *a'* = *Null* ∨
(∃ *t*. *T* = *RefT t*) ∧ *a'* ≠ *Null* ∧ *G*⊢*obj-ty* (*lookup-obj s a'*)≼*T*
apply (*unfold fits-def*)
apply (*case-tac* ∃ *pt*. *T* = *PrimT pt*)
apply *simp-all*

apply (*case-tac T*)
defer
apply (*case-tac a′ = Null*)
apply *simp-all*
apply *iprover*
done

constdefs
catch ::*prog* ⇒ *state* ⇒ *qtname* ⇒ *bool* (-,-⊢*catch* -[*61*,*61*,*61*]*60*)
G,*s*⊢*catch C*≡∃ *xc*. *abrupt s*=*Some* (*Xcpt xc*) ∧
G,*store s*⊢*Addr* (*the-Loc xc*) *fits Class C*

lemma *catch-Norm* [*simp*]: ¬*G*,*Norm s*⊢*catch tn*
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

lemma *catch-XcptLoc* [*simp*]:
G,(*Some* (*Xcpt* (*Loc a*)),*s*)⊢*catch C* = *G*,*s*⊢*Addr a fits Class C*
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

lemma *catch-Jump* [*simp*]: ¬*G*,(*Some* (*Jump j*),*s*)⊢*catch tn*
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

lemma *catch-Error* [*simp*]: ¬*G*,(*Some* (*Error e*),*s*)⊢*catch tn*
apply (*unfold catch-def*)
apply (*simp* (*no-asm*))
done

constdefs
new-xcpt-var :: *vname* ⇒ *state* ⇒ *state*
new-xcpt-var vn ≡
λ(*x*,*s*). *Norm* (*lupd*(*VName vn*↦*Addr* (*the-Loc* (*the-Xcpt* (*the x*)))) *s*)

lemma *new-xcpt-var-def2* [*simp*]:
new-xcpt-var vn (*x*,*s*) =
Norm (*lupd*(*VName vn*↦*Addr* (*the-Loc* (*the-Xcpt* (*the x*)))) *s*)
apply (*unfold new-xcpt-var-def*)
apply (*simp* (*no-asm*))
done

misc

constdefs

assign :: (′*a* ⇒ *state* ⇒ *state*) ⇒ ′*a* ⇒ *state* ⇒ *state*
assign f v ≡ λ(*x*,*s*). *let* (*x*′,*s*′) = (*if x* = *None then f v else id*) (*x*,*s*)
in (*x*′,*if x*′ = *None then s*′ *else s*)

lemma *assign-Norm-Norm* [*simp*]:
$f\ v\ (Norm\ s) = Norm\ s' \Longrightarrow assign\ f\ v\ (Norm\ s) = Norm\ s'$
by (*simp add*: *assign-def Let-def*)

lemma *assign-Norm-Some* [*simp*]:
$[\![abrupt\ (f\ v\ (Norm\ s)) = Some\ y]\!]$
$\Longrightarrow assign\ f\ v\ (Norm\ s) = (Some\ y,s)$
by (*simp add*: *assign-def Let-def split-beta*)

lemma *assign-Some* [*simp*]:
$assign\ f\ v\ (Some\ x,s) = (Some\ x,s)$
by (*simp add*: *assign-def Let-def split-beta*)

lemma *assign-Some1* [*simp*]: $\neg\ normal\ s \Longrightarrow assign\ f\ v\ s = s$
by (*auto simp add*: *assign-def Let-def split-beta*)

lemma *assign-supd* [*simp*]:
$assign\ (\lambda v.\ supd\ (f\ v))\ v\ (x,s)$
$= (x,\ if\ x = None\ then\ f\ v\ s\ else\ s)$
apply *auto*
done

lemma *assign-raise-if* [*simp*]:
$assign\ (\lambda v\ (x,s).\ ((raise\text{-}if\ (b\ s\ v)\ xcpt)\ x,\ f\ v\ s))\ v\ (x,\ s) =$
$(raise\text{-}if\ (b\ s\ v)\ xcpt\ x,\ if\ x{=}None \wedge \neg b\ s\ v\ then\ f\ v\ s\ else\ s)$
apply (*case-tac* $x = None$)
apply *auto*
done

constdefs

$init\text{-}comp\text{-}ty :: ty \Rightarrow stmt$
$init\text{-}comp\text{-}ty\ T \equiv if\ (\exists\ C.\ T = Class\ C)\ then\ Init\ (the\text{-}Class\ T)\ else\ Skip$

lemma *init-comp-ty-PrimT* [*simp*]: $init\text{-}comp\text{-}ty\ (PrimT\ pt) = Skip$
apply (*unfold init-comp-ty-def*)
apply (*simp* (*no-asm*))
done

constdefs

$invocation\text{-}class :: inv\text{-}mode \Rightarrow st \Rightarrow val \Rightarrow ref\text{-}ty \Rightarrow qtname$
$invocation\text{-}class\ m\ s\ a'\ statT$
$\equiv (case\ m\ of$
$Static \Rightarrow if\ (\exists\ statC.\ statT = ClassT\ statC)$

$\qquad\qquad$ *then the-Class* (*RefT statT*)
$\qquad\qquad$ *else Object*
$\quad$ | *SuperM* $\Rightarrow$ *the-Class* (*RefT statT*)
$\quad$ | *IntVir* $\Rightarrow$ *obj-class* (*lookup-obj s a'*))

invocation-declclass::*prog* $\Rightarrow$ *inv-mode* $\Rightarrow$ *st* $\Rightarrow$ *val* $\Rightarrow$ *ref-ty* $\Rightarrow$ *sig* $\Rightarrow$ *qtname*
invocation-declclass G m s a' statT sig
$\equiv$ *declclass* (*the* (*dynlookup G statT*
$\qquad\qquad$ (*invocation-class m s a' statT*)
$\qquad\qquad$ *sig*))

lemma *invocation-class-IntVir* [*simp*]:
invocation-class IntVir s a' statT = *obj-class* (*lookup-obj s a'*)
by (*simp add*: *invocation-class-def*)

lemma *dynclass-SuperM* [*simp*]:
invocation-class SuperM s a' statT = *the-Class* (*RefT statT*)
by (*simp add*: *invocation-class-def*)

lemma *invocation-class-Static* [*simp*]:
invocation-class Static s a' statT = (*if* ($\exists$ *statC*. *statT* = *ClassT statC*)
$\qquad\qquad$ *then the-Class* (*RefT statT*)
$\qquad\qquad$ *else Object*)
by (*simp add*: *invocation-class-def*)

constdefs
init-lvars :: *prog* $\Rightarrow$ *qtname* $\Rightarrow$ *sig* $\Rightarrow$ *inv-mode* $\Rightarrow$ *val* $\Rightarrow$ *val list* $\Rightarrow$
$\qquad$ *state* $\Rightarrow$ *state*
init-lvars G C sig mode a' pvs
$\equiv$ λ (*x*,*s*).
$\quad$ *let m* = *mthd* (*the* (*methd G C sig*));
$\qquad$ *l* = λ *k*.
$\qquad\quad$ (*case k of*
$\qquad\qquad$ *EName e*
$\qquad\qquad\ \ \Rightarrow$ (*case e of*
$\qquad\qquad\qquad$ *VNam v* $\Rightarrow$ (*empty* ((*pars m*)[$\mapsto$]*pvs*)) *v*
$\qquad\qquad\quad$ | *Res* $\Rightarrow$ *None*)
$\qquad\quad$ | *This*
$\qquad\qquad\ \ \Rightarrow$ (*if mode*=*Static then None else Some a'*))
$\quad$ *in set-lvars l* (*if mode* = *Static then x else np a' x*,*s*)

lemma *init-lvars-def2*: — better suited for simplification
init-lvars G C sig mode a' pvs (*x*,*s*) =
set-lvars
(λ *k*.
$\quad$ (*case k of*
$\qquad$ *EName e*
$\qquad\ \ \Rightarrow$ (*case e of*
$\qquad\qquad$ *VNam v*
$\qquad\qquad\ \ \Rightarrow$ (*empty* ((*pars* (*mthd* (*the* (*methd G C sig*))))[$\mapsto$]*pvs*)) *v*
$\qquad\quad$ | *Res* $\Rightarrow$ *None*)
$\quad$ | *This*
$\qquad\ \ \Rightarrow$ (*if mode*=*Static then None else Some a'*)))

(*if mode* = *Static then x else np a′ x*,*s*)
apply (*unfold init-lvars-def*)
apply (*simp* (*no-asm*) *add*: *Let-def*)
done

constdefs

```
 body :: prog ⇒ qtname ⇒ sig ⇒ expr
body G C sig ≡ let m = the (methd G C sig)
               in Body (declclass m) (stmt (mbody (mthd m)))
```

lemma *body-def2*: — better suited for simplification

```
body G C sig = Body  (declclass (the (methd G C sig)))
                     (stmt (mbody (mthd (the (methd G C sig)))))
```

apply (*unfold body-def Let-def*)
apply *auto*
done

variables

constdefs

```
 lvar :: lname ⇒ st ⇒ vvar
lvar vn s ≡ (the (locals s vn), λv. supd (lupd(vn↦v)))

 fvar :: qtname ⇒ bool ⇒ vname ⇒ val ⇒ state ⇒ vvar × state
fvar C stat fn a′ s
  ≡ let (oref,xf) = if stat then (Stat C,id)
                               else (Heap (the-Addr a′),np a′);
                 n = Inl (fn,C);
                 f = (λv. supd (upd-gobj oref n v))
     in ((the (values (the (globs (store s) oref)) n),f),abupd xf s)

 avar :: prog ⇒ val ⇒ val ⇒ state ⇒ vvar × state
avar G i′ a′ s
  ≡ let   oref = Heap (the-Addr a′);
              i = the-Intg i′;
              n = Inr i;
        (T,k,cs) = the-Arr (globs (store s) oref);
              f = (λv (x,s). (raise-if (¬G,s⊢v fits T)
                                          ArrStore x
                            ,upd-gobj oref n v s))
     in ((the (cs n),f)
        ,abupd (raise-if (¬i in-bounds k) IndOutBound ∘ np a′) s)
```

lemma *fvar-def2*: — better suited for simplification

```
fvar C stat fn a′ s =
  ((the
     (values
      (the (globs (store s) (if stat then Stat C else Heap (the-Addr a′))))
      (Inl (fn,C)))
   ,(λv. supd (upd-gobj (if stat then Stat C else Heap (the-Addr a′))
                        (Inl (fn,C))
                        v)))
  ,abupd (if stat then id else np a′) s)
```

apply (*unfold fvar-def*)
apply (*simp* (*no-asm*) *add*: *Let-def split-beta*)

done

lemma *avar-def2*: — better suited for simplification
```
avar G i' a' s =
  ((the ((snd(snd(the-Arr (globs (store s) (Heap (the-Addr a'))))))
          (Inr (the-Intg i')))
   ,(λv (x,s'). (raise-if (¬G,s'⊢v fits (fst(the-Arr (globs (store s)
                                                (Heap (the-Addr a'))))))
                              ArrStore x
                ,upd-gobj (Heap (the-Addr a'))
                                (Inr (the-Intg i')) v s')))
  ,abupd (raise-if (¬(the-Intg i') in-bounds (fst(snd(the-Arr (globs (store s)
                  (Heap (the-Addr a'))))))) IndOutBound ∘ np a')
          s)
```
apply (*unfold avar-def*)
apply (*simp* (*no-asm*) *add*: *Let-def split-beta*)
done

constdefs
```
check-field-access::
prog ⇒ qtname ⇒ qtname ⇒ vname ⇒ bool ⇒ val ⇒ state ⇒ state
check-field-access G accC statDeclC fn stat a' s
 ≡ let oref = if stat then Stat statDeclC
                       else Heap (the-Addr a');
       dynC = case oref of
                   Heap a ⇒ obj-class (the (globs (store s) oref))
                 | Stat C ⇒ C;
       f    = (the (table-of (DeclConcepts.fields G dynC) (fn,statDeclC)))
   in abupd
        (error-if (¬ G⊢Field fn (statDeclC,f) in dynC dyn-accessible-from accC)
                  AccessViolation)
        s
```

constdefs
```
check-method-access::
  prog ⇒ qtname ⇒ ref-ty ⇒ inv-mode ⇒  sig ⇒ val ⇒ state ⇒ state
check-method-access G accC statT mode sig  a' s
 ≡ let invC = invocation-class mode (store s) a' statT;
       dynM = the (dynlookup G statT invC sig)
   in abupd
        (error-if (¬ G⊢Methd sig dynM in invC dyn-accessible-from accC)
                  AccessViolation)
        s
```

evaluation judgments

inductive
  halloc :: [*prog*,*state*,*obj-tag*,*loc*,*state*]⇒*bool* (-⊢- −*halloc* -≻-→ -[*61*,*61*,*61*,*61*,*61*]*60*) **for** *G*::*prog*
where — allocating objects on the heap, cf. 12.5

```
  Abrupt:
  G⊢(Some x,s) −halloc oi≻arbitrary→ (Some x,s)

| New:   ⟦new-Addr (heap s) = Some a;
          (x,oi') = (if atleast-free (heap s) (Suc (Suc 0)) then (None,oi)
                     else (Some (Xcpt (Loc a)),CInst (SXcpt OutOfMemory)))⟧
          ⟹
          G⊢Norm s −halloc oi≻a→ (x,init-obj G oi' (Heap a) s)
```

inductive *sxalloc* :: [*prog*,*state*,*state*]⇒*bool* (-⊢- −*sxalloc*→ -[*61*,*61*,*61*]*60*) **for** *G*::*prog*
where — allocating exception objects for standard exceptions (other than OutOfMemory)

Norm: $G\vdash$ *Norm* *s* −*sxalloc*→ *Norm* *s*

| *Jmp*: $G\vdash$(*Some* (*Jump* *j*), *s*) −*sxalloc*→ (*Some* (*Jump* *j*), *s*)

| *Error*: $G\vdash$(*Some* (*Error* *e*), *s*) −*sxalloc*→ (*Some* (*Error* *e*), *s*)

| *XcptL*: $G\vdash$(*Some* (*Xcpt* (*Loc* *a*)),*s*) −*sxalloc*→ (*Some* (*Xcpt* (*Loc* *a*)),*s*)

| *SXcpt*: ⟦$G\vdash$*Norm* *s0* −*halloc* (*CInst* (*SXcpt* *xn*))≻*a*→ (*x*,*s1*)⟧ ⟹
$G\vdash$(*Some* (*Xcpt* (*Std* *xn*)),*s0*) −*sxalloc*→ (*Some* (*Xcpt* (*Loc* *a*)),*s1*)

inductive
eval :: [*prog*,*state*,*term*,*vals*,*state*]⇒*bool* (-⊢- −-≻→ ′(-, -′) [*61*,*61*,*80*,*0*,*0*]*60*)
and *exec* ::[*prog*,*state*,*stmt* ,*state*]⇒*bool*(-⊢- −-→ - [*61*,*61*,*65*, *61*]*60*)
and *evar* ::[*prog*,*state*,*var* ,*vvar*,*state*]⇒*bool*(-⊢- −-=≻-→ -[*61*,*61*,*90*,*61*,*61*]*60*)
and *eval′*::[*prog*,*state*,*expr* ,*val* ,*state*]⇒*bool*(-⊢- −-−≻-→ -[*61*,*61*,*80*,*61*,*61*]*60*)
and *evals*::[*prog*,*state*,*expr* *list* ,
val *list* ,*state*]⇒*bool*(-⊢- −-≐≻-→ -[*61*,*61*,*61*,*61*,*61*]*60*)
for *G*::*prog*
where

$G\vdash s$ −*c* → *s′* ≡ $G\vdash s$ −*In1r* *c*≻→ (◇, *s′*)
| $G\vdash s$ −*e*−≻*v* → *s′* ≡ $G\vdash s$ −*In1l* *e*≻→ (*In1* *v*, *s′*)
| $G\vdash s$ −*e*=≻*vf*→ *s′* ≡ $G\vdash s$ −*In2* *e*≻→ (*In2* *vf*, *s′*)
| $G\vdash s$ −*e*≐≻*v* → *s′* ≡ $G\vdash s$ −*In3* *e*≻→ (*In3* *v*, *s′*)

— propagation of abrupt completion

— cf. 14.1, 15.5
| *Abrupt*:
$G\vdash$(*Some* *xc*,*s*) −*t*≻→ (*arbitrary3* *t*, (*Some* *xc*, *s*))

— execution of statements

— cf. 14.5
| *Skip*: $G\vdash$*Norm* *s* −*Skip*→ *Norm* *s*

— cf. 14.7
| *Expr*: ⟦$G\vdash$*Norm* *s0* −*e*−≻*v*→ *s1*⟧ ⟹
$G\vdash$*Norm* *s0* −*Expr* *e*→ *s1*

| *Lab*: ⟦$G\vdash$*Norm* *s0* −*c* → *s1*⟧ ⟹
$G\vdash$*Norm* *s0* −*l*• *c*→ *abupd* (*absorb* *l*) *s1*
— cf. 14.2
| *Comp*: ⟦$G\vdash$*Norm* *s0* −*c1* → *s1*;
$G\vdash$ *s1* −*c2* → *s2*⟧ ⟹
$G\vdash$*Norm* *s0* −*c1*;; *c2*→ *s2*

— cf. 14.8.2
| *If*: ⟦$G\vdash$*Norm* *s0* −*e*−≻*b*→ *s1*;
$G\vdash$ *s1*−(*if* *the-Bool* *b* *then* *c1* *else* *c2*)→ *s2*⟧ ⟹
$G\vdash$*Norm* *s0* −*If*(*e*) *c1* *Else* *c2* → *s2*

— cf. 14.10, 14.10.1

— A continue jump from the while body c is handled by this rule. If a continue jump with the proper label was invoked inside c this label (Cont l) is deleted out of the abrupt component of the state before the iterative evaluation of the while statement. A break jump is handled by the Lab Statement *Lab l* (*while*...).

$$
\begin{array}{l}
|\ \mathit{Loop}{:}\ [\![G\vdash \mathit{Norm}\ s0\ -e-\succ b\rightarrow\ s1;\\
\quad\quad \mathit{if}\ \mathit{the\text{-}Bool}\ b\\
\quad\quad\quad \mathit{then}\ (G\vdash s1\ -c\rightarrow\ s2\ \wedge\\
\quad\quad\quad\quad\quad G\vdash(\mathit{abupd}\ (\mathit{absorb}\ (\mathit{Cont}\ l))\ s2)\ -l\bullet\ \mathit{While}(e)\ c\rightarrow\ s3)\\
\quad\quad\quad \mathit{else}\ s3\ =\ s1]\!]\ \Longrightarrow\\
\quad\quad\quad\quad\quad\quad\quad\quad G\vdash \mathit{Norm}\ s0\ -l\bullet\ \mathit{While}(e)\ c\rightarrow\ s3
\end{array}
$$

$$|\ \mathit{Jmp}{:}\ G\vdash \mathit{Norm}\ s\ -\mathit{Jmp}\ j\rightarrow\ (\mathit{Some}\ (\mathit{Jump}\ j),\ s)$$

— cf. 14.16

$$
\begin{array}{l}
|\ \mathit{Throw}{:}\ [\![G\vdash \mathit{Norm}\ s0\ -e-\succ a'\rightarrow\ s1]\!]\ \Longrightarrow\\
\quad\quad\quad\quad\quad\quad\quad\quad G\vdash \mathit{Norm}\ s0\ -\mathit{Throw}\ e\rightarrow\ \mathit{abupd}\ (\mathit{throw}\ a')\ s1
\end{array}
$$

— cf. 14.18.1

$$
\begin{array}{l}
|\ \mathit{Try}{:}\ [\![G\vdash \mathit{Norm}\ s0\ -c1\rightarrow\ s1;\ G\vdash s1\ -\mathit{sxalloc}\rightarrow\ s2;\\
\quad\quad \mathit{if}\ G,s2\vdash \mathit{catch}\ C\ \mathit{then}\ G\vdash \mathit{new\text{-}xcpt\text{-}var}\ vn\ s2\ -c2\rightarrow\ s3\ \mathit{else}\ s3\ =\ s2]\!]\ \Longrightarrow\\
\quad\quad\quad\quad G\vdash \mathit{Norm}\ s0\ -\mathit{Try}\ c1\ \mathit{Catch}(C\ vn)\ c2\rightarrow\ s3
\end{array}
$$

— cf. 14.18.2

$$
\begin{array}{l}
|\ \mathit{Fin}{:}\ [\![G\vdash \mathit{Norm}\ s0\ -c1\rightarrow\ (x1,s1);\\
\quad\quad G\vdash \mathit{Norm}\ s1\ -c2\rightarrow\ s2;\\
\quad\quad s3{=}(\mathit{if}\ (\exists\ \mathit{err}.\ x1{=}\mathit{Some}\ (\mathit{Error}\ \mathit{err}))\\
\quad\quad\quad \mathit{then}\ (x1,s1)\\
\quad\quad\quad \mathit{else}\ \mathit{abupd}\ (\mathit{abrupt\text{-}if}\ (x1{\neq}\mathit{None})\ x1)\ s2)\ ]\!]\\
\quad\quad \Longrightarrow\\
\quad\quad G\vdash \mathit{Norm}\ s0\ -c1\ \mathit{Finally}\ c2\rightarrow\ s3
\end{array}
$$

— cf. 12.4.2, 8.5

$$
\begin{array}{l}
|\ \mathit{Init}{:}\ [\![\mathit{the}\ (\mathit{class}\ G\ C)\ =\ c;\\
\quad\quad \mathit{if}\ \mathit{inited}\ C\ (\mathit{globs}\ s0)\ \mathit{then}\ s3\ =\ \mathit{Norm}\ s0\\
\quad\quad \mathit{else}\ (G\vdash \mathit{Norm}\ (\mathit{init\text{-}class\text{-}obj}\ G\ C\ s0)\\
\quad\quad\quad\quad -(\mathit{if}\ C\ =\ \mathit{Object}\ \mathit{then}\ \mathit{Skip}\ \mathit{else}\ \mathit{Init}\ (\mathit{super}\ c))\rightarrow\ s1\ \wedge\\
\quad\quad\quad G\vdash \mathit{set\text{-}lvars}\ \mathit{empty}\ s1\ -\mathit{init}\ c\rightarrow\ s2\ \wedge\ s3\ =\ \mathit{restore\text{-}lvars}\ s1\ s2)]\!]\\
\quad\quad\quad \Longrightarrow\\
\quad\quad\quad\quad G\vdash \mathit{Norm}\ s0\ -\mathit{Init}\ C\rightarrow\ s3
\end{array}
$$

— This class initialisation rule is a little bit inaccurate. Look at the exact sequence: (1) The current class object (the static fields) are initialised (*init-class-obj*), (2) the superclasses are initialised, (3) the static initialiser of the current class is invoked. More precisely we should expect another ordering, namely 2 1 3. But we can't just naively toggle 1 and 2. By calling *init-class-obj* before initialising the superclasses, we also implicitly record that we have started to initialise the current class (by setting an value for the class object). This becomes crucial for the completeness proof of the axiomatic semantics *AxCompl.thy*. Static initialisation requires an induction on the number of classes not yet initialised (or to be more precise, classes were the initialisation has not yet begun). So we could first assign a dummy value to the class before superclass initialisation and afterwards set the correct values. But as long as we don't take memory overflow into account when allocating class objects, we can leave things as they are for convenience.

— evaluation of expressions

— cf. 15.8.1, 12.4.1

$$
\begin{array}{l}
|\ \mathit{NewC}{:}\ [\![G\vdash \mathit{Norm}\ s0\ -\mathit{Init}\ C\rightarrow\ s1;\\
\quad\quad G\vdash\quad\ s1\ -\mathit{halloc}\ (\mathit{CInst}\ C)\succ a\rightarrow\ s2]\!]\ \Longrightarrow\\
\quad\quad\quad\quad\quad\quad\quad\quad G\vdash \mathit{Norm}\ s0\ -\mathit{NewC}\ C-\succ \mathit{Addr}\ a\rightarrow\ s2
\end{array}
$$

— cf. 15.9.1, 12.4.1

$$
\begin{array}{l}
|\ \mathit{NewA}{:}\ [\![G\vdash \mathit{Norm}\ s0\ -\mathit{init\text{-}comp\text{-}ty}\ T\rightarrow\ s1;\ G\vdash s1\ -e-\succ i'\rightarrow\ s2;\\
\quad\quad G\vdash \mathit{abupd}\ (\mathit{check\text{-}neg}\ i')\ s2\ -\mathit{halloc}\ (\mathit{Arr}\ T\ (\mathit{the\text{-}Intg}\ i'))\succ a\rightarrow\ s3]\!]\ \Longrightarrow
\end{array}
$$

$$G\vdash Norm\ s0\ -New\ T[e]-\succ Addr\ a\rightarrow\ s3$$

— cf. 15.15

$| \ Cast$: $[\![G\vdash Norm\ s0\ -e-\succ v\rightarrow\ s1;$
$s2\ =\ abupd\ (raise\text{-}if\ (\neg G,store\ s1\vdash v\ fits\ T)\ ClassCast)\ s1]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -Cast\ T\ e-\succ v\rightarrow\ s2$

— cf. 15.19.2

$| \ Inst$: $[\![G\vdash Norm\ s0\ -e-\succ v\rightarrow\ s1;$
$b\ =\ (v\neq Null\ \wedge\ G,store\ s1\vdash v\ fits\ RefT\ T)]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -e\ InstOf\ T-\succ Bool\ b\rightarrow\ s1$

— cf. 15.7.1

$| \ Lit$: $G\vdash Norm\ s\ -Lit\ v-\succ v\rightarrow\ Norm\ s$

$| \ UnOp$: $[\![G\vdash Norm\ s0\ -e-\succ v\rightarrow\ s1]\!]$
$\Longrightarrow\ G\vdash Norm\ s0\ -UnOp\ unop\ e-\succ(eval\text{-}unop\ unop\ v)\rightarrow\ s1$

$| \ BinOp$: $[\![G\vdash Norm\ s0\ -e1-\succ v1\rightarrow\ s1;$
$G\vdash s1\ -(if\ need\text{-}second\text{-}arg\ binop\ v1\ then\ (In1l\ e2)\ else\ (In1r\ Skip))$
$\succ\rightarrow\ (In1\ v2,\ s2)$
$]\!]$
$\Longrightarrow\ G\vdash Norm\ s0\ -BinOp\ binop\ e1\ e2-\succ(eval\text{-}binop\ binop\ v1\ v2)\rightarrow\ s2$

— cf. 15.10.2

$| \ Super$: $G\vdash Norm\ s\ -Super-\succ val\text{-}this\ s\rightarrow\ Norm\ s$

— cf. 15.2

$| \ Acc$: $[\![G\vdash Norm\ s0\ -va=\succ(v,f)\rightarrow\ s1]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -Acc\ va-\succ v\rightarrow\ s1$

— cf. 15.25.1

$| \ Ass$: $[\![G\vdash Norm\ s0\ -va=\succ(w,f)\rightarrow\ s1;$
$G\vdash\quad s1\ -e-\succ v\ \rightarrow\ s2]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -va{:=}e-\succ v\rightarrow\ assign\ f\ v\ s2$

— cf. 15.24

$| \ Cond$: $[\![G\vdash Norm\ s0\ -e0-\succ b\rightarrow\ s1;$
$G\vdash\quad s1\ -(if\ the\text{-}Bool\ b\ then\ e1\ else\ e2)-\succ v\rightarrow\ s2]\!]\Longrightarrow$
$G\vdash Norm\ s0\ -e0\ ?\ e1\ :\ e2-\succ v\rightarrow\ s2$

— The interplay of *Call*, *Methd* and *Body*: Method invocation is split up into these three rules:

Call Calculates the target address and evaluates the arguments of the method, and then performs dynamic or static lookup of the method, corresponding to the call mode. Then the *Methd* rule is evaluated on the calculated declaration class of the method invocation.

Methd A syntactic bridge for the folded method body. It is used by the axiomatic semantics to add the proper hypothesis for recursive calls of the method.

Body An extra syntactic entity for the unfolded method body was introduced to properly trigger class initialisation. Without class initialisation we could just evaluate the body statement.

— cf. 15.11.4.1, 15.11.4.2, 15.11.4.4, 15.11.4.5

$| \ Call$:
$[\![G\vdash Norm\ s0\ -e-\succ a'\rightarrow\ s1;\ G\vdash s1\ -args\doteq\succ vs\rightarrow\ s2;$
$D\ =\ invocation\text{-}declclass\ G\ mode\ (store\ s2)\ a'\ statT\ (\!|name{=}mn,parTs{=}pTs|\!);$
$s3{=}init\text{-}lvars\ G\ D\ (\!|name{=}mn,parTs{=}pTs|\!)\ mode\ a'\ vs\ s2;$
$s3'\ =\ check\text{-}method\text{-}access\ G\ accC\ statT\ mode\ (\!|name{=}mn,parTs{=}pTs|\!)\ a'\ s3;$

$G \vdash s3' -Methd\ D\ (|name=mn,parTs=pTs|) -\succ v \rightarrow s4]\!]$
$\Longrightarrow$
$G \vdash Norm\ s0\ -\{accC,statT,mode\}e \cdot mn(\{pTs\}args) -\succ v \rightarrow (restore\text{-}lvars\ s2\ s4)$

— The accessibility check is after *init-lvars*, to keep it simple. *init-lvars* already tests for the absence of a null-pointer reference in case of an instance method invocation.

| *Methd*: $[\![G \vdash Norm\ s0\ -body\ G\ D\ sig -\succ v \rightarrow s1]\!] \Longrightarrow$
$G \vdash Norm\ s0\ -Methd\ D\ sig -\succ v \rightarrow s1$

| *Body*: $[\![G \vdash Norm\ s0\ -Init\ D \rightarrow s1;\ G \vdash s1\ -c \rightarrow s2;$
$s3 = (if\ (\exists\ l.\ abrupt\ s2 = Some\ (Jump\ (Break\ l)) \vee$
$abrupt\ s2 = Some\ (Jump\ (Cont\ l)))$
$then\ abupd\ (\lambda\ x.\ Some\ (Error\ CrossMethodJump))\ s2$
$else\ s2)]\!] \Longrightarrow$
$G \vdash Norm\ s0\ -Body\ D\ c -\succ the\ (locals\ (store\ s2)\ Result)$
$\rightarrow abupd\ (absorb\ Ret)\ s3$

— cf. 14.15, 12.4.1

— We filter out a break/continue in *s2*, so that we can proof definite assignment correct, without the need of conformance of the state. By this the different parts of the typesafety proof can be disentangled a little.

— evaluation of variables

— cf. 15.13.1, 15.7.2

| *LVar*: $G \vdash Norm\ s\ -LVar\ vn =\succ lvar\ vn\ s \rightarrow Norm\ s$

— cf. 15.10.1, 12.4.1

| *FVar*: $[\![G \vdash Norm\ s0\ -Init\ statDeclC \rightarrow s1;\ G \vdash s1\ -e -\succ a \rightarrow s2;$
$(v,s2') = fvar\ statDeclC\ stat\ fn\ a\ s2;$
$s3 = check\text{-}field\text{-}access\ G\ accC\ statDeclC\ fn\ stat\ a\ s2'\ ]\!] \Longrightarrow$
$G \vdash Norm\ s0\ -\{accC,statDeclC,stat\}e..fn =\succ v \rightarrow s3$

— The accessibility check is after *fvar*, to keep it simple. *fvar* already tests for the absence of a null-pointer reference in case of an instance field

— cf. 15.12.1, 15.25.1

| *AVar*: $[\![G \vdash\ Norm\ s0\ -e1 -\succ a \rightarrow s1;\ G \vdash s1\ -e2 -\succ i \rightarrow s2;$
$(v,s2') = avar\ G\ i\ a\ s2]\!] \Longrightarrow$
$G \vdash Norm\ s0\ -e1.[e2] =\succ v \rightarrow s2'$

— evaluation of expression lists

— cf. 15.11.4.2

| *Nil*:
$G \vdash Norm\ s0\ -[] \doteq\succ [] \rightarrow Norm\ s0$

— cf. 15.6.4

| *Cons*: $[\![G \vdash Norm\ s0\ -e -\succ v \rightarrow s1;$
$G \vdash\ s1\ -es \doteq\succ vs \rightarrow s2]\!] \Longrightarrow$
$G \vdash Norm\ s0\ -e\#es \doteq\succ v\#vs \rightarrow s2$

ML-setup ⟪
bind-thm (*eval-induct-*, *rearrange-prems*
[*0*,*1*,*2*,*8*,*4*,*30*,*31*,*27*,*15*,*16*,
17,*18*,*19*,*20*,*21*,*3*,*5*,*25*,*26*,*23*,*6*,
7,*11*,*9*,*13*,*14*,*12*,*22*,*10*,*28*,
29,*24*] @{*thm eval.induct*})
⟫

lemmas *eval-induct* = *eval-induct-* [*split-format* **and and and and and and and and and and and and and and** *s1* **and and** *s2* **and and and and and and** *s2* **and and** *s2*]

declare *split-if* [*split del*] *split-if-asm* [*split del*]
option.split [*split del*] *option.split-asm* [*split del*]
inductive-cases *halloc-elim-cases*:
$G\vdash(Some\ xc,s)\ -halloc\ oi\succ a\rightarrow\ s'$
$G\vdash(Norm\ \ s)\ -halloc\ oi\succ a\rightarrow\ s'$

inductive-cases *sxalloc-elim-cases*:
$G\vdash\ Norm\ \ s\ -sxalloc\rightarrow\ s'$
$G\vdash(Some\ (Jump\ j),s)\ -sxalloc\rightarrow\ s'$
$G\vdash(Some\ (Error\ e),s)\ -sxalloc\rightarrow\ s'$
$G\vdash(Some\ (Xcpt\ (Loc\ a\ )),s)\ -sxalloc\rightarrow\ s'$
$G\vdash(Some\ (Xcpt\ (Std\ xn)),s)\ -sxalloc\rightarrow\ s'$
inductive-cases *sxalloc-cases*: $G\vdash s\ -sxalloc\rightarrow\ s'$

lemma *sxalloc-elim-cases2*: $[\![G\vdash s\ -sxalloc\rightarrow\ s';$
$\bigwedge s.\ \ [\![s' = Norm\ s]\!] \Longrightarrow P;$
$\bigwedge j\ s.\ [\![s' = (Some\ (Jump\ j),s)]\!] \Longrightarrow P;$
$\bigwedge e\ s.\ [\![s' = (Some\ (Error\ e),s)]\!] \Longrightarrow P;$
$\bigwedge a\ s.\ [\![s' = (Some\ (Xcpt\ (Loc\ a)),s)]\!] \Longrightarrow P$
$]\!] \Longrightarrow P$
apply *cut-tac*
apply (*erule sxalloc-cases*)
apply *blast+*
done

declare *not-None-eq* [*simp del*]
declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss => ss delloop split-all-tac*)) ⟫

inductive-cases *eval-cases*: $G\vdash s\ -t\succ\rightarrow\ (v,\ s')$

inductive-cases *eval-elim-cases* [*cases set*]:
$G\vdash(Some\ xc,s)\ -t\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1r\ Skip\ \succ\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (Jmp\ j)\ \succ\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (l\cdot\ c)\ \succ\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In3\ \ ([])\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In3\ \ (e\#es)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Lit\ w)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (UnOp\ unop\ e)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (BinOp\ binop\ e1\ e2)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In2\ \ (LVar\ vn)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Cast\ T\ e)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (e\ InstOf\ T)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Super)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Acc\ va)\ \succ\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Expr\ e)\ \succ\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (c1;;\ c2)\ \succ\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (Methd\ C\ sig)\ \succ\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (Body\ D\ c)\ \succ\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (e0\ ?\ e1\ :\ e2)\ \succ\rightarrow\ (v,\ s')$

$G\vdash Norm\ s\ -In1r\ (If(e)\ c1\ Else\ c2) \succ\to (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (l\bullet\ While(e)\ c) \succ\to (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (c1\ Finally\ c2) \succ\to (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (Throw\ e) \succ\to (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (NewC\ C) \succ\to (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (New\ T[e]) \succ\to (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Ass\ va\ e) \succ\to (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Try\ c1\ Catch(tn\ vn)\ c2) \succ\to (x,\ s')$
$G\vdash Norm\ s\ -In2\ (\{accC,statDeclC,stat\}e..fn) \succ\to (v,\ s')$
$G\vdash Norm\ s\ -In2\ (e1.[e2]) \succ\to (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (\{accC,statT,mode\}e\cdot mn(\{pT\}p)) \succ\to (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Init\ C) \succ\to (x,\ s')$
declare *not-None-eq* [*simp*]
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declaration ⟨⟨ *K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*))) ⟩⟩
declare *split-if* [*split*] *split-if-asm* [*split*]
option.split [*split*] *option.split-asm* [*split*]

lemma *eval-Inj-elim*:
$G\vdash s\ -t\succ\to (w,s')$
$\Longrightarrow case\ t\ of$
$In1\ ec \Rightarrow (case\ ec\ of$
$Inl\ e \Rightarrow (\exists v.\ w = In1\ v)$
$\mid Inr\ c \Rightarrow w = \Diamond)$
$\mid In2\ e \Rightarrow (\exists v.\ w = In2\ v)$
$\mid In3\ e \Rightarrow (\exists v.\ w = In3\ v)$
apply (*erule eval-cases*)
apply *auto*
apply (*induct-tac t*)
apply (*induct-tac a*)
apply *auto*
done

The following simplification procedures set up the proper injections of terms and their corresponding values in the evaluation relation: E.g. an expression (injection *In1l* into terms) always evaluates to ordinary values (injection *In1* into generalised values *vals*).

lemma *eval-expr-eq*: $G\vdash s\ -In1l\ t\succ\to (w,\ s') = (\exists v.\ w{=}In1\ v \wedge G\vdash s\ -t-\succ v \to s')$
by (*auto*, *frule eval-Inj-elim*, *auto*)

lemma *eval-var-eq*: $G\vdash s\ -In2\ t\succ\to (w,\ s') = (\exists vf.\ w{=}In2\ vf \wedge G\vdash s\ -t{=}\succ vf\to s')$
by (*auto*, *frule eval-Inj-elim*, *auto*)

lemma *eval-exprs-eq*: $G\vdash s\ -In3\ t\succ\to (w,\ s') = (\exists vs.\ w{=}In3\ vs \wedge G\vdash s\ -t\doteq\succ vs\to s')$
by (*auto*, *frule eval-Inj-elim*, *auto*)

lemma *eval-stmt-eq*: $G\vdash s\ -In1r\ t\succ\to (w,\ s') = (w{=}\Diamond \wedge G\vdash s\ -t \to s')$
by (*auto*, *frule eval-Inj-elim*, *auto*, *frule eval-Inj-elim*, *auto*)

simproc-setup *eval-expr* ($G\vdash s\ -In1l\ t\succ\to (w,\ s')$) = ⟨⟨
fn - => *fn* - => *fn ct* =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm eval-expr-eq*})) ⟩⟩

simproc-setup *eval-var* $(G\vdash s\ -In2\ t\succ\rightarrow (w,\ s')) = \langle\langle$
fn - => *fn* - => *fn ct* =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ (*Const* - \$ -) \$ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm eval-var-eq*})) ⟫

simproc-setup *eval-exprs* $(G\vdash s\ -In3\ t\succ\rightarrow (w,\ s')) = \langle\langle$
fn - => *fn* - => *fn ct* =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ (*Const* - \$ -) \$ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm eval-exprs-eq*})) ⟫

simproc-setup *eval-stmt* $(G\vdash s\ -In1r\ t\succ\rightarrow (w,\ s')) = \langle\langle$
fn - => *fn* - => *fn ct* =>
(*case Thm.term-of ct of*
(- \$ - \$ - \$ - \$ (*Const* - \$ -) \$ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm eval-stmt-eq*})) ⟫

ML-setup ⟪
bind-thms (*AbruptIs*, *sum3-instantiate* @{*thm eval.Abrupt*})
⟫

declare *halloc.Abrupt* [*intro*!] *eval.Abrupt* [*intro*!] *AbruptIs* [*intro*!]

Callee,*InsInitE*, *InsInitV*, *FinA* are only used in smallstep semantics, not in the bigstep semantics. So their is no valid evaluation of these terms

lemma *eval-Callee*: $G\vdash Norm\ s-Callee\ l\ e-\succ v\rightarrow\ s' = False$
proof –
{ **fix** $s\ t\ v\ s'$
assume *eval*: $G\vdash s\ -t\succ\rightarrow (v,s')$ **and**
normal: *normal s* **and**
callee: $t=In1l\ (Callee\ l\ e)$
then have *False* **by** *induct auto*
}
then show *?thesis*
by (*cases s'*) *fastsimp*
qed

lemma *eval-InsInitE*: $G\vdash Norm\ s-InsInitE\ c\ e-\succ v\rightarrow\ s' = False$
proof –
{ **fix** $s\ t\ v\ s'$
assume *eval*: $G\vdash s\ -t\succ\rightarrow (v,s')$ **and**
normal: *normal s* **and**
callee: $t=In1l\ (InsInitE\ c\ e)$
then have *False* **by** *induct auto*
}
then show *?thesis*
by (*cases s'*) *fastsimp*
qed

lemma *eval-InsInitV*: $G\vdash Norm\ s-InsInitV\ c\ w=\succ v\rightarrow\ s' = False$
proof –
{ **fix** $s\ t\ v\ s'$
assume *eval*: $G\vdash s\ -t\succ\rightarrow (v,s')$ **and**
normal: *normal s* **and**
callee: $t=In2\ (InsInitV\ c\ w)$

```
    then have False by induct auto
  }
  then show ?thesis
    by (cases s') fastsimp
qed


lemma eval-FinA: G⊢Norm s−FinA a c→ s' = False
proof −
  { fix s t v s'
    assume eval: G⊢s −t≻→ (v,s') and
           normal: normal s and
           callee: t=In1r (FinA a c)
    then have False by induct auto
  }
  then show ?thesis
    by (cases s') fastsimp
qed


lemma eval-no-abrupt-lemma:
  ⋀s s'. G⊢s −t≻→ (w,s') ⟹ normal s' ⟶ normal s
by (erule eval-cases, auto)


lemma eval-no-abrupt:
  G⊢(x,s) −t≻→ (w,Norm s') =
       (x = None ∧ G⊢Norm s −t≻→ (w,Norm s'))
apply auto
apply (frule eval-no-abrupt-lemma, auto)+
done

simproc-setup eval-no-abrupt (G⊢(x,s) −e≻→ (w,Norm s')) = ⟪
  fn - => fn - => fn ct =>
    (case Thm.term-of ct of
     (- $ - $ (Const (@{const-name Pair}, -) $ (Const (@{const-name None}, -)) $ -) $ -  $ - $ -) => NONE
    | - => SOME (mk-meta-eq @{thm eval-no-abrupt}))
⟫


lemma eval-abrupt-lemma:
  G⊢s −t≻→ (v,s') ⟹ abrupt s=Some xc ⟶ s'= s ∧ v = arbitrary3 t
by (erule eval-cases, auto)


lemma eval-abrupt:
  G⊢(Some xc,s) −t≻→ (w,s') =
     (s'=(Some xc,s) ∧ w=arbitrary3 t ∧
     G⊢(Some xc,s) −t≻→ (arbitrary3 t,(Some xc,s)))
apply auto
apply (frule eval-abrupt-lemma, auto)+
done

simproc-setup eval-abrupt (G⊢(Some xc,s) −e≻→ (w,s')) = ⟪
  fn - => fn - => fn ct =>
    (case Thm.term-of ct of
       (- $ - $ - $ - $ - $ (Const (@{const-name Pair}, -) $ (Const (@{const-name Some}, -) $ -)$ -)) =>
NONE
```

  | - => *SOME* (*mk-meta-eq* @{*thm eval-abrupt*}))
⟫

lemma *LitI*: $G\vdash s\ -Lit\ v-\succ(if\ normal\ s\ then\ v\ else\ arbitrary)\rightarrow\ s$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Lit*)

lemma *SkipI* [*intro*!]: $G\vdash s\ -Skip\rightarrow\ s$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Skip*)

lemma *ExprI*: $G\vdash s\ -e-\succ v\rightarrow\ s' \Longrightarrow G\vdash s\ -Expr\ e\rightarrow\ s'$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Expr*)

lemma *CompI*: $[\![G\vdash s\ -c1\rightarrow\ s1;\ G\vdash s1\ -c2\rightarrow\ s2]\!] \Longrightarrow G\vdash s\ -c1;;\ c2\rightarrow\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Comp*)

lemma *CondI*:
  $\bigwedge s1.\ [\![G\vdash s\ -e-\succ b\rightarrow\ s1;\ G\vdash s1\ -(if\ the\text{-}Bool\ b\ then\ e1\ else\ e2)-\succ v\rightarrow\ s2]\!] \Longrightarrow$
    $G\vdash s\ -e\ ?\ e1\ :\ e2-\succ(if\ normal\ s1\ then\ v\ else\ arbitrary)\rightarrow\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Cond*)

lemma *IfI*: $[\![G\vdash s\ -e-\succ v\rightarrow\ s1;\ G\vdash s1\ -(if\ the\text{-}Bool\ v\ then\ c1\ else\ c2)\rightarrow\ s2]\!]$
    $\Longrightarrow G\vdash s\ -If(e)\ c1\ Else\ c2\rightarrow\ s2$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.If*)

lemma *MethdI*: $G\vdash s\ -body\ G\ C\ sig-\succ v\rightarrow\ s'$
    $\Longrightarrow G\vdash s\ -Methd\ C\ sig-\succ v\rightarrow\ s'$
apply (*case-tac s*, *case-tac a* = *None*)
by (*auto intro*!: *eval.Methd*)

lemma *eval-Call*:
  $[\![G\vdash Norm\ s0\ -e-\succ a'\rightarrow\ s1;\ G\vdash s1\ -ps\doteq\succ pvs\rightarrow\ s2;$
   $D = invocation\text{-}declclass\ G\ mode\ (store\ s2)\ a'\ statT\ (\!|name=mn,parTs=pTs|\!);$
   $s3 = init\text{-}lvars\ G\ D\ (\!|name=mn,parTs=pTs|\!)\ mode\ a'\ pvs\ s2;$
   $s3' = check\text{-}method\text{-}access\ G\ accC\ statT\ mode\ (\!|name=mn,parTs=pTs|\!)\ a'\ s3;$
   $G\vdash s3'-Methd\ D\ (\!|name=mn,parTs=pTs|\!)-\succ\ v\rightarrow\ s4;$
    $s4' = restore\text{-}lvars\ s2\ s4]\!] \Longrightarrow$
    $G\vdash Norm\ s0\ -\{accC,statT,mode\}e\cdot mn(\{pTs\}ps)-\succ v\rightarrow\ s4'$
apply (*drule eval.Call*, *assumption*)
apply (*rule HOL.refl*)
apply *simp*+
done

lemma *eval-Init*:
$[\![if\ inited\ C\ (globs\ s0)\ then\ s3 = Norm\ s0$

else $G\vdash$*Norm* (*init-class-obj G C s0*)
  $-$(*if C = Object then Skip else Init* (*super* (*the* (*class G C*))))$\rightarrow$ *s1* $\wedge$
 $G\vdash$*set-lvars empty s1* $-$(*init* (*the* (*class G C*)))$\rightarrow$ *s2* $\wedge$
 s3 = *restore-lvars s1 s2*$\rrbracket \Longrightarrow$
$G\vdash$*Norm s0* $-$*Init C*$\rightarrow$ *s3*
apply (*rule eval.Init*)
apply *auto*
done

lemma *init-done*: *initd C s* $\Longrightarrow$ $G\vdash$*s* $-$*Init C*$\rightarrow$ *s*
apply (*case-tac s*, *simp*)
apply (*case-tac a*)
apply *safe*
apply (*rule eval-Init*)
apply *auto*
done

lemma *eval-StatRef*:
$G\vdash$*s* $-$*StatRef rt*$-\succ$(*if abrupt s=None then Null else arbitrary*)$\rightarrow$ *s*
apply (*case-tac s*, *simp*)
apply (*case-tac a = None*)
apply (*auto del*: *eval.Abrupt intro*!: *eval.intros*)
done

lemma *SkipD* [*dest*!]: $G\vdash$*s* $-$*Skip*$\rightarrow$ *s'* $\Longrightarrow$ *s'* = *s*
apply (*erule eval-cases*)
by *auto*

lemma *Skip-eq* [*simp*]: $G\vdash$*s* $-$*Skip*$\rightarrow$ *s'* = (*s* = *s'*)
by *auto*

lemma *init-retains-locals* [*rule-format* (*no-asm*)]: $G\vdash$*s* $-t\succ\rightarrow$ (*w*,*s'*) $\Longrightarrow$
($\forall$ *C*. *t=In1r* (*Init C*) $\longrightarrow$ *locals* (*store s*) = *locals* (*store s'*))
apply (*erule eval.induct*)
apply (*simp* (*no-asm-use*) *split del*: *split-if-asm option.split-asm*)+
apply *auto*
done

lemma *halloc-xcpt* [*dest*!]:
$\bigwedge$*s'*. $G\vdash$(*Some xc*,*s*) $-$*halloc oi*$\succ$*a*$\rightarrow$ *s'* $\Longrightarrow$ *s'*=(*Some xc*,*s*)
apply (*erule-tac halloc-elim-cases*)
by *auto*

lemma *eval-Methd*:
$G\vdash$*s* $-$*In1l*(*body G C sig*)$\succ\rightarrow$ (*w*,*s'*)
$\Longrightarrow$ $G\vdash$*s* $-$*In1l*(*Methd C sig*)$\succ\rightarrow$ (*w*,*s'*)
apply (*case-tac s*)
apply (*case-tac a*)

apply *clarsimp+*
apply (*erule eval.Methd*)
apply (*drule eval-abrupt-lemma*)
apply *force*
done

lemma *eval-Body*: ⟦*G*⊢*Norm s0* −*Init D*→ *s1*; *G*⊢*s1* −*c*→ *s2*;
res=*the* (*locals* (*store s2*) *Result*);
s3 = (*if* (∃ *l*. *abrupt s2* = *Some* (*Jump* (*Break l*)) ∨
abrupt s2 = *Some* (*Jump* (*Cont l*)))
then abupd (λ *x*. *Some* (*Error CrossMethodJump*)) *s2*
else s2);
s4=*abupd* (*absorb Ret*) *s3*⟧ ⟹
G⊢*Norm s0* −*Body D c*−≻*res*→*s4*
by (*auto elim*: *eval.Body*)

lemma *eval-binop-arg2-indep*:
¬ *need-second-arg binop v1* ⟹ *eval-binop binop v1 x* = *eval-binop binop v1 y*
by (*cases binop*)
(*simp-all add*: *need-second-arg-def*)

lemma *eval-BinOp-arg2-indepI*:
assumes *eval-e1*: *G*⊢*Norm s0* −*e1*−≻*v1*→ *s1* **and**
no-need: ¬ *need-second-arg binop v1*
shows *G*⊢*Norm s0* −*BinOp binop e1 e2*−≻(*eval-binop binop v1 v2*)→ *s1*
(**is** *?EvalBinOp v2*)
proof −
from *eval-e1*
have *?EvalBinOp Unit*
by (*rule eval.BinOp*)
(*simp add*: *no-need*)
moreover
from *no-need*
have *eval-binop binop v1 Unit* = *eval-binop binop v1 v2*
by (*simp add*: *eval-binop-arg2-indep*)
ultimately
show *?thesis*
by *simp*
qed

single valued

lemma *unique-halloc* [*rule-format* (*no-asm*)]:
G⊢*s* −*halloc oi*≻*a* → *s'* ⟹ *G*⊢*s* −*halloc oi*≻*a'* → *s''* ⟶ *a'* = *a* ∧ *s''* = *s'*
apply (*erule halloc.induct*)
apply (*auto elim*!: *halloc-elim-cases split del*: *split-if split-if-asm*)
apply (*drule trans* [*THEN sym*], *erule sym*)
defer
apply (*drule trans* [*THEN sym*], *erule sym*)
apply *auto*
done

lemma *single-valued-halloc*:

$single\text{-}valued\ \{((s,oi),(a,s')).\ G\vdash s\ -halloc\ oi\succ a \rightarrow s'\}$
apply (*unfold single-valued-def*)
by (*clarsimp*, *drule* (*1*) *unique-halloc*, *auto*)

lemma *unique-sxalloc* [*rule-format* (*no-asm*)]:
$G\vdash s\ -sxalloc\rightarrow\ s' \Longrightarrow G\vdash s\ -sxalloc\rightarrow\ s'' \longrightarrow s'' = s'$
apply (*erule sxalloc.induct*)
apply (*auto dest*: *unique-halloc elim*!: *sxalloc-elim-cases*
split del: *split-if split-if-asm*)
done

lemma *single-valued-sxalloc*: $single\text{-}valued\ \{(s,s').\ G\vdash s\ -sxalloc\rightarrow\ s'\}$
apply (*unfold single-valued-def*)
apply (*blast dest*: *unique-sxalloc*)
done

lemma *split-pairD*: $(x,y) = p \Longrightarrow x = fst\ p\ \&\ y = snd\ p$
by *auto*

lemma *unique-eval* [*rule-format* (*no-asm*)]:
$G\vdash s\ -t\succ\rightarrow\ (w,\ s') \Longrightarrow (\forall w'\ s''.\ G\vdash s\ -t\succ\rightarrow\ (w',\ s'') \longrightarrow w' = w \wedge s'' = s')$
apply (*erule eval-induct*)
apply (*tactic* ⟨⟨ *ALLGOALS* (*EVERY'*
[*strip-tac*, *rotate-tac* ~*1*, *eresolve-tac* (*thms eval-elim-cases*)]) ⟩⟩)

prefer *28*
apply (*simp* (*no-asm-use*) *only*: *split add*: *split-if-asm*)

prefer *30*
apply (*case-tac inited C* (*globs s0*), (*simp only*: *if-True if-False simp-thms*)+)
prefer *26*
apply (*simp* (*no-asm-use*) *only*: *split add*: *split-if-asm*, *blast*)

apply (*blast dest*: *unique-sxalloc unique-halloc split-pairD*)+
done

lemma *single-valued-eval*:
$single\text{-}valued\ \{((s,\ t),\ (v,\ s')).\ G\vdash s\ -t\succ\rightarrow\ (v,\ s')\}$
apply (*unfold single-valued-def*)
by (*clarify*, *drule* (*1*) *unique-eval*, *auto*)

end

Chapter 16

Example

43 Example Bali program

theory *Example* **imports** *Eval WellForm* **begin**

The following example Bali program includes:

- class and interface declarations with inheritance, hiding of fields, overriding of methods (with refined result type), array type,
- method call (with dynamic binding), parameter access, return expressions,
- expression statements, sequential composition, literal values, local assignment, local access, field assignment, type cast,
- exception generation and propagation, try and catch statement, throw statement
- instance creation and (default) static initialization

```
package java_lang

public interface HasFoo {
  public Base foo(Base z);
}

public class Base implements HasFoo {
  static boolean arr[] = new boolean[2];
  public HasFoo vee;
  public Base foo(Base z) {
    return z;
  }
}

public class Ext extends Base {
  public int vee;
  public Ext foo(Base z) {
    ((Ext)z).vee = 1;
    return null;
  }
}

public class Main {
  public static void main(String args[]) throws Throwable {
    Base e = new Ext();
    try {e.foo(null); }
    catch(NullPointerException z) {
      while(Ext.arr[2]) ;
    }
  }
}
```

declare *widen.null* [*intro*]

lemma *wf-fdecl-def2*: $\bigwedge fd.$ *wf-fdecl G P fd* = *is-acc-type G P* (*type* (*snd fd*))
apply (*unfold wf-fdecl-def*)

apply (*simp* (*no-asm*))
done

declare *wf-fdecl-def2* [*iff*]

type and expression names

datatype $tnam' = HasFoo' \mid Base' \mid Ext' \mid Main'$
datatype $vnam' = arr' \mid vee' \mid z' \mid e'$
datatype $label' = lab1'$

consts

$tnam' :: tnam' \Rightarrow tnam$
$vnam' :: vnam' \Rightarrow vname$
$label' :: label' \Rightarrow label$

axioms

inj-tnam' [*simp*]: $(tnam'\ x = tnam'\ y) = (x = y)$
inj-vnam' [*simp*]: $(vnam'\ x = vnam'\ y) = (x = y)$
inj-label' [*simp*]: $(label'\ x = label'\ y) = (x = y)$

surj-tnam': $\exists\, m.\ n = tnam'\ m$
surj-vnam': $\exists\, m.\ n = vnam'\ m$
surj-label': $\exists\, m.\ n = label'\ m$

abbreviation
HasFoo :: *qtname* **where**
$HasFoo == (\!|pid{=}java\text{-}lang, tid{=}TName\ (tnam'\ HasFoo')|\!)$

abbreviation
Base :: *qtname* **where**
$Base == (\!|pid{=}java\text{-}lang, tid{=}TName\ (tnam'\ Base')|\!)$

abbreviation
Ext :: *qtname* **where**
$Ext == (\!|pid{=}java\text{-}lang, tid{=}TName\ (tnam'\ Ext')|\!)$

abbreviation
Main :: *qtname* **where**
$Main == (\!|pid{=}java\text{-}lang, tid{=}TName\ (tnam'\ Main')|\!)$

abbreviation
arr :: *vname* **where**
$arr == (vnam'\ arr')$

abbreviation
vee :: *vname* **where**
$vee == (vnam'\ vee')$

abbreviation
z :: *vname* **where**
$z == (vnam'\ z')$

abbreviation
e :: *vname* **where**
$e == (vnam'\ e')$

abbreviation
lab1:: *label* **where**
lab1 == *label′ lab1′*

lemma *neq-Base-Object* [*simp*]: *Base*≠*Object*
by (*simp add*: *Object-def*)

lemma *neq-Ext-Object* [*simp*]: *Ext*≠*Object*
by (*simp add*: *Object-def*)

lemma *neq-Main-Object* [*simp*]: *Main*≠*Object*
by (*simp add*: *Object-def*)

lemma *neq-Base-SXcpt* [*simp*]: *Base*≠*SXcpt xn*
by (*simp add*: *SXcpt-def*)

lemma *neq-Ext-SXcpt* [*simp*]: *Ext*≠*SXcpt xn*
by (*simp add*: *SXcpt-def*)

lemma *neq-Main-SXcpt* [*simp*]: *Main*≠*SXcpt xn*
by (*simp add*: *SXcpt-def*)

classes and interfaces

defs

Object-mdecls-def: *Object-mdecls* ≡ []
SXcpt-mdecls-def: *SXcpt-mdecls* ≡ []

consts

foo :: *mname*

constdefs

foo-sig :: *sig*
foo-sig ≡ (|*name*=*foo*,*parTs*=[*Class Base*]|)

foo-mhead :: *mhead*
foo-mhead ≡ (|*access*=*Public*,*static*=*False*,*pars*=[*z*],*resT*=*Class Base*|)

constdefs

Base-foo :: *mdecl*
Base-foo ≡ (*foo-sig*, (|*access*=*Public*,*static*=*False*,*pars*=[*z*],*resT*=*Class Base*,
mbody=(|*lcls*=[],*stmt*=*Return* (!!*z*)|)|))

constdefs
Ext-foo :: *mdecl*
Ext-foo ≡ (*foo-sig*,
(|*access*=*Public*,*static*=*False*,*pars*=[*z*],*resT*=*Class Ext*,
mbody=(|*lcls*=[]

```
              ,stmt=Expr({Ext,Ext,False}Cast (Class Ext) (!!z)..vee :=
                                                  Lit (Intg 1)) ;;
                         Return (Lit Null)⦈
         ⦈)
```

constdefs

```
arr-viewed-from :: qtname ⇒ qtname ⇒ var
arr-viewed-from accC C ≡ {accC,Base,True}StatRef (ClassT C)..arr

BaseCl :: class
BaseCl ≡ ⦇access=Public,
      cfields=[(arr, ⦇access=Public,static=True ,type=PrimT Boolean.[]⦈),
            (vee, ⦇access=Public,static=False,type=Iface HasFoo    ⦈)],
      methods=[Base-foo],
      init=Expr(arr-viewed-from Base Base
            :=New (PrimT Boolean)[Lit (Intg 2)]),
      super=Object,
      superIfs=[HasFoo]⦈

ExtCl  :: class
ExtCl  ≡ ⦇access=Public,
      cfields=[(vee, ⦇access=Public,static=False,type= PrimT Integer⦈)],
      methods=[Ext-foo],
      init=Skip,
      super=Base,
      superIfs=[]⦈

MainCl :: class
MainCl ≡ ⦇access=Public,
      cfields=[],
      methods=[],
      init=Skip,
      super=Object,
      superIfs=[]⦈
```

constdefs

```
 HasFooInt :: iface
 HasFooInt ≡ ⦇access=Public,imethods=[(foo-sig, foo-mhead)],isuperIfs=[]⦈

 Ifaces ::idecl list
 Ifaces ≡ [(HasFoo,HasFooInt)]

 Classes ::cdecl list
 Classes ≡ [(Base,BaseCl),(Ext,ExtCl),(Main,MainCl)]@standard-classes
```

lemmas *table-classes-defs* =
  Classes-def standard-classes-def ObjectC-def SXcptC-def

lemma *table-ifaces* [*simp*]: *table-of Ifaces* = *empty*(*HasFoo*↦*HasFooInt*)
apply (*unfold Ifaces-def*)
apply (*simp* (*no-asm*))
done

lemma *table-classes-Object* [*simp*]:

table-of Classes Object = Some (|*access=Public,cfields=*[]
,methods=Object-mdecls
,init=Skip,super=arbitrary,superIfs=[]|)
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add:Object-def*)
done

lemma *table-classes-SXcpt* [*simp*]:
table-of Classes (*SXcpt xn*)
= *Some* (|*access=Public,cfields=*[],*methods=SXcpt-mdecls*,
init=Skip,
super=if xn = Throwable then Object else SXcpt Throwable,
superIfs=[]|)
apply (*unfold table-classes-defs*)
apply (*induct-tac xn*)
apply (*simp add*: *Object-def SXcpt-def*)+
done

lemma *table-classes-HasFoo* [*simp*]: *table-of Classes HasFoo = None*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)
done

lemma *table-classes-Base* [*simp*]: *table-of Classes Base = Some BaseCl*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)
done

lemma *table-classes-Ext* [*simp*]: *table-of Classes Ext = Some ExtCl*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)
done

lemma *table-classes-Main* [*simp*]: *table-of Classes Main = Some MainCl*
apply (*unfold table-classes-defs*)
apply (*simp* (*no-asm*) *add*: *Object-def SXcpt-def*)
done

program

abbreviation
tprg :: *prog* **where**
tprg == (|*ifaces=Ifaces,classes=Classes*|)

constdefs
test :: (*ty*)*list* ⇒ *stmt*
test pTs ≡ *e*:==*NewC Ext*;;
Try Expr({*Main,ClassT Base,IntVir*}!!*e·foo*({*pTs*}[*Lit Null*]))
Catch((*SXcpt NullPointer*) *z*)
(*lab1·* *While*(*Acc*
(*Acc* (*arr-viewed-from Main Ext*).[*Lit* (*Intg 2*)])) *Skip*)

well-structuredness

lemma *not-Object-subcls-any* [*elim*!]: (*Object*, *C*) ∈ (*subcls1 tprg*) ^+ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D*)
done

lemma *not-Throwable-subcls-SXcpt* [*elim*!]:
(*SXcpt Throwable*, *SXcpt xn*) ∈ (*subcls1 tprg*) ^+ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D*)
apply (*simp add*: *Object-def SXcpt-def*)
done

lemma *not-SXcpt-n-subcls-SXcpt-n* [*elim*!]:
(*SXcpt xn*, *SXcpt xn*) ∈ (*subcls1 tprg*) ^+ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D*)
apply (*drule rtranclD*)
apply *auto*
done

lemma *not-Base-subcls-Ext* [*elim*!]: (*Base*, *Ext*) ∈ (*subcls1 tprg*) ^+ ⟹ *R*
apply (*auto dest*!: *tranclD subcls1D simp add*: *BaseCl-def*)
done

lemma *not-TName-n-subcls-TName-n* [*rule-format* (*no-asm*), *elim*!]:
((|*pid*=*java-lang*,*tid*=*TName tn*|), (|*pid*=*java-lang*,*tid*=*TName tn*|))
∈ (*subcls1 tprg*) ^+ ⟶ *R*
apply (*rule-tac n1* = *tn* **in** *surj-tnam′* [*THEN exE*])
apply (*erule ssubst*)
apply (*rule tnam′.induct*)
apply *safe*
apply (*auto dest*!: *tranclD subcls1D simp add*: *BaseCl-def ExtCl-def MainCl-def*)
apply (*drule rtranclD*)
apply *auto*
done

lemma *ws-idecl-HasFoo*: *ws-idecl tprg HasFoo* []
apply (*unfold ws-idecl-def*)
apply (*simp* (*no-asm*))
done

lemma *ws-cdecl-Object*: *ws-cdecl tprg Object any*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Throwable*: *ws-cdecl tprg* (*SXcpt Throwable*) *Object*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-SXcpt*: *ws-cdecl tprg* (*SXcpt xn*) (*SXcpt Throwable*)
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Base*: *ws-cdecl tprg Base Object*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Ext*: *ws-cdecl tprg Ext Base*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemma *ws-cdecl-Main*: *ws-cdecl tprg Main Object*
apply (*unfold ws-cdecl-def*)
apply *auto*
done

lemmas *ws-cdecls* = *ws-cdecl-SXcpt ws-cdecl-Object ws-cdecl-Throwable*
ws-cdecl-Base ws-cdecl-Ext ws-cdecl-Main

declare *not-Object-subcls-any* [*rule del*]
not-Throwable-subcls-SXcpt [*rule del*]
not-SXcpt-n-subcls-SXcpt-n [*rule del*]
not-Base-subcls-Ext [*rule del*] *not-TName-n-subcls-TName-n* [*rule del*]

lemma *ws-idecl-all*:
G=*tprg* $\Longrightarrow$ ($\forall$ (I,i)$\in$*set Ifaces*. *ws-idecl G I* (*isuperIfs i*))
apply (*simp* (*no-asm*) *add*: *Ifaces-def HasFooInt-def*)
apply (*auto intro*!: *ws-idecl-HasFoo*)
done

lemma *ws-cdecl-all*: G=*tprg* $\Longrightarrow$ ($\forall$ (C,c)$\in$*set Classes*. *ws-cdecl G C* (*super c*))
apply (*simp* (*no-asm*) *add*: *Classes-def BaseCl-def ExtCl-def MainCl-def*)
apply (*auto intro*!: *ws-cdecls simp add*: *standard-classes-def ObjectC-def*
SXcptC-def)
done

lemma *ws-tprg*: *ws-prog tprg*
apply (*unfold ws-prog-def*)
apply (*auto intro*!: *ws-idecl-all ws-cdecl-all*)
done

misc program properties (independent of well-structuredness)

lemma *single-iface* [*simp*]: *is-iface tprg I* = (I = *HasFoo*)
apply (*unfold Ifaces-def*)
apply (*simp* (*no-asm*))
done

lemma *empty-subint1* [*simp*]: *subint1 tprg* = {}
apply (*unfold subint1-def Ifaces-def HasFooInt-def*)
apply *auto*
done

lemma *unique-ifaces*: *unique Ifaces*
apply (*unfold Ifaces-def*)
apply (*simp* (*no-asm*))
done

lemma *unique-classes*: *unique Classes*
apply (*unfold table-classes-defs*)
apply (*simp*)
done

lemma *SXcpt-subcls-Throwable* [*simp*]: *tprg*$\vdash$*SXcpt xn*$\preceq_C$ *SXcpt Throwable*
apply (*rule SXcpt-subcls-Throwable-lemma*)
apply *auto*
done

lemma *Ext-subclseq-Base* [*simp*]: *tprg*$\vdash$*Ext* $\preceq_C$ *Base*
apply (*rule subcls-direct1*)
apply (*simp* (*no-asm*) *add*: *ExtCl-def*)
apply (*simp add*: *Object-def*)
apply (*simp* (*no-asm*))
done

lemma *Ext-subcls-Base* [*simp*]: *tprg*$\vdash$*Ext* $\prec_C$ *Base*
apply (*rule subcls-direct2*)
apply (*simp* (*no-asm*) *add*: *ExtCl-def*)
apply (*simp add*: *Object-def*)
apply (*simp* (*no-asm*))
done

fields and method lookup

lemma *fields-tprg-Object* [*simp*]: *DeclConcepts.fields tprg Object* = []
by (*rule ws-tprg* [*THEN fields-emptyI*], *force*+)

lemma *fields-tprg-Throwable* [*simp*]:
DeclConcepts.fields tprg (*SXcpt Throwable*) = []
by (*rule ws-tprg* [*THEN fields-emptyI*], *force*+)

lemma *fields-tprg-SXcpt* [*simp*]: *DeclConcepts.fields tprg* (*SXcpt xn*) = []
apply (*case-tac xn* = *Throwable*)
apply (*simp* (*no-asm-simp*))
by (*rule ws-tprg* [*THEN fields-emptyI*], *force*+)

lemmas *fields-rec′* = *fields-rec* [*OF - ws-tprg*]

lemma *fields-Base* [*simp*]:

DeclConcepts.fields tprg Base
  = [((*arr*,*Base*), (|*access*=*Public*,*static*=*True* ,*type*=*PrimT Boolean*.[]|)),
    ((*vee*,*Base*), (|*access*=*Public*,*static*=*False*,*type*=*Iface HasFoo* |))]
apply (*subst fields-rec′*)
apply (*auto simp add*: *BaseCl-def*)
done

lemma *fields-Ext* [*simp*]:
 DeclConcepts.fields tprg Ext
  = [((*vee*,*Ext*), (|*access*=*Public*,*static*=*False*,*type*= *PrimT Integer*|))]
   @ *DeclConcepts.fields tprg Base*
apply (*rule trans*)
apply (*rule fields-rec′*)
apply (*auto simp add*: *ExtCl-def Object-def*)
done

lemmas *imethds-rec′* = *imethds-rec* [*OF* - *ws-tprg*]
lemmas *methd-rec′* = *methd-rec* [*OF* - *ws-tprg*]

lemma *imethds-HasFoo* [*simp*]:
 imethds tprg HasFoo = *o2s* ∘ *empty*(*foo-sig*↦(*HasFoo*, *foo-mhead*))
apply (*rule trans*)
apply (*rule imethds-rec′*)
apply (*auto simp add*: *HasFooInt-def*)
done

lemma *methd-tprg-Object* [*simp*]: *methd tprg Object* = *empty*
apply (*subst methd-rec′*)
apply (*auto simp add*: *Object-mdecls-def*)
done

lemma *methd-Base* [*simp*]:
 methd tprg Base = *table-of* [(λ(*s*,*m*). (*s*, *Base*, *m*)) *Base-foo*]
apply (*rule trans*)
apply (*rule methd-rec′*)
apply (*auto simp add*: *BaseCl-def*)
done

lemma *memberid-Base-foo-simp* [*simp*]:
 memberid (*mdecl Base-foo*) = *mid foo-sig*
by (*simp add*: *Base-foo-def*)

lemma *memberid-Ext-foo-simp* [*simp*]:
 memberid (*mdecl Ext-foo*) = *mid foo-sig*
by (*simp add*: *Ext-foo-def*)

lemma *Base-declares-foo*:
 tprg⊢*mdecl Base-foo declared-in Base*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def BaseCl-def Base-foo-def*)

lemma *foo-sig-not-undeclared-in-Base*:

$\neg$ *tprg*$\vdash$*mid foo-sig undeclared-in Base*
proof –
  from *Base-declares-foo*
  show *?thesis*
    by (*auto dest*!: *declared-not-undeclared*)
qed

lemma *Ext-declares-foo*:
  tprg$\vdash$*mdecl Ext-foo declared-in Ext*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def ExtCl-def Ext-foo-def*)

lemma *foo-sig-not-undeclared-in-Ext*:
  $\neg$ *tprg*$\vdash$*mid foo-sig undeclared-in Ext*
proof –
  from *Ext-declares-foo*
  show *?thesis*
    by (*auto dest*!: *declared-not-undeclared*)
qed

lemma *Base-foo-not-inherited-in-Ext*:
$\neg$ *tprg* $\vdash$ *Ext inherits* (*Base*,*mdecl Base-foo*)
by (*auto simp add*: *inherits-def foo-sig-not-undeclared-in-Ext*)

lemma *Ext-method-inheritance*:
filter-tab (λ*sig m*. *tprg* $\vdash$ *Ext inherits method sig m*)
    (*empty*(*fst* ((λ(*s*, *m*). (*s*, *Base*, *m*)) *Base-foo*)$\mapsto$
     snd ((λ(*s*, *m*). (*s*, *Base*, *m*)) *Base-foo*)))
  = *empty*
proof –
  from *Base-foo-not-inherited-in-Ext*
  show *?thesis*
    by (*auto intro*: *filter-tab-all-False simp add*: *Base-foo-def*)
qed

lemma *methd-Ext* [*simp*]: *methd tprg Ext* =
  table-of [(λ(*s*,*m*). (*s*, *Ext*, *m*)) *Ext-foo*]
apply (*rule trans*)
apply (*rule methd-rec'*)
apply (*auto simp add*: *ExtCl-def Object-def Ext-method-inheritance*)
done

accessibility

lemma *classesDefined*:
$[\![$*class tprg C* = *Some c*; *C*$\neq$*Object*$]\!]$ $\Longrightarrow$ $\exists$ *sc*. *class tprg* (*super c*) = *Some sc*
apply (*auto simp add*: *Classes-def standard-classes-def*
          BaseCl-def ExtCl-def MainCl-def
          SXcptC-def ObjectC-def)
done

lemma *superclassesBase* [*simp*]: *superclasses tprg Base*={*Object*}
proof –

have *ws*: *ws-prog tprg* **by** (*rule ws-tprg*)
then show *?thesis*
by (*auto simp add*: *superclasses-rec BaseCl-def*)
qed

lemma *superclassesExt* [*simp*]: *superclasses tprg Ext*={*Base*,*Object*}
proof −
have *ws*: *ws-prog tprg* **by** (*rule ws-tprg*)
then show *?thesis*
by (*auto simp add*: *superclasses-rec ExtCl-def BaseCl-def*)
qed

lemma *superclassesMain* [*simp*]: *superclasses tprg Main*={*Object*}
proof −
have *ws*: *ws-prog tprg* **by** (*rule ws-tprg*)
then show *?thesis*
by (*auto simp add*: *superclasses-rec MainCl-def*)
qed

lemma *HasFoo-accessible*[*simp*]:*tprg*⊢(*Iface HasFoo*) *accessible-in P*
by (*simp add*: *accessible-in-RefT-simp is-public-def HasFooInt-def*)

lemma *HasFoo-is-acc-iface*[*simp*]: *is-acc-iface tprg P HasFoo*
by (*simp add*: *is-acc-iface-def*)

lemma *HasFoo-is-acc-type*[*simp*]: *is-acc-type tprg P* (*Iface HasFoo*)
by (*simp add*: *is-acc-type-def*)

lemma *Base-accessible*[*simp*]:*tprg*⊢(*Class Base*) *accessible-in P*
by (*simp add*: *accessible-in-RefT-simp is-public-def BaseCl-def*)

lemma *Base-is-acc-class*[*simp*]: *is-acc-class tprg P Base*
by (*simp add*: *is-acc-class-def*)

lemma *Base-is-acc-type*[*simp*]: *is-acc-type tprg P* (*Class Base*)
by (*simp add*: *is-acc-type-def*)

lemma *Ext-accessible*[*simp*]:*tprg*⊢(*Class Ext*) *accessible-in P*
by (*simp add*: *accessible-in-RefT-simp is-public-def ExtCl-def*)

lemma *Ext-is-acc-class*[*simp*]: *is-acc-class tprg P Ext*
by (*simp add*: *is-acc-class-def*)

lemma *Ext-is-acc-type*[*simp*]: *is-acc-type tprg P* (*Class Ext*)
by (*simp add*: *is-acc-type-def*)

lemma *accmethd-tprg-Object* [*simp*]: *accmethd tprg S Object* = *empty*

apply (*unfold accmethd-def*)
apply (*simp*)
done

lemma *snd-special-simp*: *snd* $((\lambda(s,\ m).\ (s,\ a,\ m))\ x) = (a,snd\ x)$
by (*cases x*) (*auto*)

lemma *fst-special-simp*: *fst* $((\lambda(s,\ m).\ (s,\ a,\ m))\ x) = fst\ x$
by (*cases x*) (*auto*)

lemma *foo-sig-undeclared-in-Object*:
tprg$\vdash$*mid foo-sig undeclared-in Object*
by (*auto simp add*: *undeclared-in-def cdeclaredmethd-def Object-mdecls-def*)

lemma *unique-sig-Base-foo*:
tprg$\vdash$ *mdecl* (*sig*, *snd Base-foo*) *declared-in Base* $\Longrightarrow$ *sig*=*foo-sig*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def*
Base-foo-def BaseCl-def)

lemma *Base-foo-no-override*:
tprg,*sig*$\vdash$(*Base*,(*snd Base-foo*)) *overrides old* $\Longrightarrow$ *P*
apply (*drule overrides-commonD*)
apply (*clarsimp*)
apply (*frule subclsEval*)
apply (*rule ws-tprg*)
apply (*simp*)
apply (*rule classesDefined*)
apply *assumption+*
apply (*frule unique-sig-Base-foo*)
apply (*auto dest!*: *declared-not-undeclared intro*: *foo-sig-undeclared-in-Object*
dest: *unique-sig-Base-foo*)
done

lemma *Base-foo-no-stat-override*:
tprg,*sig*$\vdash$(*Base*,(*snd Base-foo*)) $overrides_S$ *old* $\Longrightarrow$ *P*
apply (*drule stat-overrides-commonD*)
apply (*clarsimp*)
apply (*frule subclsEval*)
apply (*rule ws-tprg*)
apply (*simp*)
apply (*rule classesDefined*)
apply *assumption+*
apply (*frule unique-sig-Base-foo*)
apply (*auto dest!*: *declared-not-undeclared intro*: *foo-sig-undeclared-in-Object*
dest: *unique-sig-Base-foo*)
done

lemma *Base-foo-no-hide*:
tprg,*sig*$\vdash$(*Base*,(*snd Base-foo*)) *hides old* $\Longrightarrow$ *P*
by (*auto dest*: *hidesD simp add*: *Base-foo-def member-is-static-simp*)

lemma *Ext-foo-no-hide*:
tprg,sig$\vdash$(*Ext*,(*snd Ext-foo*)) *hides old* $\Longrightarrow$ *P*
by (*auto dest*: *hidesD simp add*: *Ext-foo-def member-is-static-simp*)

lemma *unique-sig-Ext-foo*:
tprg$\vdash$ *mdecl* (*sig*, *snd Ext-foo*) *declared-in Ext* $\Longrightarrow$ *sig*=*foo-sig*
by (*auto simp add*: *declared-in-def cdeclaredmethd-def*
Ext-foo-def ExtCl-def)

lemma *Ext-foo-override*:
tprg,sig$\vdash$(*Ext*,(*snd Ext-foo*)) *overrides old*
$\Longrightarrow$ *old* = (*Base*,(*snd Base-foo*))
apply (*drule overrides-commonD*)
apply (*clarsimp*)
apply (*frule subclsEval*)
apply (*rule ws-tprg*)
apply (*simp*)
apply (*rule classesDefined*)
apply *assumption+*
apply (*frule unique-sig-Ext-foo*)
apply (*case-tac old*)
apply (*insert Base-declares-foo foo-sig-undeclared-in-Object*)
apply (*auto simp add*: *ExtCl-def Ext-foo-def*
BaseCl-def Base-foo-def Object-mdecls-def
split-paired-all
member-is-static-simp
dest: *declared-not-undeclared unique-declaration*)
done

lemma *Ext-foo-stat-override*:
tprg,sig$\vdash$(*Ext*,(*snd Ext-foo*)) *overrides*$_S$ *old*
$\Longrightarrow$ *old* = (*Base*,(*snd Base-foo*))
apply (*drule stat-overrides-commonD*)
apply (*clarsimp*)
apply (*frule subclsEval*)
apply (*rule ws-tprg*)
apply (*simp*)
apply (*rule classesDefined*)
apply *assumption+*
apply (*frule unique-sig-Ext-foo*)
apply (*case-tac old*)
apply (*insert Base-declares-foo foo-sig-undeclared-in-Object*)
apply (*auto simp add*: *ExtCl-def Ext-foo-def*
BaseCl-def Base-foo-def Object-mdecls-def
split-paired-all
member-is-static-simp
dest: *declared-not-undeclared unique-declaration*)
done

lemma *Base-foo-member-of-Base*:
tprg$\vdash$(*Base*,*mdecl Base-foo*) *member-of Base*
by (*auto intro*!: *members.Immediate Base-declares-foo*)

lemma *Base-foo-member-in-Base*:
tprg⊢(*Base*,*mdecl Base-foo*) *member-in Base*
by (*rule member-of-to-member-in* [*OF Base-foo-member-of-Base*])

lemma *Ext-foo-member-of-Ext*:
tprg⊢(*Ext*,*mdecl Ext-foo*) *member-of Ext*
by (*auto intro*!: *members.Immediate Ext-declares-foo*)

lemma *Ext-foo-member-in-Ext*:
tprg⊢(*Ext*,*mdecl Ext-foo*) *member-in Ext*
by (*rule member-of-to-member-in* [*OF Ext-foo-member-of-Ext*])

lemma *Base-foo-permits-acc*:
tprg ⊢ (*Base*, *mdecl Base-foo*) *in Base permits-acc-from S*
by (*simp add*: *permits-acc-def Base-foo-def*)

lemma *Base-foo-accessible* [*simp*]:
tprg⊢(*Base*,*mdecl Base-foo*) *of Base accessible-from S*
by (*auto intro*: *accessible-fromR.Immediate*
Base-foo-member-of-Base Base-foo-permits-acc)

lemma *Base-foo-dyn-accessible* [*simp*]:
tprg⊢(*Base*,*mdecl Base-foo*) *in Base dyn-accessible-from S*
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule Base-foo-member-in-Base*)
apply (*rule Base-foo-permits-acc*)
done

lemma *accmethd-Base* [*simp*]:
accmethd tprg S Base = *methd tprg Base*
apply (*simp add*: *accmethd-def*)
apply (*rule filter-tab-all-True*)
apply (*simp add*: *snd-special-simp fst-special-simp*)
done

lemma *Ext-foo-permits-acc*:
tprg ⊢ (*Ext*, *mdecl Ext-foo*) *in Ext permits-acc-from S*
by (*simp add*: *permits-acc-def Ext-foo-def*)

lemma *Ext-foo-accessible* [*simp*]:
tprg⊢(*Ext*,*mdecl Ext-foo*) *of Ext accessible-from S*
by (*auto intro*: *accessible-fromR.Immediate*
Ext-foo-member-of-Ext Ext-foo-permits-acc)

lemma *Ext-foo-dyn-accessible* [*simp*]:
tprg⊢(*Ext*,*mdecl Ext-foo*) *in Ext dyn-accessible-from S*
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule Ext-foo-member-in-Ext*)
apply (*rule Ext-foo-permits-acc*)
done

lemma *Ext-foo-overrides-Base-foo*:
tprg⊢(*Ext*,*Ext-foo*) *overrides* (*Base*,*Base-foo*)
proof (*rule overridesR.Direct*, *simp-all*)
  show ¬ *is-static Ext-foo*
    by (*simp add*: *member-is-static-simp Ext-foo-def*)
  show ¬ *is-static Base-foo*
    by (*simp add*: *member-is-static-simp Base-foo-def*)
  show *accmodi Ext-foo* ≠ *Private*
    by (*simp add*: *Ext-foo-def*)
  show *msig* (*Ext*, *Ext-foo*) = *msig* (*Base*, *Base-foo*)
    by (*simp add*: *Ext-foo-def Base-foo-def*)
  show *tprg*⊢*mdecl Ext-foo declared-in Ext*
    by (*auto intro*: *Ext-declares-foo*)
  show *tprg*⊢*mdecl Base-foo declared-in Base*
    by (*auto intro*: *Base-declares-foo*)
  show *tprg* ⊢(*Base*, *mdecl Base-foo*) *inheritable-in java-lang*
    by (*simp add*: *inheritable-in-def Base-foo-def*)
  show *tprg*⊢*resTy Ext-foo*≼*resTy Base-foo*
    by (*simp add*: *Ext-foo-def Base-foo-def mhead-resTy-simp*)
qed

lemma *accmethd-Ext* [*simp*]:
  accmethd tprg S Ext = *methd tprg Ext*
apply (*simp add*: *accmethd-def*)
apply (*rule filter-tab-all-True*)
apply (*auto simp add*: *snd-special-simp fst-special-simp*)
done

lemma *cls-Ext*: *class tprg Ext* = *Some ExtCl*
by *simp*

lemma *dynmethd-Ext-foo*:
dynmethd tprg Base Ext (|*name* = *foo*, *parTs* = [*Class Base*]|)
  = *Some* (*Ext*,*snd Ext-foo*)
proof −
  have *methd tprg Base* (|*name* = *foo*, *parTs* = [*Class Base*]|)
          = *Some* (*Base*,*snd Base-foo*) **and**
        methd tprg Ext (|*name* = *foo*, *parTs* = [*Class Base*]|)
          = *Some* (*Ext*,*snd Ext-foo*)
    by (*auto simp add*: *Ext-foo-def Base-foo-def foo-sig-def*)
  with *cls-Ext ws-tprg Ext-foo-overrides-Base-foo*
  show *?thesis*
    by (*auto simp add*: *dynmethd-rec simp add*: *Ext-foo-def Base-foo-def*)
qed

lemma *Base-fields-accessible*[*simp*]:
accfield tprg S Base
  = *table-of*((*map* (λ((*n*,*d*),*f*).(*n*,(*d*,*f*)))) (*DeclConcepts.fields tprg Base*))
apply (*auto simp add*: *accfield-def expand-fun-eq Let-def*
          accessible-in-RefT-simp
          is-public-def
          BaseCl-def
          permits-acc-def
          declared-in-def

cdeclaredfield-def
intro!: *filter-tab-all-True-Some filter-tab-None*
accessible-fromR.Immediate
intro: *members.Immediate*)
done

lemma *arr-member-of-Base*:
tprg⊢(*Base*, *fdecl* (*arr*,
(|*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]|)))
member-of Base
by (*auto intro*: *members.Immediate*
simp add: *declared-in-def cdeclaredfield-def BaseCl-def*)

lemma *arr-member-in-Base*:
tprg⊢(*Base*, *fdecl* (*arr*,
(|*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]|)))
member-in Base
by (*rule member-of-to-member-in* [*OF arr-member-of-Base*])

lemma *arr-member-of-Ext*:
tprg⊢(*Base*, *fdecl* (*arr*,
(|*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]|)))
member-of Ext
apply (*rule members.Inherited*)
apply (*simp add*: *inheritable-in-def*)
apply (*simp add*: *undeclared-in-def cdeclaredfield-def ExtCl-def*)
apply (*auto intro*: *arr-member-of-Base simp add*: *subcls1-def ExtCl-def*)
done

lemma *arr-member-in-Ext*:
tprg⊢(*Base*, *fdecl* (*arr*,
(|*access* = *Public*, *static* = *True*, *type* = *PrimT Boolean*.[]|)))
member-in Ext
by (*rule member-of-to-member-in* [*OF arr-member-of-Ext*])

lemma *Ext-fields-accessible*[*simp*]:
accfield tprg S Ext
= *table-of*((*map* (λ((*n*,*d*),*f*).(*n*,(*d*,*f*)))) (*DeclConcepts.fields tprg Ext*))
apply (*auto simp add*: *accfield-def expand-fun-eq Let-def*
accessible-in-RefT-simp
is-public-def
BaseCl-def
ExtCl-def
permits-acc-def
intro!: *filter-tab-all-True-Some filter-tab-None*
accessible-fromR.Immediate)
apply (*auto intro*: *members.Immediate arr-member-of-Ext*
simp add: *declared-in-def cdeclaredfield-def ExtCl-def*)
done

lemma *arr-Base-dyn-accessible* [*simp*]:
tprg⊢(*Base*, *fdecl* (*arr*, (|*access*=*Public*,*static*=*True* ,*type*=*PrimT Boolean*.[]|)))

in Base dyn-accessible-from S
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule arr-member-in-Base*)
apply (*simp add*: *permits-acc-def*)
done

lemma *arr-Ext-dyn-accessible*[*simp*]:
tprg⊢(*Base*, *fdecl* (*arr*, (|*access*=*Public*,*static*=*True* ,*type*=*PrimT Boolean*.[]|)))
in Ext dyn-accessible-from S
apply (*rule dyn-accessible-fromR.Immediate*)
apply (*rule arr-member-in-Ext*)
apply (*simp add*: *permits-acc-def*)
done

lemma *array-of-PrimT-acc* [*simp*]:
is-acc-type tprg java-lang (*PrimT t*.[])
apply (*simp add*: *is-acc-type-def accessible-in-RefT-simp*)
done

lemma *PrimT-acc* [*simp*]:
is-acc-type tprg java-lang (*PrimT t*)
apply (*simp add*: *is-acc-type-def accessible-in-RefT-simp*)
done

lemma *Object-acc* [*simp*]:
is-acc-class tprg java-lang Object
apply (*auto simp add*: *is-acc-class-def accessible-in-RefT-simp is-public-def*)
done

well-formedness

lemma *wf-HasFoo*: *wf-idecl tprg* (*HasFoo*, *HasFooInt*)
apply (*unfold wf-idecl-def HasFooInt-def*)
apply (*auto intro*!: *wf-mheadI ws-idecl-HasFoo*
simp add: *foo-sig-def foo-mhead-def mhead-resTy-simp*
member-is-static-simp)
done

declare *member-is-static-simp* [*simp*]
declare *wt.Skip* [*rule del*] *wt.Init* [*rule del*]
ML-setup ⟪ *bind-thms* (*wt-intros*, *map* (*rewrite-rule* @{*thms id-def*}) @{*thms wt.intros*}) ⟫
lemmas *wtIs* = *wt-Call wt-Super wt-FVar wt-StatRef wt-intros*
lemmas *daIs* = *assigned.select-convs da-Skip da-NewC da-Lit da-Super da.intros*

lemmas *Base-foo-defs* = *Base-foo-def foo-sig-def foo-mhead-def*
lemmas *Ext-foo-defs* = *Ext-foo-def foo-sig-def*

lemma *wf-Base-foo*: *wf-mdecl tprg Base Base-foo*
apply (*unfold Base-foo-defs*)
apply (*auto intro*!: *wf-mdeclI wf-mheadI intro*!: *wtIs*

```
          simp add: mhead-resTy-simp)

apply (rule exI)
apply (simp add: parameters-def)
apply (rule conjI)
apply  (rule da.Comp)
apply     (rule da.Expr)
apply     (rule da.AssLVar)
apply       (rule da.AccLVar)
apply         (simp)
apply        (rule assigned.select-convs)
apply       (simp)
apply      (rule assigned.select-convs)
apply     (simp)
apply    (simp)
apply    (rule da.Jmp)
apply      (simp)
apply     (rule assigned.select-convs)
apply    (simp)
apply   (rule assigned.select-convs)
apply  (simp)
apply (simp)
done


lemma wf-Ext-foo: wf-mdecl tprg Ext Ext-foo
apply (unfold Ext-foo-defs )
apply (auto intro!: wf-mdeclI wf-mheadI intro!: wtIs
          simp add: mhead-resTy-simp )
apply  (rule wt.Cast)
prefer 2
apply    simp
apply   (rule-tac [2] narrow.subcls [THEN cast.narrow])
apply   (auto intro!: wtIs)

apply (rule exI)
apply (simp add: parameters-def)
apply (rule conjI)
apply  (rule da.Comp)
apply     (rule da.Expr)
apply     (rule da.Ass)
apply       simp
apply      (rule da.FVar)
apply      (rule da.Cast)
apply      (rule da.AccLVar)
apply        simp
apply       (rule assigned.select-convs)
apply      simp
apply     (rule da-Lit)
apply     (simp)
apply    (rule da.Comp)
apply       (rule da.Expr)
apply       (rule da.AssLVar)
apply         (rule da.Lit)
apply        (rule assigned.select-convs)
apply       simp
apply      (rule da.Jmp)
apply        simp
```

apply (*rule assigned.select-convs*)
apply *simp*
apply (*rule assigned.select-convs*)
apply (*simp*)
apply (*rule assigned.select-convs*)
apply *simp*
apply *simp*
done

declare *mhead-resTy-simp* [*simp add*]
declare *member-is-static-simp* [*simp add*]

lemma *wf-BaseC*: *wf-cdecl tprg* (*Base*,*BaseCl*)
apply (*unfold wf-cdecl-def BaseCl-def arr-viewed-from-def*)
apply (*auto intro*!: *wf-Base-foo*)
apply (*auto intro*!: *ws-cdecl-Base simp add*: *Base-foo-def foo-mhead-def*)
apply (*auto intro*!: *wtIs*)

apply (*rule exI*)
apply (*rule da.Expr*)
apply (*rule da.Ass*)
apply (*simp*)
apply (*rule da.FVar*)
apply (*rule da.Cast*)
apply (*rule da-Lit*)
apply *simp*
apply (*rule da.NewA*)
apply (*rule da.Lit*)
apply (*auto simp add*: *Base-foo-defs entails-def Let-def*)
apply (*insert Base-foo-no-stat-override*, *simp add*: *Base-foo-def*,*blast*)+
apply (*insert Base-foo-no-hide* , *simp add*: *Base-foo-def*,*blast*)
done

lemma *wf-ExtC*: *wf-cdecl tprg* (*Ext*,*ExtCl*)
apply (*unfold wf-cdecl-def ExtCl-def*)
apply (*auto intro*!: *wf-Ext-foo ws-cdecl-Ext*)
apply (*auto simp add*: *entails-def snd-special-simp*)
apply (*insert Ext-foo-stat-override*)
apply (*rule exI*,*rule da.Skip*)
apply (*force simp add*: *qmdecl-def Ext-foo-def Base-foo-def*)
apply (*force simp add*: *qmdecl-def Ext-foo-def Base-foo-def*)
apply (*force simp add*: *qmdecl-def Ext-foo-def Base-foo-def*)
apply (*insert Ext-foo-no-hide*)
apply (*simp-all add*: *qmdecl-def*)
apply *blast*+
done

lemma *wf-MainC*: *wf-cdecl tprg* (*Main*,*MainCl*)
apply (*unfold wf-cdecl-def MainCl-def*)
apply (*auto intro*: *ws-cdecl-Main*)
apply (*rule exI*,*rule da.Skip*)
done

lemma *wf-idecl-all*: *p*=*tprg* $\Longrightarrow$ *Ball* (*set Ifaces*) (*wf-idecl p*)

apply (*simp* (*no-asm*) *add*: *Ifaces-def*)
apply (*simp* (*no-asm-simp*))
apply (*rule wf-HasFoo*)
done

lemma *wf-cdecl-all-standard-classes*:
Ball (*set standard-classes*) (*wf-cdecl tprg*)
apply (*unfold standard-classes-def Let-def*
ObjectC-def SXcptC-def Object-mdecls-def SXcpt-mdecls-def)
apply (*simp* (*no-asm*) *add*: *wf-cdecl-def ws-cdecls*)
apply (*auto simp add*:*is-acc-class-def accessible-in-RefT-simp SXcpt-def*
intro: *da.Skip*)
apply (*auto simp add*: *Object-def Classes-def standard-classes-def*
SXcptC-def SXcpt-def)
done

lemma *wf-cdecl-all*: $p{=}tprg \Longrightarrow$ *Ball* (*set Classes*) (*wf-cdecl p*)
apply (*simp* (*no-asm*) *add*: *Classes-def*)
apply (*simp* (*no-asm-simp*))
apply (*rule wf-BaseC* [*THEN conjI*])
apply (*rule wf-ExtC* [*THEN conjI*])
apply (*rule wf-MainC* [*THEN conjI*])
apply (*rule wf-cdecl-all-standard-classes*)
done

theorem *wf-tprg*: *wf-prog tprg*
apply (*unfold wf-prog-def Let-def*)
apply (*simp* (*no-asm*) *add*: *unique-ifaces unique-classes*)
apply (*rule conjI*)
apply ((*simp* (*no-asm*) *add*: *Classes-def standard-classes-def*))
apply (*rule conjI*)
apply (*simp add*: *Object-mdecls-def*)
apply *safe*
apply (*cut-tac xn-cases*)
apply (*simp* (*no-asm-simp*) *add*: *Classes-def standard-classes-def*)
apply (*insert wf-idecl-all*)
apply (*insert wf-cdecl-all*)
apply *auto*
done

max spec

lemma *appl-methds-Base-foo*:
appl-methds tprg S (*ClassT Base*) ⦇*name*=*foo*, *parTs*=[*NT*]⦈ =
{((*ClassT Base*, ⦇*access*=*Public*,*static*=*False*,*pars*=[*z*],*resT*=*Class Base*⦈)
,[*Class Base*])}
apply (*unfold appl-methds-def*)
apply (*simp* (*no-asm*))
apply (*subgoal-tac tprg*$\vdash$*NT*$\preceq$ *Class Base*)
apply (*auto simp add*: *cmheads-def Base-foo-defs*)
done

lemma *max-spec-Base-foo*: *max-spec tprg S* (*ClassT Base*) ⦇*name*=*foo*,*parTs*=[*NT*]⦈ =
{((*ClassT Base*, ⦇*access*=*Public*,*static*=*False*,*pars*=[*z*],*resT*=*Class Base*⦈)
, [*Class Base*])}
apply (*unfold max-spec-def*)

apply (*simp* (*no-asm*) *add*: *appl-methds-Base-foo*)
apply *auto*
done

well-typedness

lemma *wt-test*: (|*prg*=*tprg*,*cls*=*Main*,*lcl*=*empty*(*VName e*↦*Class Base*)|)⊢*test ?pTs*::√
apply (*unfold test-def arr-viewed-from-def*)

apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
prefer *4*
apply (*simp*)
defer
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*simp*)
apply (*rule wtIs*)
apply (*simp*)
apply (*rule max-spec-Base-foo*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*rule wtIs*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*simp*)
apply (*rule wtIs*)
apply (*simp*)
apply (*rule wtIs*)
done

definite assignment

lemma *da-test*: (|*prg*=*tprg*,*cls*=*Main*,*lcl*=*empty*(*VName e*↦*Class Base*)|)

⊢{} »⟨*test ?pTs*⟩» ⦇*nrm*={*VName e*},*brk*=λ *l*. *UNIV*⦈
apply (*unfold test-def arr-viewed-from-def*)
apply (*rule da.Comp*)
apply (*rule da.Expr*)
apply (*rule da.AssLVar*)
apply (*rule da.NewC*)
apply (*rule assigned.select-convs*)
apply (*simp*)
apply (*rule da.Try*)
apply (*rule da.Expr*)
apply (*rule da.Call*)
apply (*rule da.AccLVar*)
apply (*simp*)
apply (*rule assigned.select-convs*)
apply (*simp*)
apply (*rule da.Cons*)
apply (*rule da.Lit*)
apply (*rule da.Nil*)
apply (*rule da.Loop*)
apply (*rule da.Acc*)
apply (*simp*)
apply (*rule da.AVar*)
apply (*rule da.Acc*)
apply *simp*
apply (*rule da.FVar*)
apply (*rule da.Cast*)
apply (*rule da.Lit*)
apply (*rule da.Lit*)
apply (*rule da-Skip*)
apply (*simp*)
apply (*simp*,*rule assigned.select-convs*)
apply (*simp*)
apply (*simp*,*rule assigned.select-convs*)
apply (*simp*)
apply *simp*
apply *blast*
apply *simp*
apply (*simp add*: *intersect-ts-def*)
done

execution

lemma *alloc-one*: ⋀*a obj*. ⟦*the* (*new-Addr h*) = *a*; *atleast-free h* (*Suc n*)⟧ ⟹
new-Addr h = *Some a* ∧ *atleast-free* (*h*(*a*↦*obj*)) *n*
apply (*frule atleast-free-SucD*)
apply (*drule atleast-free-Suc* [*THEN iffD1*])
apply *clarsimp*
apply (*frule new-Addr-SomeI*)
apply *force*
done

declare *fvar-def2* [*simp*] *avar-def2* [*simp*] *init-lvars-def2* [*simp*]
declare *init-obj-def* [*simp*] *var-tys-def* [*simp*] *fields-table-def* [*simp*]
declare *BaseCl-def* [*simp*] *ExtCl-def* [*simp*] *Ext-foo-def* [*simp*]
Base-foo-defs [*simp*]

ML-setup ⟪ *bind-thms* (*eval-intros*, *map*
(*simplify* (*simpset*() *delsimps* @{*thms Skip-eq*}
addsimps @{*thms lvar-def*}) *o*

```
      rewrite-rule [@{thm assign-def}, @{thm Let-def}]) @{thms eval.intros}) ⟫
lemmas eval-Is = eval-Init eval-StatRef AbruptIs eval-intros

consts
  a :: loc
  b :: loc
  c :: loc

abbreviation one == Suc 0
abbreviation two == Suc one
abbreviation tree == Suc two
abbreviation four == Suc tree

syntax
  obj-a :: obj
  obj-b :: obj
  obj-c :: obj
  arr-N :: (vn, val) table
  arr-a :: (vn, val) table
  globs1 :: globs
  globs2 :: globs
  globs3 :: globs
  globs8 :: globs
  locs3 :: locals
  locs4 :: locals
  locs8 :: locals
  s0  :: state
  s0' :: state
  s9' :: state
  s1  :: state
  s1' :: state
  s2  :: state
  s2' :: state
  s3  :: state
  s3' :: state
  s4  :: state
  s4' :: state
  s6' :: state
  s7' :: state
  s8  :: state
  s8' :: state

translations
  obj-a   <= (|tag=Arr (PrimT Boolean) (CONST two)
              ,values=CONST empty(Inr 0↦Bool False)(Inr (CONST one)↦Bool False)|)
  obj-b   <= (|tag=CInst (CONST Ext)
              ,values=(CONST empty(Inl (CONST vee, CONST Base)↦Null   )
                      (Inl (CONST vee, CONST Ext )↦Intg 0))|)
  obj-c   == (|tag=CInst (SXcpt NullPointer),values=CONST empty|)
  arr-N   == CONST empty(Inl (CONST arr, CONST Base)↦Null)
  arr-a   == CONST empty(Inl (CONST arr, CONST Base)↦Addr a)
  globs1  == CONST empty(Inr (CONST Ext)   ↦(|tag=arbitrary, values=CONST empty|))
                  (Inr (CONST Base)  ↦(|tag=arbitrary, values=arr-N|))
                  (Inr Object↦(|tag=arbitrary, values=CONST empty|))
  globs2  == CONST empty(Inr (CONST Ext)   ↦(|tag=arbitrary, values=CONST empty|))
                  (Inr Object↦(|tag=arbitrary, values=CONST empty|))
                  (Inl a↦obj-a)
                  (Inr (CONST Base)  ↦(|tag=arbitrary, values=arr-a|))
  globs3  == globs2(Inl b↦obj-b)
```

globs8 == *globs3*(*Inl c*↦*obj-c*)
locs3 == *CONST empty*(*VName* (*CONST e*)↦*Addr b*)
locs4 == *CONST empty*(*VName* (*CONST z*)↦*Null*)(*Inr*()↦*Addr b*)
locs8 == *locs3*(*VName* (*CONST z*)↦*Addr c*)
s0 == *st* (*CONST empty*) (*CONST empty*)
s0' == *Norm s0*
s1 == *st globs1* (*CONST empty*)
s1' == *Norm s1*
s2 == *st globs2* (*CONST empty*)
s2' == *Norm s2*
s3 == *st globs3 locs3*
s3' == *Norm s3*
s4 == *st globs3 locs4*
s4' == *Norm s4*
s6' == (*Some* (*Xcpt* (*Std NullPointer*)), *s4*)
s7' == (*Some* (*Xcpt* (*Std NullPointer*)), *s3*)
s8 == *st globs8 locs8*
s8' == *Norm s8*
s9' == (*Some* (*Xcpt* (*Std IndOutBound*)), *s8*)

declare *Pair-eq* [*simp del*]

lemma *exec-test*:
⟦*the* (*new-Addr* (*heap s1*)) = *a*;
the (*new-Addr* (*heap ?s2*)) = *b*;
the (*new-Addr* (*heap ?s3*)) = *c*⟧ ⟹
atleast-free (*heap s0*) *four* ⟹
tprg⊢*s0'* −*test* [*Class Base*]→ *?s9'*
apply (*unfold test-def arr-viewed-from-def*)

apply (*simp* (*no-asm-use*))
apply (*drule* (*1*) *alloc-one*,*clarsimp*)
apply (*rule eval-Is*)
apply (*erule-tac V* = *the* (*new-Addr ?h*) = *c* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *new-Addr ?h* = *Some a* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *atleast-free ?h four* **in** *thin-rl*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)

apply (*erule-tac V* = *the* (*new-Addr ?h*) = *b* **in** *thin-rl*)
apply (*erule-tac V* = *atleast-free ?h tree* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *atleast-free ?h four* **in** *thin-rl*)
apply (*erule-tac* [*2*] *V* = *new-Addr ?h* = *Some a* **in** *thin-rl*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule conjI*)
prefer *2* **apply** (*rule conjI HOL.refl*)+
apply (*rule eval-Is*)
apply (*simp add*: *arr-viewed-from-def*)
apply (*rule conjI*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule conjI*, *rule HOL.refl*)+
apply (*rule HOL.refl*)
apply (*simp*)
apply (*rule conjI*, *rule-tac* [*2*] *HOL.refl*)

apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp add*: *check-field-access-def Let-def*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule halloc.New*)
apply (*simp* (*no-asm-simp*))
apply (*drule atleast-free-weaken*,*drule atleast-free-weaken*)
apply (*simp* (*no-asm-simp*))
apply (*simp add*: *upd-gobj-def*)

apply (*rule halloc.New*)
apply (*drule alloc-one*)
prefer *2* **apply** *fast*
apply (*simp* (*no-asm-simp*))
apply (*drule atleast-free-weaken*)
apply *force*
apply (*simp*)
apply (*drule alloc-one*)
apply (*simp* (*no-asm-simp*))
apply *clarsimp*
apply (*erule-tac* V = *atleast-free ?h tree* **in** *thin-rl*)
apply (*drule-tac* x = a **in** *new-AddrD2* [*THEN spec*])
apply (*simp* (*no-asm-use*))
apply (*rule eval-Is*)
apply (*rule eval-Is*)

apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp*)
apply (*subgoal-tac*
tprg⊢(*Ext*,*mdecl Ext-foo*) *in Ext dyn-accessible-from Main*)
apply (*simp add*: *check-method-access-def Let-def*
invocation-declclass-def dynlookup-def dynmethd-Ext-foo)
apply (*rule Ext-foo-dyn-accessible*)
apply (*rule eval-Is*)
apply (*simp add*: *body-def Let-def*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*simp add*: *invocation-declclass-def dynlookup-def dynmethd-Ext-foo*)
apply (*simp add*: *invocation-declclass-def dynlookup-def dynmethd-Ext-foo*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)

apply (*rule eval-Is*)
apply (*simp*)
apply (*simp split del*: *split-if*)
apply (*simp add*: *check-field-access-def Let-def*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*rule conjI*)
apply (*simp*)
apply (*rule eval-Is*)
apply (*simp*)

apply *simp*
apply (*rule sxalloc.intros*)
apply (*rule halloc.New*)
apply (*erule alloc-one* [*THEN conjunct1*])
apply (*simp* (*no-asm-simp*))
apply (*simp* (*no-asm-simp*))
apply (*simp add*: *gupd-def lupd-def obj-ty-def split del*: *split-if*)
apply (*drule alloc-one* [*THEN conjunct1*])
apply (*simp* (*no-asm-simp*))
apply (*erule-tac V* = *atleast-free ?h two* **in** *thin-rl*)
apply (*drule-tac x* = *a* **in** *new-AddrD2* [*THEN spec*])
apply *simp*
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule eval-Is*)
apply (*rule init-done*, *simp*)
apply (*rule eval-Is*)
apply (*simp*)
apply (*simp add*: *check-field-access-def Let-def*)
apply (*rule eval-Is*)
apply (*simp* (*no-asm-simp*))
apply (*auto simp add*: *in-bounds-def*)
done
declare *Pair-eq* [*simp*]

end

Chapter 17

Conform

44 Conformance notions for the type soundness proof for Java

theory *Conform* **imports** *State* **begin**

design issues:

- lconf allows for (arbitrary) inaccessible values
- "conforms" does not directly imply that the dynamic types of all objects on the heap are indeed existing classes. Yet this can be inferred for all referenced objs.

types $env' = prog \times (lname, ty)\ table$

extension of global store

constdefs

$gext\ \ :: st \Rightarrow st \Rightarrow bool \qquad (\text{-}\leq|\text{-} \qquad [71,71] \quad 70)$
$s\leq|s' \equiv \forall r.\ \forall obj\in globs\ s\ r\text{:}\ \exists obj'\in globs\ s'\ r\text{:}\ tag\ obj' = tag\ obj$

For the the proof of type soundness we will need the property that during execution, objects are not lost and moreover retain the values of their tags. So the object store grows conservatively. Note that if we considered garbage collection, we would have to restrict this property to accessible objects.

lemma *gext-objD*:
$[\![s\leq|s';\ globs\ s\ r = Some\ obj]\!]$
$\Longrightarrow \exists obj'.\ globs\ s'\ r = Some\ obj' \wedge tag\ obj' = tag\ obj$
apply (*simp only*: *gext-def*)
by *force*

lemma *rev-gext-objD*:
$[\![globs\ s\ r = Some\ obj;\ s\leq|s']\!]$
$\Longrightarrow \exists obj'.\ globs\ s'\ r = Some\ obj' \wedge tag\ obj' = tag\ obj$
by (*auto elim*: *gext-objD*)

lemma *init-class-obj-inited*:
$init\text{-}class\text{-}obj\ G\ C\ s1\leq|s2 \Longrightarrow inited\ C\ (globs\ s2)$
apply (*unfold inited-def init-obj-def*)
apply (*auto dest*!: *gext-objD*)
done

lemma *gext-refl* [*intro*!, *simp*]: $s\leq|s$
apply (*unfold gext-def*)
apply (*fast del*: *fst-splitE*)
done

lemma *gext-gupd* [*simp*, *elim*!]: $\bigwedge s.\ globs\ s\ r = None \Longrightarrow s\leq|gupd(r\mapsto x)s$
by (*auto simp*: *gext-def*)

lemma *gext-new* [*simp*, *elim*!]: $\bigwedge s.\ globs\ s\ r = None \Longrightarrow s\leq|init\text{-}obj\ G\ oi\ r\ s$
apply (*simp only*: *init-obj-def*)
apply (*erule-tac gext-gupd*)
done

lemma *gext-trans* [*elim*]: $\bigwedge X.\ [\![s{\leq}|s';\ s'{\leq}|s'']\!] \Longrightarrow s{\leq}|s''$
by (*force simp*: *gext-def*)

lemma *gext-upd-gobj* [*intro*!]: $s{\leq}|upd\text{-}gobj\ r\ n\ v\ s$
apply (*simp only*: *gext-def*)
apply *auto*
apply (*case-tac ra* = *r*)
apply *auto*
apply (*case-tac globs s r* = *None*)
apply *auto*
done

lemma *gext-cong1* [*simp*]: $set\text{-}locals\ l\ s1{\leq}|s2 = s1{\leq}|s2$
by (*auto simp*: *gext-def*)

lemma *gext-cong2* [*simp*]: $s1{\leq}|set\text{-}locals\ l\ s2 = s1{\leq}|s2$
by (*auto simp*: *gext-def*)

lemma *gext-lupd1* [*simp*]: $lupd(vn{\mapsto}v)s1{\leq}|s2 = s1{\leq}|s2$
by (*auto simp*: *gext-def*)

lemma *gext-lupd2* [*simp*]: $s1{\leq}|lupd(vn{\mapsto}v)s2 = s1{\leq}|s2$
by (*auto simp*: *gext-def*)

lemma *inited-gext*: $[\![inited\ C\ (globs\ s);\ s{\leq}|s']\!] \Longrightarrow inited\ C\ (globs\ s')$
apply (*unfold inited-def*)
apply (*auto dest*: *gext-objD*)
done

value conformance

constdefs

$conf :: prog \Rightarrow st \Rightarrow val \Rightarrow ty \Rightarrow bool \quad (\text{-,-}{\vdash}\text{-}{::}{\preceq}\text{-} \quad [71,71,71,71]\ 70)$
$\quad G,s{\vdash}v{::}{\preceq}T \equiv \exists\ T'{\in}typeof\ (\lambda a.\ option\text{-}map\ obj\text{-}ty\ (heap\ s\ a))\ v{:}G{\vdash}T'{\preceq}T$

lemma *conf-cong* [*simp*]: $G,set\text{-}locals\ l\ s{\vdash}v{::}{\preceq}T = G,s{\vdash}v{::}{\preceq}T$
by (*auto simp*: *conf-def*)

lemma *conf-lupd* [*simp*]: $G,lupd(vn{\mapsto}va)s{\vdash}v{::}{\preceq}T = G,s{\vdash}v{::}{\preceq}T$
by (*auto simp*: *conf-def*)

lemma *conf-PrimT* [*simp*]: $\forall\ dt.\ typeof\ dt\ v = Some\ (PrimT\ t) \Longrightarrow G,s{\vdash}v{::}{\preceq}PrimT\ t$
apply (*simp add*: *conf-def*)
done

lemma *conf-Boolean*: $G,s{\vdash}v{::}{\preceq}PrimT\ Boolean \Longrightarrow \exists\ b.\ v{=}Bool\ b$

by (*cases v*)
  (*auto simp*: *conf-def obj-ty-def*
    dest: *widen-Boolean2*
    split: *obj-tag.splits*)

lemma *conf-litval* [*rule-format* (*no-asm*)]:
  $typeof\ (\lambda a.\ None)\ v = Some\ T \longrightarrow G,s\vdash v::\preceq T$
apply (*unfold conf-def*)
apply (*rule val.induct*)
apply *auto*
done

lemma *conf-Null* [*simp*]: $G,s\vdash Null::\preceq T = G\vdash NT\preceq T$
by (*simp add*: *conf-def*)

lemma *conf-Addr*:
  $G,s\vdash Addr\ a::\preceq T = (\exists\, obj.\ heap\ s\ a = Some\ obj \wedge G\vdash obj\text{-}ty\ obj\preceq T)$
by (*auto simp*: *conf-def*)

lemma *conf-AddrI*: $[\![heap\ s\ a = Some\ obj;\ G\vdash obj\text{-}ty\ obj\preceq T]\!] \Longrightarrow G,s\vdash Addr\ a::\preceq T$
apply (*rule conf-Addr* [*THEN iffD2*])
by *fast*

lemma *defval-conf* [*rule-format* (*no-asm*), *elim*]:
  $is\text{-}type\ G\ T \longrightarrow G,s\vdash default\text{-}val\ T::\preceq T$
apply (*unfold conf-def*)
apply (*induct T*)
apply (*auto intro*: *prim-ty.induct*)
done

lemma *conf-widen* [*rule-format* (*no-asm*), *elim*]:
  $G\vdash T\preceq T' \Longrightarrow G,s\vdash x::\preceq T \longrightarrow ws\text{-}prog\ G \longrightarrow G,s\vdash x::\preceq T'$
apply (*unfold conf-def*)
apply (*rule val.induct*)
apply (*auto elim*: *ws-widen-trans*)
done

lemma *conf-gext* [*rule-format* (*no-asm*), *elim*]:
  $G,s\vdash v::\preceq T \longrightarrow s\le|s' \longrightarrow G,s'\vdash v::\preceq T$
apply (*unfold gext-def conf-def*)
apply (*rule val.induct*)
apply *force+*
done

lemma *conf-list-widen* [*rule-format* (*no-asm*)]:
$ws\text{-}prog\ G \Longrightarrow$
  $\forall\, Ts\ Ts'.\ list\text{-}all2\ (conf\ G\ s)\ vs\ Ts$
    $\longrightarrow\ \ G\vdash Ts[\preceq]\ Ts' \longrightarrow list\text{-}all2\ (conf\ G\ s)\ vs\ Ts'$
apply (*unfold widens-def*)

apply (*rule list-all2-trans*)
apply *auto*
done

lemma *conf-RefTD* [*rule-format* (*no-asm*)]:
$G,s \vdash a'::\preceq RefT\ T$
$\longrightarrow a' = Null \vee (\exists\, a\ obj\ T'.\ a' = Addr\ a \wedge heap\ s\ a = Some\ obj \wedge$
$obj\text{-}ty\ obj = T' \wedge G \vdash T' \preceq RefT\ T)$
apply (*unfold conf-def*)
apply (*induct-tac* a')
apply (*auto dest*: *widen-PrimT*)
done

value list conformance

constdefs

$lconf :: prog \Rightarrow st \Rightarrow ('a,\ val)\ table \Rightarrow ('a,\ ty)\ table \Rightarrow bool$
$(\text{-},\text{-}\vdash\text{-}[::\preceq]\text{-}\ [71,71,71,71]\ 70)$
$G,s \vdash vs[::\preceq]Ts \equiv \forall\, n.\ \forall\, T \in Ts\ n{:}\ \exists\, v \in vs\ n{:}\ G,s \vdash v::\preceq T$

lemma *lconfD*: $[\![G,s \vdash vs[::\preceq]Ts;\ Ts\ n = Some\ T]\!] \Longrightarrow G,s \vdash (the\ (vs\ n))::\preceq T$
by (*force simp*: *lconf-def*)

lemma *lconf-cong* [*simp*]: $\bigwedge s.\ G,set\text{-}locals\ x\ s \vdash l[::\preceq]L = G,s \vdash l[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-lupd* [*simp*]: $G,lupd(vn \mapsto v)s \vdash l[::\preceq]L = G,s \vdash l[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-new*: $[\![L\ vn = None;\ G,s \vdash l[::\preceq]L]\!] \Longrightarrow G,s \vdash l(vn \mapsto v)[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-upd*: $[\![G,s \vdash l[::\preceq]L;\ G,s \vdash v::\preceq T;\ L\ vn = Some\ T]\!] \Longrightarrow$
$G,s \vdash l(vn \mapsto v)[::\preceq]L$
by (*auto simp*: *lconf-def*)

lemma *lconf-ext*: $[\![G,s \vdash l[::\preceq]L;\ G,s \vdash v::\preceq T]\!] \Longrightarrow G,s \vdash l(vn \mapsto v)[::\preceq]L(vn \mapsto T)$
by (*auto simp*: *lconf-def*)

lemma *lconf-map-sum* [*simp*]:
$G,s \vdash l1\ (+)\ l2[::\preceq]L1\ (+)\ L2 = (G,s \vdash l1[::\preceq]L1 \wedge G,s \vdash l2[::\preceq]L2)$
apply (*unfold lconf-def*)
apply *safe*
apply (*case-tac* [3] *n*)
apply (*force split add*: *sum.split*)+
done

lemma *lconf-ext-list* [*rule-format* (*no-asm*)]:
$\bigwedge X.\ [\![G,s\vdash l[::\preceq]L]\!] \Longrightarrow$
$\forall\, vs\ Ts.\ distinct\ vns \longrightarrow length\ Ts = length\ vns$
$\longrightarrow list\text{-}all2\ (conf\ G\ s)\ vs\ Ts \longrightarrow G,s\vdash l(vns[\mapsto]vs)[::\preceq]L(vns[\mapsto]Ts)$
apply (*unfold lconf-def*)
apply (*induct-tac vns*)
apply *clarsimp*
apply *clarify*
apply (*frule list-all2-lengthD*)
apply (*clarsimp*)
done

lemma *lconf-deallocL*: $[\![G,s\vdash l[::\preceq]L(vn\mapsto T);\ L\ vn = None]\!] \Longrightarrow G,s\vdash l[::\preceq]L$
apply (*simp only*: *lconf-def*)
apply *safe*
apply (*drule spec*)
apply (*drule ospec*)
apply *auto*
done

lemma *lconf-gext* [*elim*]: $[\![G,s\vdash l[::\preceq]L;\ s\le|s']\!] \Longrightarrow G,s'\vdash l[::\preceq]L$
apply (*simp only*: *lconf-def*)
apply *fast*
done

lemma *lconf-empty* [*simp*, *intro*!]: $G,s\vdash vs[::\preceq]empty$
apply (*unfold lconf-def*)
apply *force*
done

lemma *lconf-init-vals* [*intro*!]:
$\forall\, n.\ \forall\, T\in fs\ n{:}is\text{-}type\ G\ T \Longrightarrow G,s\vdash init\text{-}vals\ fs[::\preceq]fs$
apply (*unfold lconf-def*)
apply *force*
done

weak value list conformance

Only if the value is defined it has to conform to its type. This is the contribution of the definite assignment analysis to the notion of conformance. The definite assignment analysis ensures that the program only attempts to access local variables that actually have a defined value in the state. So conformance must only ensure that the defined values are of the right type, and not also that the value is defined.

constdefs

$wlconf :: prog \Rightarrow st \Rightarrow ('a,\ val)\ table \Rightarrow ('a,\ ty)\ table \Rightarrow bool$
(-,-⊢-[∼::⪯]- [71,71,71,71] 70)
$G,s\vdash vs[\sim::\preceq]Ts \equiv \forall\, n.\ \forall\, T\in Ts\ n{:}\ \forall\ v\in vs\ n{:}\ G,s\vdash v::\preceq T$

lemma *wlconfD*: $[\![G,s\vdash vs[\sim::\preceq]Ts;\ Ts\ n = Some\ T;\ vs\ n = Some\ v]\!] \Longrightarrow G,s\vdash v::\preceq T$
by (*auto simp*: *wlconf-def*)

lemma *wlconf-cong* [*simp*]: $\bigwedge s.\ G,set\text{-}locals\ x\ s\vdash l[\sim::\preceq]L = G,s\vdash l[\sim::\preceq]L$
by (*auto simp*: *wlconf-def*)

lemma *wlconf-lupd* [*simp*]: $G,lupd(vn\mapsto v)s\vdash l[\sim::\preceq]L = G,s\vdash l[\sim::\preceq]L$
by (*auto simp*: *wlconf-def*)

lemma *wlconf-upd*: $[\![G,s\vdash l[\sim::\preceq]L;\ G,s\vdash v::\preceq T;\ L\ vn = Some\ T]\!] \Longrightarrow$
$G,s\vdash l(vn\mapsto v)[\sim::\preceq]L$
by (*auto simp*: *wlconf-def*)

lemma *wlconf-ext*: $[\![G,s\vdash l[\sim::\preceq]L;\ G,s\vdash v::\preceq T]\!] \Longrightarrow G,s\vdash l(vn\mapsto v)[\sim::\preceq]L(vn\mapsto T)$
by (*auto simp*: *wlconf-def*)

lemma *wlconf-map-sum* [*simp*]:
$G,s\vdash l1\ (+)\ l2[\sim::\preceq]L1\ (+)\ L2 = (G,s\vdash l1[\sim::\preceq]L1 \wedge G,s\vdash l2[\sim::\preceq]L2)$
apply (*unfold wlconf-def*)
apply *safe*
apply (*case-tac* [*3*] *n*)
apply (*force split add*: *sum.split*)+
done

lemma *wlconf-ext-list* [*rule-format* (*no-asm*)]:
$\bigwedge X.\ [\![G,s\vdash l[\sim::\preceq]L]\!] \Longrightarrow$
$\forall vs\ Ts.\ distinct\ vns \longrightarrow length\ Ts = length\ vns$
$\longrightarrow list\text{-}all2\ (conf\ G\ s)\ vs\ Ts \longrightarrow G,s\vdash l(vns[\mapsto]vs)[\sim::\preceq]L(vns[\mapsto]Ts)$
apply (*unfold wlconf-def*)
apply (*induct-tac vns*)
apply *clarsimp*
apply *clarify*
apply (*frule list-all2-lengthD*)
apply *clarsimp*
done

lemma *wlconf-deallocL*: $[\![G,s\vdash l[\sim::\preceq]L(vn\mapsto T);\ L\ vn = None]\!] \Longrightarrow G,s\vdash l[\sim::\preceq]L$
apply (*simp only*: *wlconf-def*)
apply *safe*
apply (*drule spec*)
apply (*drule ospec*)
defer
apply (*drule ospec*)
apply *auto*
done

lemma *wlconf-gext* [*elim*]: $[\![G,s\vdash l[\sim::\preceq]L;\ s\leq|s']\!] \Longrightarrow G,s'\vdash l[\sim::\preceq]L$
apply (*simp only*: *wlconf-def*)
apply *fast*

done

lemma *wlconf-empty* [*simp*, *intro*!]: $G,s\vdash vs[\sim::\preceq]empty$
apply (*unfold wlconf-def*)
apply *force*
done

lemma *wlconf-empty-vals*: $G,s\vdash empty[\sim::\preceq]ts$
by (*simp add*: *wlconf-def*)

lemma *wlconf-init-vals* [*intro*!]:
$\forall n.\ \forall T\in fs\ n{:}is\text{-}type\ G\ T \Longrightarrow G,s\vdash init\text{-}vals\ fs[\sim::\preceq]fs$
apply (*unfold wlconf-def*)
apply *force*
done

lemma *lconf-wlconf*:
$G,s\vdash l[::\preceq]L \Longrightarrow G,s\vdash l[\sim::\preceq]L$
by (*force simp add*: *lconf-def wlconf-def*)

object conformance

constdefs

$oconf :: prog \Rightarrow st \Rightarrow obj \Rightarrow oref \Rightarrow bool\ \ (\text{-},\text{-}\vdash\text{-}::\preceq\surd\text{-}\ \ [71,71,71,71]\ 70)$
$G,s\vdash obj::\preceq\surd r \equiv G,s\vdash values\ obj[::\preceq]var\text{-}tys\ G\ (tag\ obj)\ r \wedge$
$(case\ r\ of$
$Heap\ a \Rightarrow is\text{-}type\ G\ (obj\text{-}ty\ obj)$
$\mid Stat\ C \Rightarrow True)$

lemma *oconf-is-type*: $G,s\vdash obj::\preceq\surd Heap\ a \Longrightarrow is\text{-}type\ G\ (obj\text{-}ty\ obj)$
by (*auto simp*: *oconf-def Let-def*)

lemma *oconf-lconf*: $G,s\vdash obj::\preceq\surd r \Longrightarrow G,s\vdash values\ obj[::\preceq]var\text{-}tys\ G\ (tag\ obj)\ r$
by (*simp add*: *oconf-def*)

lemma *oconf-cong* [*simp*]: $G,set\text{-}locals\ l\ s\vdash obj::\preceq\surd r = G,s\vdash obj::\preceq\surd r$
by (*auto simp*: *oconf-def Let-def*)

lemma *oconf-init-obj-lemma*:
$[\![\bigwedge C\ c.\ class\ G\ C = Some\ c \Longrightarrow unique\ (DeclConcepts.fields\ G\ C);$
$\bigwedge C\ c\ f\ fld.\ [\![class\ G\ C = Some\ c;$
$table\text{-}of\ (DeclConcepts.fields\ G\ C)\ f = Some\ fld\ ]\!]$
$\Longrightarrow is\text{-}type\ G\ (type\ fld);$
$(case\ r\ of$
$Heap\ a \Rightarrow is\text{-}type\ G\ (obj\text{-}ty\ obj)$
$\mid Stat\ C \Rightarrow is\text{-}class\ G\ C)$
$]\!] \Longrightarrow\ \ G,s\vdash obj\ (\!|values{:=}init\text{-}vals\ (var\text{-}tys\ G\ (tag\ obj)\ r)|\!)::\preceq\surd r$
apply (*auto simp add*: *oconf-def*)
apply (*drule-tac var-tys-Some-eq* [*THEN iffD1*])

defer
apply (*subst obj-ty-cong*)
apply(*auto dest*!: *fields-table-SomeD obj-ty-CInst1 obj-ty-Arr1*
split add: *sum.split-asm obj-tag.split-asm*)
done

state conformance

constdefs

$conforms :: state \Rightarrow env' \Rightarrow bool \quad (\text{-::}\preceq\text{-} \quad [71,71] \quad 70)$
$xs{::}{\preceq}E \equiv let\ (G,\ L) = E;\ s = snd\ xs;\ l = locals\ s\ in$
$(\forall r.\ \forall obj{\in}globs\ s\ r{:}\quad G{,}s{\vdash}obj \quad {::}{\preceq}\surd r) \wedge$
$G{,}s{\vdash}l \quad [\sim{::}{\preceq}]L \quad \wedge$
$(\forall a.\ fst\ xs{=}Some(Xcpt\ (Loc\ a)) \longrightarrow G{,}s{\vdash}Addr\ a{::}{\preceq}\ Class\ (SXcpt\ Throwable)) \wedge$
$(fst\ xs{=}Some(Jump\ Ret) \longrightarrow l\ Result \neq None)$

conforms

lemma *conforms-globsD*:
$[\![(x,\ s){::}{\preceq}(G,\ L);\ globs\ s\ r = Some\ obj]\!] \Longrightarrow G{,}s{\vdash}obj{::}{\preceq}\surd r$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-localD*: $(x,\ s){::}{\preceq}(G,\ L) \Longrightarrow G{,}s{\vdash}locals\ s[\sim{::}{\preceq}]L$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-XcptLocD*: $[\![(x,\ s){::}{\preceq}(G,\ L);\ x = Some\ (Xcpt\ (Loc\ a))]\!] \Longrightarrow$
$G{,}s{\vdash}Addr\ a{::}{\preceq}\ Class\ (SXcpt\ Throwable)$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-RetD*: $[\![(x,\ s){::}{\preceq}(G,\ L);\ x = Some\ (Jump\ Ret)]\!] \Longrightarrow$
$(locals\ s)\ Result \neq None$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-RefTD*:
$[\![G{,}s{\vdash}a'{::}{\preceq}RefT\ t;\ a' \neq Null;\ (x{,}s)\ {::}{\preceq}(G,\ L)]\!] \Longrightarrow$
$\exists a\ obj.\ a' = Addr\ a \wedge globs\ s\ (Inl\ a) = Some\ obj \wedge$
$G{\vdash}obj\text{-}ty\ obj{\preceq}RefT\ t \wedge is\text{-}type\ G\ (obj\text{-}ty\ obj)$
apply (*drule-tac conf-RefTD*)
apply *clarsimp*
apply (*rule conforms-globsD* [*THEN oconf-is-type*])
apply *auto*
done

lemma *conforms-Jump* [*iff*]:
$j{=}Ret \longrightarrow locals\ s\ Result \neq None$
$\Longrightarrow ((Some\ (Jump\ j),\ s){::}{\preceq}(G,\ L)) = (Norm\ s{::}{\preceq}(G,\ L))$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-StdXcpt* [*iff*]:
$((Some\ (Xcpt\ (Std\ xn)),\ s){::}{\preceq}(G,\ L)) = (Norm\ s{::}{\preceq}(G,\ L))$
by (*auto simp*: *conforms-def*)

lemma *conforms-Err* [*iff*]:
$((\mathit{Some}\ (\mathit{Error}\ e),\ s)::\preceq(G,\ L)) = (\mathit{Norm}\ s::\preceq(G,\ L))$
by (*auto simp*: *conforms-def*)

lemma *conforms-raise-if* [*iff*]:
$((\mathit{raise\text{-}if}\ c\ xn\ x,\ s)::\preceq(G,\ L)) = ((x,\ s)::\preceq(G,\ L))$
by (*auto simp*: *abrupt-if-def*)

lemma *conforms-error-if* [*iff*]:
$((\mathit{error\text{-}if}\ c\ err\ x,\ s)::\preceq(G,\ L)) = ((x,\ s)::\preceq(G,\ L))$
by (*auto simp*: *abrupt-if-def split*: *split-if*)

lemma *conforms-NormI*: $(x,\ s)::\preceq(G,\ L) \Longrightarrow \mathit{Norm}\ s::\preceq(G,\ L)$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-absorb* [*rule-format*]:
$(a,\ b)::\preceq(G,\ L) \longrightarrow (\mathit{absorb}\ j\ a,\ b)::\preceq(G,\ L)$
apply (*rule impI*)
apply (*case-tac a*)
apply (*case-tac absorb j a*)
apply *auto*
apply (*case-tac absorb j* (*Some a*),*auto*)
apply (*erule conforms-NormI*)
done

lemma *conformsI*: $[\![\forall r.\ \forall obj\in globs\ s\ r:\ G,s\vdash obj::\preceq\surd r;$
$G,s\vdash locals\ s[\sim::\preceq]L;$
$\forall a.\ x = \mathit{Some}\ (\mathit{Xcpt}\ (\mathit{Loc}\ a)) \longrightarrow G,s\vdash \mathit{Addr}\ a::\preceq\ \mathit{Class}\ (\mathit{SXcpt}\ \mathit{Throwable});$
$x = \mathit{Some}\ (\mathit{Jump}\ \mathit{Ret})\longrightarrow locals\ s\ \mathit{Result} \neq \mathit{None}]\!] \Longrightarrow$
$(x,\ s)::\preceq(G,\ L)$
by (*auto simp*: *conforms-def Let-def*)

lemma *conforms-xconf*: $[\![(x,\ s)::\preceq(G,L);$
$\forall a.\ x' = \mathit{Some}\ (\mathit{Xcpt}\ (\mathit{Loc}\ a)) \longrightarrow G,s\vdash \mathit{Addr}\ a::\preceq\ \mathit{Class}\ (\mathit{SXcpt}\ \mathit{Throwable});$
$x' = \mathit{Some}\ (\mathit{Jump}\ \mathit{Ret}) \longrightarrow locals\ s\ \mathit{Result} \neq \mathit{None}]\!] \Longrightarrow$
$(x',s)::\preceq(G,L)$
by (*fast intro*: *conformsI elim*: *conforms-globsD conforms-localD*)

lemma *conforms-lupd*:
$[\![(x,\ s)::\preceq(G,\ L);\ L\ vn = \mathit{Some}\ T;\ G,s\vdash v::\preceq T]\!] \Longrightarrow (x,\ lupd(vn\mapsto v)s)::\preceq(G,\ L)$
by (*force intro*: *conformsI wlconf-upd dest*: *conforms-globsD conforms-localD*
conforms-XcptLocD conforms-RetD
simp: *oconf-def*)

lemmas *conforms-allocL-aux* = *conforms-localD* [*THEN wlconf-ext*]

lemma *conforms-allocL*:
$[\![(x,\ s)::\preceq(G,\ L);\ G,s\vdash v::\preceq T]\!] \Longrightarrow (x,\ lupd(vn\mapsto v)s)::\preceq(G,\ L(vn\mapsto T))$
by (*force intro*: *conformsI dest*: *conforms-globsD conforms-RetD*

elim: *conforms-XcptLocD* *conforms-allocL-aux*
simp: *oconf-def*)

lemmas *conforms-deallocL-aux* = *conforms-localD* [*THEN wlconf-deallocL*]

lemma *conforms-deallocL*: $\bigwedge s.[\![s{::}\preceq(G,\ L(vn\mapsto T));\ L\ vn = None]\!] \Longrightarrow s{::}\preceq(G,L)$
by (*fast intro*: *conformsI dest*: *conforms-globsD conforms-RetD*
elim: *conforms-XcptLocD conforms-deallocL-aux*)

lemma *conforms-gext*: $[\![(x,\ s){::}\preceq(G,L);\ s{\leq}|s';$
$\forall r.\ \forall obj{\in}globs\ s'\ r{:}\ G,s'{\vdash}obj{::}\preceq\surd r;$
$locals\ s'{=}locals\ s]\!] \Longrightarrow (x,s'){::}\preceq(G,L)$
apply (*rule conformsI*)
apply *assumption*
apply (*drule conforms-localD*) **apply** *force*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-XcptLocD*) **apply** *force*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-RetD*) **apply** *force*
done

lemma *conforms-xgext*:
$[\![(x\ ,s){::}\preceq(G,L);\ (x',\ s'){::}\preceq(G,\ L);\ s'{\leq}|s;dom\ (locals\ s') \subseteq dom\ (locals\ s)]\!]$
$\Longrightarrow (x',s){::}\preceq(G,L)$
apply (*erule-tac conforms-xconf*)
apply (*fast dest*: *conforms-XcptLocD*)
apply (*intro strip*)
apply (*drule* (*1*) *conforms-RetD*)
apply (*auto dest*: *domI*)
done

lemma *conforms-gupd*: $\bigwedge obj.\ [\![(x,\ s){::}\preceq(G,\ L);\ G,s{\vdash}obj{::}\preceq\surd r;\ s{\leq}|gupd(r{\mapsto}obj)s]\!]$
$\Longrightarrow\ (x,\ gupd(r{\mapsto}obj)s){::}\preceq(G,\ L)$
apply (*rule conforms-gext*)
apply *auto*
apply (*force dest*: *conforms-globsD simp add*: *oconf-def*)+
done

lemma *conforms-upd-gobj*: $[\![(x,s){::}\preceq(G,\ L);\ globs\ s\ r = Some\ obj;$
$var\text{-}tys\ G\ (tag\ obj)\ r\ n = Some\ T;\ G,s{\vdash}v{::}\preceq T]\!] \Longrightarrow (x,upd\text{-}gobj\ r\ n\ v\ s){::}\preceq(G,L)$
apply (*rule conforms-gext*)
apply *auto*
apply (*drule* (*1*) *conforms-globsD*)
apply (*simp add*: *oconf-def*)
apply *safe*
apply (*rule lconf-upd*)
apply *auto*
apply (*simp only*: *obj-ty-cong*)
apply (*force dest*: *conforms-globsD intro*!: *lconf-upd*
simp add: *oconf-def cong del*: *sum.weak-case-cong*)
done

lemma *conforms-set-locals*:
$[\![(x,s)::\preceq(G,\ L'); G,s\vdash l[\sim::\preceq]L; x=Some\ (Jump\ Ret) \longrightarrow l\ Result \neq None]\!]$
$\Longrightarrow (x,set\text{-}locals\ l\ s)::\preceq(G,L)$
apply (*rule conformsI*)
apply (*intro strip*)
apply *simp*
apply (*drule* (*2*) *conforms-globsD*)
apply *simp*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-XcptLocD*)
apply *simp*
apply (*intro strip*)
apply (*drule* (*1*) *conforms-RetD*)
apply *simp*
done

lemma *conforms-locals*:
$[\![(a,b)::\preceq(G,\ L); L\ x = Some\ T; locals\ b\ x \neq None]\!]$
$\Longrightarrow G,b\vdash the\ (locals\ b\ x)::\preceq T$
apply (*force simp*: *conforms-def Let-def wlconf-def*)
done

lemma *conforms-return*:
$\bigwedge s'.\ [\![(x,s)::\preceq(G,\ L); (x',s')::\preceq(G,\ L'); s\leq|s'; x'\neq Some\ (Jump\ Ret)]\!] \Longrightarrow$
$(x',set\text{-}locals\ (locals\ s)\ s')::\preceq(G,\ L)$
apply (*rule conforms-xconf*)
prefer *2* **apply** (*force dest*: *conforms-XcptLocD*)
apply (*erule conforms-gext*)
apply (*force dest*: *conforms-globsD*)+
done

end

Chapter 18

DefiniteAssignmentCorrect

45 Correctness of Definite Assignment

theory *DefiniteAssignmentCorrect* **imports** *WellForm Eval* **begin**

declare [[*simproc del*: *wt-expr wt-var wt-exprs wt-stmt*]]

lemma *sxalloc-no-jump*:
  assumes *sxalloc*: $G \vdash s0\ -sxalloc\rightarrow\ s1$ **and**
    no-jmp: $abrupt\ s0 \neq Some\ (Jump\ j)$
  shows $abrupt\ s1 \neq Some\ (Jump\ j)$
using *sxalloc no-jmp*
by *cases simp-all*

lemma *sxalloc-no-jump'*:
  assumes *sxalloc*: $G \vdash s0\ -sxalloc\rightarrow\ s1$ **and**
    jump: $abrupt\ s1 = Some\ (Jump\ j)$
  shows $abrupt\ s0 = Some\ (Jump\ j)$
using *sxalloc jump*
by *cases simp-all*

lemma *halloc-no-jump*:
  assumes *halloc*: $G \vdash s0\ -halloc\ oi \succ a\rightarrow\ s1$ **and**
    no-jmp: $abrupt\ s0 \neq Some\ (Jump\ j)$
  shows $abrupt\ s1 \neq Some\ (Jump\ j)$
using *halloc no-jmp*
by *cases simp-all*

lemma *halloc-no-jump'*:
  assumes *halloc*: $G \vdash s0\ -halloc\ oi \succ a\rightarrow\ s1$ **and**
    jump: $abrupt\ s1 = Some\ (Jump\ j)$
  shows $abrupt\ s0 = Some\ (Jump\ j)$
using *halloc jump*
by *cases simp-all*

lemma *Body-no-jump*:
  assumes *eval*: $G \vdash s0\ -Body\ D\ c-\succ v\rightarrow s1$ **and**
    jump: $abrupt\ s0 \neq Some\ (Jump\ j)$
  shows $abrupt\ s1 \neq Some\ (Jump\ j)$
proof (*cases normal s0*)
  case *True*
  with *eval* **obtain** *s0'* **where** *eval'*: $G \vdash Norm\ s0'\ -Body\ D\ c-\succ v\rightarrow s1$ **and**
        s0: $s0 = Norm\ s0'$
    by (*cases s0*) *simp*
  from *eval'* **obtain** *s2* **where**
    s1: $s1 = abupd\ (absorb\ Ret)$
      $(if\ \exists l.\ abrupt\ s2 = Some\ (Jump\ (Break\ l)) \vee$
        $abrupt\ s2 = Some\ (Jump\ (Cont\ l))$
      $then\ abupd\ (\lambda x.\ Some\ (Error\ CrossMethodJump))\ s2\ else\ s2)$
    by *cases simp*
  show *?thesis*
  proof ($cases\ \exists l.\ abrupt\ s2 = Some\ (Jump\ (Break\ l)) \vee$
        $abrupt\ s2 = Some\ (Jump\ (Cont\ l))$)
    case *True*
    with *s1* **have** $abrupt\ s1 = Some\ (Error\ CrossMethodJump)$

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      by (cases s2) simp
    thus ?thesis by simp
  next
    case False
    with s1 have s1=abupd (absorb Ret) s2
      by simp
    with False show ?thesis
      by (cases s2,cases j) (auto simp add: absorb-def)
  qed
next
  case False
  with eval obtain s0' abr where G⊢(Some abr,s0') −Body D c−≻v→s1
                                  s0 = (Some abr, s0')
    by (cases s0) fastsimp
  with this jump
  show ?thesis
    by (cases) (simp)
qed


lemma Methd-no-jump:
   assumes eval: G⊢s0 −Methd D sig−≻v→ s1 and
           jump: abrupt s0 ≠ Some (Jump j)
   shows abrupt s1 ≠ Some (Jump j)
proof (cases normal s0)
  case True
   with eval obtain s0' where G⊢Norm s0' −Methd D sig−≻v→s1
                              s0 = Norm s0'
    by (cases s0) simp
  then obtain D' body where G⊢s0 −Body D' body−≻v→s1
    by (cases) (simp add: body-def2)
  from this jump
  show ?thesis
    by (rule Body-no-jump)
next
  case False
  with eval obtain s0' abr where G⊢(Some abr,s0') −Methd D sig−≻v→s1
                                  s0 = (Some abr, s0')
    by (cases s0) fastsimp
  with this jump
  show ?thesis
    by (cases) (simp)
qed


lemma jumpNestingOkS-mono:
  assumes jumpNestingOk-l': jumpNestingOkS jmps' c
      and        subset: jmps' ⊆ jmps
 shows jumpNestingOkS jmps c
proof −
    have True and True and
       ⋀ jmps' jmps. ⟦jumpNestingOkS jmps' c; jmps' ⊆ jmps⟧ ⟹ jumpNestingOkS jmps c
       and True
  proof (induct rule: var-expr-stmt.inducts)
    case (Lab j c jmps' jmps)
    note jmpOk = ‹jumpNestingOkS jmps' (j· c)›
    note jmps = ‹jmps' ⊆ jmps›
    with jmpOk have jumpNestingOkS ({j} ∪ jmps') c by simp
    moreover from jmps have ({j} ∪ jmps') ⊆ ({j} ∪ jmps) by auto
```

```
    ultimately
    have jumpNestingOkS ({j} ∪ jmps) c
      by (rule Lab.hyps)
    thus ?case
      by simp
  next
    case (Jmp j jmps' jmps)
    thus ?case
      by (cases j) auto
  next
    case (Comp c1 c2 jmps' jmps)
    from Comp.prems
    have jumpNestingOkS jmps c1 by − (rule Comp.hyps,auto)
    moreover from Comp.prems
    have jumpNestingOkS jmps c2 by − (rule Comp.hyps,auto)
    ultimately show ?case
      by simp
  next
    case (If' e c1 c2 jmps' jmps)
    from If'.prems
    have jumpNestingOkS jmps c1 by − (rule If'.hyps,auto)
    moreover from If'.prems
    have jumpNestingOkS jmps c2 by − (rule If'.hyps,auto)
    ultimately show ?case
      by simp
  next
    case (Loop l e c jmps' jmps)
    from ‹jumpNestingOkS jmps' (l· While(e) c)›
    have jumpNestingOkS ({Cont l} ∪ jmps') c by simp
    moreover
    from ‹jmps' ⊆ jmps›
    have {Cont l} ∪ jmps'  ⊆ {Cont l} ∪ jmps by auto
    ultimately
    have jumpNestingOkS ({Cont l} ∪ jmps) c
      by (rule Loop.hyps)
    thus ?case by simp
  next
    case (TryC c1 C vn c2 jmps' jmps)
    from TryC.prems
    have jumpNestingOkS jmps c1 by − (rule TryC.hyps,auto)
    moreover from TryC.prems
    have jumpNestingOkS jmps c2 by − (rule TryC.hyps,auto)
    ultimately show ?case
      by simp
  next
    case (Fin c1 c2 jmps' jmps)
    from Fin.prems
    have jumpNestingOkS jmps c1 by − (rule Fin.hyps,auto)
    moreover from Fin.prems
    have jumpNestingOkS jmps c2 by − (rule Fin.hyps,auto)
    ultimately show ?case
      by simp
  qed (simp-all)
  with jumpNestingOk-l' subset
  show ?thesis
    by iprover
qed
```

corollary *jumpNestingOk-mono*:

```
  assumes jmpOk: jumpNestingOk jmps′ t
     and subset: jmps′ ⊆ jmps
  shows  jumpNestingOk jmps t
proof (cases t)
  case (In1 expr-stmt)
  show ?thesis
  proof (cases expr-stmt)
    case (Inl e)
    with In1 show ?thesis by simp
  next
    case (Inr s)
    with In1 jmpOk subset show ?thesis by (auto intro: jumpNestingOkS-mono)
  qed
qed (simp-all)


lemma assign-abrupt-propagation:
 assumes f-ok: abrupt (f n s) ≠ x
   and    ass: abrupt (assign f n s) = x
  shows abrupt s = x
proof (cases x)
  case None
  with ass show ?thesis
    by (cases s) (simp add: assign-def Let-def)
next
  case (Some xcpt)
  from f-ok
  obtain xf sf where f n s = (xf,sf)
    by (cases f n s)
  with Some ass f-ok show ?thesis
    by (cases s) (simp add: assign-def Let-def)
qed


lemma wt-init-comp-ty′:
is-acc-type (prg Env) (pid (cls Env)) T ⟹ Env⊢init-comp-ty T::√
apply (unfold init-comp-ty-def)
apply (clarsimp simp add: accessible-in-RefT-simp
                          is-acc-type-def is-acc-class-def)
done


lemma fvar-upd-no-jump:
      assumes upd: upd = snd (fst (fvar statDeclC stat fn a s′))
        and noJmp: abrupt s ≠ Some (Jump j)
        shows abrupt (upd val s) ≠ Some (Jump j)
proof (cases stat)
  case True
  with noJmp upd
  show ?thesis
    by (cases s) (simp add: fvar-def2)
next
  case False
  with noJmp upd
  show ?thesis
    by (cases s) (simp add: fvar-def2)
qed
```

lemma *avar-state-no-jump*:
  assumes *jmp*: *abrupt* (*snd* (*avar* G *i a s*)) = *Some* (*Jump j*)
  shows *abrupt s* = *Some* (*Jump j*)
proof (*cases normal s*)
  case *True* **with** *jmp* **show** *?thesis* **by** (*auto simp add*: *avar-def2 abrupt-if-def*)
next
  case *False* **with** *jmp* **show** *?thesis* **by** (*auto simp add*: *avar-def2 abrupt-if-def*)
qed

lemma *avar-upd-no-jump*:
  assumes *upd*: *upd* = *snd* (*fst* (*avar* G *i a s'*))
  and *noJmp*: *abrupt s* ≠ *Some* (*Jump j*)
  shows *abrupt* (*upd val s*) ≠ *Some* (*Jump j*)
using *upd noJmp*
by (*cases s*) (*simp add*: *avar-def2 abrupt-if-def*)

The next theorem expresses: If jumps (breaks, continues, returns) are nested correctly, we won't find an unexpected jump in the result state of the evaluation. For exeample, a break can't leave its enclosing loop, an return cant leave its enclosing method. To proove this, the method call is critical. Allthough the wellformedness of the whole program guarantees that the jumps (breaks,continues and returns) are nested correctly in all method bodies, the call rule alone does not guarantee that I will call a method or even a class that is part of the program due to dynamic binding! To be able to enshure this we need a kind of conformance of the state, like in the typesafety proof. But then we will redo the typesafty proof here. It would be nice if we could find an easy precondition that will guarantee that all calls will actually call classes and methods of the current program, which can be instantiated in the typesafty proof later on. To fix this problem, I have instrumented the semantic definition of a call to filter out any breaks in the state and to throw an error instead.

To get an induction hypothesis which is strong enough to perform the proof, we can't just assume *jumpNestingOk* for the empty set and conlcude, that no jump at all will be in the resulting state, because the set is altered by the statements *Lab* and *While*.

The wellformedness of the program is used to enshure that for all classinitialisations and methods the nesting of jumps is wellformed, too.

theorem *jumpNestingOk-eval*:
  assumes *eval*: $G\vdash$ *s0* $-t\succ\rightarrow$ (*v*,*s1*)
  and *jmpOk*: *jumpNestingOk jmps t*
  and *wt*: *Env*$\vdash$*t*::*T*
  and *wf*: *wf-prog G*
  and *G*: *prg Env* = *G*
  and *no-jmp*: ∀ *j*. *abrupt s0* = *Some* (*Jump j*) ⟶ *j* ∈ *jmps*
    (**is** *?Jmp jmps s0*)
  shows (∀ *j*. *fst s1* = *Some* (*Jump j*) ⟶ *j* ∈ *jmps*) ∧
    (*normal s1* ⟶
    (∀ *w upd*. *v*=*In2* (*w*,*upd*)
    ⟶ (∀ *s j val*.
    abrupt s ≠ *Some* (*Jump j*) ⟶
    abrupt (*upd val s*) ≠ *Some* (*Jump j*))))
  (**is** *?Jmp jmps s1* ∧ *?Upd v s1*)
proof −
  let *?HypObj* = λ *t s0 s1 v*.
    (∀ *jmps T Env*.
    ?Jmp jmps s0 ⟶ *jumpNestingOk jmps t* ⟶ *Env*$\vdash$*t*::*T* ⟶ *prg Env*=*G*⟶
    ?Jmp jmps s1 ∧ *?Upd v s1*)
  — Variable *?HypObj* is the following goal spelled in terms of the object logic, instead of the meta logic. It is needed in some cases of the induction were, the atomize-rulify process of induct does not work fine, because

the eval rules mix up object and meta logic. See for example the case for the loop.
from *eval*
have $\bigwedge$ *jmps T Env*. ⟦*?Jmp jmps s0*; *jumpNestingOk jmps t*; *Env*⊢*t*::*T*;*prg Env*=*G*⟧
$\Longrightarrow$ *?Jmp jmps s1* ∧ *?Upd v s1*
(**is** *PROP ?Hyp t s0 s1 v*)
— We need to abstract over *jmps* since *jmps* are extended during analysis of *Lab*. Also we need to abstract over *T* and *Env* since they are altered in various typing judgements.
proof (*induct*)
case *Abrupt* **thus** *?case* **by** *simp*
next
case *Skip* **thus** *?case* **by** *simp*
next
case *Expr* **thus** *?case* **by** (*elim wt-elim-cases*) *simp*
next
case (*Lab s0 c s1 jmp jmps T Env*)
note *jmpOK* = ‹*jumpNestingOk jmps* (*In1r* (*jmp*· *c*))›
note *G* = ‹*prg Env* = *G*›
have *wt-c*: *Env*⊢*c*::√
using *Lab.prems* **by** (*elim wt-elim-cases*)
{
fix *j*
assume *ab-s1*: *abrupt* (*abupd* (*absorb jmp*) *s1*) = *Some* (*Jump j*)
have *j*∈*jmps*
proof −
from *ab-s1* **have** *jmp-s1*: *abrupt s1* = *Some* (*Jump j*)
by (*cases s1*) (*simp add*: *absorb-def*)
note *hyp-c* = ‹*PROP ?Hyp* (*In1r c*) (*Norm s0*) *s1* ◇›
from *ab-s1* **have** *j* ≠ *jmp*
by (*cases s1*) (*simp add*: *absorb-def*)
moreover have *j* ∈ {*jmp*} ∪ *jmps*
proof −
from *jmpOK*
have *jumpNestingOk* ({*jmp*} ∪ *jmps*) (*In1r c*) **by** *simp*
with *wt-c jmp-s1 G hyp-c*
show *?thesis*
by − (*rule hyp-c* [*THEN conjunct1*,*rule-format*],*simp*)
qed
ultimately show *?thesis*
by *simp*
qed
}
thus *?case* **by** *simp*
next
case (*Comp s0 c1 s1 c2 s2 jmps T Env*)
note *jmpOk* = ‹*jumpNestingOk jmps* (*In1r* (*c1*;; *c2*))›
note *G* = ‹*prg Env* = *G*›
from *Comp.prems* **obtain**
wt-c1: *Env*⊢*c1*::√ **and** *wt-c2*: *Env*⊢*c2*::√
by (*elim wt-elim-cases*)
{
fix *j*
assume *abr-s2*: *abrupt s2* = *Some* (*Jump j*)
have *j*∈*jmps*
proof −
have *jmp*: *?Jmp jmps s1*
proof −
note *hyp-c1* = ‹*PROP ?Hyp* (*In1r c1*) (*Norm s0*) *s1* ◇›
with *wt-c1 jmpOk G*
show *?thesis* **by** *simp*

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      qed
      moreover note hyp-c2 = ‹PROP ?Hyp (In1r c2) s1 s2 (♢::vals)›
      have jmpOk': jumpNestingOk jmps (In1r c2) using jmpOk by simp
      moreover note wt-c2 G abr-s2
      ultimately show j ∈ jmps
        by (rule hyp-c2 [THEN conjunct1,rule-format (no-asm)])
    qed
  } thus ?case by simp
next
  case (If s0 e b s1 c1 c2 s2 jmps T Env)
  note jmpOk = ‹jumpNestingOk jmps (In1r (If(e) c1 Else c2))›
  note G = ‹prg Env = G›
  from If.prems obtain
           wt-e: Env⊢e::−PrimT Boolean and
    wt-then-else: Env⊢(if the-Bool b then c1 else c2)::√
    by (elim wt-elim-cases) simp
  {
    fix j
    assume jmp: abrupt s2 = Some (Jump j)
    have j∈jmps
    proof −
      note ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 b)›
      with wt-e G have ?Jmp jmps s1
        by simp
      moreover note hyp-then-else =
        ‹PROP ?Hyp (In1r (if the-Bool b then c1 else c2)) s1 s2 ♢›
      have jumpNestingOk jmps (In1r (if the-Bool b then c1 else c2))
        using jmpOk by (cases the-Bool b) simp-all
      moreover note wt-then-else G jmp
      ultimately show j∈ jmps
        by (rule hyp-then-else [THEN conjunct1,rule-format (no-asm)])
    qed
  }
  thus ?case by simp
next
  case (Loop s0 e b s1 c s2 l s3 jmps T Env)
  note jmpOk = ‹jumpNestingOk jmps (In1r (l• While(e) c))›
  note G = ‹prg Env = G›
  note wt = ‹Env⊢In1r (l• While(e) c)::T›
  then obtain
           wt-e: Env⊢e::−PrimT Boolean and
           wt-c: Env⊢c::√
    by (elim wt-elim-cases)
  {
    fix j
    assume jmp: abrupt s3 = Some (Jump j)
    have j∈jmps
    proof −
      note ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 b)›
      with wt-e G have jmp-s1: ?Jmp jmps s1
        by simp
      show ?thesis
      proof (cases the-Bool b)
        case False
        from Loop.hyps
        have s3=s1
          by (simp (no-asm-use) only: if-False False)
        with jmp-s1 jmp have j ∈ jmps by simp
        thus ?thesis by simp
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```
      next
        case True
        from Loop.hyps

        have ?HypObj (In1r c) s1 s2 (◇::vals)
          apply (simp (no-asm-use) only: True if-True )
          apply (erule conjE)+
          apply assumption
          done
        note hyp-c = this [rule-format (no-asm)]
        moreover from jmpOk have jumpNestingOk ({Cont l} ∪ jmps) (In1r c)
          by simp
        moreover from jmp-s1 have ?Jmp ({Cont l} ∪ jmps) s1 by simp
        ultimately have jmp-s2: ?Jmp ({Cont l} ∪ jmps) s2
          using wt-c G by iprover
        have ?Jmp jmps (abupd (absorb (Cont l)) s2)
        proof −
          {
            fix j'
            assume abs: abrupt (abupd (absorb (Cont l)) s2)=Some (Jump j')
            have j' ∈ jmps
            proof (cases j' = Cont l)
              case True
              with abs show ?thesis
                by (cases s2) (simp add: absorb-def)
            next
              case False
              with abs have abrupt s2 = Some (Jump j')
                by (cases s2) (simp add: absorb-def)
              with jmp-s2 False show ?thesis
                by simp
            qed
          }
          thus ?thesis by simp
        qed
        moreover
        from Loop.hyps
        have ?HypObj (In1r (l· While(e) c))
                     (abupd (absorb (Cont l)) s2) s3 (◇::vals)
          apply (simp (no-asm-use) only: True if-True)
          apply (erule conjE)+
          apply assumption
          done
        note hyp-w = this [rule-format (no-asm)]
        note jmpOk wt G jmp
        ultimately show j∈ jmps
          by (rule hyp-w [THEN conjunct1,rule-format (no-asm)])
      qed
    qed
  }
  thus ?case by simp
next
  case (Jmp s j jmps T Env) thus ?case by simp
next
  case (Throw s0 e a s1 jmps T Env)
  note jmpOk = ‹jumpNestingOk jmps (In1r (Throw e))›
  note G = ‹prg Env = G›
  from Throw.prems obtain Te where
    wt-e: Env⊢e::−Te
```

```
      by (elim wt-elim-cases)
    {
      fix j
      assume jmp: abrupt (abupd (throw a) s1) = Some (Jump j)
      have j∈jmps
      proof −
        from ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 a)›
        have ?Jmp jmps s1 using wt-e G by simp
        moreover
        from jmp
        have abrupt s1 = Some (Jump j)
          by (cases s1) (simp add: throw-def abrupt-if-def)
        ultimately show j ∈ jmps by simp
      qed
    }
    thus ?case by simp
  next
    case (Try s0 c1 s1 s2 C vn c2 s3 jmps T Env)
    note jmpOk = ‹jumpNestingOk jmps (In1r (Try c1 Catch(C vn) c2))›
    note G = ‹prg Env = G›
    from Try.prems obtain
      wt-c1: Env⊢c1::√ and
      wt-c2: Env(|lcl := lcl Env(VName vn↦Class C)|)⊢c2::√
      by (elim wt-elim-cases)
    {
      fix j
      assume jmp: abrupt s3 = Some (Jump j)
      have j∈jmps
      proof −
        note ‹PROP ?Hyp (In1r c1) (Norm s0) s1 (◇::vals)›
        with jmpOk wt-c1 G
        have jmp-s1: ?Jmp jmps s1 by simp
        note s2 = ‹G⊢s1 −sxalloc→ s2›
        show j ∈ jmps
        proof (cases G,s2⊢catch C)
          case False
          from Try.hyps have s3=s2
            by (simp (no-asm-use) only: False if-False)
          with jmp have abrupt s1 = Some (Jump j)
            using sxalloc-no-jump' [OF s2] by simp
          with jmp-s1
          show ?thesis by simp
        next
          case True
          with Try.hyps
          have ?HypObj (In1r c2) (new-xcpt-var vn s2) s3 (◇::vals)
            apply (simp (no-asm-use) only: True if-True simp-thms)
            apply (erule conjE)+
            apply assumption
            done
          note hyp-c2 = this [rule-format (no-asm)]
          from jmp-s1 sxalloc-no-jump' [OF s2]
          have ?Jmp jmps s2
            by simp
          hence ?Jmp jmps (new-xcpt-var vn s2)
            by (cases s2) simp
          moreover have jumpNestingOk jmps (In1r c2) using jmpOk by simp
          moreover note wt-c2
          moreover from G
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        have prg (Env(|lcl := lcl Env(VName vn↦Class C)|)) = G
          by simp
        moreover note jmp
        ultimately show ?thesis
          by (rule hyp-c2 [THEN conjunct1,rule-format (no-asm)])
      qed
    qed
  }
  thus ?case by simp
next
  case (Fin s0 c1 x1 s1 c2 s2 s3 jmps T Env)
  note jmpOk = ‹jumpNestingOk jmps (In1r (c1 Finally c2))›
  note G = ‹prg Env = G›
  from Fin.prems obtain
    wt-c1: Env⊢c1::√ and wt-c2: Env⊢c2::√
    by (elim wt-elim-cases)
  {
    fix j
    assume jmp: abrupt s3 = Some (Jump j)
    have j ∈ jmps
    proof (cases x1=Some (Jump j))
      case True
      note hyp-c1 = ‹PROP ?Hyp (In1r c1) (Norm s0) (x1,s1) ◇›
      with True jmpOk wt-c1 G show ?thesis
        by − (rule hyp-c1 [THEN conjunct1,rule-format (no-asm)],simp-all)
    next
      case False
      note hyp-c2 = ‹PROP ?Hyp (In1r c2) (Norm s1) s2 ◇›
      note ‹s3 = (if ∃ err. x1 = Some (Error err) then (x1, s1)
                 else abupd (abrupt-if (x1 ≠ None) x1) s2)›
      with False jmp have abrupt s2 = Some (Jump j)
        by (cases s2) (simp add: abrupt-if-def)
      with jmpOk wt-c2 G show ?thesis
        by − (rule hyp-c2 [THEN conjunct1,rule-format (no-asm)],simp-all)
    qed
  }
  thus ?case by simp
next
  case (Init C c s0 s3 s1 s2 jmps T Env)
  note ‹jumpNestingOk jmps (In1r (Init C))›
  note G = ‹prg Env = G›
  note ‹the (class G C) = c›
  with Init.prems have c: class G C = Some c
    by (elim wt-elim-cases) auto
  {
    fix j
    assume jmp: abrupt s3 = (Some (Jump j))
    have j∈jmps
    proof (cases inited C (globs s0))
      case True
      with Init.hyps have s3=Norm s0
        by simp
      with jmp
      have False by simp thus ?thesis ..
    next
      case False
      from wf c G
      have wf-cdecl: wf-cdecl G (C,c)
        by (simp add: wf-prog-cdecl)
```

```
      from Init.hyps
      have ?HypObj (In1r (if C = Object then Skip else Init (super c)))
                   (Norm ((init-class-obj G C) s0)) s1 (◇::vals)
        apply (simp (no-asm-use) only: False if-False simp-thms)
        apply (erule conjE)+
        apply assumption
        done
      note hyp-s1 = this [rule-format (no-asm)]
      from wf-cdecl G have
        wt-super: Env⊢(if C = Object then Skip else Init (super c))::√
        by (cases C=Object)
          (auto dest: wf-cdecl-supD is-acc-classD)
      from hyp-s1 [OF - - wt-super G]
      have ?Jmp jmps s1
        by simp
      hence jmp-s1: ?Jmp jmps ((set-lvars empty) s1) by (cases s1) simp
      from False Init.hyps
      have ?HypObj (In1r (init c)) ((set-lvars empty) s1) s2 (◇::vals)
        apply (simp (no-asm-use) only: False if-False simp-thms)
        apply (erule conjE)+
        apply assumption
        done
      note hyp-init-c = this [rule-format (no-asm)]
      from wf-cdecl
      have wt-init-c: ⦇prg = G, cls = C, lcl = empty⦈⊢init c::√
        by (rule wf-cdecl-wt-init)
      from wf-cdecl have jumpNestingOkS {} (init c)
        by (cases rule: wf-cdeclE)
      hence jumpNestingOkS jmps (init c)
        by (rule jumpNestingOkS-mono) simp
      moreover
      have abrupt s2 = Some (Jump j)
      proof −
        from False Init.hyps
        have s3 = (set-lvars (locals (store s1))) s2  by simp
        with jmp show ?thesis by (cases s2) simp
      qed
      ultimately show ?thesis
        using hyp-init-c [OF jmp-s1 - wt-init-c]
        by simp
    qed
  }
  thus ?case by simp
next
  case (NewC s0 C s1 a s2 jmps T Env)
  {
    fix j
    assume jmp: abrupt s2 = Some (Jump j)
    have j∈jmps
    proof −
      note ⟨prg Env = G⟩
      moreover note hyp-init = ⟨PROP ?Hyp (In1r (Init C)) (Norm s0) s1 ◇⟩
      moreover from wf NewC.prems
      have Env⊢(Init C)::√
        by (elim wt-elim-cases) (drule is-acc-classD,simp)
      moreover
      have abrupt s1 = Some (Jump j)
      proof −
        from ⟨G⊢s1 −halloc CInst C≻a→ s2⟩ and jmp show ?thesis
```

```
        by (rule halloc-no-jump')
      qed
      ultimately show j ∈ jmps
        by − (rule hyp-init [THEN conjunct1,rule-format (no-asm)],auto)
    qed
  }
  thus ?case by simp
next
  case (NewA s0 elT s1 e i s2 a s3 jmps T Env)
  {
    fix j
    assume jmp: abrupt s3 = Some (Jump j)
    have j∈jmps
    proof −
      note G = ‹prg Env = G›
      from NewA.prems
      obtain wt-init: Env⊢init-comp-ty elT::√ and
             wt-size: Env⊢e::−PrimT Integer
        by (elim wt-elim-cases) (auto dest:  wt-init-comp-ty')
      note ‹PROP ?Hyp (In1r (init-comp-ty elT)) (Norm s0) s1 ◇›
      with wt-init G
      have ?Jmp jmps s1
        by (simp add: init-comp-ty-def)
      moreover
      note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2 (In1 i)›
      have abrupt s2 = Some (Jump j)
      proof −
        note ‹G⊢abupd (check-neg i) s2−halloc Arr elT (the-Intg i)≻a→ s3›
        moreover note jmp
        ultimately
        have abrupt (abupd (check-neg i) s2) = Some (Jump j)
          by (rule halloc-no-jump')
        thus ?thesis by (cases s2) auto
      qed
      ultimately show j∈jmps using wt-size G
        by − (rule hyp-e [THEN conjunct1,rule-format (no-asm)],simp-all)
    qed
  }
  thus ?case by simp
next
  case (Cast s0 e v s1 s2 cT jmps T Env)
  {
    fix j
    assume jmp: abrupt s2 = Some (Jump j)
    have j∈jmps
    proof −
      note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
      note ‹prg Env = G›
      moreover from Cast.prems
      obtain eT where Env⊢e::−eT by (elim wt-elim-cases)
      moreover
      have abrupt s1 = Some (Jump j)
      proof −
        note ‹s2 = abupd (raise-if (¬ G,snd s1⊢v fits cT) ClassCast) s1›
        moreover note jmp
        ultimately show ?thesis by (cases s1) (simp add: abrupt-if-def)
      qed
      ultimately show ?thesis
        by − (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all)
```

```
      qed
    }
    thus ?case by simp
  next
    case (Inst s0 e v s1 b eT jmps T Env)
    {
      fix j
      assume jmp: abrupt s1 = Some (Jump j)
      have j∈jmps
      proof –
        note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
        note ‹prg Env = G›
        moreover from Inst.prems
        obtain eT where Env⊢e::–eT by (elim wt-elim-cases)
        moreover note jmp
        ultimately show j∈jmps
          by – (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all)
      qed
    }
    thus ?case by simp
  next
    case Lit thus ?case by simp
  next
    case (UnOp s0 e v s1 unop jmps T Env)
    {
      fix j
      assume jmp: abrupt s1 = Some (Jump j)
      have j∈jmps
      proof –
        note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
        note ‹prg Env = G›
        moreover from UnOp.prems
        obtain eT where Env⊢e::–eT by (elim wt-elim-cases)
        moreover note jmp
        ultimately show  j∈jmps
          by – (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all)
      qed
    }
    thus ?case by simp
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2 jmps T Env)
    {
      fix j
      assume jmp: abrupt s2 = Some (Jump j)
      have j∈jmps
      proof –
        note G = ‹prg Env = G›
        from BinOp.prems
        obtain e1T e2T where
          wt-e1: Env⊢e1::–e1T and
          wt-e2: Env⊢e2::–e2T
          by (elim wt-elim-cases)
        note ‹PROP ?Hyp (In1l e1) (Norm s0) s1 (In1 v1)›
        with G wt-e1 have jmp-s1: ?Jmp jmps s1 by simp
        note hyp-e2 =
          ‹PROP ?Hyp (if need-second-arg binop v1 then In1l e2 else In1r Skip)
                     s1 s2 (In1 v2)›
        show j∈jmps
        proof (cases need-second-arg binop v1)
```

```
        case True with jmp-s1 wt-e2 jmp G
        show ?thesis
          by − (rule hyp-e2 [THEN conjunct1,rule-format (no-asm)],simp-all)
      next
        case False with jmp-s1 jmp G
        show ?thesis
          by − (rule hyp-e2 [THEN conjunct1,rule-format (no-asm)],auto)
      qed
    qed
  }
  thus ?case by simp
next
  case Super thus ?case by simp
next
  case (Acc s0 va v f s1 jmps T Env)
  {
    fix j
    assume jmp: abrupt s1 = Some (Jump j)
    have j∈jmps
    proof −
      note hyp-va = ‹PROP ?Hyp (In2 va) (Norm s0) s1 (In2 (v,f))›
      note ‹prg Env = G›
      moreover from Acc.prems
      obtain vT where Env⊢va::=vT by (elim wt-elim-cases)
      moreover note jmp
      ultimately show j∈jmps
        by − (rule hyp-va [THEN conjunct1,rule-format (no-asm)], simp-all)
    qed
  }
  thus ?case by simp
next
  case (Ass s0 va w f s1 e v s2 jmps T Env)
  note G = ‹prg Env = G›
  from Ass.prems
  obtain vT eT where
    wt-va: Env⊢va::=vT and
     wt-e: Env⊢e::−eT
    by (elim wt-elim-cases)
  note hyp-v = ‹PROP ?Hyp (In2 va) (Norm s0) s1 (In2 (w,f))›
  note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2 (In1 v)›
  {
    fix j
    assume jmp: abrupt (assign f v s2) = Some (Jump j)
    have j∈jmps
    proof −
      have abrupt s2 = Some (Jump j)
      proof (cases normal s2)
        case True
        from ‹G⊢s1 −e−≻v→ s2› and True have nrm-s1: normal s1
          by (rule eval-no-abrupt-lemma [rule-format])
        with nrm-s1 wt-va G True
        have abrupt (f v s2) ≠ Some (Jump j)
          using hyp-v [THEN conjunct2,rule-format (no-asm)]
          by simp
        from this jmp
        show ?thesis
          by (rule assign-abrupt-propagation)
      next
        case False with jmp
```

```
      show ?thesis by (cases s2) (simp add: assign-def Let-def)
    qed
    moreover from wt-va G
    have ?Jmp jmps s1
      by - (rule hyp-v [THEN conjunct1],simp-all)
    ultimately show ?thesis using G
      by - (rule hyp-e [THEN conjunct1,rule-format (no-asm)], simp-all, rule wt-e)
  qed
}
thus ?case by simp
next
  case (Cond s0 e0 b s1 e1 e2 v s2 jmps T Env)
  note G = ‹prg Env = G›
  note hyp-e0 = ‹PROP ?Hyp (In1l e0) (Norm s0) s1 (In1 b)›
  note hyp-e1-e2 = ‹PROP ?Hyp (In1l (if the-Bool b then e1 else e2)) s1 s2 (In1 v)›
  from Cond.prems
  obtain e1T e2T
    where wt-e0: Env⊢e0::−PrimT Boolean
     and  wt-e1: Env⊢e1::−e1T
     and  wt-e2: Env⊢e2::−e2T
    by (elim wt-elim-cases)
  {
    fix j
    assume jmp: abrupt s2 = Some (Jump j)
    have j∈jmps
    proof −
      from wt-e0 G
      have jmp-s1: ?Jmp jmps s1
        by − (rule hyp-e0 [THEN conjunct1],simp-all)
      show ?thesis
      proof (cases the-Bool b)
        case True
        with jmp-s1 wt-e1 G jmp
        show ?thesis
          by−(rule hyp-e1-e2 [THEN conjunct1,rule-format (no-asm)],simp-all)
      next
        case False
        with jmp-s1 wt-e2 G jmp
        show ?thesis
          by−(rule hyp-e1-e2 [THEN conjunct1,rule-format (no-asm)],simp-all)
      qed
    qed
  }
  thus ?case by simp
next
  case (Call s0 e a s1 args vs s2 D mode statT mn pTs s3 s3' accC v s4
            jmps T Env)
  note G = ‹prg Env = G›
  from Call.prems
  obtain eT argsT
    where wt-e: Env⊢e::−eT and wt-args: Env⊢args::≐argsT
    by (elim wt-elim-cases)
  {
    fix j
    assume jmp: abrupt ((set-lvars (locals (store s2))) s4)
                  = Some (Jump j)
    have j∈jmps
    proof −
      note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 a)›
```

```
      from wt-e G
      have jmp-s1: ?Jmp jmps s1
        by − (rule hyp-e [THEN conjunct1],simp-all)
      note hyp-args = ‹PROP ?Hyp (In3 args) s1 s2 (In3 vs)›
      have abrupt s2 = Some (Jump j)
      proof −
        note ‹G⊢s3′ −Methd D (|name = mn, parTs = pTs|)−≻v→ s4›
        moreover
        from jmp have abrupt s4 = Some (Jump j)
          by (cases s4) simp
        ultimately have abrupt s3′ = Some (Jump j)
          by − (rule ccontr,drule (1) Methd-no-jump,simp)
        moreover note ‹s3′ = check-method-access G accC statT mode
                          (|name = mn, parTs = pTs|) a s3›
        ultimately have abrupt s3 = Some (Jump j)
          by (cases s3)
             (simp add: check-method-access-def abrupt-if-def Let-def)
        moreover
        note ‹s3 = init-lvars G D (|name=mn, parTs=pTs|) mode a vs s2›
        ultimately show ?thesis
          by (cases s2) (auto simp add: init-lvars-def2)
      qed
      with jmp-s1 wt-args G
      show ?thesis
        by − (rule hyp-args [THEN conjunct1,rule-format (no-asm)], simp-all)
    qed
  }
  thus ?case by simp
next
  case (Methd s0 D sig v s1 jmps T Env)
  from ‹G⊢Norm s0 −body G D sig−≻v→ s1›
  have G⊢Norm s0 −Methd D sig−≻v→ s1
    by (rule eval.Methd)
  hence ⋀ j. abrupt s1 ≠ Some (Jump j)
    by (rule Methd-no-jump) simp
  thus ?case by simp
next
  case (Body s0 D s1 c s2 s3 jmps T Env)
  have G⊢Norm s0 −Body D c−≻the (locals (store s2) Result)
         → abupd (absorb Ret) s3
    by (rule eval.Body) (rule Body)+
  hence ⋀ j. abrupt (abupd (absorb Ret) s3) ≠ Some (Jump j)
    by (rule Body-no-jump) simp
  thus ?case by simp
next
  case LVar
  thus ?case by (simp add: lvar-def Let-def)
next
  case (FVar s0 statDeclC s1 e a s2 v s2′ stat fn s3 accC jmps T Env)
  note G = ‹prg Env = G›
  from wf FVar.prems
  obtain  statC f where
    wt-e: Env⊢e::−Class statC and
    accfield: accfield (prg Env) accC statC fn = Some (statDeclC,f)
    by  (elim wt-elim-cases) simp
  have wt-init: Env⊢Init statDeclC::√
  proof −
    from wf wt-e G
    have is-class (prg Env) statC
```

```
      by (auto dest: ty-expr-is-type type-is-class)
    with wf accfield G
    have is-class (prg Env) statDeclC
      by (auto dest!: accfield-fields dest: fields-declC)
    thus ?thesis
      by simp
  qed
  note fvar = ‹(v, s2') = fvar statDeclC stat fn a s2›
  {
    fix j
    assume jmp: abrupt s3 = Some (Jump j)
    have j∈jmps
    proof −
      note hyp-init = ‹PROP ?Hyp (In1r (Init statDeclC)) (Norm s0) s1 ◇›
      from G wt-init
      have ?Jmp jmps s1
        by − (rule hyp-init [THEN conjunct1],auto)
      moreover
      note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2 (In1 a)›
      have abrupt s2 = Some (Jump j)
      proof −
        note ‹s3 = check-field-access G accC statDeclC fn stat a s2'›
        with jmp have abrupt s2' = Some (Jump j)
          by (cases s2')
             (simp add: check-field-access-def abrupt-if-def Let-def)
       with fvar show abrupt s2 =  Some (Jump j)
          by (cases s2) (simp add: fvar-def2 abrupt-if-def)
      qed
      ultimately show ?thesis
        using G wt-e
        by − (rule hyp-e [THEN conjunct1, rule-format (no-asm)],simp-all)
    qed
  }
  moreover
  from fvar obtain upd w
    where upd: upd = snd (fst (fvar statDeclC stat fn a s2)) and
          v: v=(w,upd)
    by (cases fvar statDeclC stat fn a s2) simp
  {
    fix j val fix s::state
    assume normal s3
    assume no-jmp: abrupt s ≠ Some (Jump j)
    with upd
    have abrupt (upd val s) ≠ Some (Jump j)
      by (rule fvar-upd-no-jump)
  }
  ultimately show ?case using v by simp
next
  case (AVar s0 e1 a s1 e2 i s2 v s2' jmps T Env)
  note G = ‹prg Env = G›
  from AVar.prems
  obtain  e1T e2T where
    wt-e1: Env⊢e1::−e1T and wt-e2: Env⊢e2::−e2T
    by  (elim wt-elim-cases) simp
  note avar = ‹(v, s2') = avar G i a s2›
  {
    fix j
    assume jmp: abrupt s2' = Some (Jump j)
    have j∈jmps
```

```
      proof -
        note hyp-e1 = ‹PROP ?Hyp (In1l e1) (Norm s0) s1 (In1 a)›
        from G wt-e1
        have ?Jmp jmps s1
          by - (rule hyp-e1 [THEN conjunct1], auto)
        moreover
        note hyp-e2 = ‹PROP ?Hyp (In1l e2) s1 s2 (In1 i)›
        have abrupt s2 = Some (Jump j)
        proof -
          from avar have s2' = snd (avar G i a s2)
            by (cases avar G i a s2) simp
          with jmp show ?thesis by - (rule avar-state-no-jump,simp)
        qed
        ultimately show ?thesis
          using wt-e2 G
          by - (rule hyp-e2 [THEN conjunct1, rule-format (no-asm)],simp-all)
      qed
    }
    moreover
    from avar obtain upd w
      where upd: upd = snd (fst (avar G i a s2)) and
            v: v=(w,upd)
      by (cases avar G i a s2) simp
    {
      fix j val fix s::state
      assume normal s2'
      assume no-jmp: abrupt s ≠ Some (Jump j)
      with upd
      have abrupt (upd val s) ≠ Some (Jump j)
        by (rule avar-upd-no-jump)
    }
    ultimately show ?case using v by simp
  next
    case Nil thus ?case by simp
  next
    case (Cons s0 e v s1 es vs s2 jmps T Env)
    note G = ‹prg Env = G›
    from Cons.prems obtain eT esT
      where wt-e: Env⊢e::-eT and wt-e2: Env⊢es::≐esT
      by (elim wt-elim-cases) simp
    {
      fix j
      assume jmp: abrupt s2 = Some (Jump j)
      have j∈jmps
      proof -
        note hyp-e = ‹PROP ?Hyp (In1l e) (Norm s0) s1 (In1 v)›
        from G wt-e
        have ?Jmp jmps s1
          by - (rule hyp-e [THEN conjunct1],simp-all)
        moreover
        note hyp-es = ‹PROP ?Hyp (In3 es) s1 s2 (In3 vs)›
        ultimately show ?thesis
          using wt-e2 G jmp
          by - (rule hyp-es [THEN conjunct1, rule-format (no-asm)],
                (assumption|simp (no-asm-simp))+)
      qed
    }
    thus ?case by simp
  qed
```

note *generalized* = *this*
from *no-jmp jmpOk wt G*
show *?thesis*
by (*rule generalized*)
qed

lemmas *jumpNestingOk-evalE* = *jumpNestingOk-eval* [*THEN conjE*,*rule-format*]

lemma *jumpNestingOk-eval-no-jump*:
assumes *eval*: *prg Env*$\vdash$ *s0* $-t\succ\rightarrow$ (*v*,*s1*) **and**
jmpOk: *jumpNestingOk* {} *t* **and**
no-jmp: *abrupt s0* $\neq$ *Some* (*Jump j*) **and**
wt: *Env*$\vdash$*t*::*T* **and**
wf: *wf-prog* (*prg Env*)
shows *abrupt s1* $\neq$ *Some* (*Jump j*) $\wedge$
(*normal s1* $\longrightarrow$ *v*=*In2* (*w*,*upd*)
$\longrightarrow$ *abrupt s* $\neq$ *Some* (*Jump j'*)
$\longrightarrow$ *abrupt* (*upd val s*) $\neq$ *Some* (*Jump j'*))
proof (*cases* $\exists$ *j'*. *abrupt s0* = *Some* (*Jump j'*))
case *True*
then obtain *j'* **where** *jmp*: *abrupt s0* = *Some* (*Jump j'*) **..**
with *no-jmp* **have** *j'*$\neq$*j* **by** *simp*
with *eval jmp* **have** *s1*=*s0* **by** *auto*
with *no-jmp jmp* **show** *?thesis* **by** *simp*
next
case *False*
obtain *G* **where** *G*: *prg Env* = *G*
by (*cases Env*) *simp*
from *G eval* **have** *G*$\vdash$ *s0* $-t\succ\rightarrow$ (*v*,*s1*) **by** *simp*
moreover note *jmpOk wt*
moreover from *G wf* **have** *wf-prog G* **by** *simp*
moreover note *G*
moreover from *False* **have** $\bigwedge$ *j*. *abrupt s0* = *Some* (*Jump j*) $\Longrightarrow$ *j* $\in$ {}
by *simp*
ultimately show *?thesis*
apply (*rule jumpNestingOk-evalE*)
apply *assumption*
apply *simp*
apply *fastsimp*
done
qed

lemmas *jumpNestingOk-eval-no-jumpE*
= *jumpNestingOk-eval-no-jump* [*THEN conjE*,*rule-format*]

corollary *eval-expression-no-jump*:
assumes *eval*: *prg Env*$\vdash$*s0* $-e-\succ v\rightarrow$ *s1* **and**
no-jmp: *abrupt s0* $\neq$ *Some* (*Jump j*) **and**
wt: *Env*$\vdash$*e*::$-$*T* **and**
wf: *wf-prog* (*prg Env*)
shows *abrupt s1* $\neq$ *Some* (*Jump j*)
using *eval* - *no-jmp wt wf*
by (*rule jumpNestingOk-eval-no-jumpE*, *simp-all*)

corollary *eval-var-no-jump*:
assumes *eval*: *prg Env*$\vdash$*s0* $-var{=}\succ$(*w*,*upd*)$\rightarrow$ *s1* **and**
no-jmp: *abrupt s0* $\neq$ *Some* (*Jump j*) **and**

wt: *Env*⊢*var*::=*T* **and**
wf: *wf-prog* (*prg Env*)
shows *abrupt s1* ≠ *Some* (*Jump j*) ∧
(*normal s1* ⟶
(*abrupt s* ≠ *Some* (*Jump j'*)
⟶ *abrupt* (*upd val s*) ≠ *Some* (*Jump j'*)))
apply (*rule-tac upd*=*upd* **and** *val*=*val* **and** *s*=*s* **and** *w*=*w* **and** *j'*=*j'*
in *jumpNestingOk-eval-no-jumpE* [*OF eval - no-jmp wt wf*])
by *simp-all*

lemmas *eval-var-no-jumpE* = *eval-var-no-jump* [*THEN conjE*,*rule-format*]

corollary *eval-statement-no-jump*:
assumes *eval*: *prg Env*⊢*s0* −*c*→ *s1* **and**
jmpOk: *jumpNestingOkS* {} *c* **and**
no-jmp: *abrupt s0* ≠ *Some* (*Jump j*) **and**
wt: *Env*⊢*c*::√ **and**
wf: *wf-prog* (*prg Env*)
shows *abrupt s1* ≠ *Some* (*Jump j*)
using *eval - no-jmp wt wf*
by (*rule jumpNestingOk-eval-no-jumpE*) (*simp-all add*: *jmpOk*)

corollary *eval-expression-list-no-jump*:
assumes *eval*: *prg Env*⊢*s0* −*es*≐≻*v*→ *s1* **and**
no-jmp: *abrupt s0* ≠ *Some* (*Jump j*) **and**
wt: *Env*⊢*es*::≐*T* **and**
wf: *wf-prog* (*prg Env*)
shows *abrupt s1* ≠ *Some* (*Jump j*)
using *eval - no-jmp wt wf*
by (*rule jumpNestingOk-eval-no-jumpE*, *simp-all*)

lemma *union-subseteq-elim* [*elim*]: ⟦*A* ∪ *B* ⊆ *C*; ⟦*A* ⊆ *C*; *B* ⊆ *C*⟧ ⟹ *P*⟧ ⟹ *P*
by *blast*

lemma *dom-locals-halloc-mono*:
assumes *halloc*: *G*⊢*s0*−*halloc oi*≻*a*→*s1*
shows *dom* (*locals* (*store s0*)) ⊆ *dom* (*locals* (*store s1*))
proof −
from *halloc* **show** *?thesis*
by *cases simp-all*
qed

lemma *dom-locals-sxalloc-mono*:
assumes *sxalloc*: *G*⊢*s0*−*sxalloc*→*s1*
shows *dom* (*locals* (*store s0*)) ⊆ *dom* (*locals* (*store s1*))
proof −
from *sxalloc* **show** *?thesis*
proof (*cases*)
case *Norm* **thus** *?thesis* **by** *simp*
next
case *Jmp* **thus** *?thesis* **by** *simp*
next
case *Error* **thus** *?thesis* **by** *simp*
next
case *XcptL* **thus** *?thesis* **by** *simp*

next
case *SXcpt* **thus** *?thesis*
by − (*drule dom-locals-halloc-mono*,*simp*)
qed
qed

lemma *dom-locals-assign-mono*:
assumes *f-ok*: *dom* (*locals* (*store s*)) ⊆ *dom* (*locals* (*store* (*f n s*)))
shows *dom* (*locals* (*store s*)) ⊆ *dom* (*locals* (*store* (*assign f n s*)))
proof (*cases normal s*)
case *False* **thus** *?thesis*
by (*cases s*) (*auto simp add*: *assign-def Let-def*)
next
case *True*
then obtain *s′* **where** *s′*: *s* = (*None*,*s′*)
by *auto*
moreover
obtain *x1 s1* **where** *f n s* = (*x1*,*s1*)
by (*cases f n s*)
ultimately
show *?thesis*
using *f-ok*
by (*simp add*: *assign-def Let-def*)
qed

lemma *dom-locals-lvar-mono*:
dom (*locals* (*store s*)) ⊆ *dom* (*locals* (*store* (*snd* (*lvar vn s′*) *val s*)))
by (*simp add*: *lvar-def*) *blast*

lemma *dom-locals-fvar-vvar-mono*:
dom (*locals* (*store s*))
⊆ *dom* (*locals* (*store* (*snd* (*fst* (*fvar statDeclC stat fn a s′*)) *val s*)))
proof (*cases stat*)
case *True*
thus *?thesis*
by (*cases s*) (*simp add*: *fvar-def2*)
next
case *False*
thus *?thesis*
by (*cases s*) (*simp add*: *fvar-def2*)
qed

lemma *dom-locals-fvar-mono*:
dom (*locals* (*store s*))
⊆ *dom* (*locals* (*store* (*snd* (*fvar statDeclC stat fn a s*))))
proof (*cases stat*)
case *True*
thus *?thesis*
by (*cases s*) (*simp add*: *fvar-def2*)
next
case *False*

```
  thus ?thesis
    by (cases s) (simp add: fvar-def2)
qed
```

```
lemma dom-locals-avar-vvar-mono:
dom (locals (store s))
  ⊆ dom (locals (store (snd (fst (avar G i a s')) val s)))
by (cases s, simp add: avar-def2)
```

```
lemma dom-locals-avar-mono:
dom (locals (store s))
  ⊆ dom (locals (store (snd (avar G i a s))))
by (cases s, simp add: avar-def2)
```

Since assignments are modelled as functions from states to states, we must take into account these functions. They appear only in the assignment rule and as result from evaluating a variable. Thats why we need the complicated second part of the conjunction in the goal. The reason for the very generic way to treat assignments was the aim to omit redundancy. There is only one evaluation rule for each kind of variable (locals, fields, arrays). These rules are used for both accessing variables and updating variables. Thats why the evaluation rules for variables result in a pair consisting of a value and an update function. Of course we could also think of a pair of a value and a reference in the store, instead of the generic update function. But as only array updates can cause a special exception (if the types mismatch) and not array reads we then have to introduce two different rules to handle array reads and updates

```
lemma dom-locals-eval-mono:
  assumes   eval: G⊢ s0 −t≻→ (v,s1)
  shows dom (locals (store s0)) ⊆ dom (locals (store s1)) ∧
        (∀ vv. v=In2 vv ∧ normal s1
           ⟶ (∀ s val. dom (locals (store s))
                          ⊆ dom (locals (store ((snd vv) val s)))))
proof −
  from eval show ?thesis
  proof (induct)
    case Abrupt thus ?case by simp
  next
    case Skip thus ?case by simp
  next
    case Expr thus ?case by simp
  next
    case Lab thus ?case by simp
  next
    case (Comp s0 c1 s1 c2 s2)
    from Comp.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from Comp.hyps
    have ... ⊆ dom (locals (store s2))
      by simp
    finally show ?case by simp
  next
    case (If s0 e b s1 c1 c2 s2)
    from If.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
```

also
from *If.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
by *simp*
finally show *?case* **by** *simp*
next
case (*Loop s0 e b s1 c s2 l s3*)
show *?case*
proof (*cases the-Bool b*)
case *True*
with *Loop.hyps*
obtain
s0-s1:
dom (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*)) **and**
s1-s2: *dom* (*locals* (*store s1*)) ⊆ *dom* (*locals* (*store s2*)) **and**
s2-s3: *dom* (*locals* (*store s2*)) ⊆ *dom* (*locals* (*store s3*))
by *simp*
note *s0-s1* **also note** *s1-s2* **also note** *s2-s3*
finally show *?thesis*
by *simp*
next
case *False*
with *Loop.hyps* **show** *?thesis*
by *simp*
qed
next
case *Jmp* **thus** *?case* **by** *simp*
next
case *Throw* **thus** *?case* **by** *simp*
next
case (*Try s0 c1 s1 s2 C vn c2 s3*)
then
have *s0-s1*: *dom* (*locals* (*store* ((*Norm s0*)::*state*)))
⊆ *dom* (*locals* (*store s1*)) **by** *simp*
from ‹*G*⊢*s1* −*sxalloc*→ *s2*›
have *s1-s2*: *dom* (*locals* (*store s1*)) ⊆ *dom* (*locals* (*store s2*))
by (*rule dom-locals-sxalloc-mono*)
thus *?case*
proof (*cases G*,*s2*⊢*catch C*)
case *True*
note *s0-s1* **also note** *s1-s2*
also
from *True Try.hyps*
have *dom* (*locals* (*store* (*new-xcpt-var vn s2*)))
⊆ *dom* (*locals* (*store s3*))
by *simp*
hence *dom* (*locals* (*store s2*)) ⊆ *dom* (*locals* (*store s3*))
by (*cases s2*, *simp*)
finally show *?thesis* **by** *simp*
next
case *False*
note *s0-s1* **also note** *s1-s2*
finally
show *?thesis*
using *False Try.hyps* **by** *simp*
qed
next
case (*Fin s0 c1 x1 s1 c2 s2 s3*)
show *?case*

```
    proof (cases ∃ err. x1 = Some (Error err))
      case True
      with Fin.hyps show ?thesis
        by simp
    next
      case False
      from Fin.hyps
      have dom (locals (store ((Norm s0)::state)))
             ⊆ dom (locals (store (x1, s1)))
        by simp
      hence dom (locals (store ((Norm s0)::state)))
              ⊆ dom (locals (store ((Norm s1)::state)))
        by simp
      also
      from Fin.hyps
      have ... ⊆ dom (locals (store s2))
        by simp
      finally show ?thesis
        using Fin.hyps by simp
    qed
  next
    case (Init C c s0 s3 s1 s2)
    show ?case
    proof (cases inited C (globs s0))
      case True
      with Init.hyps show ?thesis by simp
    next
      case False
      with Init.hyps
      obtain s0-s1: dom (locals (store (Norm ((init-class-obj G C) s0))))
                      ⊆ dom (locals (store s1)) and
             s3: s3 = (set-lvars (locals (snd s1))) s2
        by simp
      from s0-s1
      have dom (locals (store (Norm s0))) ⊆ dom (locals (store s1))
        by (cases s0) simp
      with s3
      have dom (locals (store (Norm s0))) ⊆ dom (locals (store s3))
        by (cases s2) simp
      thus ?thesis by simp
    qed
  next
    case (NewC s0 C s1 a s2)
    note halloc = ‹G⊢s1 −halloc CInst C≻a→ s2›
    from NewC.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from halloc
    have ...  ⊆ dom (locals (store s2)) by (rule dom-locals-halloc-mono)
    finally show ?case by simp
  next
    case (NewA s0 T s1 e i s2 a s3)
    note halloc = ‹G⊢abupd (check-neg i) s2 −halloc Arr T (the-Intg i)≻a→ s3›
    from NewA.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from NewA.hyps
```

```
    have ... ⊆ dom (locals (store s2)) by simp
    also
    from halloc
    have ... ⊆  dom (locals (store s3))
      by (rule dom-locals-halloc-mono [elim-format]) simp
    finally show ?case by simp
  next
    case Cast thus ?case by simp
  next
    case Inst thus ?case by simp
  next
    case Lit thus ?case by simp
  next
    case UnOp thus ?case by simp
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2)
    from BinOp.hyps
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    also
    from BinOp.hyps
    have ... ⊆ dom (locals (store s2)) by simp
    finally show ?case by simp
  next
    case Super thus ?case by simp
  next
    case Acc thus ?case by simp
  next
    case (Ass s0 va w f s1 e v s2)
    from Ass.hyps
    have s0-s1:
      dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by simp
    show ?case
    proof (cases normal s1)
      case True
      with Ass.hyps
      have ass-ok:
        ⋀ s val. dom (locals (store s)) ⊆ dom (locals (store (f val s)))
        by simp
      note s0-s1
      also
      from Ass.hyps
      have dom (locals (store s1)) ⊆ dom (locals (store s2))
        by simp
      also
      from ass-ok
      have ... ⊆ dom (locals (store (assign f v s2)))
        by (rule dom-locals-assign-mono)
      finally show ?thesis by simp
    next
      case False
      with ‹G⊢s1 −e−≻v→ s2›
      have s2=s1
        by auto
      with s0-s1 False
      have dom (locals (store ((Norm s0)::state)))
            ⊆ dom (locals (store (assign f v s2)))
        by simp
```

thus *?thesis*
by *simp*
qed
next
case (*Cond s0 e0 b s1 e1 e2 v s2*)
from *Cond.hyps*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
by *simp*
also
from *Cond.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
by *simp*
finally show *?case* **by** *simp*
next
case (*Call s0 e a′ s1 args vs s2 D mode statT mn pTs s3 s3′ accC v s4*)
note *s3* = ‹*s3* = *init-lvars G D* (|*name* = *mn*, *parTs* = *pTs*|) *mode a′ vs s2*›
from *Call.hyps*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
by *simp*
also
from *Call.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
by *simp*
also
have ... ⊆ *dom* (*locals* (*store* ((*set-lvars* (*locals* (*store s2*))) *s4*)))
by (*cases s4*) *simp*
finally show *?case* **by** *simp*
next
case *Methd* **thus** *?case* **by** *simp*
next
case (*Body s0 D s1 c s2 s3*)
from *Body.hyps*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
by *simp*
also
from *Body.hyps*
have ... ⊆ *dom* (*locals* (*store s2*))
by *simp*
also
have ... ⊆ *dom* (*locals* (*store* (*abupd* (*absorb Ret*) *s2*)))
by *simp*
also
have ... ⊆ *dom* (*locals* (*store* (*abupd* (*absorb Ret*) *s3*)))
proof −
from ‹*s3* =
(*if* ∃ *l*. *abrupt s2* = *Some* (*Jump* (*Break l*)) ∨
abrupt s2 = *Some* (*Jump* (*Cont l*))
then abupd (λ*x*. *Some* (*Error CrossMethodJump*)) *s2 else s2*)›
show *?thesis*
by *simp*
qed
finally show *?case* **by** *simp*
next
case *LVar*
thus *?case*
using *dom-locals-lvar-mono*
by *simp*
next
case (*FVar s0 statDeclC s1 e a s2 v s2′ stat fn s3 accC*)

from *FVar.hyps*
obtain *s2'*: *s2' = snd (fvar statDeclC stat fn a s2)* **and**
v: *v = fst (fvar statDeclC stat fn a s2)*
by (*cases fvar statDeclC stat fn a s2*) *simp*
from *v*
have ∀ *s val. dom (locals (store s))*
⊆ *dom (locals (store (snd v val s)))* (**is** *?V-ok*)
by (*simp add: dom-locals-fvar-vvar-mono*)
hence *v-ok*: (∀ *vv. In2 v = In2 vv* ∧ *normal s3* ⟶ *?V-ok*)
by − (*intro strip, simp*)
note *s3* = ‹*s3 = check-field-access G accC statDeclC fn stat a s2'*›
from *FVar.hyps*
have *dom (locals (store ((Norm s0)::state)))* ⊆ *dom (locals (store s1))*
by *simp*
also
from *FVar.hyps*
have . . . ⊆ *dom (locals (store s2))*
by *simp*
also
from *s2'*
have . . . ⊆ *dom (locals (store s2'))*
by (*simp add: dom-locals-fvar-mono*)
also
from *s3*
have . . . ⊆ *dom (locals (store s3))*
by (*simp add: check-field-access-def Let-def*)
finally
show *?case*
using *v-ok*
by *simp*
next
case (*AVar s0 e1 a s1 e2 i s2 v s2'*)
from *AVar.hyps*
obtain *s2'*: *s2' = snd (avar G i a s2)* **and**
v: *v = fst (avar G i a s2)*
by (*cases avar G i a s2*) *simp*
from *v*
have ∀ *s val. dom (locals (store s))*
⊆ *dom (locals (store (snd v val s)))* (**is** *?V-ok*)
by (*simp add: dom-locals-avar-vvar-mono*)
hence *v-ok*: (∀ *vv. In2 v = In2 vv* ∧ *normal s2'* ⟶ *?V-ok*)
by − (*intro strip, simp*)
from *AVar.hyps*
have *dom (locals (store ((Norm s0)::state)))* ⊆ *dom (locals (store s1))*
by *simp*
also
from *AVar.hyps*
have . . . ⊆ *dom (locals (store s2))*
by *simp*
also
from *s2'*
have . . . ⊆ *dom (locals (store s2'))*
by (*simp add: dom-locals-avar-mono*)
finally
show *?case* **using** *v-ok* **by** *simp*
next
case *Nil* **thus** *?case* **by** *simp*
next
case (*Cons s0 e v s1 es vs s2*)

from *Cons.hyps*
have $dom\ (locals\ (store\ ((Norm\ s0)::state))) \subseteq dom\ (locals\ (store\ s1))$
by *simp*
also
from *Cons.hyps*
have $\ldots \subseteq dom\ (locals\ (store\ s2))$
by *simp*
finally show *?case* **by** *simp*
qed
qed

lemma *dom-locals-eval-mono-elim*:
assumes $eval$: $G\vdash\ s0\ -t\succ\rightarrow\ (v,s1)$
obtains $dom\ (locals\ (store\ s0)) \subseteq dom\ (locals\ (store\ s1))$ **and**
$\bigwedge\ vv\ s\ val.\ [\![v=In2\ vv;\ normal\ s1]\!]$
$\Longrightarrow dom\ (locals\ (store\ s))$
$\subseteq dom\ (locals\ (store\ ((snd\ vv)\ val\ s)))$
using *eval* **by** (*rule dom-locals-eval-mono* [*THEN conjE*]) (*rule that*, *auto*)

lemma *halloc-no-abrupt*:
assumes *halloc*: $G\vdash s0-halloc\ oi\succ a\rightarrow s1$ **and**
normal: *normal s1*
shows *normal s0*
proof −
from *halloc normal* **show** *?thesis*
by *cases simp-all*
qed

lemma *sxalloc-mono-no-abrupt*:
assumes *sxalloc*: $G\vdash s0-sxalloc\rightarrow s1$ **and**
normal: *normal s1*
shows *normal s0*
proof −
from *sxalloc normal* **show** *?thesis*
by *cases simp-all*
qed

lemma *union-subseteqI*: $[\![A \cup B \subseteq C;\ A' \subseteq A;\ B' \subseteq B]\!] \Longrightarrow A' \cup B' \subseteq C$
by *blast*

lemma *union-subseteqIl*: $[\![A \cup B \subseteq C;\ A' \subseteq A]\!] \Longrightarrow A' \cup B \subseteq C$
by *blast*

lemma *union-subseteqIr*: $[\![A \cup B \subseteq C;\ B' \subseteq B]\!] \Longrightarrow A \cup B' \subseteq C$
by *blast*

lemma *subseteq-union-transl* [*trans*]: $[\![A \subseteq B;\ B \cup C \subseteq D]\!] \Longrightarrow A \cup C \subseteq D$
by *blast*

lemma *subseteq-union-transr* [*trans*]: $[\![A \subseteq B;\ C \cup B \subseteq D]\!] \Longrightarrow A \cup C \subseteq D$
by *blast*

lemma *union-subseteq-weaken*: $\llbracket A \cup B \subseteq C;\ \llbracket A \subseteq C;\ B \subseteq C\rrbracket \Longrightarrow P\ \rrbracket \Longrightarrow P$
  by *blast*

lemma *assigns-good-approx*:
  assumes
      eval: $G\vdash s0\ -t\succ\rightarrow (v,s1)$ **and**
    normal: *normal s1*
  shows *assigns t* $\subseteq$ *dom* (*locals* (*store s1*))
proof −
  from *eval normal* **show** *?thesis*
  proof (*induct*)
    case *Abrupt* **thus** *?case* **by** *simp*
  next — For statements its trivial, since then *assigns t* = {}
    case *Skip* **show** *?case* **by** *simp*
  next
    case *Expr* **show** *?case* **by** *simp*
  next
    case *Lab* **show** *?case* **by** *simp*
  next
    case *Comp* **show** *?case* **by** *simp*
  next
    case *If* **show** *?case* **by** *simp*
  next
    case *Loop* **show** *?case* **by** *simp*
  next
    case *Jmp* **show** *?case* **by** *simp*
  next
    case *Throw* **show** *?case* **by** *simp*
  next
    case *Try* **show** *?case* **by** *simp*
  next
    case *Fin* **show** *?case* **by** *simp*
  next
    case *Init* **show** *?case* **by** *simp*
  next
    case *NewC* **show** *?case* **by** *simp*
  next
    case (*NewA s0 T s1 e i s2 a s3*)
    note *halloc* = ‹$G\vdash$*abupd* (*check-neg i*) *s2* −*halloc Arr T* (*the-Intg i*)$\succ a\rightarrow$ *s3*›
    have *assigns* (*In1l e*) $\subseteq$ *dom* (*locals* (*store s2*))
    proof −
      from *NewA*
      have *normal* (*abupd* (*check-neg i*) *s2*)
        by − (*erule halloc-no-abrupt* [*rule-format*])
      hence *normal s2* **by** (*cases s2*) *simp*
      with *NewA.hyps*
      show *?thesis* **by** *iprover*
    qed
    also
    from *halloc*
    have ... $\subseteq$ *dom* (*locals* (*store s3*))
      by (*rule dom-locals-halloc-mono* [*elim-format*]) *simp*
    finally show *?case* **by** *simp*
  next
    case (*Cast s0 e v s1 s2 T*)
    hence *normal s1* **by** (*cases s1*,*simp*)

```
    with Cast.hyps
    have assigns (In1l e) ⊆ dom (locals (store s1))
      by simp
    also
    from Cast.hyps
    have ... ⊆ dom (locals (store s2))
      by simp
    finally
    show ?case
      by simp
  next
    case Inst thus ?case by simp
  next
    case Lit thus ?case by simp
  next
    case UnOp thus ?case by simp
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2)
    hence normal s1 by − (erule eval-no-abrupt-lemma [rule-format])
    with BinOp.hyps
    have assigns (In1l e1) ⊆ dom (locals (store s1))
      by iprover
    also
    have ...  ⊆ dom (locals (store s2))
    proof −
      note ‹G⊢s1 −(if need-second-arg binop v1 then In1l e2
                     else In1r Skip)≻→ (In1 v2, s2)›
      thus ?thesis
        by (rule dom-locals-eval-mono-elim)
    qed
    finally have s2: assigns (In1l e1) ⊆ dom (locals (store s2)) .
    show ?case
    proof (cases binop=CondAnd ∨ binop=CondOr)
      case True
      with s2 show ?thesis by simp
    next
      case False
      with BinOp
      have assigns (In1l e2) ⊆ dom (locals (store s2))
        by (simp add: need-second-arg-def)
      with s2
      show ?thesis using False by (simp add: Un-subset-iff)
    qed
  next
    case Super thus ?case by simp
  next
    case Acc thus ?case by simp
  next
    case (Ass s0 va w f s1 e v s2)
    note nrm-ass-s2 = ‹normal (assign f v s2)›
    hence nrm-s2: normal s2
      by (cases s2, simp add: assign-def Let-def)
    with Ass.hyps
    have nrm-s1: normal s1
      by − (erule eval-no-abrupt-lemma [rule-format])
    with Ass.hyps
    have assigns (In2 va) ⊆ dom (locals (store s1))
      by iprover
    also
```

```
    from Ass.hyps
    have ... ⊆ dom (locals (store s2))
      by − (erule dom-locals-eval-mono-elim)
    also
    from nrm-s2 Ass.hyps
    have assigns (In1l e) ⊆ dom (locals (store s2))
      by iprover
    ultimately
    have assigns (In2 va) ∪ assigns (In1l e) ⊆  dom (locals (store s2))
      by (rule Un-least)
    also
    from Ass.hyps nrm-s1
    have ...  ⊆ dom (locals (store (f v s2)))
      by − (erule dom-locals-eval-mono-elim, cases s2,simp)
    then
    have dom (locals (store s2))  ⊆ dom (locals (store (assign f v s2)))
      by (rule dom-locals-assign-mono)
    finally
    have va-e:  assigns (In2 va) ∪ assigns (In1l e)
             ⊆ dom (locals (snd (assign f v s2))) .
    show ?case
    proof (cases ∃ n. va = LVar n)
      case False
      with va-e show ?thesis
        by (simp add: Un-assoc)
    next
      case True
      then obtain n where va: va = LVar n
        by blast
      with Ass.hyps
      have G⊢Norm s0 −LVar n=≻(w,f)→ s1
        by simp
      hence (w,f) = lvar n s0
        by (rule eval-elim-cases) simp
      with nrm-ass-s2
      have n ∈ dom (locals (store (assign f v s2)))
        by (cases s2) (simp add: assign-def Let-def lvar-def)
      with va-e True va
      show ?thesis by (simp add: Un-assoc)
    qed
  next
    case (Cond s0 e0 b s1 e1 e2 v s2)
    hence normal s1
      by − (erule eval-no-abrupt-lemma [rule-format])
    with Cond.hyps
    have assigns (In1l e0) ⊆ dom (locals (store s1))
      by iprover
    also from Cond.hyps
    have ... ⊆ dom (locals (store s2))
      by − (erule dom-locals-eval-mono-elim)
    finally have e0: assigns (In1l e0) ⊆ dom (locals (store s2)) .
    show ?case
    proof (cases the-Bool b)
      case True
      with Cond
      have assigns (In1l e1) ⊆ dom (locals (store s2))
        by simp
      hence assigns (In1l e1) ∩ assigns (In1l e2) ⊆ ...
        by blast
```

```
    with e0
    have assigns (In1l e0) ∪ assigns (In1l e1) ∩ assigns (In1l e2)
          ⊆ dom (locals (store s2))
      by (rule Un-least)
    thus ?thesis using True by simp
  next
    case False
    with Cond
    have  assigns (In1l e2) ⊆ dom (locals (store s2))
      by simp
    hence assigns (In1l e1) ∩ assigns (In1l e2) ⊆ ...
      by blast
    with e0
    have assigns (In1l e0) ∪ assigns (In1l e1) ∩ assigns (In1l e2)
          ⊆ dom (locals (store s2))
      by (rule Un-least)
    thus ?thesis using False by simp
  qed
next
  case (Call s0 e a' s1 args vs s2 D mode statT mn pTs s3 s3' accC v s4)
  have nrm-s2: normal s2
  proof −
    from ‹normal ((set-lvars (locals (snd s2))) s4)›
    have normal-s4: normal s4 by simp
    hence normal s3' using Call.hyps
      by − (erule eval-no-abrupt-lemma [rule-format])
    moreover note
     ‹s3' = check-method-access G accC statT mode (|name=mn, parTs=pTs|) a' s3›
    ultimately have normal s3
      by (cases s3) (simp add: check-method-access-def Let-def)
    moreover
    note s3 = ‹s3 = init-lvars G D (|name = mn, parTs = pTs|) mode a' vs s2›
    ultimately show normal s2
      by (cases s2) (simp add: init-lvars-def2)
  qed
  hence normal s1 using Call.hyps
    by − (erule eval-no-abrupt-lemma [rule-format])
  with Call.hyps
  have assigns (In1l e) ⊆ dom (locals (store s1))
    by iprover
  also from Call.hyps
  have ... ⊆  dom (locals (store s2))
    by − (erule dom-locals-eval-mono-elim)
  also
  from nrm-s2 Call.hyps
  have assigns (In3 args) ⊆ dom (locals (store s2))
    by iprover
  ultimately have assigns (In1l e) ∪ assigns (In3 args) ⊆ ...
    by (rule Un-least)
  also
  have ... ⊆ dom (locals (store ((set-lvars (locals (store s2))) s4)))
    by (cases s4) simp
  finally show ?case
    by simp
next
  case Methd thus ?case by simp
next
  case Body thus ?case by simp
next
```

case *LVar* **thus** *?case* **by** *simp*
next
case (*FVar s0 statDeclC s1 e a s2 v s2′ stat fn s3 accC*)
note *s3* = ‹*s3* = *check-field-access G accC statDeclC fn stat a s2′*›
note *avar* = ‹(*v*, *s2′*) = *fvar statDeclC stat fn a s2*›
have *nrm-s2*: *normal s2*
proof −
note ‹*normal s3*›
with *s3* **have** *normal s2′*
by (*cases s2′*) (*simp add*: *check-field-access-def Let-def*)
with *avar* **show** *normal s2*
by (*cases s2*) (*simp add*: *fvar-def2*)
qed
with *FVar.hyps*
have *assigns* (*In1l e*) ⊆ *dom* (*locals* (*store s2*))
by *iprover*
also
have . . . ⊆ *dom* (*locals* (*store s2′*))
proof −
from *avar*
have *s2′* = *snd* (*fvar statDeclC stat fn a s2*)
by (*cases fvar statDeclC stat fn a s2*) *simp*
thus *?thesis*
by *simp* (*rule dom-locals-fvar-mono*)
qed
also from *s3*
have . . . ⊆ *dom* (*locals* (*store s3*))
by (*cases s2′*) (*simp add*: *check-field-access-def Let-def*)
finally show *?case*
by *simp*
next
case (*AVar s0 e1 a s1 e2 i s2 v s2′*)
note *avar* = ‹(*v*, *s2′*) = *avar G i a s2*›
have *nrm-s2*: *normal s2*
proof −
from *avar* **and** ‹*normal s2′*›
show *?thesis* **by** (*cases s2*) (*simp add*: *avar-def2*)
qed
with *AVar.hyps*
have *normal s1*
by − (*erule eval-no-abrupt-lemma* [*rule-format*])
with *AVar.hyps*
have *assigns* (*In1l e1*) ⊆ *dom* (*locals* (*store s1*))
by *iprover*
also from *AVar.hyps*
have . . . ⊆ *dom* (*locals* (*store s2*))
by − (*erule dom-locals-eval-mono-elim*)
also
from *AVar.hyps nrm-s2*
have *assigns* (*In1l e2*) ⊆ *dom* (*locals* (*store s2*))
by *iprover*
ultimately
have *assigns* (*In1l e1*) ∪ *assigns* (*In1l e2*) ⊆ . . .
by (*rule Un-least*)
also
have *dom* (*locals* (*store s2*)) ⊆ *dom* (*locals* (*store s2′*))
proof −
from *avar* **have** *s2′* = *snd* (*avar G i a s2*)
by (*cases avar G i a s2*) *simp*

```
      thus ?thesis
        by simp (rule dom-locals-avar-mono)
    qed
    finally
    show ?case
      by simp
  next
    case Nil show ?case by simp
  next
    case (Cons s0 e v s1 es vs s2)
    have assigns (In1l e) ⊆ dom (locals (store s1))
    proof −
      from Cons
      have normal s1 by − (erule eval-no-abrupt-lemma [rule-format])
      with Cons.hyps show ?thesis by iprover
    qed
    also from Cons.hyps
    have . . . ⊆  dom (locals (store s2))
      by − (erule dom-locals-eval-mono-elim)
    also from Cons
    have assigns (In3 es) ⊆ dom (locals (store s2))
      by iprover
    ultimately
    have assigns (In1l e) ∪ assigns (In3 es) ⊆ dom (locals (store s2))
      by (rule Un-least)
    thus ?case
      by simp
  qed
qed

corollary assignsE-good-approx:
  assumes
      eval: prg Env⊢ s0 −e−≻v→ s1 and
    normal: normal s1
  shows assignsE e ⊆ dom (locals (store s1))
proof −
from eval normal show ?thesis
  by (rule assigns-good-approx [elim-format]) simp
qed

corollary assignsV-good-approx:
  assumes
     eval: prg Env⊢ s0 −v=≻vf→ s1 and
   normal: normal s1
  shows assignsV v ⊆ dom (locals (store s1))
proof −
from eval normal show ?thesis
  by (rule assigns-good-approx [elim-format]) simp
qed

corollary assignsEs-good-approx:
  assumes
      eval: prg Env⊢ s0 −es≐≻vs→ s1 and
    normal: normal s1
  shows assignsEs es ⊆ dom (locals (store s1))
proof −
from eval normal show ?thesis
  by (rule assigns-good-approx [elim-format]) simp
qed
```

```
lemma constVal-eval:
 assumes const: constVal e = Some c and
         eval: G⊢Norm s0 −e−≻v→ s
  shows v = c ∧ normal s
proof −
  have  True and
       ⋀ c v s0 s. ⟦ constVal e = Some c; G⊢Norm s0 −e−≻v→ s⟧
                  ⟹ v = c ∧ normal s
       and True and True
  proof (induct  rule: var-expr-stmt.inducts)
    case NewC hence False by simp thus ?case ..
  next
    case NewA hence False by simp thus ?case ..
  next
    case Cast hence False by simp thus ?case ..
  next
    case Inst hence False by simp thus ?case ..
  next
    case (Lit val c v s0 s)
    note ‹constVal (Lit val) = Some c›
    moreover
    from ‹G⊢Norm s0 −Lit val−≻v→ s›
    obtain v=val and normal s
      by cases simp
    ultimately show v=c ∧ normal s by simp
  next
    case (UnOp unop e c v s0 s)
    note const = ‹constVal (UnOp unop e) = Some c›
    then obtain ce where ce: constVal e = Some ce by simp
    from ‹G⊢Norm s0 −UnOp unop e−≻v→ s›
    obtain ve where ve: G⊢Norm s0 −e−≻ve→ s and
                    v: v = eval-unop unop ve
      by cases simp
    from ce ve
    obtain eq-ve-ce: ve=ce and nrm-s: normal s
      by (rule UnOp.hyps [elim-format]) iprover
    from eq-ve-ce const ce v
    have v=c
      by simp
    from this nrm-s
    show ?case ..
  next
    case (BinOp binop e1 e2 c v s0 s)
    note const = ‹constVal (BinOp binop e1 e2) = Some c›
    then obtain c1 c2 where c1: constVal e1 = Some c1 and
                            c2: constVal e2 = Some c2 and
                             c: c = eval-binop binop c1 c2
      by simp
    from ‹G⊢Norm s0 −BinOp binop e1 e2−≻v→ s›
    obtain v1 s1 v2
      where v1: G⊢Norm s0 −e1−≻v1→ s1 and
            v2: G⊢s1 −(if need-second-arg binop v1 then In1l e2
                          else In1r Skip)≻→ (In1 v2, s) and
             v: v = eval-binop binop v1 v2
      by cases simp
    from c1 v1
    obtain eq-v1-c1: v1 = c1 and
```

```
          nrm-s1: normal s1
    by (rule BinOp.hyps [elim-format]) iprover
  show ?case
  proof (cases need-second-arg binop v1)
    case True
    with v2 nrm-s1 obtain s1'
      where G⊢Norm s1' −e2−≻v2→ s
      by (cases s1) simp
    with c2 obtain v2 = c2 normal s
      by (rule BinOp.hyps [elim-format]) iprover
    with c c1 c2 eq-v1-c1 v
    show ?thesis by simp
  next
    case False
    with nrm-s1 v2
    have s=s1
      by (cases s1) (auto elim!: eval-elim-cases)
    moreover
    from False c v eq-v1-c1
    have v = c
      by (simp add: eval-binop-arg2-indep)
    ultimately
    show ?thesis
      using nrm-s1 by simp
  qed
next
  case Super hence False by simp thus ?case ..
next
  case Acc hence False by simp thus ?case ..
next
  case Ass hence False by simp thus ?case ..
next
  case (Cond b e1 e2 c v s0 s)
  note c = ‹constVal (b ? e1 : e2) = Some c›
  then obtain cb c1 c2 where
    cb: constVal b  = Some cb and
    c1: constVal e1 = Some c1 and
    c2: constVal e2 = Some c2
    by (auto split: bool.splits)
  from ‹G⊢Norm s0 −b ? e1 : e2−≻v→ s›
  obtain vb s1
    where     vb: G⊢Norm s0 −b−≻vb→ s1 and
          eval-v: G⊢s1 −(if the-Bool vb then e1 else e2)−≻v→ s
    by cases simp
  from cb vb
  obtain eq-vb-cb: vb = cb and nrm-s1: normal s1
    by (rule Cond.hyps [elim-format]) iprover
  show ?case
  proof (cases the-Bool vb)
    case True
    with c cb c1 eq-vb-cb
    have c = c1
      by simp
    moreover
    from True eval-v nrm-s1 obtain s1'
      where G⊢Norm s1' −e1−≻v→ s
      by (cases s1) simp
    with c1 obtain c1 = v normal s
      by (rule Cond.hyps [elim-format]) iprover
```

```
      ultimately show ?thesis by simp
    next
      case False
      with c cb c2 eq-vb-cb
      have c = c2
        by simp
      moreover
      from False eval-v nrm-s1 obtain s1'
        where G⊢Norm s1' −e2−≻v→ s
        by (cases s1) simp
      with c2 obtain c2 = v normal s
        by (rule Cond.hyps [elim-format]) iprover
      ultimately show ?thesis by simp
    qed
  next
    case Call hence False by simp thus ?case ..
  qed simp-all
  with const eval
  show ?thesis
    by iprover
qed

lemmas constVal-eval-elim = constVal-eval [THEN conjE]


lemma eval-unop-type:
  typeof dt (eval-unop unop v) = Some (PrimT (unop-type unop))
  by (cases unop) simp-all


lemma eval-binop-type:
  typeof dt (eval-binop binop v1 v2) = Some (PrimT (binop-type binop))
  by (cases binop) simp-all


lemma constVal-Boolean:
 assumes const: constVal e = Some c and
           wt: Env⊢e::−PrimT Boolean
  shows typeof empty-dt c = Some (PrimT Boolean)
proof −
  have True and
      ⋀ c. ⟦constVal e = Some c;Env⊢e::−PrimT Boolean⟧
              ⟹ typeof empty-dt c = Some (PrimT Boolean)
       and True and True
  proof (induct rule: var-expr-stmt.inducts)
    case NewC hence False by simp thus ?case ..
  next
    case NewA hence False by simp thus ?case ..
  next
    case Cast hence False by simp thus ?case ..
  next
    case Inst hence False by simp thus ?case ..
  next
    case (Lit v c)
    from ‹constVal (Lit v) = Some c›
    have c=v by simp
    moreover
    from ‹Env⊢Lit v::−PrimT Boolean›
    have typeof empty-dt v = Some (PrimT Boolean)
```

```
    by cases simp
  ultimately show ?case by simp
next
  case (UnOp unop e c)
  from ‹Env⊢UnOp unop e::−PrimT Boolean›
  have Boolean = unop-type unop by cases simp
  moreover
  from ‹constVal (UnOp unop e) = Some c›
  obtain ce where c = eval-unop unop ce by auto
  ultimately show ?case by (simp add: eval-unop-type)
next
  case (BinOp binop e1 e2 c)
  from ‹Env⊢BinOp binop e1 e2::−PrimT Boolean›
  have Boolean = binop-type binop by cases simp
  moreover
  from ‹constVal (BinOp binop e1 e2) = Some c›
  obtain c1 c2 where c = eval-binop binop c1 c2 by auto
  ultimately show ?case by (simp add: eval-binop-type)
next
  case Super hence False by simp thus ?case ..
next
  case Acc hence False by simp thus ?case ..
next
  case Ass hence False by simp thus ?case ..
next
  case (Cond b e1 e2 c)
  note c = ‹constVal (b ? e1 : e2) = Some c›
  then obtain cb c1 c2 where
    cb: constVal b  = Some cb and
    c1: constVal e1 = Some c1 and
    c2: constVal e2 = Some c2
    by (auto split: bool.splits)
  note wt = ‹Env⊢b ? e1 : e2::−PrimT Boolean›
  then
  obtain T1 T2
    where Env⊢b::−PrimT Boolean and
          wt-e1: Env⊢e1::−PrimT Boolean and
          wt-e2: Env⊢e2::−PrimT Boolean
    by cases (auto dest: widen-Boolean2)
  show ?case
  proof (cases the-Bool cb)
    case True
    from c1 wt-e1
    have  typeof empty-dt c1 = Some (PrimT Boolean)
      by (rule Cond.hyps)
    with True c cb c1 show ?thesis by simp
  next
    case False
    from c2 wt-e2
    have typeof empty-dt c2 = Some (PrimT Boolean)
      by (rule Cond.hyps)
    with False c cb c2 show ?thesis by simp
  qed
next
  case Call hence False by simp thus ?case ..
qed simp-all
with const wt
show ?thesis
  by iprover
```

qed

lemma *assigns-if-good-approx*:
  assumes
    eval: *prg Env*⊢ *s0* −*e*−≻*b*→ *s1* **and**
   normal: *normal s1* **and**
    bool: *Env*⊢ *e*::−*PrimT Boolean*
  shows *assigns-if* (*the-Bool b*) *e* ⊆ *dom* (*locals* (*store s1*))
proof −
  — To properly perform induction on the evaluation relation we have to generalize the lemma to terms not only expressions.
  { fix *t val*
   assume *eval′*: *prg Env*⊢ *s0* −*t*≻→ (*val*,*s1*)
   assume *bool′*: *Env*⊢ *t*::*Inl* (*PrimT Boolean*)
   assume *expr*: ∃ *expr*. *t*=*In1l expr*
   have *assigns-if* (*the-Bool* (*the-In1 val*)) (*the-In1l t*)
        ⊆ *dom* (*locals* (*store s1*))
   using *eval′ normal bool′ expr*
   proof (*induct*)
    case *Abrupt* **thus** *?case* **by** *simp*
   next
    case (*NewC s0 C s1 a s2*)
    from ‹*Env*⊢*NewC C*::−*PrimT Boolean*›
    have *False*
     by *cases simp*
    thus *?case* **..**
   next
    case (*NewA s0 T s1 e i s2 a s3*)
    from ‹*Env*⊢*New T*[*e*]::−*PrimT Boolean*›
    have *False*
     by *cases simp*
    thus *?case* **..**
   next
    case (*Cast s0 e b s1 s2 T*)
    note *s2* = ‹*s2* = *abupd* (*raise-if* (¬ *prg Env*,*snd s1*⊢*b fits T*) *ClassCast*) *s1*›
    have *assigns-if* (*the-Bool b*) *e* ⊆ *dom* (*locals* (*store s1*))
    proof −
     from *s2* **and** ‹*normal s2*›
     have *normal s1*
      by (*cases s1*) *simp*
     moreover
     from ‹*Env*⊢*Cast T e*::−*PrimT Boolean*›
     have *Env*⊢*e*::− *PrimT Boolean*
      by *cases* (*auto dest*: *cast-Boolean2*)
     ultimately show *?thesis*
      by (*rule Cast.hyps* [*elim-format*]) *auto*
    qed
    also from *s2*
    have . . . ⊆ *dom* (*locals* (*store s2*))
     by *simp*
    finally show *?case* **by** *simp*
   next
    case (*Inst s0 e v s1 b T*)
    from ‹*prg Env*⊢*Norm s0* −*e*−≻*v*→ *s1*› **and** ‹*normal s1*›
    have *assignsE e* ⊆ *dom* (*locals* (*store s1*))
     by (*rule assignsE-good-approx*)
    thus *?case*
     by *simp*

```
next
  case (Lit s v)
  from ‹Env⊢Lit v::−PrimT Boolean›
  have typeof empty-dt v = Some (PrimT Boolean)
    by cases simp
  then obtain b where v=Bool b
    by (cases v) (simp-all add: empty-dt-def)
  thus ?case
    by simp
next
  case (UnOp s0 e v s1 unop)
  note bool = ‹Env⊢UnOp unop e::−PrimT Boolean›
  hence bool-e: Env⊢e::−PrimT Boolean
    by cases (cases unop,simp-all)
  show ?case
  proof (cases constVal (UnOp unop e))
    case None
    note ‹normal s1›
    moreover note bool-e
    ultimately have assigns-if (the-Bool v) e ⊆ dom (locals (store s1))
      by (rule UnOp.hyps [elim-format]) auto
    moreover
    from bool have unop = UNot
      by cases (cases unop, simp-all)
    moreover note None
    ultimately
    have assigns-if (the-Bool (eval-unop unop v)) (UnOp unop e)
          ⊆ dom (locals (store s1))
      by simp
    thus ?thesis by simp
  next
    case (Some c)
    moreover
    from ‹prg Env⊢Norm s0 −e−≻v→ s1›
    have prg Env⊢Norm s0 −UnOp unop e−≻eval-unop unop v→ s1
      by (rule eval.UnOp)
    with Some
    have eval-unop unop v=c
      by (rule constVal-eval-elim) simp
    moreover
    from Some bool
    obtain b where c=Bool b
      by (rule constVal-Boolean [elim-format])
         (cases c, simp-all add: empty-dt-def)
    ultimately
    have assigns-if (the-Bool (eval-unop unop v)) (UnOp unop e) = {}
      by simp
    thus ?thesis by simp
  qed
next
  case (BinOp s0 e1 v1 s1 binop e2 v2 s2)
  note bool = ‹Env⊢BinOp binop e1 e2::−PrimT Boolean›
  show ?case
  proof (cases constVal (BinOp binop e1 e2))
    case (Some c)
    moreover
    from BinOp.hyps
    have
      prg Env⊢Norm s0 −BinOp binop e1 e2−≻eval-binop binop v1 v2→ s2
```

```
      by − (rule eval.BinOp)
    with Some
    have eval-binop binop v1 v2=c
      by (rule constVal-eval-elim) simp
    moreover
    from Some bool
    obtain b where c = Bool b
      by (rule constVal-Boolean [elim-format])
         (cases c, simp-all add: empty-dt-def)
    ultimately
    have assigns-if (the-Bool (eval-binop binop v1 v2)) (BinOp binop e1 e2)
          = {}
      by simp
    thus ?thesis by simp
  next
    case None
    show ?thesis
    proof (cases binop=CondAnd ∨ binop=CondOr)
      case True
      from bool obtain bool-e1: Env⊢e1::−PrimT Boolean and
                       bool-e2: Env⊢e2::−PrimT Boolean
        using True by cases auto
      have assigns-if (the-Bool v1) e1 ⊆ dom (locals (store s1))
      proof −
        from BinOp have normal s1
          by − (erule eval-no-abrupt-lemma [rule-format])
        from this bool-e1
        show ?thesis
          by (rule BinOp.hyps [elim-format]) auto
      qed
      also
      from BinOp.hyps
      have ... ⊆ dom (locals (store s2))
        by − (erule dom-locals-eval-mono-elim,simp)
      finally
      have e1-s2: assigns-if (the-Bool v1) e1 ⊆ dom (locals (store s2)).
      from True show ?thesis
      proof
        assume condAnd: binop = CondAnd
        show ?thesis
        proof (cases the-Bool (eval-binop binop v1 v2))
          case True
          with condAnd
          have need-second: need-second-arg binop v1
            by (simp add: need-second-arg-def)
          from ‹normal s2›
          have assigns-if (the-Bool v2) e2 ⊆ dom (locals (store s2))
            by (rule BinOp.hyps [elim-format])
               (simp add: need-second bool-e2)+
          with e1-s2
          have assigns-if (the-Bool v1) e1 ∪ assigns-if (the-Bool v2) e2
                 ⊆ dom (locals (store s2))
            by (rule Un-least)
          with True condAnd None show ?thesis
            by simp
        next
          case False
          note binop-False = this
          show ?thesis
```

proof (*cases need-second-arg binop v1*)
case *True*
with *binop-False condAnd*
obtain *the-Bool v1=True* **and** *the-Bool v2 = False*
by (*simp add*: *need-second-arg-def*)
moreover
from ⟨*normal s2*⟩
have *assigns-if* (*the-Bool v2*) *e2* ⊆ *dom* (*locals* (*store s2*))
by (*rule BinOp.hyps* [*elim-format*]) (*simp add*: *True bool-e2*)+
with *e1-s2*
have *assigns-if* (*the-Bool v1*) *e1* ∪ *assigns-if* (*the-Bool v2*) *e2*
⊆ *dom* (*locals* (*store s2*))
by (*rule Un-least*)
moreover note *binop-False condAnd None*
ultimately show *?thesis*
by *auto*
next
case *False*
with *binop-False condAnd*
have *the-Bool v1=False*
by (*simp add*: *need-second-arg-def*)
with *e1-s2*
show *?thesis*
using *binop-False condAnd None* **by** *auto*
qed
qed
next
assume *condOr*: *binop = CondOr*
show *?thesis*
proof (*cases the-Bool* (*eval-binop binop v1 v2*))
case *False*
with *condOr*
have *need-second*: *need-second-arg binop v1*
by (*simp add*: *need-second-arg-def*)
from ⟨*normal s2*⟩
have *assigns-if* (*the-Bool v2*) *e2* ⊆ *dom* (*locals* (*store s2*))
by (*rule BinOp.hyps* [*elim-format*])
(*simp add*: *need-second bool-e2*)+
with *e1-s2*
have *assigns-if* (*the-Bool v1*) *e1* ∪ *assigns-if* (*the-Bool v2*) *e2*
⊆ *dom* (*locals* (*store s2*))
by (*rule Un-least*)
with *False condOr None* **show** *?thesis*
by *simp*
next
case *True*
note *binop-True = this*
show *?thesis*
proof (*cases need-second-arg binop v1*)
case *True*
with *binop-True condOr*
obtain *the-Bool v1=False* **and** *the-Bool v2 = True*
by (*simp add*: *need-second-arg-def*)
moreover
from ⟨*normal s2*⟩
have *assigns-if* (*the-Bool v2*) *e2* ⊆ *dom* (*locals* (*store s2*))
by (*rule BinOp.hyps* [*elim-format*]) (*simp add*: *True bool-e2*)+
with *e1-s2*
have *assigns-if* (*the-Bool v1*) *e1* ∪ *assigns-if* (*the-Bool v2*) *e2*

```
              ⊆ dom (locals (store s2))
            by (rule Un-least)
          moreover note binop-True condOr None
          ultimately show ?thesis
            by auto
        next
          case False
          with binop-True condOr
          have the-Bool v1=True
            by (simp add: need-second-arg-def)
          with e1-s2
          show ?thesis
            using binop-True condOr None by auto
        qed
      qed
    qed
  next
    case False
    note ‹¬ (binop = CondAnd ∨ binop = CondOr)›
    from BinOp.hyps
    have
      prg Env⊢Norm s0 −BinOp binop e1 e2−≻eval-binop binop v1 v2→ s2
      by − (rule eval.BinOp)
    moreover note ‹normal s2›
    ultimately
    have assignsE (BinOp binop e1 e2) ⊆ dom (locals (store s2))
      by (rule assignsE-good-approx)
    with False None
    show ?thesis
      by simp
  qed
qed
next
  case Super
  note ‹Env⊢Super::−PrimT Boolean›
  hence False
    by cases simp
  thus ?case ..
next
  case (Acc s0 va v f s1)
  from ‹prg Env⊢Norm s0 −va=≻(v, f)→ s1› and ‹normal s1›
  have assignsV va ⊆ dom (locals (store s1))
    by (rule assignsV-good-approx)
  thus ?case by simp
next
  case (Ass s0 va w f s1 e v s2)
  hence prg Env⊢Norm s0 −va := e−≻v→ assign f v s2
    by − (rule eval.Ass)
  moreover note ‹normal (assign f v s2)›
  ultimately
  have assignsE (va := e) ⊆ dom (locals (store (assign f v s2)))
    by (rule assignsE-good-approx)
  thus ?case by simp
next
  case (Cond s0 e0 b s1 e1 e2 v s2)
  from ‹Env⊢e0 ? e1 : e2::−PrimT Boolean›
  obtain wt-e1: Env⊢e1::−PrimT Boolean and
         wt-e2: Env⊢e2::−PrimT Boolean
    by cases (auto dest: widen-Boolean2)
```

```
note eval-e0 = ‹prg Env⊢Norm s0 −e0−≻b→ s1›
have e0-s2: assignsE e0 ⊆ dom (locals (store s2))
proof −
  note eval-e0
  moreover
  from Cond.hyps and ‹normal s2› have normal s1
    by − (erule eval-no-abrupt-lemma [rule-format],simp)
  ultimately
  have assignsE e0 ⊆ dom (locals (store s1))
    by (rule assignsE-good-approx)
  also
  from Cond
  have ... ⊆ dom (locals (store s2))
    by − (erule dom-locals-eval-mono [elim-format],simp)
  finally show ?thesis .
qed
show ?case
proof (cases constVal e0)
  case None
  have assigns-if (the-Bool v) e1 ∩ assigns-if (the-Bool v) e2
         ⊆ dom (locals (store s2))
  proof (cases the-Bool b)
    case True
    from ‹normal s2›
    have assigns-if (the-Bool v) e1 ⊆ dom (locals (store s2))
      by (rule Cond.hyps [elim-format]) (simp-all add: wt-e1 True)
    thus ?thesis
      by blast
  next
    case False
    from ‹normal s2›
    have assigns-if (the-Bool v) e2 ⊆ dom (locals (store s2))
      by (rule Cond.hyps [elim-format]) (simp-all add: wt-e2 False)
    thus ?thesis
      by blast
  qed
  with e0-s2
  have assignsE e0 ∪
        (assigns-if (the-Bool v) e1 ∩ assigns-if (the-Bool v) e2)
          ⊆ dom (locals (store s2))
    by (rule Un-least)
  with None show ?thesis
    by simp
next
  case (Some c)
  from this eval-e0 have eq-b-c: b=c
    by (rule constVal-eval-elim)
  show ?thesis
  proof (cases the-Bool c)
    case True
    from ‹normal s2›
    have assigns-if (the-Bool v) e1 ⊆ dom (locals (store s2))
      by (rule Cond.hyps [elim-format]) (simp-all add: eq-b-c True wt-e1)
    with e0-s2
    have assignsE e0 ∪ assigns-if (the-Bool v) e1 ⊆ ...
      by (rule Un-least)
    with Some True show ?thesis
      by simp
  next
```

```
        case False
        from ‹normal s2›
        have assigns-if (the-Bool v) e2 ⊆ dom (locals (store s2))
          by (rule Cond.hyps [elim-format]) (simp-all add: eq-b-c False wt-e2)
        with e0-s2
        have assignsE e0 ∪ assigns-if (the-Bool v) e2  ⊆ ...
          by (rule Un-least)
        with Some False show ?thesis
          by simp
      qed
    qed
  next
    case (Call s0 e a s1 args vs s2 D mode statT mn pTs s3 s3' accC v s4)
    hence
    prg Env⊢Norm s0 −({accC,statT,mode}e·mn( {pTs}args))−≻v→
                 (set-lvars (locals (store s2)) s4)
      by − (rule eval.Call)
    hence assignsE ({accC,statT,mode}e·mn( {pTs}args))
             ⊆  dom (locals (store ((set-lvars (locals (store s2))) s4)))
      using ‹normal ((set-lvars (locals (snd s2))) s4)›
      by (rule assignsE-good-approx)
    thus ?case by simp
  next
    case Methd show ?case by simp
  next
    case Body show ?case by simp
  qed simp+ — all the statements and variables
 }
 note generalized = this
 from eval bool show ?thesis
   by (rule generalized [elim-format]) simp+
qed


lemma assigns-if-good-approx':
  assumes   eval: G⊢s0 −e−≻b→ s1
     and  normal: normal s1
     and    bool: (|prg=G,cls=C,lcl=L|)⊢e::− (PrimT Boolean)
  shows assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
proof −
  from eval have prg (|prg=G,cls=C,lcl=L|)⊢s0 −e−≻b→ s1 by simp
  from this normal bool show ?thesis
    by (rule assigns-if-good-approx)
qed


lemma subset-Intl: A ⊆ C ⟹ A ∩ B ⊆ C
  by blast


lemma subset-Intr: B ⊆ C ⟹ A ∩ B ⊆ C
  by blast


lemma da-good-approx:
  assumes  eval: prg Env⊢s0 −t≻→ (v,s1) and
          wt: Env⊢t::T     (is ?Wt Env t T) and
          da: Env⊢ dom (locals (store s0)) »t» A  (is ?Da Env s0 t A) and
```

```
      wf: wf-prog (prg Env)
    shows (normal s1 ⟶ (nrm A ⊆ dom (locals (store s1)))) ∧
        (∀ l. abrupt s1 = Some (Jump (Break l)) ∧ normal s0
                ⟶ (brk A l ⊆ dom (locals (store s1)))) ∧
        (abrupt s1 = Some (Jump Ret) ∧ normal s0
                ⟶ Result ∈ dom (locals (store s1)))
    (is ?NormalAssigned s1 A ∧ ?BreakAssigned s0 s1 A ∧ ?ResAssigned s0 s1)
proof −
  note inj-term-simps [simp]
  obtain G where G: prg Env = G by (cases Env) simp
  with eval have eval: G⊢s0 −t≻→ (v,s1) by simp
  from G wf have wf: wf-prog G by simp
  let ?HypObj = λ t s0 s1.
      ∀ Env T A. ?Wt Env t T ⟶ ?Da Env s0 t A ⟶ prg Env = G
       ⟶ ?NormalAssigned s1 A ∧ ?BreakAssigned s0 s1 A ∧ ?ResAssigned s0 s1
  — Goal in object logic variant
  let ?Hyp = λt s0 s1. (⋀ Env T A. ⟦?Wt Env t T; ?Da Env s0 t A; prg Env = G⟧
        ⟹ ?NormalAssigned s1 A ∧ ?BreakAssigned s0 s1 A ∧ ?ResAssigned s0 s1)
  from eval and wt da G
  show ?thesis
  proof (induct arbitrary: Env T A)
    case (Abrupt xc s t Env T A)
    have da: Env⊢ dom (locals s) »t» A using Abrupt.prems by simp
    have ?NormalAssigned (Some xc,s) A
      by simp
    moreover
    have ?BreakAssigned (Some xc,s) (Some xc,s) A
      by simp
    moreover have ?ResAssigned (Some xc,s) (Some xc,s)
      by simp
    ultimately show ?case by (intro conjI)
  next
    case (Skip s Env T A)
    have da: Env⊢ dom (locals (store (Norm s))) »⟨Skip⟩» A
      using Skip.prems by simp
    hence nrm A = dom (locals (store (Norm s)))
      by (rule da-elim-cases) simp
    hence ?NormalAssigned (Norm s) A
      by auto
    moreover
    have ?BreakAssigned (Norm s) (Norm s) A
      by simp
    moreover have ?ResAssigned (Norm s) (Norm s)
      by simp
    ultimately show ?case by (intro conjI)
  next
    case (Expr s0 e v s1 Env T A)
    from Expr.prems
    show ?NormalAssigned s1 A ∧ ?BreakAssigned (Norm s0) s1 A
          ∧ ?ResAssigned (Norm s0) s1
      by (elim wt-elim-cases da-elim-cases)
         (rule Expr.hyps, auto)
  next
    case (Lab s0 c s1 j Env T A)
    note G = ‹prg Env = G›
    from Lab.prems
    obtain C l where
      da-c: Env⊢ dom (locals (snd (Norm s0))) »⟨c⟩» C and
         A: nrm A = nrm C ∩ (brk C) l brk A = rmlab l (brk C) and
```

```
      j: j = Break l
    by − (erule da-elim-cases, simp)
  from Lab.prems
  have wt-c: Env⊢c::√
    by − (erule wt-elim-cases, simp)
  from wt-c da-c G and Lab.hyps
  have norm-c: ?NormalAssigned s1 C and
       brk-c: ?BreakAssigned (Norm s0) s1 C and
       res-c: ?ResAssigned (Norm s0) s1
    by simp-all
  have ?NormalAssigned (abupd (absorb j) s1) A
  proof
    assume normal: normal (abupd (absorb j) s1)
    show nrm A ⊆ dom (locals (store (abupd (absorb j) s1)))
    proof (cases abrupt s1)
      case None
      with norm-c A
      show ?thesis
        by auto
    next
      case Some
      with normal j
      have abrupt s1 = Some (Jump (Break l))
        by (auto dest: absorb-Some-NoneD)
      with brk-c A
      show ?thesis
        by auto
    qed
  qed
  moreover
  have ?BreakAssigned (Norm s0) (abupd (absorb j) s1) A
  proof −
    {
      fix l′
      assume break: abrupt (abupd (absorb j) s1) = Some (Jump (Break l′))
      with j
      have l≠l′
        by (cases s1) (auto dest!: absorb-Some-JumpD)
      hence (rmlab l (brk C)) l′= (brk C) l′
        by (simp)
      with break brk-c A
      have
        (brk A l′ ⊆ dom (locals (store (abupd (absorb j) s1))))
        by (cases s1) auto
    }
    then show ?thesis
      by simp
  qed
  moreover
  from res-c have ?ResAssigned (Norm s0) (abupd (absorb j) s1)
    by (cases s1) (simp add: absorb-def)
  ultimately show ?case by (intro conjI)
next
  case (Comp s0 c1 s1 c2 s2 Env T A)
  note G = ‹prg Env = G›
  from Comp.prems
  obtain C1 C2
    where da-c1: Env⊢ dom (locals (snd (Norm s0))) »⟨c1⟩» C1 and
          da-c2: Env⊢ nrm C1 »⟨c2⟩» C2 and
```

```
          A: nrm A = nrm C2 brk A = (brk C1) ⇒∩ (brk C2)
    by (elim da-elim-cases) simp
  from Comp.prems
  obtain wt-c1: Env⊢c1::√ and
         wt-c2: Env⊢c2::√
    by (elim wt-elim-cases) simp
  note ‹PROP ?Hyp (In1r c1) (Norm s0) s1›
  with wt-c1 da-c1 G
  obtain nrm-c1: ?NormalAssigned s1 C1 and
         brk-c1: ?BreakAssigned (Norm s0) s1 C1 and
         res-c1: ?ResAssigned (Norm s0) s1
    by simp
  show ?case
  proof (cases normal s1)
    case True
    with nrm-c1 have nrm C1 ⊆ dom (locals (snd s1)) by iprover
    with da-c2 obtain C2'
      where  da-c2': Env⊢ dom (locals (snd s1)) »⟨c2⟩» C2' and
             nrm-c2: nrm C2 ⊆ nrm C2'                   and
             brk-c2: ∀ l. brk C2 l ⊆ brk C2' l
      by (rule da-weakenE) iprover
    note ‹PROP ?Hyp (In1r c2) s1 s2›
    with wt-c2 da-c2' G
    obtain nrm-c2': ?NormalAssigned s2 C2' and
           brk-c2': ?BreakAssigned s1 s2 C2' and
           res-c2 : ?ResAssigned s1 s2
      by simp
    from nrm-c2' nrm-c2 A
    have ?NormalAssigned s2 A
      by blast
    moreover from brk-c2' brk-c2 A
    have ?BreakAssigned s1 s2 A
      by fastsimp
    with True
    have ?BreakAssigned (Norm s0) s2 A by simp
    moreover from res-c2 True
    have ?ResAssigned (Norm s0) s2
      by simp
    ultimately show ?thesis by (intro conjI)
  next
    case False
    with ‹G⊢s1 −c2→ s2›
    have eq-s1-s2: s2=s1 by auto
    with False have ?NormalAssigned s2 A by blast
    moreover
    have ?BreakAssigned (Norm s0) s2 A
    proof (cases ∃ l. abrupt s1 = Some (Jump (Break l)))
      case True
      then obtain l where l: abrupt s1 = Some (Jump (Break l)) ..
      with brk-c1
      have brk C1 l ⊆ dom (locals (store s1))
        by simp
      with A eq-s1-s2
      have brk A l ⊆ dom (locals (store s2))
        by auto
      with l eq-s1-s2
      show ?thesis by simp
    next
      case False
```

```
      with eq-s1-s2 show ?thesis by simp
    qed
    moreover from False res-c1 eq-s1-s2
    have ?ResAssigned (Norm s0) s2
      by simp
    ultimately show ?thesis by (intro conjI)
  qed
next
  case (If s0 e b s1 c1 c2 s2 Env T A)
  note G = ‹prg Env = G›
  with If.hyps have eval-e: prg Env ⊢Norm s0 −e−≻b→ s1 by simp
  from If.prems
  obtain E C1 C2 where
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E and
    da-c1: Env⊢ (dom (locals (store ((Norm s0)::state)))
              ∪ assigns-if True e) »⟨c1⟩» C1 and
    da-c2: Env⊢ (dom (locals (store ((Norm s0)::state)))
              ∪ assigns-if False e) »⟨c2⟩» C2 and
    A: nrm A = nrm C1 ∩ nrm C2  brk A = brk C1 ⇒∩ brk C2
    by (elim da-elim-cases)
  from If.prems
  obtain
     wt-e:  Env⊢e::− PrimT Boolean and
     wt-c1: Env⊢c1::√ and
     wt-c2: Env⊢c2::√
    by (elim wt-elim-cases)
  from If.hyps have
   s0-s1:dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
    by (elim dom-locals-eval-mono-elim)
  show ?case
  proof (cases normal s1)
    case True
    note normal-s1 = this
    show ?thesis
    proof (cases the-Bool b)
      case True
      from eval-e normal-s1 wt-e
      have assigns-if True e ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx [elim-format]) (simp add: True)
      with s0-s1
      have dom (locals (store ((Norm s0)::state))) ∪ assigns-if True e ⊆ ...
        by (rule Un-least)
      with da-c1 obtain C1'
        where da-c1': Env⊢ dom (locals (store s1)) »⟨c1⟩» C1' and
              nrm-c1: nrm C1 ⊆ nrm C1'                    and
              brk-c1: ∀ l. brk C1 l ⊆ brk C1' l
        by (rule da-weakenE) iprover
      from If.hyps True have PROP ?Hyp (In1r c1) s1 s2 by simp
      with wt-c1 da-c1'
      obtain nrm-c1': ?NormalAssigned s2 C1' and
             brk-c1': ?BreakAssigned s1 s2 C1' and
             res-c1: ?ResAssigned s1 s2
        using G by simp
      from nrm-c1' nrm-c1 A
      have ?NormalAssigned s2 A
        by blast
      moreover from brk-c1' brk-c1 A
      have ?BreakAssigned s1 s2 A
        by fastsimp
```

```
      with normal-s1
      have ?BreakAssigned (Norm s0) s2 A by simp
      moreover from res-c1 normal-s1 have ?ResAssigned (Norm s0) s2
        by simp
      ultimately show ?thesis by (intro conjI)
    next
      case False
      from eval-e normal-s1 wt-e
      have assigns-if False e ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx [elim-format]) (simp add: False)
      with s0-s1
      have dom (locals (store ((Norm s0)::state)))∪ assigns-if False e ⊆ . . .
        by (rule Un-least)
      with da-c2 obtain C2'
        where da-c2': Env⊢ dom (locals (store s1)) »⟨c2⟩» C2' and
              nrm-c2: nrm C2 ⊆ nrm C2'                 and
              brk-c2: ∀ l. brk C2 l ⊆ brk C2' l
        by (rule da-weakenE) iprover
      from If.hyps False have PROP ?Hyp (In1r c2) s1 s2 by simp
      with wt-c2 da-c2'
      obtain nrm-c2': ?NormalAssigned s2 C2' and
             brk-c2': ?BreakAssigned s1 s2 C2' and
             res-c2: ?ResAssigned s1 s2
        using G by simp
      from nrm-c2' nrm-c2 A
      have ?NormalAssigned s2 A
        by blast
      moreover from brk-c2' brk-c2 A
      have ?BreakAssigned s1 s2 A
        by fastsimp
      with normal-s1
      have ?BreakAssigned (Norm s0) s2 A by simp
      moreover from res-c2 normal-s1 have ?ResAssigned (Norm s0) s2
        by simp
      ultimately show ?thesis by (intro conjI)
    qed
  next
    case False
    then obtain abr where abr: abrupt s1 = Some abr
      by (cases s1) auto
    moreover
    from eval-e - wt-e have ⋀ j. abrupt s1 ≠ Some (Jump j)
      by (rule eval-expression-no-jump) (simp-all add: G wf)
    moreover
    have s2 = s1
    proof −
      from abr and ‹G⊢s1 −(if the-Bool b then c1 else c2)→ s2›
      show ?thesis
        by (cases s1) simp
    qed
    ultimately show ?thesis by simp
  qed
next
  case (Loop s0 e b s1 c s2 l s3 Env T A)
  note G = ‹prg Env = G›
  with Loop.hyps have eval-e: prg Env⊢Norm s0 −e−≻b→ s1
    by (simp (no-asm-simp))
  from Loop.prems
  obtain E C where
```

```
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E and
    da-c: Env⊢ (dom (locals (store ((Norm s0)::state)))
               ∪ assigns-if True e) »⟨c⟩» C and
   A: nrm A = nrm C ∩
           (dom (locals (store ((Norm s0)::state))) ∪ assigns-if False e)
      brk A = brk C
    by (elim da-elim-cases)
  from Loop.prems
  obtain
     wt-e: Env⊢e::−PrimT Boolean and
     wt-c: Env⊢c::√
    by (elim wt-elim-cases)
  from wt-e da-e G
  obtain res-s1: ?ResAssigned (Norm s0) s1
    by (elim Loop.hyps [elim-format]) simp+
  from Loop.hyps have
    s0-s1:dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
    by (elim dom-locals-eval-mono-elim)
  show ?case
  proof (cases normal s1)
    case True
    note normal-s1 = this
    show ?thesis
    proof (cases the-Bool b)
      case True
      with Loop.hyps obtain
        eval-c: G⊢s1 −c→ s2 and
        eval-while: G⊢abupd (absorb (Cont l)) s2 −l· While(e) c→ s3
        by simp
      from Loop.hyps True
      have ?HypObj (In1r c) s1 s2 by simp
      note hyp-c = this [rule-format]
      from Loop.hyps True
      have ?HypObj (In1r (l· While(e) c)) (abupd (absorb (Cont l)) s2) s3
        by simp
      note hyp-while = this [rule-format]
      from eval-e normal-s1 wt-e
      have assigns-if True e ⊆ dom (locals (store s1))
        by (rule assigns-if-good-approx [elim-format]) (simp add: True)
      with s0-s1
      have dom (locals (store ((Norm s0)::state))) ∪ assigns-if True e ⊆ . . .
        by (rule Un-least)
      with da-c obtain C′
        where da-c′: Env⊢ dom (locals (store s1)) »⟨c⟩» C′ and
          nrm-C-C′: nrm C ⊆ nrm C′                    and
          brk-C-C′: ∀ l. brk C l ⊆ brk C′ l
        by (rule da-weakenE) iprover
      from hyp-c wt-c da-c′
      obtain nrm-C′: ?NormalAssigned s2 C′ and
        brk-C′: ?BreakAssigned s1 s2 C′ and
        res-s2: ?ResAssigned s1 s2
        using G by simp
      show ?thesis
      proof (cases normal s2 ∨ abrupt s2 = Some (Jump (Cont l)))
        case True
        from Loop.prems obtain
          wt-while: Env⊢In1r (l· While(e) c)::T and
          da-while: Env⊢ dom (locals (store ((Norm s0)::state)))
                       »⟨l· While(e) c⟩» A
```

```
      by simp
    have dom (locals (store ((Norm s0)::state)))
           ⊆ dom (locals (store (abupd (absorb (Cont l)) s2)))
    proof –
      note s0-s1
      also from eval-c
      have dom (locals (store s1)) ⊆ dom (locals (store s2))
        by (rule dom-locals-eval-mono-elim)
      also have . . . ⊆ dom (locals (store (abupd (absorb (Cont l)) s2)))
        by simp
      finally show ?thesis .
    qed
    with da-while obtain A′
      where
      da-while′: Env⊢ dom (locals (store (abupd (absorb (Cont l)) s2)))
               »⟨l• While(e) c⟩» A′
      and  nrm-A-A′: nrm A ⊆ nrm A′
      and  brk-A-A′: ∀ l. brk A l ⊆ brk A′ l
      by (rule da-weakenE) simp
    with wt-while hyp-while
    obtain nrm-A′: ?NormalAssigned s3 A′ and
           brk-A′: ?BreakAssigned (abupd (absorb (Cont l)) s2) s3 A′ and
           res-s3: ?ResAssigned (abupd (absorb (Cont l)) s2) s3
      using G by simp
    from nrm-A-A′ nrm-A′
    have ?NormalAssigned s3 A
      by blast
    moreover
    have ?BreakAssigned (Norm s0) s3 A
    proof –
      from brk-A-A′ brk-A′
      have ?BreakAssigned (abupd (absorb (Cont l)) s2) s3 A
        by fastsimp
      moreover
      from True have normal (abupd (absorb (Cont l)) s2)
        by (cases s2) auto
      ultimately show ?thesis
        by simp
    qed
    moreover from res-s3 True have ?ResAssigned (Norm s0) s3
      by auto
    ultimately show ?thesis by (intro conjI)
  next
    case False
    then obtain abr where
      abrupt s2 = Some abr and
      abrupt (abupd (absorb (Cont l)) s2) = Some abr
      by auto
    with eval-while
    have eq-s3-s2: s3=s2
      by auto
    with nrm-C-C′ nrm-C′ A
    have ?NormalAssigned s3 A
      by fastsimp
    moreover
    from eq-s3-s2 brk-C-C′ brk-C′ normal-s1 A
    have ?BreakAssigned (Norm s0) s3 A
      by fastsimp
    moreover
```

```
      from eq-s3-s2 res-s2 normal-s1 have ?ResAssigned (Norm s0) s3
        by simp
      ultimately show ?thesis by (intro conjI)
    qed
  next
    case False
    with Loop.hyps have eq-s3-s1: s3=s1
      by simp
    from eq-s3-s1 res-s1
    have res-s3: ?ResAssigned (Norm s0) s3
      by simp
    from eval-e True wt-e
    have assigns-if False e ⊆ dom (locals (store s1))
      by (rule assigns-if-good-approx [elim-format]) (simp add: False)
    with s0-s1
    have dom (locals (store ((Norm s0)::state)))∪assigns-if False e ⊆ ...
      by (rule Un-least)
    hence nrm C ∩
           (dom (locals (store ((Norm s0)::state))) ∪ assigns-if False e)
           ⊆ dom (locals (store s1))
      by (rule subset-Intr)
    with normal-s1 A eq-s3-s1
    have ?NormalAssigned s3 A
      by simp
    moreover
    from normal-s1 eq-s3-s1
    have ?BreakAssigned (Norm s0) s3 A
      by simp
    moreover note res-s3
    ultimately show ?thesis by (intro conjI)
  qed
next
  case False
  then obtain abr where abr: abrupt s1 = Some abr
    by (cases s1) auto
  moreover
  from eval-e - wt-e have no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
    by (rule eval-expression-no-jump) (simp-all add: wf G)
  moreover
  have eq-s3-s1: s3=s1
  proof (cases the-Bool b)
    case True
    with Loop.hyps obtain
      eval-c: G⊢s1 −c→ s2 and
      eval-while: G⊢abupd (absorb (Cont l)) s2 −l• While(e) c→ s3
      by simp
    from eval-c abr have s2=s1 by auto
    moreover from calculation no-jmp have abupd (absorb (Cont l)) s2=s2
      by (cases s1) (simp add: absorb-def)
    ultimately show ?thesis
      using eval-while abr
      by auto
  next
    case False
    with Loop.hyps show ?thesis by simp
  qed
  moreover
  from eq-s3-s1 res-s1
  have res-s3: ?ResAssigned (Norm s0) s3
```

```
      by simp
    ultimately show ?thesis
      by simp
  qed
next
  case (Jmp s j Env T A)
  have ?NormalAssigned (Some (Jump j),s) A by simp
  moreover
  from Jmp.prems
  obtain ret: j = Ret ⟶ Result ∈ dom (locals (store (Norm s))) and
         brk: brk A = (case j of
                         Break l ⇒ λ k. if k=l
                                     then dom (locals (store ((Norm s)::state)))
                                     else UNIV
                       | Cont l  ⇒ λ k. UNIV
                       | Ret     ⇒ λ k. UNIV)
    by (elim da-elim-cases) simp
  from brk have ?BreakAssigned (Norm s) (Some (Jump j),s) A
    by simp
  moreover from ret have ?ResAssigned (Norm s) (Some (Jump j),s)
    by simp
  ultimately show ?case by (intro conjI)
next
  case (Throw s0 e a s1 Env T A)
  note G = ‹prg Env = G›
  from Throw.prems obtain E where
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E
    by (elim da-elim-cases)
  from Throw.prems
    obtain eT where wt-e: Env⊢e::−eT
      by (elim wt-elim-cases)
  have ?NormalAssigned (abupd (throw a) s1) A
    by (cases s1) (simp add: throw-def)
  moreover
  have ?BreakAssigned (Norm s0) (abupd (throw a) s1) A
  proof −
    from G Throw.hyps have eval-e: prg Env⊢Norm s0 −e−≻a→ s1
      by (simp (no-asm-simp))
    from eval-e - wt-e
    have ⋀ l. abrupt s1 ≠ Some (Jump (Break l))
      by (rule eval-expression-no-jump) (simp-all add: wf G)
    hence ⋀ l. abrupt (abupd (throw a) s1) ≠ Some (Jump (Break l))
      by (cases s1) (simp add: throw-def abrupt-if-def)
    thus ?thesis
      by simp
  qed
  moreover
  from wt-e da-e G have ?ResAssigned (Norm s0) s1
    by (elim Throw.hyps [elim-format]) simp+
  hence ?ResAssigned (Norm s0) (abupd (throw a) s1)
    by (cases s1) (simp add: throw-def abrupt-if-def)
  ultimately show ?case by (intro conjI)
next
  case (Try s0 c1 s1 s2 C vn c2 s3 Env T A)
  note G = ‹prg Env = G›
  from Try.prems obtain C1 C2 where
    da-c1: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨c1⟩» C1 and
    da-c2:
     Env⦇lcl := lcl Env(VName vn↦Class C)⦈
```

```
      ⊢ (dom (locals (store ((Norm s0)::state))) ∪ {VName vn}) »⟨c2⟩» C2 and
    A: nrm A = nrm C1 ∩ nrm C2 brk A = brk C1 ⇒∩ brk C2
    by (elim da-elim-cases) simp
  from Try.prems obtain
    wt-c1: Env⊢c1::√ and
    wt-c2: Env(|lcl := lcl Env(VName vn↦Class C)|)⊢c2::√
    by (elim wt-elim-cases)
  have sxalloc: prg Env⊢s1 −sxalloc→ s2 using Try.hyps G
    by (simp (no-asm-simp))
  note ‹PROP ?Hyp (In1r c1) (Norm s0) s1›
  with wt-c1 da-c1 G
  obtain nrm-C1: ?NormalAssigned s1 C1 and
         brk-C1: ?BreakAssigned (Norm s0) s1 C1 and
         res-s1: ?ResAssigned (Norm s0) s1
    by simp
  show ?case
  proof (cases normal s1)
    case True
    with nrm-C1 have nrm C1 ∩ nrm C2 ⊆ dom (locals (store s1))
      by auto
    moreover
    have s3=s1
    proof −
      from sxalloc True have eq-s2-s1: s2=s1
        by (cases s1) (auto elim: sxalloc-elim-cases)
      with True have ¬ G,s2⊢catch C
        by (simp add: catch-def)
      with Try.hyps have s3=s2
        by simp
      with eq-s2-s1 show ?thesis by simp
    qed
    ultimately show ?thesis
      using True A res-s1 by simp
  next
    case False
    note not-normal-s1 = this
    show ?thesis
    proof (cases ∃ l. abrupt s1 = Some (Jump (Break l)))
      case True
      then obtain l where l: abrupt s1 = Some (Jump (Break l))
        by auto
      with brk-C1 have (brk C1 ⇒∩ brk C2) l ⊆ dom (locals (store s1))
        by auto
      moreover have s3=s1
      proof −
        from sxalloc l have eq-s2-s1: s2=s1
          by (cases s1) (auto elim: sxalloc-elim-cases)
        with l have ¬ G,s2⊢catch C
          by (simp add: catch-def)
        with Try.hyps have s3=s2
          by simp
        with eq-s2-s1 show ?thesis by simp
      qed
      ultimately show ?thesis
        using l A res-s1 by simp
    next
      case False
      note abrupt-no-break = this
      show ?thesis
```

```
proof (cases G,s2⊢catch C)
  case True
  with Try.hyps have ?HypObj (In1r c2) (new-xcpt-var vn s2) s3
    by simp
  note hyp-c2 = this [rule-format]
  have (dom (locals (store ((Norm s0)::state))) ∪ {VName vn})
          ⊆ dom (locals (store (new-xcpt-var vn s2)))
  proof −
    from ‹G⊢Norm s0 −c1→ s1›
    have dom (locals (store ((Norm s0)::state)))
            ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim)
    also
    from sxalloc
    have ... ⊆ dom (locals (store s2))
      by (rule dom-locals-sxalloc-mono)
    also
    have ... ⊆ dom (locals (store (new-xcpt-var vn s2)))
      by (cases s2) (simp add: new-xcpt-var-def, blast)
    also
    have {VName vn} ⊆ ...
      by (cases s2) simp
    ultimately show ?thesis
      by (rule Un-least)
  qed
  with da-c2
  obtain C2' where
    da-C2': Env(|lcl := lcl Env(VName vn↦Class C)|)
             ⊢ dom (locals (store (new-xcpt-var vn s2))) »⟨c2⟩» C2'
   and nrm-C2': nrm C2 ⊆ nrm C2'
   and brk-C2': ∀ l. brk C2 l ⊆ brk C2' l
    by (rule da-weakenE) simp
  from wt-c2 da-C2' G and hyp-c2
  obtain nrmAss-C2: ?NormalAssigned s3 C2' and
         brkAss-C2: ?BreakAssigned (new-xcpt-var vn s2) s3 C2' and
         resAss-s3: ?ResAssigned (new-xcpt-var vn s2) s3
    by simp
  from nrmAss-C2 nrm-C2' A
  have ?NormalAssigned s3 A
    by auto
  moreover
  have ?BreakAssigned (Norm s0) s3 A
  proof −
    from brkAss-C2 have ?BreakAssigned (Norm s0) s3 C2'
      by (cases s2) (auto simp add: new-xcpt-var-def)
    with brk-C2' A show ?thesis
      by fastsimp
  qed
  moreover
  from resAss-s3 have ?ResAssigned (Norm s0) s3
    by (cases s2) ( simp add: new-xcpt-var-def)
  ultimately show ?thesis by (intro conjI)
next
  case False
  with Try.hyps
  have eq-s3-s2: s3=s2 by simp
  moreover from sxalloc not-normal-s1 abrupt-no-break
  obtain ¬ normal s2
         ∀ l. abrupt s2 ≠ Some (Jump (Break l))
```

```
        by − (rule sxalloc-cases,auto)
      ultimately obtain
        ?NormalAssigned s3 A and ?BreakAssigned (Norm s0) s3 A
        by (cases s2) auto
      moreover have ?ResAssigned (Norm s0) s3
      proof (cases abrupt s1 = Some (Jump Ret))
        case True
        with sxalloc have s2=s1
          by (elim sxalloc-cases) auto
        with res-s1 eq-s3-s2 show ?thesis by simp
      next
        case False
        with sxalloc
        have abrupt s2 ≠ Some (Jump Ret)
          by (rule sxalloc-no-jump)
        with eq-s3-s2 show ?thesis
          by simp
      qed
      ultimately show ?thesis by (intro conjI)
    qed
  qed
 qed
next
 case (Fin s0 c1 x1 s1 c2 s2 s3 Env T A)
 note G = ‹prg Env = G›
 from Fin.prems obtain C1 C2 where
   da-C1: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨c1⟩» C1 and
   da-C2: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨c2⟩» C2 and
   nrm-A: nrm A = nrm C1 ∪ nrm C2 and
   brk-A: brk A = ((brk C1) ⇒∪∀ (nrm C2)) ⇒∩ (brk C2)
   by (elim da-elim-cases) simp
 from Fin.prems obtain
   wt-c1: Env⊢c1::√ and
   wt-c2: Env⊢c2::√
   by (elim wt-elim-cases)
 note ‹PROP ?Hyp (In1r c1) (Norm s0) (x1,s1)›
 with wt-c1 da-C1 G
 obtain nrmAss-C1: ?NormalAssigned (x1,s1) C1 and
        brkAss-C1: ?BreakAssigned (Norm s0) (x1,s1) C1 and
        resAss-s1: ?ResAssigned (Norm s0) (x1,s1)
   by simp
 obtain nrmAss-C2: ?NormalAssigned s2 C2 and
        brkAss-C2: ?BreakAssigned (Norm s1) s2 C2 and
        resAss-s2: ?ResAssigned (Norm s1) s2
 proof −
   from Fin.hyps
   have dom (locals (store ((Norm s0)::state)))
          ⊆ dom (locals (store (x1,s1)))
     by − (rule dom-locals-eval-mono-elim)
   with da-C2 obtain C2'
     where
     da-C2': Env⊢ dom (locals (store (x1,s1))) »⟨c2⟩» C2' and
     nrm-C2': nrm C2 ⊆ nrm C2' and
     brk-C2': ∀ l. brk C2 l ⊆ brk C2' l
     by (rule da-weakenE) simp
   note ‹PROP ?Hyp (In1r c2) (Norm s1) s2›
   with wt-c2 da-C2' G
   obtain nrmAss-C2': ?NormalAssigned s2 C2' and
          brkAss-C2': ?BreakAssigned (Norm s1) s2 C2' and
```

```
        resAss-s2': ?ResAssigned (Norm s1) s2
    by simp
  from nrmAss-C2' nrm-C2' have ?NormalAssigned s2 C2
    by blast
  moreover
  from brkAss-C2' brk-C2' have ?BreakAssigned (Norm s1) s2 C2
    by fastsimp
  ultimately
  show ?thesis
    using that resAss-s2' by simp
qed
note s3 = ‹s3 = (if ∃ err. x1 = Some (Error err) then (x1, s1)
                 else abupd (abrupt-if (x1 ≠ None) x1) s2)›
have s1-s2: dom (locals s1) ⊆ dom (locals (store s2))
proof −
  from ‹G⊢Norm s1 −c2→ s2›
  show ?thesis
    by (rule dom-locals-eval-mono-elim) simp
qed

have ?NormalAssigned s3 A
proof
  assume normal-s3: normal s3
  show nrm A ⊆ dom (locals (snd s3))
  proof −
    have nrm C1 ⊆ dom (locals (snd s3))
    proof −
      from normal-s3 s3
      have normal (x1,s1)
        by (cases s2) (simp add: abrupt-if-def)
      with normal-s3 nrmAss-C1 s3 s1-s2
      show ?thesis
        by fastsimp
    qed
    moreover
    have nrm C2 ⊆ dom (locals (snd s3))
    proof −
      from normal-s3 s3
      have normal s2
        by (cases s2) (simp add: abrupt-if-def)
      with normal-s3 nrmAss-C2 s3 s1-s2
      show ?thesis
        by fastsimp
    qed
    ultimately have nrm C1 ∪ nrm C2 ⊆ . . .
      by (rule Un-least)
    with nrm-A show ?thesis
      by simp
  qed
qed
moreover
{
  fix l assume brk-s3: abrupt s3 = Some (Jump (Break l))
  have brk A l ⊆ dom (locals (store s3))
  proof (cases normal s2)
    case True
    with brk-s3 s3
    have s2-s3: dom (locals (store s2)) ⊆ dom (locals (store s3))
      by simp
```

```
      have brk C1 l ⊆ dom (locals (store s3))
      proof -
        from True brk-s3 s3 have x1=Some (Jump (Break l))
          by (cases s2) (simp add: abrupt-if-def)
        with brkAss-C1 s1-s2 s2-s3
        show ?thesis
          by simp
      qed
      moreover from True nrmAss-C2 s2-s3
      have nrm C2 ⊆ dom (locals (store s3))
        by - (rule subset-trans, simp-all)
      ultimately
      have ((brk C1) ⇒∪∀ (nrm C2)) l ⊆ ...
        by blast
      with brk-A show ?thesis
        by simp blast
    next
      case False
      note not-normal-s2 = this
      have s3=s2
      proof (cases normal (x1,s1))
        case True with not-normal-s2 s3 show ?thesis
          by (cases s2) (simp add: abrupt-if-def)
      next
        case False with not-normal-s2 s3 brk-s3 show ?thesis
          by (cases s2) (simp add: abrupt-if-def)
      qed
      with brkAss-C2 brk-s3
      have brk C2 l ⊆ dom (locals (store s3))
        by simp
      with brk-A show ?thesis
        by simp blast
    qed
  }
  hence ?BreakAssigned (Norm s0) s3 A
    by simp
  moreover
  {
    assume abr-s3: abrupt s3 = Some (Jump Ret)
    have Result ∈ dom (locals (store s3))
    proof (cases x1 = Some (Jump Ret))
      case True
      note ret-x1 = this
      with resAss-s1 have res-s1: Result ∈ dom (locals s1)
        by simp
      moreover have dom (locals (store ((Norm s1)::state)))
                      ⊆ dom (locals (store s2))
        by (rule dom-locals-eval-mono-elim) (rule Fin.hyps)
      ultimately have Result ∈ dom (locals (store s2))
        by - (rule subsetD,auto)
      with res-s1 s3 show ?thesis
        by simp
    next
      case False
      with s3 abr-s3 obtain  abrupt s2 = Some (Jump Ret) and s3=s2
        by (cases s2) (simp add: abrupt-if-def)
      with resAss-s2 show ?thesis
        by simp
    qed
```

```
    }
    hence ?ResAssigned (Norm s0) s3
      by simp
    ultimately show ?case by (intro conjI)
  next
    case (Init C c s0 s3 s1 s2 Env T A)
    note G = ‹prg Env = G›
    from Init.hyps
    have eval: prg Env⊢ Norm s0 −Init C→ s3
      apply (simp only: G)
      apply (rule eval.Init, assumption)
      apply (cases inited C (globs s0) )
      apply   simp
      apply (simp only: if-False )
      apply (elim conjE,intro conjI,assumption+,simp)
      done
    from Init.prems and ‹the (class G C) = c›
    have c: class G C = Some c
      by (elim wt-elim-cases) auto
    from Init.prems obtain
       nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
      by (elim da-elim-cases) simp
    show ?case
    proof (cases inited C (globs s0))
      case True
      with Init.hyps have s3=Norm s0 by simp
      thus ?thesis
        using nrm-A by simp
    next
      case False
      from Init.hyps False G
      obtain eval-initC:
              prg Env⊢Norm ((init-class-obj G C) s0)
                     −(if C = Object then Skip else Init (super c))→ s1 and
             eval-init: prg Env⊢(set-lvars empty) s1 −init c→ s2 and
             s3: s3=(set-lvars (locals (store s1))) s2
        by simp
      have ?NormalAssigned s3 A
      proof
        show nrm A ⊆ dom (locals (store s3))
        proof −
          from nrm-A have nrm A ⊆ dom (locals (init-class-obj G C s0))
            by simp
          also from eval-initC have ... ⊆ dom (locals (store s1))
            by (rule dom-locals-eval-mono-elim) simp
          also from s3 have ... ⊆ dom (locals (store s3))
            by (cases s1) (cases s2, simp add: init-lvars-def2)
          finally show ?thesis .
        qed
      qed
      moreover
      from eval
      have ⋀ j. abrupt s3 ≠ Some (Jump j)
        by (rule eval-statement-no-jump) (auto simp add: wf c G)
      then obtain ?BreakAssigned (Norm s0) s3 A
             and ?ResAssigned (Norm s0) s3
        by simp
      ultimately show ?thesis by (intro conjI)
    qed
```

```
next
  case (NewC s0 C s1 a s2 Env T A)
  note G = ‹prg Env = G›
  from NewC.prems
  obtain A: nrm A = dom (locals (store ((Norm s0)::state)))
            brk A = (λ l. UNIV)
    by (elim da-elim-cases) simp
  from wf NewC.prems
  have wt-init: Env⊢(Init C)::√
    by  (elim wt-elim-cases) (drule is-acc-classD,simp)
  have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s2))
  proof −
    have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim) (rule NewC.hyps)
    also
    have . . . ⊆ dom (locals (store s2))
      by (rule dom-locals-halloc-mono) (rule NewC.hyps)
    finally show ?thesis .
  qed
  with A have ?NormalAssigned s2 A
    by simp
  moreover
  {
    fix j have abrupt s2 ≠ Some (Jump j)
    proof −
      have eval: prg Env⊢ Norm s0 −NewC C−≻Addr a→ s2
        unfolding G by (rule eval.NewC NewC.hyps)+
      from NewC.prems
      obtain T′ where  T=Inl T′
        by (elim wt-elim-cases) simp
      with NewC.prems have Env⊢NewC C::−T′
        by simp
      from eval - this
      show ?thesis
        by (rule eval-expression-no-jump) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (NewA s0 elT s1 e i s2 a s3 Env T A)
  note G = ‹prg Env = G›
  from NewA.prems obtain
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
    by (elim da-elim-cases)
  from NewA.prems obtain
    wt-init: Env⊢init-comp-ty elT::√ and
    wt-size: Env⊢e::−PrimT Integer
    by (elim wt-elim-cases) (auto dest:  wt-init-comp-ty′)
  note halloc = ‹G⊢abupd (check-neg i) s2−halloc Arr elT (the-Intg i)≻a→s3›
  have dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
    by (rule dom-locals-eval-mono-elim) (rule NewA.hyps)
  with da-e obtain A′ where
             da-e′: Env⊢ dom (locals (store s1)) »⟨e⟩» A′
      and  nrm-A-A′: nrm A ⊆ nrm A′
      and  brk-A-A′: ∀ l. brk A l ⊆ brk A′ l
    by (rule da-weakenE) simp
  note ‹PROP ?Hyp (In1l e) s1 s2›
```

```
    with wt-size da-e' G obtain
      nrmAss-A': ?NormalAssigned s2 A' and
      brkAss-A': ?BreakAssigned s1 s2 A'
      by simp
    have s2-s3: dom (locals (store s2)) ⊆ dom (locals (store s3))
    proof −
      have dom (locals (store s2))
             ⊆ dom (locals (store (abupd (check-neg i) s2)))
        by (simp)
      also have ... ⊆ dom (locals (store s3))
        by (rule dom-locals-halloc-mono) (rule NewA.hyps)
      finally show ?thesis .
    qed
    have ?NormalAssigned s3 A
    proof
      assume normal-s3: normal s3
      show nrm A ⊆ dom (locals (store s3))
      proof −
        from halloc normal-s3
        have normal (abupd (check-neg i) s2)
          by cases simp-all
        hence normal s2
          by (cases s2) simp
        with nrmAss-A' nrm-A-A' s2-s3 show ?thesis
          by blast
      qed
    qed
    moreover
    {
      fix j have abrupt s3 ≠ Some (Jump j)
      proof −
        have eval: prg Env⊢ Norm s0 −New elT[e]−≻Addr a→ s3
          unfolding G by (rule eval.NewA NewA.hyps)+
        from NewA.prems
        obtain T' where  T=Inl T'
          by (elim wt-elim-cases) simp
        with NewA.prems have Env⊢New elT[e]::−T'
          by simp
        from eval - this
        show ?thesis
          by (rule eval-expression-no-jump) (simp-all add: G wf)
      qed
    }
    hence ?BreakAssigned (Norm s0) s3 A and ?ResAssigned (Norm s0) s3
      by simp-all
    ultimately show ?case by (intro conjI)
  next
    case (Cast s0 e v s1 s2 cT Env T A)
    note G = ‹prg Env = G›
    from Cast.prems obtain
      da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
      by (elim da-elim-cases)
    from Cast.prems obtain eT where
      wt-e: Env⊢e::−eT
      by (elim wt-elim-cases)
    note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
    with wt-e da-e G obtain
      nrmAss-A: ?NormalAssigned s1 A and
      brkAss-A: ?BreakAssigned (Norm s0) s1 A
```

```
    by simp
  note s2 = ‹s2 = abupd (raise-if (¬ G,snd s1⊢v fits cT) ClassCast) s1›
  hence s1-s2: dom (locals (store s1)) ⊆ dom (locals (store s2))
    by simp
  have ?NormalAssigned s2 A
  proof
    assume normal s2
    with s2 have normal s1
      by (cases s1) simp
    with nrmAss-A s1-s2
    show nrm A ⊆ dom (locals (store s2))
      by blast
  qed
  moreover
  {
    fix j have abrupt s2 ≠ Some (Jump j)
    proof −
      have eval: prg Env⊢ Norm s0 −Cast cT e−≻v→ s2
        unfolding G by (rule eval.Cast Cast.hyps)+
      from Cast.prems
      obtain T' where  T=Inl T'
        by (elim wt-elim-cases) simp
      with Cast.prems have Env⊢Cast cT e::−T'
        by simp
      from eval - this
      show ?thesis
        by (rule eval-expression-no-jump) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (Inst s0 e v s1 b iT Env T A)
  note G = ‹prg Env = G›
  from Inst.prems obtain
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
    by (elim da-elim-cases)
  from Inst.prems obtain eT where
    wt-e: Env⊢e::−eT
    by (elim wt-elim-cases)
  note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
  with wt-e da-e G obtain
    ?NormalAssigned s1 A and
    ?BreakAssigned (Norm s0) s1 A and
    ?ResAssigned (Norm s0) s1
    by simp
  thus ?case by (intro conjI)
next
  case (Lit s v Env T A)
  from Lit.prems
  have nrm A = dom (locals (store ((Norm s)::state)))
    by (elim da-elim-cases) simp
  thus ?case by simp
next
  case (UnOp s0 e v s1 unop Env T A)
  note G = ‹prg Env = G›
  from UnOp.prems obtain
    da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» A
```

```
      by (elim da-elim-cases)
    from UnOp.prems obtain eT where
      wt-e: Env⊢e::−eT
      by (elim wt-elim-cases)
    note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
    with wt-e da-e G obtain
      ?NormalAssigned s1 A and
      ?BreakAssigned (Norm s0) s1 A and
      ?ResAssigned (Norm s0) s1
      by simp
    thus ?case by (intro conjI)
  next
    case (BinOp s0 e1 v1 s1 binop e2 v2 s2 Env T A)
    note G = ‹prg Env = G›
    from BinOp.hyps
    have
      eval: prg Env⊢Norm s0 −BinOp binop e1 e2−≻(eval-binop binop v1 v2)→ s2
      by (simp only: G) (rule eval.BinOp)
    have s0-s1: dom (locals (store ((Norm s0)::state)))
                  ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim) (rule BinOp)
    also have s1-s2: dom (locals (store s1)) ⊆  dom (locals (store s2))
      by (rule dom-locals-eval-mono-elim) (rule BinOp)
    finally
    have s0-s2: dom (locals (store ((Norm s0)::state)))
                  ⊆ dom (locals (store s2)) .
    from BinOp.prems obtain e1T e2T
      where wt-e1: Env⊢e1::−e1T
       and  wt-e2: Env⊢e2::−e2T
       and  wt-binop: wt-binop (prg Env) binop e1T e2T
       and  T: T=Inl (PrimT (binop-type binop))
      by (elim wt-elim-cases) simp
    have ?NormalAssigned s2 A
    proof
      assume normal-s2: normal s2
      have   normal-s1: normal s1
        by (rule eval-no-abrupt-lemma [rule-format]) (rule BinOp.hyps, rule normal-s2)
      show nrm A ⊆ dom (locals (store s2))
      proof (cases binop=CondAnd)
        case True
        note CondAnd = this
        from BinOp.prems obtain
           nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
                          ∪ (assigns-if True (BinOp CondAnd e1 e2) ∩
                               assigns-if False (BinOp CondAnd e1 e2))
          by (elim da-elim-cases) (simp-all add: CondAnd)
        from T BinOp.prems CondAnd
        have Env⊢BinOp binop e1 e2::−PrimT Boolean
          by (simp)
        with eval normal-s2
        have ass-if: assigns-if (the-Bool (eval-binop binop v1 v2))
                               (BinOp binop e1 e2)
                        ⊆ dom (locals (store s2))
          by (rule assigns-if-good-approx)
        have (assigns-if True (BinOp CondAnd e1 e2) ∩
                        assigns-if False (BinOp CondAnd e1 e2)) ⊆ ...
        proof (cases the-Bool (eval-binop binop v1 v2))
          case True
          with ass-if CondAnd
```

```
      have assigns-if True (BinOp CondAnd e1 e2)
             ⊆ dom (locals (store s2))
        by simp
      thus ?thesis by blast
    next
      case False
      with ass-if CondAnd
      have assigns-if False (BinOp CondAnd e1 e2)
             ⊆ dom (locals (store s2))
        by (simp only: False)
      thus ?thesis by blast
    qed
    with s0-s2
    have dom (locals (store ((Norm s0)::state)))
           ∪ (assigns-if True (BinOp CondAnd e1 e2) ∩
              assigns-if False (BinOp CondAnd e1 e2)) ⊆ ...
      by (rule Un-least)
    thus ?thesis by (simp only: nrm-A)
  next
    case False
    note notCondAnd = this
    show ?thesis
    proof (cases binop=CondOr)
      case True
      note CondOr = this
      from BinOp.prems obtain
        nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
                       ∪ (assigns-if True (BinOp CondOr e1 e2) ∩
                          assigns-if False (BinOp CondOr e1 e2))
        by (elim da-elim-cases) (simp-all add: CondOr)
      from T BinOp.prems CondOr
      have Env⊢BinOp binop e1 e2::−PrimT Boolean
        by (simp)
      with eval normal-s2
      have ass-if: assigns-if (the-Bool (eval-binop binop v1 v2))
                          (BinOp binop e1 e2)
                     ⊆ dom (locals (store s2))
        by (rule assigns-if-good-approx)
      have (assigns-if True (BinOp CondOr e1 e2) ∩
                     assigns-if False (BinOp CondOr e1 e2)) ⊆ ...
      proof (cases the-Bool (eval-binop binop v1 v2))
        case True
        with ass-if CondOr
        have assigns-if True (BinOp CondOr e1 e2)
               ⊆ dom (locals (store s2))
          by (simp)
        thus ?thesis by blast
      next
        case False
        with ass-if CondOr
        have assigns-if False (BinOp CondOr e1 e2)
               ⊆ dom (locals (store s2))
          by (simp)
        thus ?thesis by blast
      qed
      with s0-s2
      have dom (locals (store ((Norm s0)::state)))
             ∪ (assigns-if True (BinOp CondOr e1 e2) ∩
                assigns-if False (BinOp CondOr e1 e2)) ⊆ ...
```

```
        by (rule Un-least)
      thus ?thesis by (simp only: nrm-A)
    next
      case False
      with notCondAnd obtain notAndOr: binop≠CondAnd binop≠CondOr
        by simp
      from BinOp.prems obtain E1
        where da-e1: Env⊢ dom (locals (snd (Norm s0))) »⟨e1⟩» E1
         and  da-e2: Env⊢ nrm E1 »⟨e2⟩» A
        by (elim da-elim-cases) (simp-all add: notAndOr)
      note ‹PROP ?Hyp (In1l e1) (Norm s0) s1›
      with wt-e1 da-e1 G normal-s1
      obtain ?NormalAssigned s1 E1
        by simp
      with normal-s1 have nrm E1 ⊆ dom (locals (store s1)) by iprover
      with da-e2 obtain A'
        where  da-e2': Env⊢ dom (locals (store s1)) »⟨e2⟩» A' and
             nrm-A-A': nrm A ⊆ nrm A'
        by (rule da-weakenE) iprover
      from notAndOr have need-second-arg binop v1 by simp
      with BinOp.hyps
      have PROP ?Hyp (In1l e2) s1 s2 by simp
      with wt-e2 da-e2' G
      obtain ?NormalAssigned s2 A'
        by simp
      with nrm-A-A' normal-s2
      show nrm A ⊆ dom (locals (store s2))
        by blast
    qed
  qed
qed
moreover
{
  fix j have abrupt s2 ≠ Some (Jump j)
  proof –
    from BinOp.prems T
    have Env⊢In1l (BinOp binop e1 e2)::Inl (PrimT (binop-type binop))
      by simp
    from eval - this
    show ?thesis
      by (rule eval-expression-no-jump) (simp-all add: G wf)
  qed
}
hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
  by simp-all
ultimately show ?case by (intro conjI)
next
case (Super s Env T A)
from Super.prems
have nrm A = dom (locals (store ((Norm s)::state)))
  by (elim da-elim-cases) simp
thus ?case by simp
next
case (Acc s0 v w upd s1 Env T A)
show ?case
proof (cases ∃ vn. v = LVar vn)
  case True
  then obtain vn where vn: v=LVar vn..
  from Acc.prems
```

```
    have nrm A = dom (locals (store ((Norm s0)::state)))
      by (simp only: vn) (elim da-elim-cases,simp-all)
    moreover
    from ‹G⊢Norm s0 −v=≻(w, upd)→ s1›
    have s1=Norm s0
      by (simp only: vn) (elim eval-elim-cases,simp)
    ultimately show ?thesis by simp
  next
    case False
    note G = ‹prg Env = G›
    from False Acc.prems
    have da-v: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨v⟩» A
      by (elim da-elim-cases) simp-all
    from Acc.prems obtain vT where
      wt-v: Env⊢v::=vT
      by (elim wt-elim-cases)
    note ‹PROP ?Hyp (In2 v) (Norm s0) s1›
    with wt-v da-v G obtain
      ?NormalAssigned s1 A and
      ?BreakAssigned (Norm s0) s1 A and
      ?ResAssigned (Norm s0) s1
      by simp
    thus ?thesis by (intro conjI)
  qed
next
  case (Ass s0 var w upd s1 e v s2 Env T A)
  note G = ‹prg Env = G›
  from Ass.prems obtain varT eT where
    wt-var: Env⊢var::=varT and
    wt-e:   Env⊢e::−eT
    by (elim wt-elim-cases) simp
  have eval-var: prg Env⊢Norm s0 −var=≻(w, upd)→ s1
    using Ass.hyps by (simp only: G)
  have ?NormalAssigned (assign upd v s2) A
  proof
    assume normal-ass-s2: normal (assign upd v s2)
    from normal-ass-s2
    have normal-s2: normal s2
      by (cases s2) (simp add: assign-def Let-def)
    hence normal-s1: normal s1
      by − (rule eval-no-abrupt-lemma [rule-format], rule Ass.hyps)
    note hyp-var = ‹PROP ?Hyp (In2 var) (Norm s0) s1›
    note hyp-e = ‹PROP ?Hyp (In1l e) s1 s2›
    show nrm A ⊆ dom (locals (store (assign upd v s2)))
    proof (cases ∃ vn. var = LVar vn)
      case True
      then obtain vn where vn: var=LVar vn..
      from Ass.prems obtain E where
        da-e: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨e⟩» E and
        nrm-A: nrm A = nrm E ∪ {vn}
        by (elim da-elim-cases) (insert vn,auto)
      obtain E' where
        da-e': Env⊢ dom (locals (store s1)) »⟨e⟩» E' and
        E-E': nrm E ⊆ nrm E'
      proof −
        have dom (locals (store ((Norm s0)::state)))
               ⊆ dom (locals (store s1))
          by (rule dom-locals-eval-mono-elim) (rule Ass.hyps)
        with da-e show thesis
```

```
      by (rule da-weakenE) (rule that)
    qed
    from G eval-var vn
    have eval-lvar: G⊢Norm s0 −LVar vn=≻(w, upd)→ s1
      by simp
    then have upd: upd = snd (lvar vn (store s1))
      by cases (cases lvar vn (store s1),simp)
    have nrm E ⊆ dom (locals (store (assign upd v s2)))
    proof −
      from hyp-e wt-e da-e' G normal-s2
      have nrm E' ⊆ dom (locals (store s2))
        by simp
      also
      from upd
      have dom (locals (store s2))  ⊆ dom (locals (store (upd v s2)))
        by (simp add: lvar-def) blast
      hence dom (locals (store s2))
               ⊆ dom (locals (store (assign upd v s2)))
        by (rule dom-locals-assign-mono)
      finally
      show ?thesis using E-E'
        by blast
    qed
    moreover
    from upd normal-s2
    have {vn} ⊆ dom (locals (store (assign upd v s2)))
      by (auto simp add: assign-def Let-def lvar-def upd split: split-split)
    ultimately
    show nrm A ⊆ . . .
      by (rule Un-least [elim-format]) (simp add: nrm-A)
  next
    case False
    from Ass.prems obtain V where
      da-var: Env⊢ dom (locals (store ((Norm s0)::state))) »⟨var⟩» V and
      da-e:   Env⊢ nrm V »⟨e⟩» A
      by (elim da-elim-cases) (insert False,simp+)
    from hyp-var wt-var da-var G normal-s1
    have nrm V ⊆ dom (locals (store s1))
      by simp
    with da-e obtain A'
      where  da-e': Env⊢ dom (locals (store s1)) »⟨e⟩» A' and
             nrm-A-A': nrm A ⊆ nrm A'
      by (rule da-weakenE) iprover
    from hyp-e wt-e da-e' G normal-s2
    obtain nrm A' ⊆ dom (locals (store s2))
      by simp
    with nrm-A-A' have nrm A ⊆ . . .
      by blast
    also have . . . ⊆ dom (locals (store (assign upd v s2)))
    proof −
      from eval-var normal-s1
      have dom (locals (store s2)) ⊆ dom (locals (store (upd v s2)))
        by (cases rule: dom-locals-eval-mono-elim)
           (cases s2, simp)
      thus ?thesis
        by (rule dom-locals-assign-mono)
    qed
    finally show ?thesis .
  qed
```

```
  qed
  moreover
  {
    fix j have abrupt (assign upd v s2) ≠ Some (Jump j)
    proof -
      have eval: prg Env⊢Norm s0 −var:=e−≻v→ (assign upd v s2)
        by (simp only: G) (rule eval.Ass Ass.hyps)+
      from Ass.prems
      obtain T' where  T=Inl T'
        by (elim wt-elim-cases) simp
      with Ass.prems have Env⊢var:=e::−T' by simp
      from eval - this
      show ?thesis
        by (rule eval-expression-no-jump) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) (assign upd v s2) A
    and ?ResAssigned (Norm s0) (assign upd v s2)
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (Cond s0 e0 b s1 e1 e2 v s2 Env T A)
  note G = ‹prg Env = G›
  have ?NormalAssigned s2 A
  proof
    assume normal-s2: normal s2
    show nrm A ⊆ dom (locals (store s2))
    proof (cases Env⊢(e0 ? e1 : e2)::−(PrimT Boolean))
      case True
      with Cond.prems
      have nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
                          ∪ (assigns-if True  (e0 ? e1 : e2) ∩
                              assigns-if False (e0 ? e1 : e2))
        by (elim da-elim-cases) simp-all
      have eval: prg Env⊢Norm s0 −(e0 ? e1 : e2)−≻v→ s2
        unfolding G by (rule eval.Cond Cond.hyps)+
      from eval
      have dom (locals (store ((Norm s0)::state)))⊆dom (locals (store s2))
        by (rule dom-locals-eval-mono-elim)
      moreover
      from eval normal-s2 True
      have ass-if: assigns-if (the-Bool v) (e0 ? e1 : e2)
                    ⊆ dom (locals (store s2))
        by (rule assigns-if-good-approx)
      have assigns-if True  (e0 ? e1:e2) ∩ assigns-if False (e0 ? e1:e2)
            ⊆ dom (locals (store s2))
      proof (cases the-Bool v)
        case True
        from ass-if
        have assigns-if True  (e0 ? e1:e2) ⊆ dom (locals (store s2))
          by (simp only: True)
        thus ?thesis by blast
      next
        case False
        from ass-if
        have assigns-if False  (e0 ? e1:e2) ⊆ dom (locals (store s2))
          by (simp only: False)
        thus ?thesis by blast
      qed
```

```
      ultimately show nrm A ⊆ dom (locals (store s2))
        by (simp only: nrm-A) (rule Un-least)
    next
      case False
      with Cond.prems obtain E1 E2 where
       da-e1: Env⊢ (dom (locals (store ((Norm s0)::state)))
                      ∪ assigns-if True e0) »⟨e1⟩» E1 and
       da-e2: Env⊢ (dom (locals (store ((Norm s0)::state)))
                     ∪ assigns-if False e0) »⟨e2⟩» E2 and
       nrm-A: nrm A = nrm E1 ∩ nrm E2
        by (elim da-elim-cases) simp-all
      from Cond.prems obtain e1T e2T where
        wt-e0: Env⊢e0::− PrimT Boolean and
        wt-e1: Env⊢e1::−e1T and
        wt-e2: Env⊢e2::−e2T
        by (elim wt-elim-cases)
      have s0-s1: dom (locals (store ((Norm s0)::state)))
                    ⊆ dom (locals (store s1))
        by (rule dom-locals-eval-mono-elim) (rule Cond.hyps)
      have eval-e0: prg Env⊢Norm s0 −e0−≻b→ s1
        unfolding G by (rule Cond.hyps)
      have normal-s1: normal s1
        by (rule eval-no-abrupt-lemma [rule-format]) (rule Cond.hyps, rule normal-s2)
      show ?thesis
      proof (cases the-Bool b)
        case True
        from True Cond.hyps have PROP ?Hyp (In1l e1) s1 s2 by simp
        moreover
        from eval-e0 normal-s1 wt-e0
        have assigns-if True e0 ⊆ dom (locals (store s1))
          by (rule assigns-if-good-approx [elim-format]) (simp only: True)
        with s0-s1
        have dom (locals (store ((Norm s0)::state)))
              ∪ assigns-if True e0 ⊆ . . .
          by (rule Un-least)
        with da-e1 obtain E1' where
              da-e1': Env⊢ dom (locals (store s1)) »⟨e1⟩» E1' and
           nrm-E1-E1': nrm E1 ⊆ nrm E1'
          by (rule da-weakenE) iprover
        ultimately have nrm E1' ⊆ dom (locals (store s2))
          using wt-e1 G normal-s2 by simp
        with nrm-E1-E1' show ?thesis
          by (simp only: nrm-A) blast
      next
        case False
        from False Cond.hyps have PROP ?Hyp (In1l e2) s1 s2 by simp
        moreover
        from eval-e0 normal-s1 wt-e0
        have assigns-if False e0 ⊆ dom (locals (store s1))
          by (rule assigns-if-good-approx [elim-format]) (simp only: False)
        with s0-s1
        have dom (locals (store ((Norm s0)::state)))
              ∪ assigns-if False e0 ⊆ . . .
          by (rule Un-least)
        with da-e2 obtain E2' where
              da-e2': Env⊢ dom (locals (store s1)) »⟨e2⟩» E2' and
           nrm-E2-E2': nrm E2 ⊆ nrm E2'
          by (rule da-weakenE) iprover
        ultimately have nrm E2' ⊆ dom (locals (store s2))
```

```
        using wt-e2 G normal-s2 by simp
      with nrm-E2-E2' show ?thesis
        by (simp only: nrm-A) blast
    qed
  qed
qed
moreover
{
  fix j have abrupt s2 ≠ Some (Jump j)
  proof −
    have eval: prg Env⊢Norm s0 −e0 ? e1 : e2−≻v→ s2
      unfolding G by (rule eval.Cond Cond.hyps)+
    from Cond.prems
    obtain T' where  T=Inl T'
      by (elim wt-elim-cases) simp
    with Cond.prems have Env⊢e0 ? e1 : e2::−T' by simp
    from eval - this
    show ?thesis
      by (rule eval-expression-no-jump) (simp-all add: G wf)
  qed
}
hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
  by simp-all
ultimately show ?case by (intro conjI)
next
  case (Call s0 e a s1 args vs s2 D mode statT mn pTs s3 s3' accC v s4
        Env T A)
  note G = ‹prg Env = G›
  have ?NormalAssigned (restore-lvars s2 s4) A
  proof
    assume normal-restore-lvars: normal (restore-lvars s2 s4)
    show nrm A ⊆ dom (locals (store (restore-lvars s2 s4)))
    proof −
      from Call.prems obtain E where
           da-e: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e⟩» E and
        da-args: Env⊢ nrm E »⟨args⟩» A
        by (elim da-elim-cases)
      from Call.prems obtain eT argsT where
           wt-e: Env⊢e::−eT and
        wt-args: Env⊢args::≐argsT
        by (elim wt-elim-cases)
      note s3 = ‹s3 = init-lvars G D (|name = mn, parTs = pTs|) mode a vs s2›
      note s3' = ‹s3' = check-method-access G accC statT mode
                                    (|name=mn,parTs=pTs|) a s3›
      have normal-s2: normal s2
      proof −
        from normal-restore-lvars have normal s4
          by simp
        then have normal s3'
          by − (rule eval-no-abrupt-lemma [rule-format], rule Call.hyps)
        with s3' have normal s3
          by (cases s3) (simp add: check-method-access-def Let-def)
        with s3 show normal s2
          by (cases s2) (simp add: init-lvars-def Let-def)
      qed
      then have normal-s1: normal s1
        by  − (rule eval-no-abrupt-lemma [rule-format], rule Call.hyps)
      note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
      with da-e wt-e G normal-s1
```

```
      have nrm E ⊆ dom (locals (store s1))
        by simp
      with da-args obtain A' where
        da-args': Env⊢ dom (locals (store s1)) »⟨args⟩» A' and
        nrm-A-A': nrm A ⊆ nrm A'
        by (rule da-weakenE) iprover
      note ‹PROP ?Hyp (In3 args) s1 s2›
      with da-args' wt-args G normal-s2
      have nrm A' ⊆ dom (locals (store s2))
        by simp
      with nrm-A-A' have nrm A ⊆ dom (locals (store s2))
        by blast
      also have ... ⊆ dom (locals (store (restore-lvars s2 s4)))
        by (cases s4) simp
      finally show ?thesis .
    qed
  qed
  moreover
  {
    fix j have abrupt (restore-lvars s2 s4) ≠ Some (Jump j)
    proof -
      have eval: prg Env⊢Norm s0 −({accC,statT,mode}e·mn( {pTs}args))−≻v
                    → (restore-lvars s2 s4)
        unfolding G by (rule eval.Call Call)+
      from Call.prems
      obtain T' where  T=Inl T'
        by (elim wt-elim-cases) simp
      with Call.prems have Env⊢({accC,statT,mode}e·mn( {pTs}args))::−T'
        by simp
      from eval - this
      show ?thesis
        by (rule eval-expression-no-jump) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) (restore-lvars s2 s4) A
    and ?ResAssigned (Norm s0) (restore-lvars s2 s4)
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (Methd s0 D sig v s1 Env T A)
  note G = ‹prg Env = G›
  from Methd.prems obtain m where
     m:       methd (prg Env) D sig = Some m and
     da-body: Env⊢(dom (locals (store ((Norm s0)::state))))
                 »⟨Body (declclass m) (stmt (mbody (mthd m)))⟩» A
    by − (erule da-elim-cases)
  from Methd.prems m obtain
    isCls: is-class (prg Env) D and
    wt-body: Env ⊢In1l (Body (declclass m) (stmt (mbody (mthd m))))::T
    by − (erule wt-elim-cases,simp)
  note ‹PROP ?Hyp (In1l (body G D sig)) (Norm s0) s1›
  moreover
  from wt-body have Env⊢In1l (body G D sig)::T
    using isCls m G by (simp add: body-def2)
  moreover
  from da-body have Env⊢(dom (locals (store ((Norm s0)::state))))
                       »⟨body G D sig⟩» A
    using isCls m G by (simp add: body-def2)
  ultimately show ?case
```

```
      using G by simp
  next
    case (Body s0 D s1 c s2 s3 Env T A)
    note G = ‹prg Env = G›
    from Body.prems
    have nrm-A: nrm A = dom (locals (store ((Norm s0)::state)))
      by (elim da-elim-cases) simp
    have eval: prg Env⊢Norm s0 −Body D c−≻the (locals (store s2) Result)
                       →abupd (absorb Ret) s3
      unfolding G by (rule eval.Body Body.hyps)+
    hence nrm A ⊆ dom (locals (store (abupd (absorb Ret) s3)))
      by (simp only: nrm-A) (rule dom-locals-eval-mono-elim)
    hence ?NormalAssigned (abupd (absorb Ret) s3) A
      by simp
    moreover
    from eval have ⋀ j. abrupt (abupd (absorb Ret) s3) ≠ Some (Jump j)
      by (rule Body-no-jump) simp
    hence ?BreakAssigned (Norm s0) (abupd (absorb Ret) s3) A and
          ?ResAssigned (Norm s0) (abupd (absorb Ret) s3)
      by simp-all
    ultimately show ?case by (intro conjI)
  next
    case (LVar s vn Env T A)
    from LVar.prems
    have nrm A = dom (locals (store ((Norm s)::state)))
      by (elim da-elim-cases) simp
    thus ?case by simp
  next
    case (FVar s0 statDeclC s1 e a s2 v s2′ stat fn s3 accC Env T A)
    note G = ‹prg Env = G›
    have ?NormalAssigned s3 A
    proof
      assume normal-s3: normal s3
      show nrm A ⊆ dom (locals (store s3))
      proof −
        note fvar = ‹(v, s2′) = fvar statDeclC stat fn a s2› and
          s3 = ‹s3 = check-field-access G accC statDeclC fn stat a s2′›
        from FVar.prems
        have da-e: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e⟩» A
          by (elim da-elim-cases)
        from FVar.prems obtain eT where
          wt-e: Env⊢e::−eT
          by (elim wt-elim-cases)
        have (dom (locals (store ((Norm s0)::state))))
               ⊆ dom (locals (store s1))
          by (rule dom-locals-eval-mono-elim) (rule FVar.hyps)
        with da-e obtain A′ where
          da-e′: Env⊢ dom (locals (store s1)) »⟨e⟩» A′ and
          nrm-A-A′: nrm A ⊆ nrm A′
          by (rule da-weakenE) iprover
        have normal-s2: normal s2
        proof −
          from normal-s3 s3
          have normal s2′
            by (cases s2′) (simp add: check-field-access-def Let-def)
          with fvar
          show normal s2
            by (cases s2) (simp add: fvar-def2)
        qed
```

```
      note ‹PROP ?Hyp (In1l e) s1 s2›
      with da-e' wt-e G normal-s2
      have nrm A' ⊆ dom (locals (store s2))
        by simp
      with nrm-A-A' have nrm A ⊆ dom (locals (store s2))
        by blast
      also have ... ⊆ dom (locals (store s3))
      proof -
        from fvar have s2' = snd (fvar statDeclC stat fn a s2)
          by (cases fvar statDeclC stat fn a s2) simp
        hence dom (locals (store s2)) ⊆  dom (locals (store s2'))
          by (simp) (rule dom-locals-fvar-mono)
        also from s3 have ... ⊆ dom (locals (store s3))
          by (cases s2') (simp add: check-field-access-def Let-def)
        finally show ?thesis .
      qed
      finally show ?thesis .
    qed
  qed
  moreover
  {
    fix j have abrupt s3 ≠ Some (Jump j)
    proof -
      obtain w upd where v: (w,upd)=v
        by (cases v) auto
      have eval: prg Env⊢Norm s0−({accC,statDeclC,stat}e..fn)=≻(w,upd)→s3
        by (simp only: G v) (rule eval.FVar FVar.hyps)+
      from FVar.prems
      obtain T' where  T=Inl T'
        by (elim wt-elim-cases) simp
      with FVar.prems have Env⊢({accC,statDeclC,stat}e..fn)::=T'
        by simp
      from eval - this
      show ?thesis
        by (rule eval-var-no-jump [THEN conjunct1]) (simp-all add: G wf)
    qed
  }
  hence ?BreakAssigned (Norm s0) s3 A and ?ResAssigned (Norm s0) s3
    by simp-all
  ultimately show ?case by (intro conjI)
next
  case (AVar s0 e1 a s1 e2 i s2 v s2' Env T A)
  note G = ‹prg Env = G›
  have ?NormalAssigned s2' A
  proof
    assume normal-s2': normal s2'
    show nrm A ⊆ dom (locals (store s2'))
    proof -
      note avar = ‹(v, s2') = avar G i a s2›
      from AVar.prems obtain E1 where
        da-e1: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e1⟩» E1 and
        da-e2: Env⊢ nrm E1 »⟨e2⟩» A
        by (elim da-elim-cases)
      from AVar.prems obtain e1T e2T where
           wt-e1: Env⊢e1::−e1T and
           wt-e2: Env⊢e2::−e2T
        by (elim wt-elim-cases)
      from avar normal-s2'
      have normal-s2: normal s2
```

```
      by (cases s2) (simp add: avar-def2)
    hence normal s1
      by - (rule eval-no-abrupt-lemma [rule-format], rule AVar, rule normal-s2)
    moreover note ‹PROP ?Hyp (In1l e1) (Norm s0) s1›
    ultimately have nrm E1 ⊆ dom (locals (store s1))
      using da-e1 wt-e1 G by simp
    with da-e2 obtain A' where
      da-e2': Env⊢ dom (locals (store s1)) »⟨e2⟩» A' and
      nrm-A-A': nrm A ⊆ nrm A'
      by (rule da-weakenE) iprover
    note ‹PROP ?Hyp (In1l e2) s1 s2›
    with da-e2' wt-e2 G normal-s2
    have nrm A' ⊆ dom (locals (store s2))
      by simp
    with nrm-A-A' have nrm A ⊆ dom (locals (store s2))
      by blast
    also have ... ⊆ dom (locals (store s2'))
    proof -
       from avar have s2' = snd (avar G i a s2)
        by (cases (avar G i a s2)) simp
      thus dom (locals (store s2)) ⊆  dom (locals (store s2'))
        by (simp) (rule dom-locals-avar-mono)
    qed
    finally show ?thesis .
  qed
qed
moreover
{
  fix j have abrupt s2' ≠ Some (Jump j)
  proof -
    obtain w upd where v: (w,upd)=v
      by (cases v) auto
    have eval: prg Env⊢Norm s0−(e1.[e2])=≻(w,upd)→s2'
      unfolding G v by (rule eval.AVar AVar.hyps)+
    from AVar.prems
    obtain T' where  T=Inl T'
      by (elim wt-elim-cases) simp
    with AVar.prems have Env⊢(e1.[e2])::=T'
      by simp
    from eval - this
    show ?thesis
      by (rule eval-var-no-jump [THEN conjunct1]) (simp-all add: G wf)
  qed
}
hence ?BreakAssigned (Norm s0) s2' A and ?ResAssigned (Norm s0) s2'
  by simp-all
ultimately show ?case by (intro conjI)
next
  case (Nil s0 Env T A)
  from Nil.prems
  have nrm A = dom (locals (store ((Norm s0)::state)))
    by (elim da-elim-cases) simp
  thus ?case by simp
next
  case (Cons s0 e v s1 es vs s2 Env T A)
  note G = ‹prg Env = G›
  have ?NormalAssigned s2 A
  proof
    assume normal-s2: normal s2
```

```
      show nrm A ⊆ dom (locals (store s2))
      proof −
        from Cons.prems obtain E where
          da-e: Env⊢ (dom (locals (store ((Norm s0)::state))))»⟨e⟩» E and
          da-es: Env⊢ nrm E »⟨es⟩» A
          by (elim da-elim-cases)
        from Cons.prems obtain eT esT where
            wt-e: Env⊢e::−eT and
            wt-es: Env⊢es::≐esT
          by (elim wt-elim-cases)
        have normal s1
          by − (rule eval-no-abrupt-lemma [rule-format], rule Cons.hyps, rule normal-s2)
        moreover note ‹PROP ?Hyp (In1l e) (Norm s0) s1›
        ultimately have nrm E ⊆ dom (locals (store s1))
          using da-e wt-e G by simp
        with da-es obtain A' where
          da-es': Env⊢ dom (locals (store s1)) »⟨es⟩» A' and
          nrm-A-A': nrm A ⊆ nrm A'
          by (rule da-weakenE) iprover
        note ‹PROP ?Hyp (In3 es) s1 s2›
        with da-es' wt-es G normal-s2
        have nrm A' ⊆ dom (locals (store s2))
          by simp
        with nrm-A-A' show nrm A ⊆ dom (locals (store s2))
          by blast
      qed
    qed
    moreover
    {
      fix j have abrupt s2 ≠ Some (Jump j)
      proof −
        have eval: prg Env⊢Norm s0−(e # es)≐≻v#vs→s2
          unfolding G by (rule eval.Cons Cons.hyps)+
        from Cons.prems
        obtain T' where  T=Inr T'
          by (elim wt-elim-cases) simp
        with Cons.prems have Env⊢(e # es)::≐T'
          by simp
        from eval - this
        show ?thesis
          by (rule eval-expression-list-no-jump) (simp-all add: G wf)
      qed
    }
    hence ?BreakAssigned (Norm s0) s2 A and ?ResAssigned (Norm s0) s2
      by simp-all
    ultimately show ?case by (intro conjI)
  qed
qed


lemma da-good-approxE:
  assumes
    prg Env⊢s0 −t≻→ (v, s1) and Env⊢t::T and
    Env⊢ dom (locals (store s0)) »t» A and wf-prog (prg Env)
  obtains
    normal s1 ⟹ nrm A ⊆ dom (locals (store s1)) and
    ⋀ l. ⟦abrupt s1 = Some (Jump (Break l)); normal s0⟧
         ⟹ brk A l ⊆ dom (locals (store s1)) and
    ⟦abrupt s1 = Some (Jump Ret);normal s0⟧⟹Result ∈ dom (locals (store s1))
```

using *prems* **by** − (*drule* (*3*) *da-good-approx*, *simp*)

lemma *da-good-approxE′*:
assumes *eval*: $G \vdash s0 \ -t\succ\rightarrow \ (v,\ s1)$
and *wt*: $(\!|prg{=}G,cls{=}C,lcl{=}L|\!) \vdash t{::}T$
and *da*: $(\!|prg{=}G,cls{=}C,lcl{=}L|\!) \vdash$ *dom* (*locals* (*store s0*)) »*t*» *A*
and *wf*: *wf-prog G*
obtains *normal s1* $\Longrightarrow$ *nrm A* $\subseteq$ *dom* (*locals* (*store s1*)) **and**
$\bigwedge$ *l*. ⟦*abrupt s1* = *Some* (*Jump* (*Break l*)); *normal s0*⟧
$\Longrightarrow$ *brk A l* $\subseteq$ *dom* (*locals* (*store s1*)) **and**
⟦*abrupt s1* = *Some* (*Jump Ret*);*normal s0*⟧
$\Longrightarrow$ *Result* $\in$ *dom* (*locals* (*store s1*))
proof −
from *eval* **have** *prg* $(\!|prg{=}G,cls{=}C,lcl{=}L|\!) \vdash s0 \ -t\succ\rightarrow \ (v,\ s1)$ **by** *simp*
moreover note *wt da*
moreover from *wf* **have** *wf-prog* (*prg* $(\!|prg{=}G,cls{=}C,lcl{=}L|\!)$) **by** *simp*
ultimately show *thesis*
using *that* **by** (*rule da-good-approxE*) *iprover*+
qed

declare [[*simproc add*: *wt-expr wt-var wt-exprs wt-stmt*]]

end

Chapter 19

TypeSafe

46 The type soundness proof for Java

```
theory TypeSafe
imports DefiniteAssignmentCorrect Conform
begin
```

error free

```
hide const field


lemma error-free-halloc:
  assumes halloc: G⊢s0 −halloc oi≻a→ s1 and
          error-free-s0: error-free s0
  shows error-free s1
proof −
  from halloc error-free-s0
  obtain abrupt0 store0 abrupt1 store1
    where eqs: s0=(abrupt0,store0) s1=(abrupt1,store1) and
          halloc': G⊢(abrupt0,store0) −halloc oi≻a→ (abrupt1,store1) and
          error-free-s0': error-free (abrupt0,store0)
    by (cases s0,cases s1) auto
  from halloc' error-free-s0'
  have error-free (abrupt1,store1)
  proof (induct)
    case Abrupt
    then show ?case .
  next
    case New
    then show ?case
      by (auto split: split-if-asm)
  qed
  with eqs
  show ?thesis
    by simp
qed


lemma error-free-sxalloc:
  assumes sxalloc: G⊢s0 −sxalloc→ s1 and error-free-s0: error-free s0
  shows error-free s1
proof −
  from sxalloc error-free-s0
  obtain abrupt0 store0 abrupt1 store1
    where eqs: s0=(abrupt0,store0) s1=(abrupt1,store1) and
          sxalloc': G⊢(abrupt0,store0) −sxalloc→ (abrupt1,store1) and
          error-free-s0': error-free (abrupt0,store0)
    by (cases s0,cases s1) auto
  from sxalloc' error-free-s0'
  have error-free (abrupt1,store1)
  proof (induct)
  qed (auto)
  with eqs
  show ?thesis
    by simp
qed


lemma error-free-check-field-access-eq:
```

error-free (*check-field-access G accC statDeclC fn stat a s*)
$\Longrightarrow$ (*check-field-access G accC statDeclC fn stat a s*) = *s*
apply (*cases s*)
apply (*auto simp add*: *check-field-access-def Let-def error-free-def*
abrupt-if-def
split: *split-if-asm*)
done

lemma *error-free-check-method-access-eq*:
error-free (*check-method-access G accC statT mode sig a′ s*)
$\Longrightarrow$ (*check-method-access G accC statT mode sig a′ s*) = *s*
apply (*cases s*)
apply (*auto simp add*: *check-method-access-def Let-def error-free-def*
abrupt-if-def
split: *split-if-asm*)
done

lemma *error-free-FVar-lemma*:
error-free s
$\Longrightarrow$ *error-free* (*abupd* (*if stat then id else np a*) *s*)
by (*case-tac s*) (*auto split*: *split-if*)

lemma *error-free-init-lvars* [*simp*,*intro*]:
error-free s $\Longrightarrow$
error-free (*init-lvars G C sig mode a pvs s*)
by (*cases s*) (*auto simp add*: *init-lvars-def Let-def split*: *split-if*)

lemma *error-free-LVar-lemma*:
error-free s $\Longrightarrow$ *error-free* (*assign* (λv. *supd lupd*($vn \mapsto v$)) *w s*)
by (*cases s*) *simp*

lemma *error-free-throw* [*simp*,*intro*]:
error-free s $\Longrightarrow$ *error-free* (*abupd* (*throw x*) *s*)
by (*cases s*) (*simp add*: *throw-def*)

result conformance

constdefs
assign-conforms :: *st* $\Rightarrow$ (*val* $\Rightarrow$ *state* $\Rightarrow$ *state*) $\Rightarrow$ *ty* $\Rightarrow$ *env′* $\Rightarrow$ *bool*
(-$\leq$|-$\preceq$-::$\preceq$- [*71*,*71*,*71*,*71*] *70*)
$s\leq|f\preceq T::\preceq E \equiv$
($\forall s'$ *w*. *Norm* $s'::\preceq E \longrightarrow$ *fst* $E,s'\vdash w::\preceq T \longrightarrow s\leq|s' \longrightarrow$ *assign f w* (*Norm* s')$::\preceq E$) $\wedge$
($\forall s'$ *w*. *error-free* $s' \longrightarrow$ (*error-free* (*assign f w* s')))

constdefs
rconf :: *prog* $\Rightarrow$ *lenv* $\Rightarrow$ *st* $\Rightarrow$ *term* $\Rightarrow$ *vals* $\Rightarrow$ *tys* $\Rightarrow$ *bool*
(-,-,-$\vdash$-$\succ$-::$\preceq$- [*71*,*71*,*71*,*71*,*71*,*71*] *70*)
$G,L,s\vdash t\succ v::\preceq T$
$\equiv$ *case T of*
Inl T $\Rightarrow$ *if* ($\exists$ *var*. *t*=*In2 var*)
then ($\forall$ *n*. (*the-In2 t*) = *LVar n*
$\longrightarrow$ (*fst* (*the-In2 v*) = *the* (*locals s n*)) $\wedge$
(*locals s n* $\neq$ *None* $\longrightarrow$ $G,s\vdash$*fst* (*the-In2 v*)$::\preceq T$)) $\wedge$

$(\neg\ (\exists\ n.\ \textit{the-In2}\ t{=}\textit{LVar}\ n) \longrightarrow (G,s{\vdash}\textit{fst}\ (\textit{the-In2}\ v){::}{\preceq}T)){\wedge}$
$(s{\leq}|\textit{snd}\ (\textit{the-In2}\ v){\preceq}T{::}{\preceq}(G,L))$
else $G,s{\vdash}\textit{the-In1}\ v{::}{\preceq}T$
$|\ \textit{Inr}\ \textit{Ts} \Rightarrow \textit{list-all2}\ (\textit{conf}\ G\ s)\ (\textit{the-In3}\ v)\ \textit{Ts}$

With *rconf* we describe the conformance of the result value of a term. This definition gets rather complicated because of the relations between the injections of the different terms, types and values. The main case distinction is between single values and value lists. In case of value lists, every value has to conform to its type. For single values we have to do a further case distinction, between values of variables $\exists\,\textit{var}.\ t = \textit{In2 var}$ and ordinary values. Values of variables are modelled as pairs consisting of the current value and an update function which will perform an assignment to the variable. This stems form the decision, that we only have one evaluation rule for each kind of variable. The decision if we read or write to the variable is made by syntactic enclosing rules. So conformance of variable-values must ensure that both the current value and an update will conform to the type. With the introduction of definite assignment of local variables we have to do another case distinction. For the notion of conformance local variables are allowed to be *None*, since the definedness is not ensured by conformance but by definite assignment. Field and array variables must contain a value.

lemma *rconf-In1* [*simp*]:
$G,L,s{\vdash}\textit{In1}\ ec{\succ}\textit{In1}\ v\ {::}{\preceq}\textit{Inl}\ T\ =\ G,s{\vdash}v{::}{\preceq}T$
apply (*unfold rconf-def*)
apply (*simp* (*no-asm*))
done

lemma *rconf-In2-no-LVar* [*simp*]:
$\forall\ n.\ va{\neq}\textit{LVar}\ n \Longrightarrow$
$G,L,s{\vdash}\textit{In2}\ va{\succ}\textit{In2}\ vf{::}{\preceq}\textit{Inl}\ T\ = (G,s{\vdash}\textit{fst}\ vf{::}{\preceq}T \wedge s{\leq}|\textit{snd}\ vf{\preceq}T{::}{\preceq}(G,L))$
apply (*unfold rconf-def*)
apply *auto*
done

lemma *rconf-In2-LVar* [*simp*]:
$va{=}\textit{LVar}\ n \Longrightarrow$
$G,L,s{\vdash}\textit{In2}\ va{\succ}\textit{In2}\ vf{::}{\preceq}\textit{Inl}\ T$
$= ((\textit{fst}\ vf\ =\ \textit{the}\ (\textit{locals}\ s\ n))\ \wedge$
$(\textit{locals}\ s\ n \neq \textit{None} \longrightarrow G,s{\vdash}\textit{fst}\ vf{::}{\preceq}T) \wedge s{\leq}|\textit{snd}\ vf{\preceq}T{::}{\preceq}(G,L))$
apply (*unfold rconf-def*)
by *simp*

lemma *rconf-In3* [*simp*]:
$G,L,s{\vdash}\textit{In3}\ es{\succ}\textit{In3}\ vs{::}{\preceq}\textit{Inr}\ \textit{Ts} = \textit{list-all2}\ (\lambda v\ T.\ G,s{\vdash}v{::}{\preceq}T)\ vs\ \textit{Ts}$
apply (*unfold rconf-def*)
apply (*simp* (*no-asm*))
done

fits and conf

lemma *conf-fits*: $G,s{\vdash}v{::}{\preceq}T \Longrightarrow G,s{\vdash}v\ \textit{fits}\ T$
apply (*unfold fits-def*)
apply *clarify*
apply (*erule contrapos-np*, *simp* (*no-asm-use*))
apply (*drule conf-RefTD*)
apply *auto*
done

lemma *fits-conf*:
$[\![G,s\vdash v::\preceq T;\ G\vdash T\preceq?\ T';\ G,s\vdash v\ \mathit{fits}\ T';\ \mathit{ws\text{-}prog}\ G]\!] \Longrightarrow G,s\vdash v::\preceq T'$
apply (*auto dest*!: *fitsD cast-PrimT2 cast-RefT2*)
apply (*force dest*: *conf-RefTD intro*: *conf-AddrI*)
done

lemma *fits-Array*:
$[\![G,s\vdash v::\preceq T;\ G\vdash T'.[]\preceq T.[];\ G,s\vdash v\ \mathit{fits}\ T';\ \mathit{ws\text{-}prog}\ G]\!] \Longrightarrow G,s\vdash v::\preceq T'$
apply (*auto dest*!: *fitsD widen-ArrayPrimT widen-ArrayRefT*)
apply (*force dest*: *conf-RefTD intro*: *conf-AddrI*)
done

gext

lemma *halloc-gext*: $\bigwedge s1\ s2.\ G\vdash s1\ -\mathit{halloc}\ oi\succ a\rightarrow\ s2 \Longrightarrow \mathit{snd}\ s1\le|\mathit{snd}\ s2$
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*erule halloc.induct*)
apply (*auto dest*!: *new-AddrD*)
done

lemma *sxalloc-gext*: $\bigwedge s1\ s2.\ G\vdash s1\ -\mathit{sxalloc}\rightarrow\ s2 \Longrightarrow \mathit{snd}\ s1\le|\mathit{snd}\ s2$
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*erule sxalloc.induct*)
apply (*auto dest*!: *halloc-gext*)
done

lemma *eval-gext-lemma* [*rule-format* (*no-asm*)]:
$G\vdash s\ -t\succ\rightarrow\ (w,s') \Longrightarrow \mathit{snd}\ s\le|\mathit{snd}\ s' \wedge (\mathit{case}\ w\ \mathit{of}$
$\quad \mathit{In1}\ v \Rightarrow \mathit{True}$
$\ \mid \mathit{In2}\ \mathit{vf} \Rightarrow \mathit{normal}\ s \longrightarrow (\forall\, v\ x\ s.\ s\le|\mathit{snd}\ (\mathit{assign}\ (\mathit{snd}\ \mathit{vf})\ v\ (x,s)))$
$\ \mid \mathit{In3}\ \mathit{vs} \Rightarrow \mathit{True})$
apply (*erule eval-induct*)
prefer *26*
  apply (*case-tac inited C* (*globs s0*), *clarsimp*, *erule thin-rl*)
apply (*auto del*: *conjI dest*!: *not-initedD gext-new sxalloc-gext halloc-gext*
 simp add: *lvar-def fvar-def2 avar-def2 init-lvars-def2*
  check-field-access-def check-method-access-def Let-def
 split del: *split-if-asm split add*: *sum3.split*)

apply *force*+
done

lemma *evar-gext-f*:
$G\vdash \mathit{Norm}\ s1\ -e{=}\!\succ \mathit{vf}\ \rightarrow\ s2 \Longrightarrow s\le|\mathit{snd}\ (\mathit{assign}\ (\mathit{snd}\ \mathit{vf})\ v\ (x,s))$
apply (*drule eval-gext-lemma* [*THEN conjunct2*])
apply *auto*
done

lemmas *eval-gext* = *eval-gext-lemma* [*THEN conjunct1*]

lemma *eval-gext'*: $G\vdash(x1,s1)\ -t\succ\rightarrow\ (w,(x2,s2)) \Longrightarrow s1\le|s2$
apply (*drule eval-gext*)

apply *auto*
done

lemma *init-yields-initd*: $G\vdash Norm\ s1\ -Init\ C\rightarrow\ s2 \Longrightarrow initd\ C\ s2$
apply (*erule eval-cases* , *auto split del*: *split-if-asm*)
apply (*case-tac inited C* (*globs s1*))
apply (*clarsimp split del*: *split-if-asm*)+
apply (*drule eval-gext'*)+
apply (*drule init-class-obj-inited*)
apply (*erule inited-gext*)
apply (*simp* (*no-asm-use*))
done

Lemmas

lemma *obj-ty-obj-class1*:
$[\![wf\text{-}prog\ G;\ is\text{-}type\ G\ (obj\text{-}ty\ obj)]\!] \Longrightarrow is\text{-}class\ G\ (obj\text{-}class\ obj)$
apply (*case-tac tag obj*)
apply (*auto simp add*: *obj-ty-def obj-class-def*)
done

lemma *oconf-init-obj*:
$[\![wf\text{-}prog\ G;$
$(case\ r\ of\ Heap\ a \Rightarrow is\text{-}type\ G\ (obj\text{-}ty\ obj)\ |\ Stat\ C \Rightarrow is\text{-}class\ G\ C)$
$]\!] \Longrightarrow G,s\vdash obj\ (\!|values:=init\text{-}vals\ (var\text{-}tys\ G\ (tag\ obj)\ r)|\!)::\preceq\surd r$
apply (*auto intro*!: *oconf-init-obj-lemma unique-fields*)
done

lemma *conforms-newG*: $[\![globs\ s\ oref = None;\ (x,\ s)::\preceq(G,L);$
$wf\text{-}prog\ G;\ case\ oref\ of\ Heap\ a \Rightarrow is\text{-}type\ G\ (obj\text{-}ty\ (\!|tag=oi,values=vs|\!))$
$|\ Stat\ C \Rightarrow is\text{-}class\ G\ C]\!] \Longrightarrow$
$(x,\ init\text{-}obj\ G\ oi\ oref\ s)::\preceq(G,\ L)$
apply (*unfold init-obj-def*)
apply (*auto elim*!: *conforms-gupd dest*!: *oconf-init-obj*
)
done

lemma *conforms-init-class-obj*:
$[\![(x,s)::\preceq(G,\ L);\ wf\text{-}prog\ G;\ class\ G\ C=Some\ y;\ \neg\ inited\ C\ (globs\ s)]\!] \Longrightarrow$
$(x,init\text{-}class\text{-}obj\ G\ C\ s)::\preceq(G,\ L)$
apply (*rule not-initedD* [*THEN conforms-newG*])
apply (*auto*)
done

lemma *fst-init-lvars*[*simp*]:
$fst\ (init\text{-}lvars\ G\ C\ sig\ (invmode\ m\ e)\ a'\ pvs\ (x,s)) =$
$(if\ is\text{-}static\ m\ then\ x\ else\ (np\ a')\ x)$
apply (*simp* (*no-asm*) *add*: *init-lvars-def2*)
done

lemma *halloc-conforms*: $\bigwedge s1.\ [\![G\vdash s1\ -halloc\ oi\succ a\rightarrow\ s2;\ wf\text{-}prog\ G;\ s1::\preceq(G,\ L);$

is-type G (obj-ty (|tag=oi,values=fs|))⟧ ⟹ *s2*::⪯(*G*, *L*)
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*case-tac aa*)
apply (*auto elim*!: *halloc-elim-cases dest*!: *new-AddrD*
    intro!: *conforms-newG* [*THEN conforms-xconf*] *conf-AddrI*)
done

lemma *halloc-type-sound*:
⋀*s1*. ⟦*G*⊢*s1* −*halloc oi*≻*a*→ (*x*,*s*); *wf-prog G*; *s1*::⪯(*G*, *L*);
  T = *obj-ty* (|*tag*=*oi*,*values*=*fs*|); *is-type G T*⟧ ⟹
  (*x*,*s*)::⪯(*G*, *L*) ∧ (*x* = *None* ⟶ *G*,*s*⊢*Addr a*::⪯*T*)
apply (*auto elim*!: *halloc-conforms*)
apply (*case-tac aa*)
apply (*subst obj-ty-eq*)
apply (*auto elim*!: *halloc-elim-cases dest*!: *new-AddrD intro*!: *conf-AddrI*)
done

lemma *sxalloc-type-sound*:
⋀*s1 s2*. ⟦*G*⊢*s1* −*sxalloc*→ *s2*; *wf-prog G*⟧ ⟹
  case fst s1 of
    None ⇒ *s2* = *s1*
  | *Some abr* ⇒ (*case abr of*
        Xcpt x ⇒ (∃ *a*. *fst s2* = *Some*(*Xcpt* (*Loc a*)) ∧
              (∀ *L*. *s1*::⪯(*G*,*L*) ⟶ *s2*::⪯(*G*,*L*)))
      | *Jump j* ⇒ *s2* = *s1*
      | *Error e* ⇒ *s2* = *s1*)
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*erule sxalloc.induct*)
apply *auto*
apply (*rule halloc-conforms* [*THEN conforms-xconf*])
apply (*auto elim*!: *halloc-elim-cases dest*!: *new-AddrD intro*!: *conf-AddrI*)
done

lemma *wt-init-comp-ty*:
is-acc-type G (*pid C*) *T* ⟹ (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*init-comp-ty T*::√
apply (*unfold init-comp-ty-def*)
apply (*clarsimp simp add*: *accessible-in-RefT-simp*
        is-acc-type-def is-acc-class-def)
done

declare *fun-upd-same* [*simp*]

declare *fun-upd-apply* [*simp del*]

constdefs
  DynT-prop::[*prog*,*inv-mode*,*qtname*,*ref-ty*] ⇒ *bool*
          (-⊢-→-⪯-[*71*,*71*,*71*,*71*]*70*)
 G⊢*mode*→*D*⪯*t* ≡ *mode* = *IntVir* ⟶ *is-class G D* ∧
     (*if* (∃ *T*. *t*=*ArrayT T*) *then D*=*Object else G*⊢*Class D*⪯*RefT t*)

lemma *DynT-propI*:
⟦(*x*,*s*)::⪯(*G*, *L*); *G*,*s*⊢*a*′::⪯*RefT statT*; *wf-prog G*; *mode* = *IntVir* ⟶ *a*′ ≠ *Null*⟧
 ⟹ *G*⊢*mode*→*invocation-class mode s a*′ *statT*⪯*statT*

proof (*unfold DynT-prop-def*)
assume *state-conform*: (*x*,*s*)::≼(*G*, *L*)
and *statT-a′*: *G*,*s*⊢*a′*::≼*RefT statT*
and *wf*: *wf-prog G*
and *mode*: *mode* = *IntVir* ⟶ *a′* ≠ *Null*
let *?invCls* = (*invocation-class mode s a′ statT*)
let *?IntVir* = *mode* = *IntVir*
let *?Concl* = λ*invCls*. *is-class G invCls* ∧
(*if* ∃ *T*. *statT* = *ArrayT T*
then invCls = *Object*
else G⊢*Class invCls*≼*RefT statT*)
show *?IntVir* ⟶ *?Concl ?invCls*
proof
assume *modeIntVir*: *?IntVir*
with *mode* **have** *not-Null*: *a′* ≠ *Null* **..**
from *statT-a′ not-Null state-conform*
obtain *a obj*
where *obj-props*: *a′* = *Addr a globs s* (*Inl a*) = *Some obj*
G⊢*obj-ty obj*≼*RefT statT is-type G* (*obj-ty obj*)
by (*blast dest*: *conforms-RefTD*)
show *?Concl ?invCls*
proof (*cases tag obj*)
case *CInst*
with *modeIntVir obj-props*
show *?thesis*
by (*auto dest*!: *widen-Array2 split add*: *split-if*)
next
case *Arr*
from *Arr* **obtain** *T* **where** *obj-ty obj* = *T*.[] **by** (*blast dest*: *obj-ty-Arr1*)
moreover from *Arr* **have** *obj-class obj* = *Object*
by (*blast dest*: *obj-class-Arr1*)
moreover note *modeIntVir obj-props wf*
ultimately show *?thesis* **by** (*auto dest*!: *widen-Array*)
qed
qed
qed

lemma *invocation-methd*:
⟦*wf-prog G*; *statT* ≠ *NullT*;
(∀ *statC*. *statT* = *ClassT statC* ⟶ *is-class G statC*);
(∀ *I*. *statT* = *IfaceT I* ⟶ *is-iface G I* ∧ *mode* ≠ *SuperM*);
(∀ *T*. *statT* = *ArrayT T* ⟶ *mode* ≠ *SuperM*);
G⊢*mode*→*invocation-class mode s a′ statT*≼*statT*;
dynlookup G statT (*invocation-class mode s a′ statT*) *sig* = *Some m* ⟧
⟹ *methd G* (*invocation-declclass G mode s a′ statT sig*) *sig* = *Some m*
proof −
assume *wf*: *wf-prog G*
and *not-NullT*: *statT* ≠ *NullT*
and *statC-prop*: (∀ *statC*. *statT* = *ClassT statC* ⟶ *is-class G statC*)
and *statI-prop*: (∀ *I*. *statT* = *IfaceT I* ⟶ *is-iface G I* ∧ *mode* ≠ *SuperM*)
and *statA-prop*: (∀ *T*. *statT* = *ArrayT T* ⟶ *mode* ≠ *SuperM*)
and *invC-prop*: *G*⊢*mode*→*invocation-class mode s a′ statT*≼*statT*
and *dynlookup*: *dynlookup G statT* (*invocation-class mode s a′ statT*) *sig*
= *Some m*
show *?thesis*
proof (*cases statT*)
case *NullT*
with *not-NullT* **show** *?thesis* **by** *simp*

```
next
  case IfaceT
  with statI-prop obtain I
    where    statI: statT = IfaceT I and
          is-iface: is-iface G I     and
        not-SuperM: mode ≠ SuperM by blast

  show ?thesis
  proof (cases mode)
    case Static
    with wf dynlookup statI is-iface
    show ?thesis
      by (auto simp add: invocation-declclass-def dynlookup-def
                         dynimethd-def dynmethd-C-C
                 intro: dynmethd-declclass
                 dest!: wf-imethdsD
                  dest: table-of-map-SomeI
                 split: split-if-asm)
  next
    case SuperM
    with not-SuperM show ?thesis ..
  next
    case IntVir
    with wf dynlookup IfaceT invC-prop show ?thesis
      by (auto simp add: invocation-declclass-def dynlookup-def dynimethd-def
                         DynT-prop-def
                 intro: methd-declclass dynmethd-declclass
                 split: split-if-asm)
  qed
next
  case ClassT
  show ?thesis
  proof (cases mode)
    case Static
    with wf ClassT dynlookup statC-prop
    show ?thesis by (auto simp add: invocation-declclass-def dynlookup-def
                          intro: dynmethd-declclass)
  next
    case SuperM
    with wf ClassT dynlookup statC-prop
    show ?thesis by (auto simp add: invocation-declclass-def dynlookup-def
                          intro: dynmethd-declclass)
  next
    case IntVir
    with wf ClassT dynlookup statC-prop invC-prop
    show ?thesis
      by (auto simp add: invocation-declclass-def dynlookup-def dynimethd-def
                         DynT-prop-def
                 intro: dynmethd-declclass)
  qed
next
  case ArrayT
  show ?thesis
  proof (cases mode)
    case Static
    with wf ArrayT dynlookup show ?thesis
      by (auto simp add: invocation-declclass-def dynlookup-def
                         dynimethd-def dynmethd-C-C
                 intro: dynmethd-declclass
```

```
                    dest: table-of-map-SomeI)
    next
      case SuperM
      with ArrayT statA-prop show ?thesis by blast
    next
      case IntVir
      with wf ArrayT dynlookup invC-prop show ?thesis
        by (auto simp add: invocation-declclass-def dynlookup-def dynimethd-def
                          DynT-prop-def dynmethd-C-C
                   intro: dynmethd-declclass
                    dest: table-of-map-SomeI)
    qed
  qed
qed
```

```
lemma DynT-mheadsD:
⟦G⊢invmode sm e→invC≼statT;
  wf-prog G; (|prg=G,cls=C,lcl=L|)⊢e::−RefT statT;
  (statDeclT,sm) ∈ mheads G C statT sig;
  invC = invocation-class (invmode sm e) s a′ statT;
  declC =invocation-declclass G (invmode sm e) s a′ statT sig
 ⟧ ⟹
  ∃ dm.
  methd G declC sig = Some dm ∧ dynlookup G statT invC sig = Some dm ∧
  G⊢resTy (mthd dm)≼resTy sm ∧
  wf-mdecl G declC (sig, mthd dm) ∧
  declC = declclass dm ∧
  is-static dm = is-static sm ∧
  is-class G invC ∧ is-class G declC ∧ G⊢invC≼C declC ∧
  (if invmode sm e = IntVir
      then (∀ statC. statT=ClassT statC ⟶ G⊢invC ≼C statC)
      else ( (∃ statC. statT=ClassT statC ∧ G⊢statC≼C declC)
            ∨ (∀ statC. statT≠ClassT statC ∧ declC=Object)) ∧
            statDeclT = ClassT (declclass dm))
proof −
  assume invC-prop: G⊢invmode sm e→invC≼statT
     and        wf: wf-prog G
     and      wt-e: (|prg=G,cls=C,lcl=L|)⊢e::−RefT statT
     and        sm: (statDeclT,sm) ∈ mheads G C statT sig
     and      invC: invC = invocation-class (invmode sm e) s a′ statT
     and     declC: declC =
                   invocation-declclass G (invmode sm e) s a′ statT sig
  from wt-e wf have type-statT: is-type G (RefT statT)
    by (auto dest: ty-expr-is-type)
  from sm have not-Null: statT ≠ NullT by auto
  from type-statT
  have wf-C: (∀ statC. statT = ClassT statC ⟶ is-class G statC)
    by (auto)
  from type-statT wt-e
  have wf-I: (∀I. statT = IfaceT I ⟶ is-iface G I ∧
                                  invmode sm e ≠ SuperM)
    by (auto dest: invocationTypeExpr-noClassD)
  from wt-e
  have wf-A: (∀     T. statT = ArrayT T ⟶ invmode sm e ≠ SuperM)
    by (auto dest: invocationTypeExpr-noClassD)
  show ?thesis
  proof (cases invmode sm e = IntVir)
    case True
```

with *invC-prop not-Null*
have *invC-prop'*: *is-class G invC* ∧
(*if* (∃ *T*. *statT*=*ArrayT T*) *then invC*=*Object*
else G⊢*Class invC*≼*RefT statT*)
by (*auto simp add*: *DynT-prop-def*)
from *True*
have ¬ *is-static sm*
by (*simp add*: *invmode-IntVir-eq member-is-static-simp*)
with *invC-prop' not-Null*
have *G*,*statT* ⊢ *invC valid-lookup-cls-for* (*is-static sm*)
by (*cases statT*) *auto*
with *sm wf type-statT* **obtain** *dm* **where**
dm: *dynlookup G statT invC sig* = *Some dm* **and**
resT-dm: *G*⊢*resTy* (*mthd dm*)≼*resTy sm* **and**
static: *is-static dm* = *is-static sm*
by − (*drule dynamic-mheadsD*,*force*+)
with *declC invC not-Null*
have *declC'*: *declC* = (*declclass dm*)
by (*auto simp add*: *invocation-declclass-def*)
with *wf invC declC not-Null wf-C wf-I wf-A invC-prop dm*
have *dm'*: *methd G declC sig* = *Some dm*
by − (*drule invocation-methd*,*auto*)
from *wf dm invC-prop' declC' type-statT*
have *declC-prop*: *G*⊢*invC* $\preceq_C$ *declC* ∧ *is-class G declC*
by (*auto dest*: *dynlookup-declC*)
from *wf dm' declC-prop declC'*
have *wf-dm*: *wf-mdecl G declC* (*sig*,(*mthd dm*))
by (*auto dest*: *methd-wf-mdecl*)
from *invC-prop'*
have *statC-prop*: (∀ *statC*. *statT*=*ClassT statC* ⟶ *G*⊢*invC* $\preceq_C$ *statC*)
by *auto*
from *True dm' resT-dm wf-dm invC-prop' declC-prop statC-prop declC' static dm*
show *?thesis* **by** *auto*
next
case *False*
with *type-statT wf invC not-Null wf-I wf-A*
have *invC-prop'*: *is-class G invC* ∧
((∃ *statC*. *statT*=*ClassT statC* ∧ *invC*=*statC*) ∨
(∀ *statC*. *statT*≠*ClassT statC* ∧ *invC*=*Object*))
by (*case-tac statT*) (*auto simp add*: *invocation-class-def*
split: *inv-mode.splits*)
with *not-Null wf*
have *dynlookup-static*: *dynlookup G statT invC sig* = *methd G invC sig*
by (*case-tac statT*) (*auto simp add*: *dynlookup-def dynmethd-C-C*
dynimethd-def)
from *sm wf wt-e not-Null False invC-prop'* **obtain** *dm* **where**
dm: *methd G invC sig* = *Some dm* **and**
eq-declC-sm-dm:*statDeclT* = *ClassT* (*declclass dm*) **and**
eq-mheads:*sm*=*mhead* (*mthd dm*)
by − (*drule static-mheadsD*, (*force dest*: *accmethd-SomeD*)+)
then have *static*: *is-static dm* = *is-static sm* **by** − (*auto*)
with *declC invC dynlookup-static dm*
have *declC'*: *declC* = (*declclass dm*)
by (*auto simp add*: *invocation-declclass-def*)
from *invC-prop' wf declC' dm*
have *dm'*: *methd G declC sig* = *Some dm*
by (*auto intro*: *methd-declclass*)
from *dynlookup-static dm*

have dm'': *dynlookup G statT invC sig = Some dm*
by *simp*
from *wf dm invC-prop' declC' type-statT*
have *declC-prop*: $G \vdash invC \preceq_C declC \wedge$ *is-class G declC*
by (*auto dest*: *methd-declC*)
then have *declC-prop1*: *invC=Object* $\longrightarrow$ *declC=Object* **by** *auto*
from *wf dm' declC-prop declC'*
have *wf-dm*: *wf-mdecl G declC* (*sig*,(*mthd dm*))
by (*auto dest*: *methd-wf-mdecl*)
from *invC-prop' declC-prop declC-prop1*
have *statC-prop*: ($(\exists\ statC.\ statT=ClassT\ statC \wedge G \vdash statC \preceq_C declC)$
$\vee\ (\forall\ statC.\ statT \neq ClassT\ statC \wedge declC=Object)$)
by *auto*
from *False dm' dm'' wf-dm invC-prop' declC-prop statC-prop declC'*
eq-declC-sm-dm eq-mheads static
show *?thesis* **by** *auto*
qed
qed

corollary *DynT-mheadsE* [*consumes 7*]:
— Same as *DynT-mheadsD* but better suited for application in typesafety proof
assumes *invC-compatible*: $G \vdash mode \rightarrow invC \preceq statT$
and *wf*: *wf-prog G*
and *wt-e*: $(\!|prg=G,cls=C,lcl=L|\!) \vdash e::-RefT\ statT$
and *mheads*: $(statDeclT,sm) \in$ *mheads G C statT sig*
and *mode*: *mode=invmode sm e*
and *invC*: *invC = invocation-class mode s a' statT*
and *declC*: *declC =invocation-declclass G mode s a' statT sig*
and *dm*: $\bigwedge$ *dm*. $[\![$*methd G declC sig = Some dm*;
dynlookup G statT invC sig = Some dm;
$G \vdash resTy\ (mthd\ dm) \preceq resTy\ sm$;
wf-mdecl G declC (*sig*, *mthd dm*);
declC = declclass dm;
is-static dm = is-static sm;
is-class G invC; *is-class G declC*; $G \vdash invC \preceq_C declC$;
(*if invmode sm e = IntVir*
then $(\forall\ statC.\ statT=ClassT\ statC \longrightarrow G \vdash invC \preceq_C statC)$
else ($(\exists\ statC.\ statT=ClassT\ statC \wedge G \vdash statC \preceq_C declC)$
$\vee\ (\forall\ statC.\ statT \neq ClassT\ statC \wedge declC=Object)) \wedge$
statDeclT = ClassT (*declclass dm*))$]\!] \Longrightarrow P$
shows *P*
proof −
from *invC-compatible mode* **have** $G \vdash invmode\ sm\ e \rightarrow invC \preceq statT$ **by** *simp*
moreover note *wf wt-e mheads*
moreover from *invC mode*
have *invC = invocation-class* (*invmode sm e*) *s a' statT* **by** *simp*
moreover from *declC mode*
have *declC =invocation-declclass G* (*invmode sm e*) *s a' statT sig* **by** *simp*
ultimately show *?thesis*
by (*rule DynT-mheadsD* [*THEN exE*,*rule-format*])
(*elim conjE*,*rule dm*)
qed

lemma *DynT-conf*: $[\![G \vdash$*invocation-class mode s a' statT* $\preceq_C$ *declC*; *wf-prog G*;
isrtype G (*statT*);
$G,s \vdash a'::\preceq RefT\ statT$; *mode = IntVir* $\longrightarrow a' \neq Null$;
mode $\neq$ *IntVir* $\longrightarrow$ $(\exists\ statC.\ statT=ClassT\ statC \wedge G \vdash statC \preceq_C declC)$

$\vee$ ($\forall$ *statC*. *statT*$\neq$*ClassT statC* $\wedge$ *declC*=*Object*)$]\!]$
$\Longrightarrow$*G*,*s*$\vdash$*a*$'$::$\preceq$ *Class declC*
apply (*case-tac mode* = *IntVir*)
apply (*drule conf-RefTD*)
apply (*force intro*!: *conf-AddrI*
simp add: *obj-class-def split add*: *obj-tag.split-asm*)
apply *clarsimp*
apply *safe*
apply (*erule* (*1*) *widen.subcls* [*THEN conf-widen*])
apply (*erule wf-ws-prog*)

apply (*frule widen-Object*) **apply** (*erule wf-ws-prog*)
apply (*erule* (*1*) *conf-widen*) **apply** (*erule wf-ws-prog*)
done

lemma *Ass-lemma*:
$[\![$ *G*$\vdash$*Norm s0* $-$*var*=$\succ$(*w*, *f*)$\rightarrow$ *Norm s1*; *G*$\vdash$*Norm s1* $-$*e*$-\succ$*v*$\rightarrow$ *Norm s2*;
G,*s2*$\vdash$*v*::$\preceq$*eT*;*s1*$\leq$|*s2* $\longrightarrow$ *assign f v* (*Norm s2*)::$\preceq$(*G*, *L*)$]\!]$
$\Longrightarrow$ *assign f v* (*Norm s2*)::$\preceq$(*G*, *L*) $\wedge$
(*normal* (*assign f v* (*Norm s2*)) $\longrightarrow$ *G*,*store* (*assign f v* (*Norm s2*))$\vdash$*v*::$\preceq$*eT*)
apply (*drule-tac x* = *None* **and** *s* = *s2* **and** *v* = *v* **in** *evar-gext-f*)
apply (*drule eval-gext*$'$, *clarsimp*)
apply (*erule conf-gext*)
apply *simp*
done

lemma *Throw-lemma*: $[\![$*G*$\vdash$*tn*$\preceq_C$ *SXcpt Throwable*; *wf-prog G*; (*x1*,*s1*)::$\preceq$(*G*, *L*);
x1 = *None* $\longrightarrow$ *G*,*s1*$\vdash$*a*$'$::$\preceq$ *Class tn*$]\!]$ $\Longrightarrow$ (*throw a*$'$ *x1*, *s1*)::$\preceq$(*G*, *L*)
apply (*auto split add*: *split-abrupt-if simp add*: *throw-def2*)
apply (*erule conforms-xconf*)
apply (*frule conf-RefTD*)
apply (*auto elim*: *widen.subcls* [*THEN conf-widen*])
done

lemma *Try-lemma*: $[\![$*G*$\vdash$*obj-ty* (*the* (*globs s1*$'$ (*Heap a*)))$\preceq$ *Class tn*;
(*Some* (*Xcpt* (*Loc a*)), *s1*$'$)::$\preceq$(*G*, *L*); *wf-prog G*$]\!]$
$\Longrightarrow$ *Norm* (*lupd*(*vn*$\mapsto$*Addr a*) *s1*$'$)::$\preceq$(*G*, *L*(*vn*$\mapsto$*Class tn*))
apply (*rule conforms-allocL*)
apply (*erule conforms-NormI*)
apply (*drule conforms-XcptLocD* [*THEN conf-RefTD*],*rule HOL.refl*)
apply (*auto intro*!: *conf-AddrI*)
done

lemma *Fin-lemma*:
$[\![$*G*$\vdash$*Norm s1* $-$*c2*$\rightarrow$ (*x2*,*s2*); *wf-prog G*; (*Some a*, *s1*)::$\preceq$(*G*, *L*); (*x2*,*s2*)::$\preceq$(*G*, *L*);
dom (*locals s1*) $\subseteq$ *dom* (*locals s2*)$]\!]$
$\Longrightarrow$ (*abrupt-if True* (*Some a*) *x2*, *s2*)::$\preceq$(*G*, *L*)
apply (*auto elim*: *eval-gext*$'$ *conforms-xgext split add*: *split-abrupt-if*)
done

lemma *FVar-lemma1*:
$[\![$*table-of* (*DeclConcepts.fields G statC*) (*fn*, *statDeclC*) = *Some f* ;
x2 = *None* $\longrightarrow$ *G*,*s2*$\vdash$*a*::$\preceq$ *Class statC*; *wf-prog G*; *G*$\vdash$*statC*$\preceq_C$ *statDeclC*;
statDeclC $\neq$ *Object*;

class G statDeclC = Some y; (*x2*,*s2*)::$\preceq$(*G*, *L*); *s1*$\leq$|*s2*;
inited statDeclC (*globs s1*);
(*if static f then id else np a*) *x2* = *None*$]\!]$
$\Longrightarrow$
$\exists$ *obj*. *globs s2* (*if static f then Inr statDeclC else Inl* (*the-Addr a*))
= *Some obj* $\wedge$
var-tys G (*tag obj*) (*if static f then Inr statDeclC else Inl*(*the-Addr a*))
(*Inl*(*fn*,*statDeclC*)) = *Some* (*type f*)
apply (*drule initedD*)
apply (*frule subcls-is-class2*, *simp* (*no-asm-simp*))
apply (*case-tac static f*)
apply *clarsimp*
apply (*drule* (*1*) *rev-gext-objD*, *clarsimp*)
apply (*frule fields-declC*, *erule wf-ws-prog*, *simp* (*no-asm-simp*))
apply (*rule var-tys-Some-eq* [*THEN iffD2*])
apply *clarsimp*
apply (*erule fields-table-SomeI*, *simp* (*no-asm*))
apply *clarsimp*
apply (*drule conf-RefTD*, *clarsimp*, *rule var-tys-Some-eq* [*THEN iffD2*])
apply (*auto dest*!: *widen-Array split add*: *obj-tag.split*)
apply (*rule fields-table-SomeI*)
apply (*auto elim*!: *fields-mono subcls-is-class2*)
done

lemma *FVar-lemma2*: *error-free state*
$\Longrightarrow$ *error-free*
(*assign*
(λ*v*. *supd*
(*upd-gobj*
(*if static field then Inr statDeclC*
else Inl (*the-Addr a*))
(*Inl* (*fn*, *statDeclC*)) *v*))
w state)
proof −
assume *error-free*: *error-free state*
obtain *a s* **where** *state*=(*a*,*s*)
by (*cases state*)
with *error-free*
show *?thesis*
by (*cases a*) *auto*
qed

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declare *split-if* [*split del*] *split-if-asm* [*split del*]
option.split [*split del*] *option.split-asm* [*split del*]
declaration $\langle\!\langle$ *K* (*Simplifier.map-ss* (*fn ss* => *ss delloop split-all-tac*)) $\rangle\!\rangle$
declaration $\langle\!\langle$ *K* (*Classical.map-cs* (*fn cs* => *cs delSWrapper split-all-tac*)) $\rangle\!\rangle$

lemma *FVar-lemma*:
$[\![$((*v*, *f*), *Norm s2'*) = *fvar statDeclC* (*static field*) *fn a* (*x2*, *s2*);
G$\vdash$*statC*$\preceq_C$ *statDeclC*;
table-of (*DeclConcepts.fields G statC*) (*fn*, *statDeclC*) = *Some field*;
wf-prog G;
x2 = *None* $\longrightarrow$ *G*,*s2*$\vdash$*a*::$\preceq$*Class statC*;
statDeclC $\neq$ *Object*; *class G statDeclC* = *Some y*;
(*x2*, *s2*)::$\preceq$(*G*, *L*); *s1*$\leq$|*s2*; *inited statDeclC* (*globs s1*)$]\!]$ $\Longrightarrow$
G,*s2'*$\vdash$*v*::$\preceq$*type field* $\wedge$ *s2'*$\leq$|*f*$\preceq$*type field*::$\preceq$(*G*, *L*)

```
apply (unfold assign-conforms-def)
apply (drule sym)
apply (clarsimp simp add: fvar-def2)
apply (drule (9) FVar-lemma1)
apply (clarsimp)
apply (drule (2) conforms-globsD [THEN oconf-lconf, THEN lconfD])
apply clarsimp
apply (rule conjI)
apply   clarsimp
apply   (drule (1) rev-gext-objD)
apply   (force elim!: conforms-upd-gobj)

apply   (blast intro: FVar-lemma2)
done
declare split-paired-All [simp] split-paired-Ex [simp]
declare split-if     [split] split-if-asm     [split]
        option.split [split] option.split-asm [split]
declaration ⟪ K (Classical.map-cs (fn cs => cs addSbefore (split-all-tac, split-all-tac))) ⟫
declaration ⟪ K (Simplifier.map-ss (fn ss => ss addloop (split-all-tac, split-all-tac))) ⟫
```

```
lemma AVar-lemma1: ⟦globs s (Inl a) = Some obj;tag obj=Arr ty i;
 the-Intg i' in-bounds i; wf-prog G; G⊢ty.[]≼Tb.[]; Norm s::≼(G, L)
⟧ ⟹ G,s⊢the ((values obj) (Inr (the-Intg i')))::≼Tb
apply (erule widen-Array-Array [THEN conf-widen])
apply  (erule-tac [2] wf-ws-prog)
apply (drule (1) conforms-globsD [THEN oconf-lconf, THEN lconfD])
defer apply assumption
apply (force intro: var-tys-Some-eq [THEN iffD2])
done
```

```
lemma obj-split: ∃ t vs. obj = (|tag=t,values=vs|)
  by (cases obj) auto
```

```
lemma AVar-lemma2: error-free state
       ⟹ error-free
           (assign
             (λv (x, s').
                 ((raise-if (¬ G,s'⊢v fits T) ArrStore) x,
                  upd-gobj (Inl a) (Inr (the-Intg i)) v s'))
             w state)
proof −
  assume error-free: error-free state
  obtain a s where state=(a,s)
    by (cases state)
  with error-free
  show ?thesis
    by (cases a) auto
qed
```

```
lemma AVar-lemma: ⟦wf-prog G; G⊢(x1, s1) −e2−≻i→ (x2, s2);
  ((v,f), Norm s2') = avar G i a (x2, s2); x1 = None ⟶ G,s1⊢a::≼Ta.[];
  (x2, s2)::≼(G, L); s1≤|s2⟧ ⟹ G,s2'⊢v::≼Ta ∧ s2'≤|f≼Ta::≼(G, L)
apply (unfold assign-conforms-def)
apply (drule sym)
```

apply (*clarsimp simp add*: *avar-def2*)
apply (*drule* (*1*) *conf-gext*)
apply (*drule conf-RefTD*, *clarsimp*)
apply (*subgoal-tac* ∃ *t vs*. *obj* = (|*tag*=*t*,*values*=*vs*|))
defer
apply (*rule obj-split*)
apply *clarify*
apply (*frule obj-ty-widenD*)
apply (*auto dest*!: *widen-Class*)
apply (*force dest*: *AVar-lemma1*)

apply (*force elim*!: *fits-Array dest*: *gext-objD*
intro: *var-tys-Some-eq* [*THEN iffD2*] *conforms-upd-gobj*)
done

Call

lemma *conforms-init-lvars-lemma*: ⟦*wf-prog G*;
wf-mhead G P sig mh;
list-all2 (*conf G s*) *pvs pTsa*; *G*⊢*pTsa*[≼](*parTs sig*)⟧ ⟹
G,*s*⊢*empty* (*pars mh*[↦]*pvs*)
[∼::≼]*table-of lvars*(*pars mh*[↦]*parTs sig*)
apply (*unfold wf-mhead-def*)
apply *clarify*
apply (*erule* (*1*) *wlconf-empty-vals* [*THEN wlconf-ext-list*])
apply (*drule wf-ws-prog*)
apply (*erule* (*2*) *conf-list-widen*)
done

lemma *lconf-map-lname* [*simp*]:
G,*s*⊢(*lname-case l1 l2*)[::≼](*lname-case L1 L2*)
=
(*G*,*s*⊢*l1*[::≼]*L1* ∧ *G*,*s*⊢(λ*x*::*unit* . *l2*)[::≼](λ*x*::*unit*. *L2*))
apply (*unfold lconf-def*)
apply (*auto split add*: *lname.splits*)
done

lemma *wlconf-map-lname* [*simp*]:
G,*s*⊢(*lname-case l1 l2*)[∼::≼](*lname-case L1 L2*)
=
(*G*,*s*⊢*l1*[∼::≼]*L1* ∧ *G*,*s*⊢(λ*x*::*unit* . *l2*)[∼::≼](λ*x*::*unit*. *L2*))
apply (*unfold wlconf-def*)
apply (*auto split add*: *lname.splits*)
done

lemma *lconf-map-ename* [*simp*]:
G,*s*⊢(*ename-case l1 l2*)[::≼](*ename-case L1 L2*)
=
(*G*,*s*⊢*l1*[::≼]*L1* ∧ *G*,*s*⊢(λ*x*::*unit*. *l2*)[::≼](λ*x*::*unit*. *L2*))
apply (*unfold lconf-def*)
apply (*auto split add*: *ename.splits*)
done

lemma *wlconf-map-ename* [*simp*]:

$G,s\vdash(\textit{ename-case } l1\ l2)[\sim::\preceq](\textit{ename-case } L1\ L2)$
$=$
$(G,s\vdash l1[\sim::\preceq]L1 \wedge G,s\vdash(\lambda x::\textit{unit}.\ l2)[\sim::\preceq](\lambda x::\textit{unit}.\ L2))$
apply (*unfold wlconf-def*)
apply (*auto split add*: *ename.splits*)
done

lemma *defval-conf1* [*rule-format* (*no-asm*), *elim*]:
is-type G T $\longrightarrow$ ($\exists$ $v\in$*Some* (*default-val T*): $G,s\vdash v::\preceq T$)
apply (*unfold conf-def*)
apply (*induct T*)
apply (*auto intro*: *prim-ty.induct*)
done

lemma *np-no-jump*: $x\neq$*Some* (*Jump j*) $\Longrightarrow$ (*np a'*) $x \neq$ *Some* (*Jump j*)
by (*auto simp add*: *abrupt-if-def*)

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declare *split-if* [*split del*] *split-if-asm* [*split del*]
option.split [*split del*] *option.split-asm* [*split del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss delloop split-all-tac*)) ⟫
declaration ⟪ *K* (*Classical.map-cs* (*fn cs* => *cs delSWrapper split-all-tac*)) ⟫

lemma *conforms-init-lvars*:
⟦*wf-mhead G* (*pid declC*) *sig* (*mhead* (*mthd dm*)); *wf-prog G*;
list-all2 (*conf G s*) *pvs pTsa*; $G\vdash pTsa[\preceq](parTs\ sig)$;
$(x, s)::\preceq(G, L)$;
methd G declC sig = *Some dm*;
isrtype G statT;
$G\vdash invC\preceq_C declC$;
$G,s\vdash a'::\preceq RefT\ statT$;
invmode (*mhd sm*) *e* = *IntVir* $\longrightarrow a' \neq$ *Null*;
invmode (*mhd sm*) *e* $\neq$ *IntVir* $\longrightarrow$
($\exists$ *statC*. *statT*=*ClassT statC* $\wedge$ $G\vdash statC\preceq_C declC$)
$\vee$ ($\forall$ *statC*. *statT*$\neq$*ClassT statC* $\wedge$ *declC*=*Object*);
invC = *invocation-class* (*invmode* (*mhd sm*) *e*) *s a' statT*;
declC = *invocation-declclass G* (*invmode* (*mhd sm*) *e*) *s a' statT sig*;
$x\neq$*Some* (*Jump Ret*)
⟧ $\Longrightarrow$
init-lvars G declC sig (*invmode* (*mhd sm*) *e*) *a'*
pvs $(x,s)::\preceq(G,\lambda\ k.$
(*case k of*
EName e $\Rightarrow$ (*case e of*
VNam v
$\Rightarrow$ (*table-of* (*lcls* (*mbody* (*mthd dm*)))
(*pars* (*mthd dm*)[$\mapsto$]*parTs sig*)) *v*
| *Res* $\Rightarrow$ *Some* (*resTy* (*mthd dm*)))
| *This* $\Rightarrow$ *if* (*is-static* (*mthd sm*))
then None else Some (*Class declC*)))
apply (*simp add*: *init-lvars-def2*)
apply (*rule conforms-set-locals*)
apply (*simp* (*no-asm-simp*) *split add*: *split-if*)
apply (*drule* (*4*) *DynT-conf*)
apply *clarsimp*

apply (*drule* (*3*) *conforms-init-lvars-lemma*
[**where** *?lvars*=(*lcls* (*mbody* (*mthd dm*)))])
apply (*case-tac dm*,*simp*)
apply (*rule conjI*)
apply (*unfold wlconf-def*, *clarify*)
apply (*clarsimp simp add*: *wf-mhead-def is-acc-type-def*)
apply (*case-tac is-static sm*)
apply *simp*
apply *simp*

apply *simp*
apply (*case-tac is-static sm*)
apply *simp*
apply (*simp add*: *np-no-jump*)
done
declare *split-paired-All* [*simp*] *split-paired-Ex* [*simp*]
declare *split-if* [*split*] *split-if-asm* [*split*]
option.split [*split*] *option.split-asm* [*split*]
declaration ⟪ *K* (*Classical.map-cs* (*fn cs* => *cs addSbefore* (*split-all-tac*, *split-all-tac*))) ⟫
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*))) ⟫

47 accessibility

theorem *dynamic-field-access-ok*:
assumes *wf*: *wf-prog G* **and**
not-Null: ¬ *stat* ⟶ *a*≠*Null* **and**
conform-a: *G*,(*store s*)⊢*a*::≼ *Class statC* **and**
conform-s: *s*::≼(*G*, *L*) **and**
normal-s: *normal s* **and**
wt-e: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*e*::−*Class statC* **and**
f: *accfield G accC statC fn* = *Some f* **and**
dynC: *if stat then dynC*=*declclass f*
else dynC=*obj-class* (*lookup-obj* (*store s*) *a*) **and**
stat: *if stat then* (*is-static f*) *else* (¬ *is-static f*)
shows *table-of* (*DeclConcepts.fields G dynC*) (*fn*,*declclass f*) = *Some* (*fld f*)∧
G⊢*Field fn f in dynC dyn-accessible-from accC*
proof (*cases stat*)
case *True*
with *stat* **have** *static*: (*is-static f*) **by** *simp*
from *True dynC*
have *dynC′*: *dynC*=*declclass f* **by** *simp*
with *f*
have *table-of* (*DeclConcepts.fields G statC*) (*fn*,*declclass f*) = *Some* (*fld f*)
by (*auto simp add*: *accfield-def Let-def intro*!: *table-of-remap-SomeD*)
moreover
from *wt-e wf* **have** *is-class G statC*
by (*auto dest*!: *ty-expr-is-type*)
moreover note *wf dynC′*
ultimately have
table-of (*DeclConcepts.fields G dynC*) (*fn*,*declclass f*) = *Some* (*fld f*)
by (*auto dest*: *fields-declC*)
with *dynC′ f static wf*
show *?thesis*
by (*auto dest*: *static-to-dynamic-accessible-from-static*
dest!: *accfield-accessibleD*)
next
case *False*
with *wf conform-a not-Null conform-s dynC*

obtain *subclseq*: $G \vdash dynC \preceq_C statC$ **and**
is-class G dynC
by (*auto dest*!: *conforms-RefTD* [*of - - - -* (*fst s*) *L*]
dest: *obj-ty-obj-class1*
simp add: *obj-ty-obj-class*)
with *wf f*
have *table-of* (*DeclConcepts.fields G dynC*) (*fn*,*declclass f*) = *Some* (*fld f*)
by (*auto simp add*: *accfield-def Let-def*
dest: *fields-mono*
dest!: *table-of-remap-SomeD*)
moreover
from *f subclseq*
have *G⊢Field fn f in dynC dyn-accessible-from accC*
by (*auto intro*!: *static-to-dynamic-accessible-from wf*
dest: *accfield-accessibleD*)
ultimately show *?thesis*
by *blast*
qed

lemma *error-free-field-access*:
assumes *accfield*: *accfield G accC statC fn* = *Some* (*statDeclC*, *f*) **and**
wt-e: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*e*::−*Class statC* **and**
eval-init: *G⊢Norm s0* −*Init statDeclC*→ *s1* **and**
eval-e: *G⊢s1* −*e*−≻*a*→ *s2* **and**
conf-s2: *s2*::≼(*G*, *L*) **and**
conf-a: *normal s2* ⟹ *G*, *store s2*⊢*a*::≼*Class statC* **and**
fvar: (*v*,*s2′*)=*fvar statDeclC* (*is-static f*) *fn a s2* **and**
wf: *wf-prog G*
shows *check-field-access G accC statDeclC fn* (*is-static f*) *a s2′* = *s2′*
proof −
from *fvar*
have *store-s2′*: *store s2′*=*store s2*
by (*cases s2*) (*simp add*: *fvar-def2*)
with *fvar conf-s2*
have *conf-s2′*: *s2′*::≼(*G*, *L*)
by (*cases s2*,*cases is-static f*) (*auto simp add*: *fvar-def2*)
from *eval-init*
have *initd-statDeclC-s1*: *initd statDeclC s1*
by (*rule init-yields-initd*)
with *eval-e store-s2′*
have *initd-statDeclC-s2′*: *initd statDeclC s2′*
by (*auto dest*: *eval-gext intro*: *inited-gext*)
show *?thesis*
proof (*cases normal s2′*)
case *False*
then show *?thesis*
by (*auto simp add*: *check-field-access-def Let-def*)
next
case *True*
with *fvar store-s2′*
have *not-Null*: ¬ (*is-static f*) ⟶ *a*≠*Null*
by (*cases s2*) (*auto simp add*: *fvar-def2*)
from *True fvar store-s2′*
have *normal s2*
by (*cases s2*,*cases is-static f*) (*auto simp add*: *fvar-def2*)
with *conf-a store-s2′*
have *conf-a′*: *G*,*store s2′*⊢*a*::≼*Class statC*
by *simp*

from *conf-a′ conf-s2′ True initd-statDeclC-s2′*
dynamic-field-access-ok [*OF wf not-Null conf-a′ conf-s2′*
True wt-e accfield]
show *?thesis*
by (*cases is-static f*)
(*auto dest*!: *initedD*
simp add: *check-field-access-def Let-def*)
qed
qed

lemma *call-access-ok*:
assumes *invC-prop*: $G \vdash$*invmode statM e*$\rightarrow$*invC*$\preceq$*statT*
and *wf*: *wf-prog G*
and *wt-e*: (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)$\vdash$*e*::−*RefT statT*
and *statM*: (*statDeclT*,*statM*) $\in$ *mheads G accC statT sig*
and *invC*: *invC* = *invocation-class* (*invmode statM e*) *s a statT*
shows $\exists$ *dynM*. *dynlookup G statT invC sig* = *Some dynM* $\wedge$
G$\vdash$*Methd sig dynM in invC dyn-accessible-from accC*
proof −
from *wt-e wf* **have** *type-statT*: *is-type G* (*RefT statT*)
by (*auto dest*: *ty-expr-is-type*)
from *statM* **have** *not-Null*: *statT* $\neq$ *NullT* **by** *auto*
from *type-statT wt-e*
have *wf-I*: ($\forall$ *I*. *statT* = *IfaceT I* $\longrightarrow$ *is-iface G I* $\wedge$
invmode statM e $\neq$ *SuperM*)
by (*auto dest*: *invocationTypeExpr-noClassD*)
from *wt-e*
have *wf-A*: ($\forall$ *T*. *statT* = *ArrayT T* $\longrightarrow$ *invmode statM e* $\neq$ *SuperM*)
by (*auto dest*: *invocationTypeExpr-noClassD*)
show *?thesis*
proof (*cases invmode statM e* = *IntVir*)
case *True*
with *invC-prop not-Null*
have *invC-prop′*: *is-class G invC* $\wedge$
(*if* ($\exists$ *T*. *statT*=*ArrayT T*) *then invC*=*Object*
else G$\vdash$*Class invC*$\preceq$*RefT statT*)
by (*auto simp add*: *DynT-prop-def*)
with *True not-Null*
have *G*,*statT* $\vdash$ *invC valid-lookup-cls-for is-static statM*
by (*cases statT*) (*auto simp add*: *invmode-def*)
with *statM type-statT wf*
show *?thesis*
by − (*rule dynlookup-access*,*auto*)
next
case *False*
with *type-statT wf invC not-Null wf-I wf-A*
have *invC-prop′*: *is-class G invC* $\wedge$
(($\exists$ *statC*. *statT*=*ClassT statC* $\wedge$ *invC*=*statC*) $\vee$
($\forall$ *statC*. *statT*$\neq$*ClassT statC* $\wedge$ *invC*=*Object*))
by (*case-tac statT*) (*auto simp add*: *invocation-class-def*
split: *inv-mode.splits*)
with *not-Null wf*
have *dynlookup-static*: *dynlookup G statT invC sig* = *methd G invC sig*
by (*case-tac statT*) (*auto simp add*: *dynlookup-def dynmethd-C-C*
dynimethd-def)
from *statM wf wt-e not-Null False invC-prop′* **obtain** *dynM* **where**
accmethd G accC invC sig = *Some dynM*
by (*auto dest*!: *static-mheadsD*)

from *invC-prop′ False not-Null wf-I*
have *G,statT* ⊢ *invC valid-lookup-cls-for is-static statM*
by (*cases statT*) (*auto simp add*: *invmode-def*)
with *statM type-statT wf*
show *?thesis*
by − (*rule dynlookup-access,auto*)
qed
qed

lemma *error-free-call-access*:
assumes
eval-args: $G\vdash s1\ -args\dot{\doteq}\succ vs\rightarrow\ s2$ **and**
wt-e: $(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash e{::}-(RefT\ statT)$ **and**
statM: $max\text{-}spec\ G\ accC\ statT\ (\!|name = mn,\ parTs = pTs|\!)$
$= \{((statDeclT,\ statM),\ pTs')\}$ **and**
conf-s2: $s2{::}\preceq(G,\ L)$ **and**
conf-a: $normal\ s1 \Longrightarrow G,\ store\ s1\vdash a{::}\preceq RefT\ statT$ **and**
invProp: $normal\ s3 \Longrightarrow$
$G\vdash invmode\ statM\ e\rightarrow invC\preceq statT$ **and**
s3: $s3{=}init\text{-}lvars\ G\ invDeclC\ (\!|name = mn,\ parTs = pTs'|\!)$
$(invmode\ statM\ e)\ a\ vs\ s2$ **and**
invC: $invC = invocation\text{-}class\ (invmode\ statM\ e)\ (store\ s2)\ a\ statT$**and**
invDeclC: $invDeclC = invocation\text{-}declclass\ G\ (invmode\ statM\ e)\ (store\ s2)$
$a\ statT\ (\!|name = mn,\ parTs = pTs'|\!)$ **and**
wf: *wf-prog G*
shows $check\text{-}method\text{-}access\ G\ accC\ statT\ (invmode\ statM\ e)\ (\!|name{=}mn,parTs{=}pTs'|\!)\ a\ s3$
$= s3$
proof (*cases normal s2*)
case *False*
with *s3*
have *abrupt s3* = *abrupt s2*
by (*auto simp add*: *init-lvars-def2*)
with *False*
show *?thesis*
by (*auto simp add*: *check-method-access-def Let-def*)
next
case *True*
note *normal-s2* = *True*
with *eval-args*
have *normal-s1*: *normal s1*
by (*cases normal s1*) *auto*
with *conf-a eval-args*
have *conf-a-s2*: $G,\ store\ s2\vdash a{::}\preceq RefT\ statT$
by (*auto dest*: *eval-gext intro*: *conf-gext*)
show *?thesis*
proof (*cases a=Null* ⟶ (*is-static statM*))
case *False*
then obtain ¬ *is-static statM a=Null*
by *blast*
with *normal-s2 s3*
have *abrupt s3* = *Some* (*Xcpt* (*Std NullPointer*))
by (*auto simp add*: *init-lvars-def2*)
then show *?thesis*
by (*auto simp add*: *check-method-access-def Let-def*)
next
case *True*
from *statM*
obtain

```
    statM': (statDeclT,statM)∈mheads G accC statT (|name=mn,parTs=pTs'|)
      by (blast dest: max-spec2mheads)
    from True normal-s2 s3
    have normal s3
      by (auto simp add: init-lvars-def2)
    then have G⊢invmode statM e→invC≼statT
      by (rule invProp)
    with wt-e statM' wf invC
    obtain dynM where
      dynM: dynlookup G statT invC (|name=mn,parTs=pTs'|) = Some dynM and
      acc-dynM: G ⊢Methd (|name=mn,parTs=pTs'|) dynM
                        in invC dyn-accessible-from accC
      by (force dest!: call-access-ok)
    moreover
    from s3 invC
    have invC': invC=(invocation-class (invmode statM e) (store s3) a statT)
      by (cases s2,cases invmode statM e)
         (simp add: init-lvars-def2 del: invmode-Static-eq)+
    ultimately
    show ?thesis
      by (auto simp add: check-method-access-def Let-def)
  qed
qed


lemma map-upds-eq-length-append-simp:
  ⋀ tab qs. length ps = length qs ⟹ tab(ps[↦]qs@zs) = tab(ps[↦]qs)
proof (induct ps)
  case Nil thus ?case by simp
next
  case (Cons p ps tab qs)
  from ‹length (p#ps) = length qs›
  obtain q qs' where qs: qs=q#qs' and eq-length: length ps=length qs'
    by (cases qs) auto
  from eq-length have (tab(p↦q))(ps[↦]qs'@zs)=(tab(p↦q))(ps[↦]qs')
    by (rule Cons.hyps)
  with qs show ?case
    by simp
qed


lemma map-upds-upd-eq-length-simp:
  ⋀ tab qs x y. length ps = length qs
                ⟹ tab(ps[↦]qs)(x↦y) = tab(ps@[x][↦]qs@[y])
proof (induct ps)
  case Nil thus ?case by simp
next
  case (Cons p ps tab qs x y)
  from ‹length (p#ps) = length qs›
  obtain q qs' where qs: qs=q#qs' and eq-length: length ps=length qs'
    by (cases qs) auto
  from eq-length
  have (tab(p↦q))(ps[↦]qs')(x↦y) = (tab(p↦q))(ps@[x][↦]qs'@[y])
    by (rule Cons.hyps)
  with qs show ?case
    by simp
qed
```

lemma *map-upd-cong*: $tab{=}tab' \Longrightarrow tab(x{\mapsto}y) = tab'(x{\mapsto}y)$
by *simp*

lemma *map-upd-cong-ext*: $tab\ z{=}tab'\ z \Longrightarrow (tab(x{\mapsto}y))\ z = (tab'(x{\mapsto}y))\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upds-cong*: $tab{=}tab' \Longrightarrow tab(xs[{\mapsto}]ys) = tab'(xs[{\mapsto}]ys)$
by (*cases xs*) *simp+*

lemma *map-upds-cong-ext*:
$\bigwedge tab\ tab'\ ys.\ tab\ z{=}tab'\ z \Longrightarrow (tab(xs[{\mapsto}]ys))\ z = (tab'(xs[{\mapsto}]ys))\ z$
proof (*induct xs*)
  case *Nil* **thus** *?case* **by** *simp*
next
  case (*Cons x xs tab tab' ys*)
  note *Hyps* = *this*
  show *?case*
  proof (*cases ys*)
    case *Nil*
    with *Hyps*
    show *?thesis* **by** *simp*
  next
    case (*Cons y ys'*)
    have $(tab(x{\mapsto}y)(xs[{\mapsto}]ys'))\ z = (tab'(x{\mapsto}y)(xs[{\mapsto}]ys'))\ z$
      by (*iprover intro*: *Hyps map-upd-cong-ext*)
    with *Cons* **show** *?thesis*
      by *simp*
  qed
qed

lemma *map-upd-override*: $(tab(x{\mapsto}y))\ x = (tab'(x{\mapsto}y))\ x$
  by *simp*

lemma *map-upds-eq-length-suffix*: $\bigwedge tab\ qs.$
    $length\ ps = length\ qs \Longrightarrow tab(ps@xs[{\mapsto}]qs) = tab(ps[{\mapsto}]qs)(xs[{\mapsto}][])$
proof (*induct ps*)
  case *Nil* **thus** *?case* **by** *simp*
next
  case (*Cons p ps tab qs*)
  then obtain *q qs'* **where** *qs*: $qs{=}q\#qs'$ **and** *eq-length*: $length\ ps{=}length\ qs'$
    by (*cases qs*) *auto*
  from *eq-length*
  have $tab(p{\mapsto}q)(ps\ @\ xs[{\mapsto}]qs') = tab(p{\mapsto}q)(ps[{\mapsto}]qs')(xs[{\mapsto}][])$
    by (*rule Cons.hyps*)
  with *qs* **show** *?case*
    by *simp*
qed

lemma *map-upds-upds-eq-length-prefix-simp*:
  $\bigwedge tab\ qs.\ length\ ps = length\ qs$
    $\Longrightarrow tab(ps[{\mapsto}]qs)(xs[{\mapsto}]ys) = tab(ps@xs[{\mapsto}]qs@ys)$

proof (*induct ps*)
  case *Nil* **thus** *?case* **by** *simp*
next
  case (*Cons p ps tab qs*)
  then obtain *q qs′* **where** *qs*: *qs=q#qs′* **and** *eq-length*: *length ps=length qs′*
    by (*cases qs*) *auto*
  from *eq-length*
  have $tab(p{\mapsto}q)(ps[{\mapsto}]qs')(xs[{\mapsto}]ys) = tab(p{\mapsto}q)(ps\ @\ xs[{\mapsto}](qs'\ @\ ys))$
    by (*rule Cons.hyps*)
  with *qs*
  show *?case* **by** *simp*
qed

lemma *map-upd-cut-irrelevant*:
$[\![(tab(x{\mapsto}y))\ vn = Some\ el;\ (tab'(x{\mapsto}y))\ vn = None]\!]$
  $\Longrightarrow tab\ vn = Some\ el$
by (*cases tab′ vn = None*) (*simp add*: *fun-upd-def*)+

lemma *map-upd-Some-expand*:
$[\![tab\ vn = Some\ z]\!]$
  $\Longrightarrow \exists\ z.\ (tab(x{\mapsto}y))\ vn = Some\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upds-Some-expand*:
$\bigwedge tab\ ys\ z.\ [\![tab\ vn = Some\ z]\!]$
  $\Longrightarrow \exists\ z.\ (tab(xs[{\mapsto}]ys))\ vn = Some\ z$
proof (*induct xs*)
  case *Nil* **thus** *?case* **by** *simp*
next
  case (*Cons x xs tab ys z*)
  note *z* = ‹*tab vn = Some z*›
  show *?case*
  proof (*cases ys*)
    case *Nil*
    with *z* **show** *?thesis* **by** *simp*
  next
    case (*Cons y ys′*)
    note *ys* = ‹*ys = y#ys′*›
    from *z* **obtain** *z′* **where** $(tab(x{\mapsto}y))\ vn = Some\ z'$
      by (*rule map-upd-Some-expand* [*of tab,elim-format*]) *blast*
    hence $\exists z.\ ((tab(x{\mapsto}y))(xs[{\mapsto}]ys'))\ vn = Some\ z$
      by (*rule Cons.hyps*)
    with *ys* **show** *?thesis*
      by *simp*
  qed
qed

lemma *map-upd-Some-swap*:
$(tab(r{\mapsto}w)(u{\mapsto}v))\ vn = Some\ z \Longrightarrow \exists\ z.\ (tab(u{\mapsto}v)(r{\mapsto}w))\ vn = Some\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upd-None-swap*:
$(tab(r{\mapsto}w)(u{\mapsto}v))\ vn = None \Longrightarrow (tab(u{\mapsto}v)(r{\mapsto}w))\ vn = None$

by (*simp add*: *fun-upd-def*)

lemma *map-eq-upd-eq*: $tab\ vn = tab'\ vn \Longrightarrow (tab(x \mapsto y))\ vn = (tab'(x \mapsto y))\ vn$
by (*simp add*: *fun-upd-def*)

lemma *map-upd-in-expansion-map-swap*:
$[\![(tab(x \mapsto y))\ vn = Some\ z; tab\ vn \neq Some\ z]\!]$
$\Longrightarrow\ (tab'(x \mapsto y))\ vn = Some\ z$
by (*simp add*: *fun-upd-def*)

lemma *map-upds-in-expansion-map-swap*:
$\bigwedge tab\ tab'\ ys\ z.\ [\![(tab(xs[\mapsto]ys))\ vn = Some\ z; tab\ vn \neq Some\ z]\!]$
$\Longrightarrow\ (tab'(xs[\mapsto]ys))\ vn = Some\ z$
proof (*induct xs*)
  case *Nil* **thus** *?case* **by** *simp*
next
  case (*Cons x xs tab tab' ys z*)
  note *some* = ‹$(tab(x \# xs[\mapsto]ys))\ vn = Some\ z$›
  note *tab-not-z* = ‹$tab\ vn \neq Some\ z$›
  show *?case*
  proof (*cases ys*)
    case *Nil* **with** *some tab-not-z* **show** *?thesis* **by** *simp*
  next
    case (*Cons y tl*)
    note *ys* = ‹$ys = y\#tl$›
    show *?thesis*
    proof (*cases* $(tab(x \mapsto y))\ vn \neq Some\ z$)
      case *True*
      with *some ys* **have** $(tab'(x \mapsto y)(xs[\mapsto]tl))\ vn = Some\ z$
        by (*fastsimp intro*: *Cons.hyps*)
      with *ys* **show** *?thesis*
        by *simp*
    next
      case *False*
      hence *tabx-z*: $(tab(x \mapsto y))\ vn = Some\ z$ **by** *blast*
      moreover
      from *tabx-z tab-not-z*
      have $(tab'(x \mapsto y))\ vn = Some\ z$
        by (*rule map-upd-in-expansion-map-swap*)
      ultimately
      have $(tab(x \mapsto y))\ vn =(tab'(x \mapsto y))\ vn$
        by *simp*
      hence $(tab(x \mapsto y)(xs[\mapsto]tl))\ vn = (tab'(x \mapsto y)(xs[\mapsto]tl))\ vn$
        by (*rule map-upds-cong-ext*)
      with *some ys*
      show *?thesis*
        by *simp*
    qed
  qed
qed

lemma *map-upds-Some-swap*:
 assumes *r-u*: $(tab(r \mapsto w)(u \mapsto v)(xs[\mapsto]ys))\ vn = Some\ z$
  shows $\exists\ z.\ (tab(u \mapsto v)(r \mapsto w)(xs[\mapsto]ys))\ vn = Some\ z$

```
proof (cases (tab(r↦w)(u↦v)) vn = Some z)
  case True
  then obtain z' where (tab(u↦v)(r↦w)) vn = Some z'
    by (rule map-upd-Some-swap [elim-format]) blast
  thus ∃ z. (tab(u↦v)(r↦w)(xs[↦]ys)) vn = Some z
    by (rule map-upds-Some-expand)
next
  case False
  with r-u
  have (tab(u↦v)(r↦w)(xs[↦]ys)) vn = Some z
    by (rule map-upds-in-expansion-map-swap)
  thus ?thesis
    by simp
qed


lemma map-upds-Some-insert:
  assumes z: (tab(xs[↦]ys)) vn = Some z
    shows ∃ z. (tab(u↦v)(xs[↦]ys)) vn = Some z
proof (cases ∃ z. tab vn = Some z)
  case True
  then obtain z' where tab vn = Some z' by blast
  then obtain z'' where (tab(u↦v)) vn = Some z''
    by (rule map-upd-Some-expand [elim-format]) blast
  thus ?thesis
    by (rule map-upds-Some-expand)
next
  case False
  hence tab vn ≠ Some z by simp
  with z
  have (tab(u↦v)(xs[↦]ys)) vn = Some z
    by (rule map-upds-in-expansion-map-swap)
  thus ?thesis ..
qed


lemma map-upds-None-cut:
assumes expand-None: (tab(xs[↦]ys)) vn = None
  shows tab vn = None
proof (cases tab vn = None)
  case True thus ?thesis by simp
next
  case False then obtain z where tab vn = Some z by blast
  then obtain z' where (tab(xs[↦]ys)) vn = Some z'
    by (rule map-upds-Some-expand [where ?tab=tab,elim-format]) blast
  with expand-None show ?thesis
    by simp
qed


lemma map-upds-cut-irrelevant:
⋀ tab tab' ys. ⟦(tab(xs[↦]ys)) vn = Some el; (tab'(xs[↦]ys)) vn = None⟧
                  ⟹ tab vn = Some el
proof (induct xs)
  case Nil thus ?case by simp
next
  case (Cons x xs tab tab' ys)
  note tab-vn = ‹(tab(x # xs[↦]ys)) vn = Some el›
```

```
  note tab'-vn = ‹(tab'(x # xs[↦]ys)) vn = None›
  show ?case
  proof (cases ys)
    case Nil
    with tab-vn show ?thesis by simp
  next
    case (Cons y tl)
    note ys = ‹ys=y#tl›
    with tab-vn tab'-vn
    have (tab(x↦y)) vn = Some el
      by − (rule Cons.hyps,auto)
    moreover from tab'-vn ys
    have (tab'(x↦y)(xs[↦]tl)) vn = None
      by simp
    hence (tab'(x↦y)) vn = None
      by (rule map-upds-None-cut)
    ultimately show tab vn = Some el
      by (rule map-upd-cut-irrelevant)
  qed
qed


lemma dom-vname-split:
 dom (lname-case (ename-case (tab(x↦y)(xs[↦]ys)) a) b)
   = dom (lname-case (ename-case (tab(x↦y)) a) b) ∪
     dom (lname-case (ename-case (tab(xs[↦]ys)) a) b)
  (is ?List x xs y ys = ?Hd x y ∪ ?Tl xs ys)
proof
  show ?List x xs y ys ⊆ ?Hd x y ∪ ?Tl xs ys
  proof
    fix el
    assume el-in-list: el ∈ ?List x xs y ys
    show el ∈  ?Hd x y ∪ ?Tl xs ys
    proof (cases el)
      case This
      with el-in-list show ?thesis by (simp add: dom-def)
    next
      case (EName en)
      show ?thesis
      proof (cases en)
        case Res
        with EName el-in-list show ?thesis by (simp add: dom-def)
      next
        case (VNam vn)
        with EName el-in-list show ?thesis
          by (auto simp add: dom-def dest: map-upds-cut-irrelevant)
      qed
    qed
  qed
next
  show ?Hd x y ∪ ?Tl xs ys  ⊆ ?List x xs y ys
  proof (rule subsetI)
    fix el
    assume  el-in-hd-tl: el ∈  ?Hd x y ∪ ?Tl xs ys
    show el ∈ ?List x xs y ys
    proof (cases el)
      case This
      with el-in-hd-tl show ?thesis by (simp add: dom-def)
```

```
    next
      case (EName en)
      show ?thesis
      proof (cases en)
        case Res
        with EName el-in-hd-tl show ?thesis by (simp add: dom-def)
      next
        case (VNam vn)
        with EName el-in-hd-tl show ?thesis
          by (auto simp add: dom-def intro: map-upds-Some-expand
                                            map-upds-Some-insert)
      qed
    qed
  qed
qed


lemma dom-map-upd: ⋀ tab. dom (tab(x↦y)) = dom tab ∪ {x}
by (auto simp add: dom-def fun-upd-def)


lemma dom-map-upds: ⋀ tab ys. length xs = length ys
  ⟹ dom (tab(xs[↦]ys)) = dom tab ∪ set xs
proof (induct xs)
  case Nil thus ?case by (simp add: dom-def)
next
  case (Cons x xs tab ys)
  note Hyp = Cons.hyps
  note len = ‹length (x#xs)=length ys›
  show ?case
  proof (cases ys)
    case Nil with len show ?thesis by simp
  next
    case (Cons y tl)
    with len have dom (tab(x↦y)(xs[↦]tl)) = dom (tab(x↦y)) ∪ set xs
      by − (rule Hyp,simp)
    moreover
    have dom (tab(x↦hd ys)) = dom tab ∪ {x}
      by (rule dom-map-upd)
    ultimately
    show ?thesis using Cons
      by simp
  qed
qed


lemma dom-ename-case-None-simp:
 dom (ename-case vname-tab None) = VNam ‘ (dom vname-tab)
  apply (auto simp add: dom-def image-def )
  apply (case-tac x)
  apply auto
  done


lemma dom-ename-case-Some-simp:
 dom (ename-case vname-tab (Some a)) = VNam ‘ (dom vname-tab) ∪ {Res}
  apply (auto simp add: dom-def image-def )
  apply (case-tac x)
  apply auto
```

done

lemma *dom-lname-case-None-simp*:
dom (*lname-case ename-tab None*) = *EName* ‘ (*dom ename-tab*)
apply (*auto simp add*: *dom-def image-def*)
apply (*case-tac x*)
apply *auto*
done

lemma *dom-lname-case-Some-simp*:
dom (*lname-case ename-tab* (*Some a*)) = *EName* ‘ (*dom ename-tab*) ∪ {*This*}
apply (*auto simp add*: *dom-def image-def*)
apply (*case-tac x*)
apply *auto*
done

lemmas *dom-lname-ename-case-simps* =
dom-ename-case-None-simp dom-ename-case-Some-simp
dom-lname-case-None-simp dom-lname-case-Some-simp

lemma *image-comp*:
f ‘ *g* ‘ *A* = (*f* ∘ *g*) ‘ *A*
by (*auto simp add*: *image-def*)

lemma *dom-locals-init-lvars*:
assumes *m*: *m*=(*mthd* (*the* (*methd G C sig*)))
assumes *len*: *length* (*pars m*) = *length pvs*
shows *dom* (*locals* (*store* (*init-lvars G C sig* (*invmode m e*) *a pvs s*)))
= *parameters m*
proof −
from *m*
have *static-m′*: *is-static m* = *static m*
by *simp*
from *len*
have *dom-vnames*: *dom* (*empty*(*pars m*[↦]*pvs*))=*set* (*pars m*)
by (*simp add*: *dom-map-upds*)
show *?thesis*
proof (*cases static m*)
case *True*
with *static-m′ dom-vnames m*
show *?thesis*
by (*cases s*) (*simp add*: *init-lvars-def Let-def parameters-def*
dom-lname-ename-case-simps image-comp)
next
case *False*
with *static-m′ dom-vnames m*
show *?thesis*
by (*cases s*) (*simp add*: *init-lvars-def Let-def parameters-def*
dom-lname-ename-case-simps image-comp)
qed
qed

```
lemma da-e2-BinOp:
  assumes da: (|prg=G,cls=accC,lcl=L|)
                ⊢dom (locals (store s0)) »⟨BinOp binop e1 e2⟩e» A
    and wt-e1: (|prg=G,cls=accC,lcl=L|)⊢e1::−e1T
    and wt-e2: (|prg=G,cls=accC,lcl=L|)⊢e2::−e2T
    and wt-binop: wt-binop G binop e1T e2T
    and conf-s0: s0::≼(G,L)
    and normal-s1: normal s1
    and eval-e1: G⊢s0 −e1−≻v1→ s1
    and conf-v1: G,store s1⊢v1::≼e1T
    and wf: wf-prog G
  shows ∃ E2. (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1))
        »(if need-second-arg binop v1 then ⟨e2⟩e else ⟨Skip⟩s)» E2
proof −
  note inj-term-simps [simp]
  from da obtain E1 where
    da-e1: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨e1⟩e» E1
    by cases simp+
  obtain E2 where
    (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1))
      »(if need-second-arg binop v1 then ⟨e2⟩e else ⟨Skip⟩s)» E2
  proof (cases need-second-arg binop v1)
    case False
    obtain S where
      daSkip: (|prg=G,cls=accC,lcl=L|)
                ⊢ dom (locals (store s1)) »⟨Skip⟩s» S
      by (auto intro: da-Skip [simplified] assigned.select-convs)
    thus ?thesis
      using that by (simp add: False)
  next
    case True
    from eval-e1 have
      s0-s1:dom (locals (store s0)) ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim)
    {
      assume condAnd: binop=CondAnd
      have ?thesis
      proof −
        from da obtain E2' where
          (|prg=G,cls=accC,lcl=L|)
            ⊢ dom (locals (store s0)) ∪ assigns-if True e1 »⟨e2⟩e» E2'
          by cases (simp add: condAnd)+
        moreover
        have dom (locals (store s0))
          ∪ assigns-if True e1 ⊆ dom (locals (store s1))
        proof −
          from condAnd wt-binop have e1T: e1T=PrimT Boolean
            by simp
          with normal-s1 conf-v1 obtain b where v1=Bool b
            by (auto dest: conf-Boolean)
          with True condAnd
          have v1: v1=Bool True
            by simp
          from eval-e1 normal-s1
          have assigns-if True e1 ⊆ dom (locals (store s1))
            by (rule assigns-if-good-approx' [elim-format])
               (insert wt-e1, simp-all add: e1T v1)
          with s0-s1 show ?thesis by (rule Un-least)
        qed
```

```
      ultimately
      show ?thesis
        using that by (cases rule: da-weakenE) (simp add: True)
    qed
  }
  moreover
  {
    assume condOr: binop=CondOr
    have ?thesis

    proof -
      from da obtain E2' where
        (|prg=G,cls=accC,lcl=L|)
           ⊢ dom (locals (store s0)) ∪ assigns-if False e1 »⟨e2⟩e» E2'
        by cases (simp add: condOr)+
      moreover
      have dom (locals (store s0))
                ∪ assigns-if False e1 ⊆ dom (locals (store s1))
      proof -
        from condOr wt-binop have e1T: e1T=PrimT Boolean
          by simp
        with normal-s1 conf-v1 obtain b where v1=Bool b
          by (auto dest: conf-Boolean)
        with True condOr
        have v1: v1=Bool False
          by simp
        from eval-e1 normal-s1
        have assigns-if False e1 ⊆ dom (locals (store s1))
          by (rule assigns-if-good-approx' [elim-format])
             (insert wt-e1, simp-all add: e1T v1)
        with s0-s1 show ?thesis by (rule Un-least)
      qed
      ultimately
      show ?thesis
        using that by (rule da-weakenE) (simp add: True)
    qed
  }
  moreover
  {
    assume notAndOr: binop≠CondAnd binop≠CondOr
    have ?thesis
    proof -
      from da notAndOr obtain E1' where
        da-e1: (|prg=G,cls=accC,lcl=L|)
                ⊢ dom (locals (store s0)) »⟨e1⟩e» E1'
        and da-e2: (|prg=G,cls=accC,lcl=L|)⊢ nrm E1' »In1l e2» A
        by cases simp+
      from eval-e1 wt-e1 da-e1 wf normal-s1
      have nrm E1' ⊆ dom (locals (store s1))
        by (cases rule: da-good-approxE') iprover
      with da-e2 show ?thesis
        using that by (rule da-weakenE) (simp add: True)
    qed
  }
  ultimately show ?thesis
    by (cases binop) auto
qed
thus ?thesis ..
qed
```

main proof of type safety

```
lemma eval-type-sound:
  assumes  eval: G⊢s0 −t≻→ (v,s1)
   and       wt: (|prg=G,cls=accC,lcl=L|)⊢t::T
   and       da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»t»A
   and       wf: wf-prog G
   and conf-s0: s0::≼(G,L)
  shows s1::≼(G,L) ∧  (normal s1 ⟶ G,L,store s1⊢t≻v::≼T) ∧
         (error-free s0 = error-free s1)
proof −
  note inj-term-simps [simp]
  let ?TypeSafeObj = λ s0 s1 t v.
         ∀  L accC T A. s0::≼(G,L) ⟶ (|prg=G,cls=accC,lcl=L|)⊢t::T
                      ⟶ (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»t»A
                      ⟶ s1::≼(G,L) ∧ (normal s1 ⟶ G,L,store s1⊢t≻v::≼T)
                         ∧ (error-free s0 = error-free s1)
  from eval
  have ⋀ L accC T A. ⟦s0::≼(G,L);(|prg=G,cls=accC,lcl=L|)⊢t::T;
                      (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»t»A⟧
        ⟹ s1::≼(G,L) ∧ (normal s1 ⟶ G,L,store s1⊢t≻v::≼T)
            ∧ (error-free s0 = error-free s1)
   (is PROP ?TypeSafe s0 s1 t v
    is ⋀ L accC T A. ?Conform L s0 ⟹ ?WellTyped L accC T t
                 ⟹ ?DefAss L accC s0 t A
                 ⟹ ?Conform L s1 ∧ ?ValueTyped L T s1 t v ∧
                     ?ErrorFree s0 s1)
  proof (induct)
    case (Abrupt xc s t L accC T A)
    from ‹(Some xc, s)::≼(G,L)›
    show (Some xc, s)::≼(G,L) ∧
      (normal (Some xc, s)
      ⟶ G,L,store (Some xc,s)⊢t≻arbitrary3 t::≼T) ∧
      (error-free (Some xc, s) = error-free (Some xc, s))
      by simp
  next
    case (Skip s L accC T A)
    from ‹Norm s::≼(G, L)› and
      ‹(|prg = G, cls = accC, lcl = L|)⊢In1r Skip::T›
    show Norm s::≼(G, L) ∧
              (normal (Norm s) ⟶ G,L,store (Norm s)⊢In1r Skip≻◇::≼T) ∧
              (error-free (Norm s) = error-free (Norm s))
      by simp
  next
    case (Expr s0 e v s1 L accC T A)
    note ‹G⊢Norm s0 −e−≻v→ s1›
    note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    moreover
    note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1r (Expr e)::T›
    then obtain eT
      where (|prg = G, cls = accC, lcl = L|)⊢In1l e::eT
      by (rule wt-elim-cases) blast
    moreover
    from Expr.prems obtain E where
      (|prg=G,cls=accC, lcl=L|)⊢dom (locals (store ((Norm s0)::state)))»In1l e»E
      by (elim da-elim-cases) simp
    ultimately
    obtain s1::≼(G, L) and error-free s1
```

```
    by (rule hyp [elim-format]) simp
  with wt
  show s1::≼(G, L) ∧
       (normal s1 ⟶ G,L,store s1⊢In1r (Expr e)≻♢::≼T) ∧
       (error-free (Norm s0) = error-free s1)
    by (simp)
next
  case (Lab s0 c s1 l L accC T A)
  note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1r c) ♢›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  moreover
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1r (l· c)::T›
  then have (|prg = G, cls = accC, lcl = L|)⊢c::√
    by (rule wt-elim-cases) blast
  moreover from Lab.prems obtain C where
   (|prg=G,cls=accC, lcl=L|)⊢dom (locals (store ((Norm s0)::state)))»In1r c»C
    by (elim da-elim-cases) simp
  ultimately
  obtain       conf-s1: s1::≼(G, L) and
         error-free-s1: error-free s1
    by (rule hyp [elim-format]) simp
  from conf-s1 have abupd (absorb l) s1::≼(G, L)
    by (cases s1) (auto intro: conforms-absorb)
  with wt error-free-s1
  show abupd (absorb l) s1::≼(G, L) ∧
       (normal (abupd (absorb l) s1)
        ⟶ G,L,store (abupd (absorb l) s1)⊢In1r (l· c)≻♢::≼T) ∧
       (error-free (Norm s0) = error-free (abupd (absorb l) s1))
    by (simp)
next
  case (Comp s0 c1 s1 c2 s2 L accC T A)
  note eval-c1 = ‹G⊢Norm s0 −c1→ s1›
  note eval-c2 = ‹G⊢s1 −c2→ s2›
  note hyp-c1 = ‹PROP ?TypeSafe (Norm s0) s1 (In1r c1) ♢›
  note hyp-c2 = ‹PROP ?TypeSafe s1        s2 (In1r c2) ♢›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1r (c1;; c2)::T›
  then obtain wt-c1: (|prg = G, cls = accC, lcl = L|)⊢c1::√ and
              wt-c2: (|prg = G, cls = accC, lcl = L|)⊢c2::√
    by (rule wt-elim-cases) blast
  from Comp.prems
  obtain C1 C2
    where da-c1: (|prg=G, cls=accC, lcl=L|)⊢
                   dom (locals (store ((Norm s0)::state))) »In1r c1» C1 and
          da-c2: (|prg=G, cls=accC, lcl=L|)⊢  nrm C1 »In1r c2» C2
    by (elim da-elim-cases) simp
  from conf-s0 wt-c1 da-c1
  obtain conf-s1: s1::≼(G, L) and
         error-free-s1: error-free s1
    by (rule hyp-c1 [elim-format]) simp
  show s2::≼(G, L) ∧
       (normal s2 ⟶ G,L,store s2⊢In1r (c1;; c2)≻♢::≼T) ∧
       (error-free (Norm s0) = error-free s2)
  proof (cases normal s1)
    case False
    with eval-c2 have s2=s1 by auto
    with conf-s1 error-free-s1 False wt show ?thesis
      by simp
  next
```

case *True*
obtain $C2'$ **where**
$(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash\ dom\ (locals\ (store\ s1))\ \gg In1r\ c2\gg\ C2'$
proof $-$
from *eval-c1 wt-c1 da-c1 wf True*
have $nrm\ C1 \subseteq dom\ (locals\ (store\ s1))$
by (*cases rule*: *da-good-approxE′*) *iprover*
with *da-c2* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-c2*
obtain $s2{::}\preceq(G,\ L)$ **and** *error-free s2*
by (*rule hyp-c2* [*elim-format*]) (*simp add*: *error-free-s1*)
thus *?thesis*
using *wt* **by** *simp*
qed
next
case (*If s0 e b s1 c1 c2 s2 L accC T A*)
note $eval\text{-}e = \langle G\vdash Norm\ s0\ -e-\succ b\rightarrow\ s1\rangle$
note $eval\text{-}then\text{-}else = \langle G\vdash s1\ -(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)\rightarrow\ s2\rangle$
note $hyp\text{-}e = \langle PROP\ ?TypeSafe\ (Norm\ s0)\ s1\ (In1l\ e)\ (In1\ b)\rangle$
note *hyp-then-else* $=$
$\langle PROP\ ?TypeSafe\ s1\ s2\ (In1r\ (if\ the\text{-}Bool\ b\ then\ c1\ else\ c2))\ \Diamond\rangle$
note $conf\text{-}s0 = \langle Norm\ s0{::}\preceq(G,\ L)\rangle$
note $wt = \langle(\!|prg = G,\ cls = accC,\ lcl = L|\!)\vdash In1r\ (If(e)\ c1\ Else\ c2){::}T\rangle$
then obtain
$wt\text{-}e$: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash e{::}-PrimT\ Boolean$ **and**
$wt\text{-}then\text{-}else$: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2){::}\surd$

by (*rule wt-elim-cases*) (*auto split add*: *split-if*)
from *If.prems* **obtain** *E C* **where**
$da\text{-}e$: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash\ dom\ (locals\ (store\ ((Norm\ s0){::}state)))$
$\gg In1l\ e\gg\ E$ **and**
da-then-else:
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash$
$(dom\ (locals\ (store\ ((Norm\ s0){::}state)))\ \cup\ assigns\text{-}if\ \ (the\text{-}Bool\ b)\ e)$
$\gg In1r\ (if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)\gg\ C$

by (*elim da-elim-cases*) (*cases the-Bool b*,*auto*)
from *conf-s0 wt-e da-e*
obtain $conf\text{-}s1$: $s1{::}\preceq(G,\ L)$ **and** $error\text{-}free\text{-}s1$: *error-free s1*
by (*rule hyp-e* [*elim-format*]) *simp*
show $s2{::}\preceq(G,\ L)\ \wedge$
$(normal\ s2\ \longrightarrow\ G{,}L{,}store\ s2\vdash In1r\ (If(e)\ c1\ Else\ c2)\succ\Diamond{::}\preceq T)\ \wedge$
$(error\text{-}free\ (Norm\ s0) = error\text{-}free\ s2)$
proof (*cases normal s1*)
case *False*
with *eval-then-else* **have** $s2{=}s1$ **by** *auto*
with *conf-s1 error-free-s1 False wt* **show** *?thesis*
by *simp*
next
case *True*
obtain C' **where**
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash$
$(dom\ (locals\ (store\ s1)))\gg In1r\ (if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)\gg\ C'$
proof $-$
from *eval-e* **have**
$dom\ (locals\ (store\ ((Norm\ s0){::}state)))\ \subseteq\ dom\ (locals\ (store\ s1))$
by (*rule dom-locals-eval-mono-elim*)

moreover
from *eval-e True wt-e*
have *assigns-if* (*the-Bool b*) *e* ⊆ *dom* (*locals* (*store s1*))
by (*rule assigns-if-good-approx′*)
ultimately
have *dom* (*locals* (*store* ((*Norm s0*)::*state*)))
∪ *assigns-if* (*the-Bool b*) *e* ⊆ *dom* (*locals* (*store s1*))
by (*rule Un-least*)
with *da-then-else* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-then-else*
obtain *s2*::≼(*G*, *L*) **and** *error-free s2*
by (*rule hyp-then-else* [*elim-format*]) (*simp add*: *error-free-s1*)
with *wt* **show** *?thesis*
by *simp*
qed

— Note that we don't have to show that *b* really is a boolean value. With *the-Bool* we enforce to get a value of boolean type. So execution will be type safe, even if b would be a string, for example. We might not expect such a behaviour to be called type safe. To remedy the situation we would have to change the evaulation rule, so that it only has a type safe evaluation if we actually get a boolean value for the condition. That b is actually a boolean value is part of *hyp-e*. See also Loop

next
case (*Loop s0 e b s1 c s2 l s3 L accC T A*)
note *eval-e* = ‹*G*⊢*Norm s0* −*e*−≻*b*→ *s1*›
note *hyp-e* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l e*) (*In1 b*)›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹(|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*In1r* (*l*· *While*(*e*) *c*)::*T*›
then obtain *wt-e*: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*e*::−*PrimT Boolean* **and**
wt-c: (|*prg* = *G*, *cls* = *accC*, *lcl* = *L*|)⊢*c*::√
by (*rule wt-elim-cases*) *blast*
note *da* = ‹(|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)
⊢ *dom* (*locals*(*store* ((*Norm s0*)::*state*))) »*In1r* (*l*· *While*(*e*) *c*)» *A*›
then
obtain *E C* **where**
da-e: (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1l e*» *E* **and**
da-c: (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*)))
∪ *assigns-if True e*) »*In1r c*» *C*
by (*rule da-elim-cases*) *simp*
from *conf-s0 wt-e da-e*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and** *error-free-s1*: *error-free s1*
by (*rule hyp-e* [*elim-format*]) *simp*
show *s3*::≼(*G*, *L*) ∧
(*normal s3* ⟶ *G*,*L*,*store s3*⊢*In1r* (*l*· *While*(*e*) *c*)≻◇::≼*T*) ∧
(*error-free* (*Norm s0*) = *error-free s3*)
proof (*cases normal s1*)
case *True*
note *normal-s1* = *this*
show *?thesis*
proof (*cases the-Bool b*)
case *True*
with *Loop.hyps* **obtain**
eval-c: *G*⊢*s1* −*c*→ *s2* **and**
eval-while: *G*⊢*abupd* (*absorb* (*Cont l*)) *s2* −*l*· *While*(*e*) *c*→ *s3*
by *simp*
have *?TypeSafeObj s1 s2* (*In1r c*) ◇
using *Loop.hyps True* **by** *simp*

```
  note hyp-c = this [rule-format]
  have ?TypeSafeObj (abupd (absorb (Cont l)) s2)
    s3 (In1r (l• While(e) c)) ♢
    using Loop.hyps True by simp
  note hyp-w = this [rule-format]
  from eval-e have
    s0-s1: dom (locals (store ((Norm s0)::state)))
              ⊆ dom (locals (store s1))
    by (rule dom-locals-eval-mono-elim)
  obtain C' where
    (|prg=G, cls=accC, lcl=L|)⊢(dom (locals (store s1)))»In1r c» C'
  proof –
    note s0-s1
    moreover
    from eval-e normal-s1 wt-e
    have assigns-if True e ⊆ dom (locals (store s1))
      by (rule assigns-if-good-approx' [elim-format]) (simp add: True)
    ultimately
    have dom (locals (store ((Norm s0)::state)))
           ∪ assigns-if True e ⊆ dom (locals (store s1))
      by (rule Un-least)
    with da-c show thesis
      by (rule da-weakenE) (rule that)
  qed
  with conf-s1 wt-c
  obtain conf-s2:  s2::≼(G, L) and error-free-s2: error-free s2
    by (rule hyp-c [elim-format]) (simp add: error-free-s1)
  from error-free-s2
  have error-free-ab-s2: error-free (abupd (absorb (Cont l)) s2)
    by simp
  from conf-s2 have abupd (absorb (Cont l)) s2 ::≼(G, L)
    by (cases s2) (auto intro: conforms-absorb)
  moreover note wt
  moreover
  obtain A' where
    (|prg=G,cls=accC,lcl=L|)⊢
        dom (locals(store (abupd (absorb (Cont l)) s2)))
          »In1r (l• While(e) c)» A'
  proof –
    note s0-s1
    also from eval-c
    have dom (locals (store s1)) ⊆ dom (locals (store s2))
      by (rule dom-locals-eval-mono-elim)
    also have . . . ⊆ dom (locals (store (abupd (absorb (Cont l)) s2)))
      by simp
    finally
    have dom (locals (store ((Norm s0)::state))) ⊆ . . . .
    with da show thesis
      by (rule da-weakenE) (rule that)
  qed
  ultimately obtain s3::≼(G, L) and error-free s3
    by (rule hyp-w [elim-format]) (simp add: error-free-ab-s2)
  with wt show ?thesis
    by simp
next
  case False
  with Loop.hyps have s3=s1 by simp
  with conf-s1 error-free-s1 wt
  show ?thesis
```

```
        by simp
      qed
    next
      case False
      have s3=s1
      proof −
        from False obtain abr where abr: abrupt s1 = Some abr
          by (cases s1) auto
        from eval-e - wt-e have no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
          by (rule eval-expression-no-jump
                [where ?Env=(|prg=G,cls=accC,lcl=L|),simplified])
             (simp-all add: wf)

        show ?thesis
        proof (cases the-Bool b)
          case True
          with Loop.hyps obtain
            eval-c: G⊢s1 −c→ s2 and
            eval-while: G⊢abupd (absorb (Cont l)) s2 −l• While(e) c→ s3
            by simp
          from eval-c abr have s2=s1 by auto
          moreover from calculation no-jmp have abupd (absorb (Cont l)) s2=s2
            by (cases s1) (simp add: absorb-def)
          ultimately show ?thesis
            using eval-while abr
            by auto
        next
          case False
          with Loop.hyps show ?thesis by simp
        qed
      qed
      with conf-s1 error-free-s1 wt
      show ?thesis
        by simp
    qed
  next
    case (Jmp s j L accC T A)
    note ‹Norm s::≼(G, L)›
    moreover
    from Jmp.prems
    have j=Ret ⟶ Result ∈ dom (locals (store ((Norm s)::state)))
      by (elim da-elim-cases)
    ultimately have (Some (Jump j), s)::≼(G, L) by auto
    then
    show (Some (Jump j), s)::≼(G, L) ∧
           (normal (Some (Jump j), s)
           ⟶ G,L,store (Some (Jump j), s)⊢In1r (Jmp j)≻◇::≼T) ∧
           (error-free (Norm s) = error-free (Some (Jump j), s))
      by simp
  next
    case (Throw s0 e a s1 L accC T A)
    note ‹G⊢Norm s0 −e−≻a→ s1›
    note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 a)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1r (Throw e)::T›
    then obtain tn
      where      wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−Class tn and
            throwable: G⊢tn≼C SXcpt Throwable
      by (rule wt-elim-cases) (auto)
```

from *Throw.prems* **obtain** *E* **where**
da-e: (|*prg=G,cls=accC,lcl=L*|)
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1l e*» *E*
by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-e da-e* **obtain**
s1::≼(*G*, *L*) **and**
(*normal s1* ⟶ *G*,*store s1*⊢*a*::≼*Class tn*) **and**
error-free-s1: *error-free s1*
by (*rule hyp* [*elim-format*]) *simp*
with *wf throwable*
have *abupd* (*throw a*) *s1*::≼(*G*, *L*)
by (*cases s1*) (*auto dest*: *Throw-lemma*)
with *wt error-free-s1*
show *abupd* (*throw a*) *s1*::≼(*G*, *L*) ∧
(*normal* (*abupd* (*throw a*) *s1*) ⟶
G,*L*,*store* (*abupd* (*throw a*) *s1*)⊢*In1r* (*Throw e*)≻◇::≼*T*) ∧
(*error-free* (*Norm s0*) = *error-free* (*abupd* (*throw a*) *s1*))
by *simp*
next
case (*Try s0 c1 s1 s2 catchC vn c2 s3 L accC T A*)
note *eval-c1* = ‹*G*⊢*Norm s0* −*c1*→ *s1*›
note *sx-alloc* = ‹*G*⊢*s1* −*sxalloc*→ *s2*›
note *hyp-c1* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1r c1*) ◇›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹(|*prg=G,cls=accC,lcl=L*|)⊢*In1r* (*Try c1 Catch*(*catchC vn*) *c2*)::*T*›
then obtain
wt-c1: (|*prg=G,cls=accC,lcl=L*|)⊢*c1*::√ **and**
wt-c2: (|*prg=G,cls=accC,lcl=L*(*VName vn*↦*Class catchC*)|)⊢*c2*::√ **and**
fresh-vn: *L*(*VName vn*)=*None*
by (*rule wt-elim-cases*) *simp*
from *Try.prems* **obtain** *C1 C2* **where**
da-c1: (|*prg=G,cls=accC,lcl=L*|)
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1r c1*» *C1* **and**
da-c2:
(|*prg=G,cls=accC,lcl=L*(*VName vn*↦*Class catchC*)|)
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))) ∪ {*VName vn*}) »*In1r c2*» *C2*
by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-c1 da-c1*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and** *error-free-s1*: *error-free s1*
by (*rule hyp-c1* [*elim-format*]) *simp*
from *conf-s1 sx-alloc wf*
have *conf-s2*: *s2*::≼(*G*, *L*)
by (*auto dest*: *sxalloc-type-sound split*: *option.splits abrupt.splits*)
from *sx-alloc error-free-s1*
have *error-free-s2*: *error-free s2*
by (*rule error-free-sxalloc*)
show *s3*::≼(*G*, *L*) ∧
(*normal s3* ⟶ *G*,*L*,*store s3*⊢*In1r* (*Try c1 Catch*(*catchC vn*) *c2*)≻◇::≼*T*)∧
(*error-free* (*Norm s0*) = *error-free s3*)
proof (*cases* ∃ *x*. *abrupt s1* = *Some* (*Xcpt x*))
case *False*
from *sx-alloc wf*
have *eq-s2-s1*: *s2*=*s1*
by (*rule sxalloc-type-sound* [*elim-format*])
(*insert False*, *auto split*: *option.splits abrupt.splits*)
with *False*
have ¬ *G*,*s2*⊢*catch catchC*
by (*simp add*: *catch-def*)
with *Try*

have *s3=s2*
by *simp*
with *wt conf-s1 error-free-s1 eq-s2-s1*
show *?thesis*
by *simp*
next
case *True*
note *exception-s1 = this*
show *?thesis*
proof (*cases G*,*s2*⊢*catch catchC*)
case *False*
with *Try*
have *s3=s2*
by *simp*
with *wt conf-s2 error-free-s2*
show *?thesis*
by *simp*
next
case *True*
with *Try* **have** *G*⊢*new-xcpt-var vn s2 −c2→ s3* **by** *simp*
from *True Try.hyps*
have *?TypeSafeObj* (*new-xcpt-var vn s2*) *s3* (*In1r c2*) ♢
by *simp*
note *hyp-c2 = this* [*rule-format*]
from *exception-s1 sx-alloc wf*
obtain *a*
where *xcpt-s2*: *abrupt s2 = Some* (*Xcpt* (*Loc a*))
by (*auto dest*!: *sxalloc-type-sound split*: *option.splits abrupt.splits*)
with *True*
have *G*⊢*obj-ty* (*the* (*globs* (*store s2*) (*Heap a*)))≼*Class catchC*
by (*cases s2*) *simp*
with *xcpt-s2 conf-s2 wf*
have *new-xcpt-var vn s2* ::≼(*G*, *L*(*VName vn*↦*Class catchC*))
by (*auto dest*: *Try-lemma*)
moreover note *wt-c2*
moreover
obtain *C2′* **where**
⦇*prg=G*,*cls=accC*,*lcl=L*(*VName vn*↦*Class catchC*)⦈
⊢ (*dom* (*locals* (*store* (*new-xcpt-var vn s2*)))) »*In1r c2*» *C2′*
proof −
have (*dom* (*locals* (*store* ((*Norm s0*)::*state*))) ∪ {*VName vn*})
⊆ *dom* (*locals* (*store* (*new-xcpt-var vn s2*)))
proof −
from ‹*G*⊢*Norm s0* −*c1*→ *s1*›
have *dom* (*locals* (*store* ((*Norm s0*)::*state*)))
⊆ *dom* (*locals* (*store s1*))
by (*rule dom-locals-eval-mono-elim*)
also
from *sx-alloc*
have . . . ⊆ *dom* (*locals* (*store s2*))
by (*rule dom-locals-sxalloc-mono*)
also
have . . . ⊆ *dom* (*locals* (*store* (*new-xcpt-var vn s2*)))
by (*cases s2*) (*simp add*: *new-xcpt-var-def*, *blast*)
also
have {*VName vn*} ⊆ . . .
by (*cases s2*) *simp*
ultimately show *?thesis*
by (*rule Un-least*)

```
        qed
        with da-c2 show thesis
          by (rule da-weakenE) (rule that)
      qed
      ultimately
      obtain     conf-s3: s3::≼(G, L(VName vn↦Class catchC)) and
             error-free-s3: error-free s3
        by (rule hyp-c2 [elim-format])
           (cases s2, simp add: xcpt-s2 error-free-s2)
      from conf-s3 fresh-vn
      have s3::≼(G,L)
        by (blast intro: conforms-deallocL)
      with wt error-free-s3
      show ?thesis
        by simp
    qed
  qed
next
  case (Fin s0 c1 x1 s1 c2 s2 s3 L accC T A)
  note eval-c1 = ‹G⊢Norm s0 −c1→ (x1, s1)›
  note eval-c2 = ‹G⊢Norm s1 −c2→ s2›
  note s3 = ‹s3 = (if ∃ err. x1 = Some (Error err)
                  then (x1, s1)
                  else abupd (abrupt-if (x1 ≠ None) x1) s2)›
  note hyp-c1 = ‹PROP ?TypeSafe (Norm s0) (x1,s1) (In1r c1) ♢›
  note hyp-c2 = ‹PROP ?TypeSafe (Norm s1) s2      (In1r c2) ♢›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1r (c1 Finally c2)::T›
  then obtain
    wt-c1: (|prg=G,cls=accC,lcl=L|)⊢c1::√ and
    wt-c2: (|prg=G,cls=accC,lcl=L|)⊢c2::√
    by (rule wt-elim-cases) blast
  from Fin.prems obtain C1 C2 where
    da-c1: (|prg=G,cls=accC,lcl=L|)
          ⊢ dom (locals (store ((Norm s0)::state))) »In1r c1» C1 and
    da-c2: (|prg=G,cls=accC,lcl=L|)
          ⊢ dom (locals (store ((Norm s0)::state))) »In1r c2» C2
    by (elim da-elim-cases) simp
  from conf-s0 wt-c1 da-c1
  obtain conf-s1: (x1,s1)::≼(G, L) and error-free-s1: error-free (x1,s1)
    by (rule hyp-c1 [elim-format]) simp
  from conf-s1 have Norm s1::≼(G, L)
    by (rule conforms-NormI)
  moreover note wt-c2
  moreover obtain C2'
    where (|prg=G,cls=accC,lcl=L|)
          ⊢ dom (locals (store ((Norm s1)::state))) »In1r c2» C2'
  proof −
    from eval-c1
    have dom (locals (store ((Norm s0)::state)))
          ⊆ dom (locals (store (x1,s1)))
      by (rule dom-locals-eval-mono-elim)
    hence dom (locals (store ((Norm s0)::state)))
           ⊆ dom (locals (store ((Norm s1)::state)))
      by simp
    with da-c2 show thesis
      by (rule da-weakenE) (rule that)
  qed
  ultimately
```

```
    obtain conf-s2: s2::≼(G, L) and error-free-s2: error-free s2
      by (rule hyp-c2 [elim-format]) simp
    from error-free-s1 s3
    have s3': s3=abupd (abrupt-if (x1 ≠ None) x1) s2
      by simp
    show s3::≼(G, L) ∧
          (normal s3 ⟶ G,L,store s3 ⊢In1r (c1 Finally c2)≻◇::≼T) ∧
          (error-free (Norm s0) = error-free s3)
    proof (cases x1)
      case None with conf-s2 s3' wt error-free-s2
      show ?thesis by auto
    next
      case (Some x)
      from eval-c2 have
        dom (locals (store ((Norm s1)::state))) ⊆ dom (locals (store s2))
        by (rule dom-locals-eval-mono-elim)
      with Some eval-c2 wf conf-s1 conf-s2
      have conf: (abrupt-if True (Some x) (abrupt s2), store s2)::≼(G, L)
        by (cases s2) (auto dest: Fin-lemma)
      from Some error-free-s1
      have ¬ (∃ err. x=Error err)
        by (simp add: error-free-def)
      with error-free-s2
      have error-free (abrupt-if True (Some x) (abrupt s2), store s2)
        by (cases s2) simp
      with Some wt conf s3' show ?thesis
        by (cases s2) auto
    qed
  next
    case (Init C c s0 s3 s1 s2 L accC T A)
    note cls = ‹the (class G C) = c›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹⦇prg = G, cls = accC, lcl = L⦈⊢In1r (Init C)::T›
    with cls
    have cls-C: class G C = Some c
      by − (erule wt-elim-cases, auto)
    show s3::≼(G, L) ∧ (normal s3 ⟶ G,L,store s3⊢In1r (Init C)≻◇::≼T) ∧
          (error-free (Norm s0) = error-free s3)
    proof (cases inited C (globs s0))
      case True
      with Init.hyps have s3 = Norm s0
        by simp
      with conf-s0 wt show ?thesis
        by simp
    next
      case False
      with Init.hyps obtain
           eval-init-super:
           G⊢Norm ((init-class-obj G C) s0)
              −(if C = Object then Skip else Init (super c))→ s1 and
        eval-init: G⊢(set-lvars empty) s1 −init c→ s2 and
        s3: s3 = (set-lvars (locals (store s1))) s2
        by simp
      have ?TypeSafeObj (Norm ((init-class-obj G C) s0)) s1
                    (In1r (if C = Object then Skip else Init (super c))) ◇
        using False Init.hyps by simp
      note hyp-init-super = this [rule-format]
      have ?TypeSafeObj ((set-lvars empty) s1) s2 (In1r (init c)) ◇
        using False Init.hyps by simp
```

note *hyp-init-c* = *this* [*rule-format*]
from *conf-s0 wf cls-C False*
have (*Norm* ((*init-class-obj G C*) *s0*))::⪯(*G*, *L*)
  by (*auto dest*: *conforms-init-class-obj*)
moreover from *wf cls-C* **have**
  wt-init-super: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈
      ⊢(*if C* = *Object then Skip else Init* (*super c*))::√
  by (*cases C*=*Object*)
    (*auto dest*: *wf-prog-cdecl wf-cdecl-supD is-acc-classD*)
moreover
obtain *S* **where**
  da-init-super:
  ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
   ⊢ *dom* (*locals* (*store* ((*Norm* ((*init-class-obj G C*) *s0*))::*state*)))
      »*In1r* (*if C* = *Object then Skip else Init* (*super c*))» *S*
proof (*cases C*=*Object*)
  case *True*
  with *da-Skip* **show** *?thesis*
    using *that* **by** (*auto intro*: *assigned.select-convs*)
next
  case *False*
  with *da-Init* **show** *?thesis*
    by − (*rule that*, *auto intro*: *assigned.select-convs*)
qed
ultimately
obtain *conf-s1*: *s1*::⪯(*G*, *L*) **and** *error-free-s1*: *error-free s1*
  by (*rule hyp-init-super* [*elim-format*]) *simp*
from *eval-init-super wt-init-super wf*
have *s1-no-ret*: ⋀ *j*. *abrupt s1* ≠ *Some* (*Jump j*)
  by − (*rule eval-statement-no-jump* [**where** *?Env*=⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈],
    auto)
with *conf-s1*
have (*set-lvars empty*) *s1*::⪯(*G*, *empty*)
  by (*cases s1*) (*auto intro*: *conforms-set-locals*)
moreover
from *error-free-s1*
have *error-free-empty*: *error-free* ((*set-lvars empty*) *s1*)
  by *simp*
from *cls-C wf* **have** *wt-init-c*: ⦇*prg*=*G*, *cls*=*C*,*lcl*=*empty*⦈⊢(*init c*)::√
  by (*rule wf-prog-cdecl* [*THEN wf-cdecl-wt-init*])
moreover from *cls-C wf* **obtain** *I*
  where ⦇*prg*=*G*,*cls*=*C*,*lcl*=*empty*⦈⊢ {} »*In1r* (*init c*)» *I*
  by (*rule wf-prog-cdecl* [*THEN wf-cdeclE*,*simplified*]) *blast*

then obtain *I′* **where**
  ⦇*prg*=*G*,*cls*=*C*,*lcl*=*empty*⦈⊢*dom* (*locals* (*store* ((*set-lvars empty*) *s1*)))
    »*In1r* (*init c*)» *I′*
    by (*rule da-weakenE*) *simp*
ultimately
obtain *conf-s2*: *s2*::⪯(*G*, *empty*) **and** *error-free-s2*: *error-free s2*
  by (*rule hyp-init-c* [*elim-format*]) (*simp add*: *error-free-empty*)
have *abrupt s2* ≠ *Some* (*Jump Ret*)
proof −
  from *s1-no-ret*
  have ⋀ *j*. *abrupt* ((*set-lvars empty*) *s1*) ≠ *Some* (*Jump j*)
    by *simp*
  moreover
  from *cls-C wf* **have** *jumpNestingOkS* {} (*init c*)
    by (*rule wf-prog-cdecl* [*THEN wf-cdeclE*])

```
        ultimately
        show ?thesis
          using eval-init wt-init-c wf
          by − (rule eval-statement-no-jump
                    [where ?Env=(|prg=G,cls=C,lcl=empty|)],simp+)
      qed
      with conf-s2 s3 conf-s1 eval-init
      have s3::≼(G, L)
        by (cases s2,cases s1) (force dest: conforms-return eval-gext')
      moreover from error-free-s2 s3
      have error-free s3
        by simp
      moreover note wt
      ultimately show ?thesis
        by simp
    qed
  next
    case (NewC s0 C s1 a s2 L accC T A)
    note ‹G⊢Norm s0 −Init C→ s1›
    note halloc = ‹G⊢s1 −halloc CInst C≻a→ s2›
    note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1r (Init C)) ♢›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    moreover
    note wt = ‹(|prg=G, cls=accC, lcl=L|)⊢In1l (NewC C)::T›
    then obtain is-cls-C: is-class G C and
                  T: T=Inl (Class C)
      by (rule wt-elim-cases) (auto dest: is-acc-classD)
    hence (|prg=G, cls=accC, lcl=L|)⊢Init C::√ by auto
    moreover obtain I where
      (|prg=G,cls=accC,lcl=L|)
         ⊢ dom (locals (store ((Norm s0)::state))) »In1r (Init C)» I
      by (auto intro: da-Init [simplified] assigned.select-convs)

    ultimately
    obtain conf-s1: s1::≼(G, L) and error-free-s1: error-free s1
      by (rule hyp [elim-format]) simp
    from conf-s1 halloc wf is-cls-C
    obtain halloc-type-safe: s2::≼(G, L)
                              (normal s2 ⟶ G,store s2⊢Addr a::≼Class C)
      by (cases s2) (auto dest!: halloc-type-sound)
    from halloc error-free-s1
    have error-free s2
      by (rule error-free-halloc)
    with halloc-type-safe T
    show s2::≼(G, L) ∧
         (normal s2 ⟶ G,L,store s2⊢In1l (NewC C)≻In1 (Addr a)::≼T)  ∧
         (error-free (Norm s0) = error-free s2)
      by auto
  next
    case (NewA s0 elT s1 e i s2 a s3 L accC T A)
    note eval-init = ‹G⊢Norm s0 −init-comp-ty elT→ s1›
    note eval-e = ‹G⊢s1 −e−≻i→ s2›
    note halloc = ‹G⊢abupd (check-neg i) s2−halloc Arr elT (the-Intg i)≻a→ s3›
    note hyp-init = ‹PROP ?TypeSafe (Norm s0) s1 (In1r (init-comp-ty elT)) ♢›
    note hyp-size = ‹PROP ?TypeSafe s1 s2 (In1l e) (In1 i)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (New elT[e])::T›
    then obtain
      wt-init: (|prg = G, cls = accC, lcl = L|)⊢init-comp-ty elT::√ and
```

wt-size: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e*::−*PrimT Integer* **and**
elT: *is-type G elT* **and**
T: *T*=*Inl* (*elT*.[])
by (*rule wt-elim-cases*) (*auto intro*: *wt-init-comp-ty dest*: *is-acc-typeD*)
from *NewA.prems*
have *da-e*:⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1l e*» *A*
by (*elim da-elim-cases*) *simp*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and** *error-free-s1*: *error-free s1*
proof −
note *conf-s0 wt-init*
moreover obtain *I* **where**
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1r* (*init-comp-ty elT*)» *I*
proof (*cases* ∃ *C*. *elT* = *Class C*)
case *True*
thus *?thesis*
by − (*rule that*, (*auto intro*: *da-Init* [*simplified*]
assigned.select-convs
simp add: *init-comp-ty-def*))

next
case *False*
thus *?thesis*
by − (*rule that*, (*auto intro*: *da-Skip* [*simplified*]
assigned.select-convs
simp add: *init-comp-ty-def*))

qed
ultimately show *thesis*
by (*rule hyp-init* [*elim-format*]) (*auto intro*: *that*)
qed
obtain *conf-s2*: *s2*::≼(*G*, *L*) **and** *error-free-s2*: *error-free s2*
proof −
from *eval-init*
have *dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
by (*rule dom-locals-eval-mono-elim*)
with *da-e*
obtain *A′* **where**
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store s1*)) »*In1l e*» *A′*
by (*rule da-weakenE*)
with *conf-s1 wt-size*
show *?thesis*
by (*rule hyp-size* [*elim-format*]) (*simp add*: *that error-free-s1*)
qed
from *conf-s2* **have** *abupd* (*check-neg i*) *s2*::≼(*G*, *L*)
by (*cases s2*) *auto*
with *halloc wf elT*
have *halloc-type-safe*:
s3::≼(*G*, *L*) ∧ (*normal s3* ⟶ *G*,*store s3*⊢*Addr a*::≼*elT*.[])
by (*cases s3*) (*auto dest*!: *halloc-type-sound*)
from *halloc error-free-s2*
have *error-free s3*
by (*auto dest*: *error-free-halloc*)
with *halloc-type-safe T*
show *s3*::≼(*G*, *L*) ∧
(*normal s3* ⟶ *G*,*L*,*store s3*⊢*In1l* (*New elT*[*e*])≻*In1* (*Addr a*)::≼*T*) ∧
(*error-free* (*Norm s0*) = *error-free s3*)

```
    by simp
  next
    case (Cast s0 e v s1 s2 castT L accC T A)
    note ‹G⊢Norm s0 −e−≻v→ s1›
    note s2 = ‹s2 = abupd (raise-if (¬ G,store s1⊢v fits castT) ClassCast) s1›
    note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (Cast castT e)::T›
    then obtain eT
      where wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT and
              eT: G⊢eT≼? castT and
               T: T=Inl castT
      by (rule wt-elim-cases) auto
    from Cast.prems
    have (|prg=G,cls=accC,lcl=L|)
             ⊢ dom (locals (store ((Norm s0)::state))) »In1l e» A
      by (elim da-elim-cases) simp
    with conf-s0 wt-e
    obtain conf-s1: s1::≼(G, L) and
           v-ok: normal s1 ⟶ G,store s1⊢v::≼eT and
      error-free-s1: error-free s1
      by (rule hyp [elim-format]) simp
    from conf-s1 s2
    have conf-s2: s2::≼(G, L)
      by (cases s1) simp
    from error-free-s1 s2
    have error-free-s2: error-free s2
      by simp
    {
      assume norm-s2: normal s2
      have G,L,store s2⊢In1l (Cast castT e)≻In1 v::≼T
      proof −
        from s2 norm-s2 have normal s1
          by (cases s1) simp
        with v-ok
        have G,store s1⊢v::≼eT
          by simp
        with eT wf s2 T norm-s2
        show ?thesis
          by (cases s1) (auto dest: fits-conf)
      qed
    }
    with conf-s2 error-free-s2
    show s2::≼(G, L) ∧
           (normal s2 ⟶ G,L,store s2⊢In1l (Cast castT e)≻In1 v::≼T)  ∧
           (error-free (Norm s0) = error-free s2)
      by blast
  next
    case (Inst s0 e v s1 b instT L accC T A)
    note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    from Inst.prems obtain eT
    where wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−RefT eT  and
             T: T=Inl (PrimT Boolean)
      by (elim wt-elim-cases) simp
    from Inst.prems
    have da-e: (|prg=G,cls=accC,lcl=L|)
             ⊢ dom (locals (store ((Norm s0)::state))) »In1l e» A
      by (elim da-elim-cases) simp
```

from *conf-s0 wt-e da-e*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and**
v-ok: *normal s1* ⟶ *G*,*store s1*⊢*v*::≼*RefT eT* **and**
error-free-s1: *error-free s1*
by (*rule hyp* [*elim-format*]) *simp*
with *T* **show** *?case*
by *simp*
next
case (*Lit s v L accC T A*)
then show *?case*
by (*auto elim*!: *wt-elim-cases*
intro: *conf-litval simp add*: *empty-dt-def*)
next
case (*UnOp s0 e v s1 unop L accC T A*)
note *hyp* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l e*) (*In1 v*)›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*In1l* (*UnOp unop e*)::*T*›
then obtain *eT*
where *wt-e*: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e*::−*eT* **and**
wt-unop: *wt-unop unop eT* **and**
T: *T*=*Inl* (*PrimT* (*unop-type unop*))
by (*auto elim*!: *wt-elim-cases*)
from *UnOp.prems* **obtain** *A* **where**
da-e: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1l e*» *A*
by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-e da-e*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and**
wt-v: *normal s1* ⟶ *G*,*store s1*⊢*v*::≼*eT* **and**
error-free-s1: *error-free s1*
by (*rule hyp* [*elim-format*]) *simp*
from *wt-v T wt-unop*
have *normal s1*⟶*G*,*L*,*snd s1*⊢*In1l* (*UnOp unop e*)≻*In1* (*eval-unop unop v*)::≼*T*
by (*cases unop*) *auto*
with *conf-s1 error-free-s1*
show *s1*::≼(*G*, *L*) ∧
(*normal s1* ⟶ *G*,*L*,*snd s1*⊢*In1l* (*UnOp unop e*)≻*In1* (*eval-unop unop v*)::≼*T*) ∧
error-free (*Norm s0*) = *error-free s1*
by *simp*
next
case (*BinOp s0 e1 v1 s1 binop e2 v2 s2 L accC T A*)
note *eval-e1* = ‹*G*⊢*Norm s0* −*e1*−≻*v1*→ *s1*›
note *eval-e2* = ‹*G*⊢*s1* −(*if need-second-arg binop v1 then In1l e2*
else In1r Skip)≻→ (*In1 v2*, *s2*)›
note *hyp-e1* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l e1*) (*In1 v1*)›
note *hyp-e2* = ‹*PROP ?TypeSafe* *s1* *s2*
(*if need-second-arg binop v1 then In1l e2 else In1r Skip*)
(*In1 v2*)›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*In1l* (*BinOp binop e1 e2*)::*T*›
then obtain *e1T e2T* **where**
wt-e1: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e1*::−*e1T* **and**
wt-e2: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e2*::−*e2T* **and**
wt-binop: *wt-binop G binop e1T e2T* **and**
T: *T*=*Inl* (*PrimT* (*binop-type binop*))
by (*elim wt-elim-cases*) *simp*
have *wt-Skip*: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*Skip*::√
by *simp*
obtain *S* **where**

daSkip: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store s1*)) »*In1r Skip*» *S*
by (*auto intro*: *da-Skip* [*simplified*] *assigned.select-convs*)
note *da* = ‹⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *dom* (*locals* (*store* ((*Norm s0*::*state*))))
»⟨*BinOp binop e1 e2*⟩$_e$» *A*›
then obtain *E1* **where**
da-e1: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1l e1*» *E1*
by (*elim da-elim-cases*) *simp*+
from *conf-s0 wt-e1 da-e1*
obtain *conf-s1*: *s1*::≼(*G*, *L*) **and**
wt-v1: *normal s1* ⟶ *G*,*store s1*⊢*v1*::≼*e1T* **and**
error-free-s1: *error-free s1*
by (*rule hyp-e1* [*elim-format*]) *simp*
from *wt-binop T*
have *conf-v*:
G,*L*,*snd s2*⊢*In1l* (*BinOp binop e1 e2*)≻*In1* (*eval-binop binop v1 v2*)::≼*T*
by (*cases binop*) *auto*

— Note that we don't use the information that v1 really is compatible with the expected type e1T and v2 is compatible with e2T, because *eval-binop* will anyway produce an output of the right type. So evaluating the addition of an integer with a string is type safe. This is a little bit annoying since we may regard such a behaviour as not type safe. If we want to avoid this we can redefine *eval-binop* so that it only produces a output of proper type if it is assigned to values of the expected types, and arbitrary if the inputs have unexpected types. The proof can easily be adapted since we have the hypothesis that the values have a proper type. This also applies to unary operations.

from *eval-e1* **have**
s0-s1:*dom* (*locals* (*store* ((*Norm s0*)::*state*))) ⊆ *dom* (*locals* (*store s1*))
by (*rule dom-locals-eval-mono-elim*)
show *s2*::≼(*G*, *L*) ∧
(*normal s2* ⟶
G,*L*,*snd s2*⊢*In1l* (*BinOp binop e1 e2*)≻*In1* (*eval-binop binop v1 v2*)::≼*T*) ∧
error-free (*Norm s0*) = *error-free s2*
proof (*cases normal s1*)
case *False*
with *eval-e2* **have** *s2*=*s1* **by** *auto*
with *conf-s1 error-free-s1 False* **show** *?thesis*
by *auto*
next
case *True*
note *normal-s1* = *this*
show *?thesis*
proof (*cases need-second-arg binop v1*)
case *False*
with *normal-s1 eval-e2* **have** *s2*=*s1*
by (*cases s1*) (*simp*, *elim eval-elim-cases*,*simp*)
with *conf-s1 conf-v error-free-s1*
show *?thesis* **by** *simp*
next
case *True*
note *need-second-arg* = *this*
with *hyp-e2*
have *hyp-e2'*: *PROP ?TypeSafe s1 s2* (*In1l e2*) (*In1 v2*) **by** *simp*
from *da wt-e1 wt-e2 wt-binop conf-s0 normal-s1 eval-e1*
wt-v1 [*rule-format*,*OF normal-s1*] *wf*
obtain *E2* **where**
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *dom* (*locals* (*store s1*)) »*In1l e2*» *E2*
by (*rule da-e2-BinOp* [*elim-format*])
(*auto simp add*: *need-second-arg*)
with *conf-s1 wt-e2*

```
      obtain s2::≼(G, L) and error-free s2
        by (rule hyp-e2' [elim-format]) (simp add: error-free-s1)
      with conf-v show ?thesis by simp
    qed
  qed
next
  case (Super s L accC T A)
  note conf-s = ‹Norm s::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l Super::T›
  then obtain C c where
          C: L This = Some (Class C) and
      neq-Obj: C≠Object and
        cls-C: class G C = Some c and
            T: T=Inl (Class (super c))
    by (rule wt-elim-cases) auto
  from Super.prems
  obtain This ∈ dom (locals s)
    by (elim da-elim-cases) simp
  with conf-s C  have G,s⊢val-this s::≼Class C
    by (auto dest: conforms-localD [THEN wlconfD])
  with neq-Obj cls-C wf
  have G,s⊢val-this s::≼Class (super c)
    by (auto intro: conf-widen
              dest: subcls-direct[THEN widen.subcls])
  with T conf-s
  show Norm s::≼(G, L) ∧
         (normal (Norm s) ⟶
            G,L,store (Norm s)⊢In1l Super≻In1 (val-this s)::≼T) ∧
         (error-free (Norm s) = error-free (Norm s))
    by simp
next
  case (Acc s0 v w upd s1 L accC T A)
  note hyp = ‹PROP ?TypeSafe (Norm s0) s1 (In2 v) (In2 (w,upd))›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  from Acc.prems obtain vT where
    wt-v: (|prg = G, cls = accC, lcl = L|)⊢v::=vT and
       T: T=Inl vT
    by (elim wt-elim-cases) simp
  from Acc.prems obtain V where
    da-v: (|prg=G,cls=accC,lcl=L|)
               ⊢ dom (locals (store ((Norm s0)::state))) »In2 v» V
    by (cases ∃ n. v=LVar n) (insert da.LVar, auto elim!: da-elim-cases)
  {
    fix n assume lvar: v=LVar n
    have locals (store s1) n ≠ None
    proof −
      from Acc.prems lvar have
        n ∈ dom (locals s0)
        by (cases ∃ n. v=LVar n) (auto elim!: da-elim-cases)
      also
      have dom (locals s0) ⊆ dom (locals (store s1))
      proof −
        from ‹G⊢Norm s0 −v=≻(w, upd)→ s1›
        show ?thesis
          by (rule dom-locals-eval-mono-elim) simp
      qed
      finally show ?thesis
        by blast
    qed
```

```
  } note lvar-in-locals = this
  from conf-s0 wt-v da-v
  obtain conf-s1: s1::≼(G, L)
    and  conf-var: (normal s1 ⟶ G,L,store s1⊢In2 v≻In2 (w, upd)::≼Inl vT)
    and  error-free-s1: error-free s1
    by (rule hyp [elim-format]) simp
  from lvar-in-locals conf-var T
  have (normal s1 ⟶ G,L,store s1⊢In1l (Acc v)≻In1 w::≼T)
    by (cases ∃ n. v=LVar n) auto
  with conf-s1 error-free-s1 show ?case
    by simp
next
  case (Ass s0 var w upd s1 e v s2 L accC T A)
  note eval-var = ‹G⊢Norm s0 −var=≻(w, upd)→ s1›
  note eval-e = ‹G⊢s1 −e−≻v→ s2›
  note hyp-var = ‹PROP ?TypeSafe (Norm s0) s1 (In2 var) (In2 (w,upd))›
  note hyp-e = ‹PROP ?TypeSafe s1 s2 (In1l e) (In1 v)›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (var:=e)::T›
  then obtain varT eT where
       wt-var: (|prg = G, cls = accC, lcl = L|)⊢var::=varT and
         wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT and
       widen: G⊢eT≼varT and
           T: T=Inl eT
    by (rule wt-elim-cases) auto
  show assign upd v s2::≼(G, L) ∧
         (normal (assign upd v s2) ⟶
          G,L,store (assign upd v s2)⊢In1l (var:=e)≻In1 v::≼T) ∧
   (error-free (Norm s0) = error-free (assign upd v s2))
  proof (cases ∃ vn. var=LVar vn)
    case False
    with Ass.prems
    obtain V E where
      da-var: (|prg=G,cls=accC,lcl=L|)
                ⊢ dom (locals (store ((Norm s0)::state))) »In2 var» V and
      da-e:   (|prg=G,cls=accC,lcl=L|) ⊢ nrm V »In1l e» E
      by (elim da-elim-cases) simp+
    from conf-s0 wt-var da-var
    obtain conf-s1: s1::≼(G, L)
      and  conf-var: normal s1
                      ⟶ G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
      and  error-free-s1: error-free s1
      by (rule hyp-var [elim-format]) simp
    show ?thesis
    proof (cases normal s1)
      case False
      with eval-e have s2=s1 by auto
      with False have assign upd v s2=s1
        by simp
      with conf-s1 error-free-s1 False show ?thesis
        by auto
    next
      case True
      note normal-s1=this
      obtain A' where (|prg=G,cls=accC,lcl=L|)
                    ⊢ dom (locals (store s1)) »In1l e» A'
      proof −
        from eval-var wt-var da-var wf normal-s1
        have nrm V ⊆  dom (locals (store s1))
```

```
        by (cases rule: da-good-approxE') iprover
      with da-e show thesis
        by (rule da-weakenE) (rule that)
    qed
    with conf-s1 wt-e
    obtain conf-s2: s2::≼(G, L) and
      conf-v: normal s2 ⟶ G,store s2⊢v::≼eT and
      error-free-s2: error-free s2
      by (rule hyp-e [elim-format]) (simp add: error-free-s1)
    show ?thesis
    proof (cases normal s2)
      case False
      with conf-s2 error-free-s2
      show ?thesis
        by auto
    next
      case True
      from True conf-v
      have conf-v-eT: G,store s2⊢v::≼eT
        by simp
      with widen wf
      have conf-v-varT: G,store s2⊢v::≼varT
        by (auto intro: conf-widen)
      from normal-s1 conf-var
      have G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
        by simp
      then
      have conf-assign:  store s1≤|upd≼varT::≼(G, L)
        by (simp add: rconf-def)
      from conf-v-eT conf-v-varT conf-assign normal-s1 True wf eval-var
        eval-e T conf-s2 error-free-s2
      show ?thesis
        by (cases s1, cases s2)
           (auto dest!: Ass-lemma simp add: assign-conforms-def)
    qed
  qed
next
  case True
  then obtain vn where vn: var=LVar vn
    by blast
  with Ass.prems
  obtain E where
    da-e: (|prg=G,cls=accC,lcl=L|)
           ⊢ dom (locals (store ((Norm s0)::state))) »In1l e» E
    by (elim da-elim-cases) simp+
  from da.LVar vn obtain V where
    da-var: (|prg=G,cls=accC,lcl=L|)
           ⊢ dom (locals (store ((Norm s0)::state))) »In2 var» V
    by auto
  obtain E' where
    da-e': (|prg=G,cls=accC,lcl=L|)
           ⊢ dom (locals (store s1)) »In1l e» E'
  proof -
    have dom (locals (store ((Norm s0)::state)))
           ⊆ dom (locals (store (s1)))
      by (rule dom-locals-eval-mono-elim) (rule Ass.hyps)
    with da-e show thesis
      by (rule da-weakenE) (rule that)
  qed
```

```
      from conf-s0 wt-var da-var
      obtain conf-s1: s1::≼(G, L)
        and  conf-var: normal s1
                  ⟶ G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
        and  error-free-s1: error-free s1
        by (rule hyp-var [elim-format]) simp
      show ?thesis
      proof (cases normal s1)
        case False
        with eval-e have s2=s1 by auto
        with False have assign upd v s2=s1
          by simp
        with conf-s1 error-free-s1 False show ?thesis
          by auto
      next
        case True
        note normal-s1 = this
        from conf-s1 wt-e da-e'
        obtain conf-s2: s2::≼(G, L) and
          conf-v: normal s2 ⟶ G,store s2⊢v::≼eT and
          error-free-s2: error-free s2
          by (rule hyp-e [elim-format]) (simp add: error-free-s1)
        show ?thesis
        proof (cases normal s2)
          case False
          with conf-s2 error-free-s2
          show ?thesis
            by auto
        next
          case True
          from True conf-v
          have conf-v-eT: G,store s2⊢v::≼eT
            by simp
          with widen wf
          have conf-v-varT: G,store s2⊢v::≼varT
            by (auto intro: conf-widen)
          from normal-s1 conf-var
          have G,L,store s1⊢In2 var≻In2 (w, upd)::≼Inl varT
            by simp
          then
          have conf-assign:  store s1≤|upd≼varT::≼(G, L)
            by (simp add: rconf-def)
          from conf-v-eT conf-v-varT conf-assign normal-s1 True wf eval-var
            eval-e T conf-s2 error-free-s2
          show ?thesis
            by (cases s1, cases s2)
               (auto dest!: Ass-lemma simp add: assign-conforms-def)
        qed
      qed
    qed
  next
    case (Cond s0 e0 b s1 e1 e2 v s2 L accC T A)
    note eval-e0 = ‹G⊢Norm s0 −e0−≻b→ s1›
    note eval-e1-e2 = ‹G⊢s1 −(if the-Bool b then e1 else e2)−≻v→ s2›
    note hyp-e0 = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e0) (In1 b)›
    note hyp-if = ‹PROP ?TypeSafe s1 s2
                  (In1l (if the-Bool b then e1 else e2)) (In1 v)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In1l (e0 ? e1 : e2)::T›
```

```
then obtain T1 T2 statT where
  wt-e0: ⦇prg = G, cls = accC, lcl = L⦈⊢e0::−PrimT Boolean and
  wt-e1: ⦇prg = G, cls = accC, lcl = L⦈⊢e1::−T1 and
  wt-e2: ⦇prg = G, cls = accC, lcl = L⦈⊢e2::−T2 and
  statT: G⊢T1≼T2 ∧ statT = T2  ∨  G⊢T2≼T1 ∧ statT =  T1 and
  T    : T=Inl statT
  by (rule wt-elim-cases) auto
with Cond.prems obtain E0 E1 E2 where
     da-e0: ⦇prg=G,cls=accC,lcl=L⦈
              ⊢ dom (locals (store ((Norm s0)::state)))
                 »In1l e0» E0 and
     da-e1: ⦇prg=G,cls=accC,lcl=L⦈
              ⊢ (dom (locals (store ((Norm s0)::state)))
                   ∪ assigns-if True e0) »In1l e1» E1 and
     da-e2: ⦇prg=G,cls=accC,lcl=L⦈
              ⊢ (dom (locals (store ((Norm s0)::state)))
                  ∪ assigns-if False e0) »In1l e2» E2
   by (elim da-elim-cases) simp+
from conf-s0 wt-e0 da-e0
obtain conf-s1: s1::≼(G, L) and error-free-s1: error-free s1
  by (rule hyp-e0 [elim-format]) simp
show s2::≼(G, L) ∧
       (normal s2 ⟶ G,L,store s2⊢In1l (e0 ? e1 : e2)≻In1 v::≼T) ∧
       (error-free (Norm s0) = error-free s2)
proof (cases normal s1)
  case False
  with eval-e1-e2 have s2=s1 by auto
  with conf-s1 error-free-s1 False show ?thesis
    by auto
next
  case True
  have s0-s1: dom (locals (store ((Norm s0)::state)))
                ∪ assigns-if (the-Bool b) e0 ⊆ dom (locals (store s1))
  proof −
    from eval-e0 have
      dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
      by (rule dom-locals-eval-mono-elim)
    moreover
    from eval-e0 True wt-e0
    have assigns-if (the-Bool b) e0 ⊆ dom (locals (store s1))
      by (rule assigns-if-good-approx')
    ultimately show ?thesis by (rule Un-least)
  qed
  show ?thesis
  proof (cases the-Bool b)
    case True
    with hyp-if have hyp-e1: PROP ?TypeSafe s1 s2 (In1l e1) (In1 v)
      by simp
    from da-e1 s0-s1 True obtain E1' where
      ⦇prg=G,cls=accC,lcl=L⦈⊢ (dom (locals (store s1)))»In1l e1» E1'
      by − (rule da-weakenE, auto iff del: Un-subset-iff)
    with conf-s1 wt-e1
    obtain
      s2::≼(G, L)
      (normal s2 ⟶ G,L,store s2⊢In1l e1≻In1 v::≼Inl T1)
      error-free s2
      by (rule hyp-e1 [elim-format]) (simp add: error-free-s1)
    moreover
    from statT
```

```
    have G⊢T1≼statT
      by auto
    ultimately show ?thesis
      using T wf by auto
  next
    case False
    with hyp-if have hyp-e2: PROP ?TypeSafe s1 s2 (In1l e2) (In1 v)
      by simp
    from da-e2 s0-s1 False obtain E2′ where
      ⦇prg=G,cls=accC,lcl=L⦈⊢ (dom (locals (store s1)))»In1l e2» E2′
      by − (rule da-weakenE, auto iff del: Un-subset-iff)
    with conf-s1 wt-e2
    obtain
      s2::≼(G, L)
      (normal s2 ⟶ G,L,store s2⊢In1l e2≻In1 v::≼Inl T2)
      error-free s2
      by (rule hyp-e2 [elim-format]) (simp add: error-free-s1)
    moreover
    from statT
    have G⊢T2≼statT
      by auto
    ultimately show ?thesis
      using T wf by auto
  qed
 qed
next
 case (Call s0 e a s1 args vs s2 invDeclC mode statT mn pTs′ s3 s3′ accC′
       v s4 L accC T A)
 note eval-e = ‹G⊢Norm s0 −e−≻a→ s1›
 note eval-args = ‹G⊢s1 −args≐≻vs→ s2›
 note invDeclC = ‹invDeclC
                  = invocation-declclass G mode (store s2) a statT
                      ⦇name = mn, parTs = pTs′⦈›
 note init-lvars =
  ‹s3 = init-lvars G invDeclC ⦇name = mn, parTs = pTs′⦈ mode a vs s2›
 note check = ‹s3′ =
    check-method-access G accC′ statT mode ⦇name = mn, parTs = pTs′⦈ a s3›
 note eval-methd =
  ‹G⊢s3′ −Methd invDeclC ⦇name = mn, parTs = pTs′⦈−≻v→ s4›
 note hyp-e = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 a)›
 note hyp-args = ‹PROP ?TypeSafe s1 s2 (In3 args) (In3 vs)›
 note hyp-methd = ‹PROP ?TypeSafe s3′ s4
    (In1l (Methd invDeclC ⦇name = mn, parTs = pTs′⦈)) (In1 v)›
 note conf-s0 = ‹Norm s0::≼(G, L)›
 note wt = ‹⦇prg=G, cls=accC, lcl=L⦈
    ⊢In1l ({accC′,statT,mode}e·mn( {pTs′}args))::T›
 from wt obtain pTs statDeclT statM where
             wt-e: ⦇prg=G, cls=accC, lcl=L⦈⊢e::−RefT statT and
          wt-args: ⦇prg=G, cls=accC, lcl=L⦈⊢args::≐pTs and
            statM: max-spec G accC statT ⦇name=mn,parTs=pTs⦈
                    = {((statDeclT,statM),pTs′)} and
             mode: mode = invmode statM e and
                T: T =Inl (resTy statM) and
    eq-accC-accC′: accC=accC′
   by (rule wt-elim-cases) fastsimp+
 from Call.prems obtain E where
  da-e: ⦇prg=G,cls=accC,lcl=L⦈
         ⊢ (dom (locals (store ((Norm s0)::state))))»In1l e» E and
  da-args: ⦇prg=G,cls=accC,lcl=L⦈⊢ nrm E »In3 args» A
```

by (*elim da-elim-cases*) *simp*
from *conf-s0 wt-e da-e*
obtain *conf-s1*: *s1*::⪯(*G*, *L*) **and**
conf-a: *normal s1* ⟹ *G*, *store s1*⊢*a*::⪯*RefT statT* **and**
error-free-s1: *error-free s1*
by (*rule hyp-e* [*elim-format*]) *simp*
{
assume *abnormal-s2*: ¬ *normal s2*
have *set-lvars* (*locals* (*store s2*)) *s4* = *s2*
proof −
from *abnormal-s2 init-lvars*
obtain *keep-abrupt*: *abrupt s3* = *abrupt s2* **and**
store s3 = *store* (*init-lvars G invDeclC* (|*name* = *mn*, *parTs* = *pTs′*|)
mode a vs s2)
by (*auto simp add*: *init-lvars-def2*)
moreover
from *keep-abrupt abnormal-s2 check*
have *eq-s3′-s3*: *s3′*=*s3*
by (*auto simp add*: *check-method-access-def Let-def*)
moreover
from *eq-s3′-s3 abnormal-s2 keep-abrupt eval-methd*
have *s4*=*s3′*
by *auto*
ultimately show
set-lvars (*locals* (*store s2*)) *s4* = *s2*
by (*cases s2*,*cases s3*) (*simp add*: *init-lvars-def2*)
qed
} **note** *propagate-abnormal-s2* = *this*
show (*set-lvars* (*locals* (*store s2*))) *s4*::⪯(*G*, *L*) ∧
(*normal* ((*set-lvars* (*locals* (*store s2*))) *s4*) ⟶
G,*L*,*store* ((*set-lvars* (*locals* (*store s2*))) *s4*)
⊢*In1l* ({*accC′*,*statT*,*mode*}*e*·*mn*({*pTs′*}*args*))≻*In1 v*::⪯*T*) ∧
(*error-free* (*Norm s0*) =
error-free ((*set-lvars* (*locals* (*store s2*))) *s4*))
proof (*cases normal s1*)
case *False*
with *eval-args* **have** *s2*=*s1* **by** *auto*
with *False propagate-abnormal-s2 conf-s1 error-free-s1*
show *?thesis*
by *auto*
next
case *True*
note *normal-s1* = *this*
obtain *A′* **where**
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢ *dom* (*locals* (*store s1*)) »*In3 args*» *A′*
proof −
from *eval-e wt-e da-e wf normal-s1*
have *nrm E* ⊆ *dom* (*locals* (*store s1*))
by (*cases rule*: *da-good-approxE′*) *iprover*
with *da-args* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-args*
obtain *conf-s2*: *s2*::⪯(*G*, *L*) **and**
conf-args: *normal s2*
⟹ *list-all2* (*conf G* (*store s2*)) *vs pTs* **and**
error-free-s2: *error-free s2*
by (*rule hyp-args* [*elim-format*]) (*simp add*: *error-free-s1*)
from *error-free-s2 init-lvars*

```
have error-free-s3: error-free s3
  by (auto simp add: init-lvars-def2)
from statM
obtain
  statM': (statDeclT,statM)∈mheads G accC statT (|name=mn,parTs=pTs'|) and
  pTs-widen: G⊢pTs[≼]pTs'
  by (blast dest: max-spec2mheads)
from check
have eq-store-s3'-s3: store s3'=store s3
  by (cases s3) (simp add: check-method-access-def Let-def)
obtain invC
  where invC: invC = invocation-class mode (store s2) a statT
  by simp
with init-lvars
have invC': invC = (invocation-class mode (store s3) a statT)
  by (cases s2,cases mode) (auto simp add: init-lvars-def2 )
show ?thesis
proof (cases normal s2)
  case False
  with propagate-abnormal-s2 conf-s2 error-free-s2
  show ?thesis
    by auto
next
  case True
  note normal-s2 = True
  with normal-s1 conf-a eval-args
  have conf-a-s2: G, store s2⊢a::≼RefT statT
    by (auto dest: eval-gext intro: conf-gext)
  show ?thesis
  proof (cases a=Null ⟶ is-static statM)
    case False
    then obtain not-static: ¬ is-static statM and Null: a=Null
      by blast
    with normal-s2 init-lvars mode
    obtain np: abrupt s3 = Some (Xcpt (Std NullPointer)) and
               store s3 = store (init-lvars G invDeclC
                              (|name = mn, parTs = pTs'|) mode a vs s2)
      by (auto simp add: init-lvars-def2)
    moreover
    from np check
    have eq-s3'-s3: s3'=s3
      by (auto simp add: check-method-access-def Let-def)
    moreover
    from eq-s3'-s3 np eval-methd
    have s4=s3'
      by auto
    ultimately have
      set-lvars (locals (store s2)) s4
      = (Some (Xcpt (Std NullPointer)),store s2)
      by (cases s2,cases s3) (simp add: init-lvars-def2)
    with conf-s2 error-free-s2
    show ?thesis
      by (cases s2) (auto dest: conforms-NormI)
  next
    case True
    with mode have notNull: mode = IntVir ⟶ a ≠ Null
      by (auto dest!: Null-staticD)
    with conf-s2 conf-a-s2 wf invC
    have dynT-prop: G⊢mode→invC≼statT
```

```
  by (cases s2) (auto intro: DynT-propI)
with wt-e statM' invC mode wf
obtain dynM where
  dynM: dynlookup G statT invC  (|name=mn,parTs=pTs'|) = Some dynM and
  acc-dynM: G ⊢Methd  (|name=mn,parTs=pTs'|) dynM
                 in invC dyn-accessible-from accC
  by (force dest!: call-access-ok)
with invC' check eq-accC-accC'
have eq-s3'-s3: s3'=s3
  by (auto simp add: check-method-access-def Let-def)
from dynT-prop wf wt-e statM' mode invC invDeclC dynM
obtain
  wf-dynM: wf-mdecl G invDeclC ((|name=mn,parTs=pTs'|),mthd dynM) and
    dynM': methd G invDeclC (|name=mn,parTs=pTs'|) = Some dynM and
  iscls-invDeclC: is-class G invDeclC and
       invDeclC': invDeclC = declclass dynM and
    invC-widen: G⊢invC≼C invDeclC and
   resTy-widen: G⊢resTy dynM≼resTy statM and
  is-static-eq: is-static dynM = is-static statM and
  involved-classes-prop:
   (if invmode statM e = IntVir
     then ∀ statC. statT = ClassT statC ⟶ G⊢invC≼C statC
     else ((∃ statC. statT = ClassT statC ∧ G⊢statC≼C invDeclC) ∨
           (∀ statC. statT ≠ ClassT statC ∧ invDeclC = Object)) ∧
            statDeclT = ClassT invDeclC)
  by (cases rule: DynT-mheadsE) simp
obtain L' where
 L':L'=(λ k.
       (case k of
          EName e
          ⇒ (case e of
                VNam v
               ⇒(table-of (lcls (mbody (mthd dynM)))
                  (pars (mthd dynM)[↦]pTs')) v
             | Res ⇒ Some (resTy dynM))
        | This ⇒ if is-static statM
                  then None else Some (Class invDeclC)))
  by simp
from wf-dynM [THEN wf-mdeclD1, THEN conjunct1] normal-s2 conf-s2 wt-e
  wf eval-args conf-a mode notNull wf-dynM involved-classes-prop
have conf-s3: s3::≼(G,L')
  apply −

  apply (drule conforms-init-lvars [of G invDeclC
          (|name=mn,parTs=pTs'|) dynM store s2 vs pTs abrupt s2
          L statT invC a (statDeclT,statM) e])
  apply (rule wf)
  apply (rule conf-args,assumption)
  apply (simp add: pTs-widen)
  apply (cases s2,simp)
  apply (rule dynM')
  apply (force dest: ty-expr-is-type)
  apply (rule invC-widen)
  apply (force intro: conf-gext dest: eval-gext)
  apply simp
  apply simp
  apply (simp add: invC)
  apply (simp add: invDeclC)
  apply (simp add: normal-s2)
```

```
      apply (cases s2, simp add: L' init-lvars
                    cong add: lname.case-cong ename.case-cong)
      done
    with eq-s3'-s3
    have conf-s3': s3'::≼(G,L') by simp
    moreover
    from  is-static-eq wf-dynM L'
    obtain mthdT where
      (|prg=G,cls=invDeclC,lcl=L'|)
        ⊢Body invDeclC (stmt (mbody (mthd dynM)))::−mthdT and
      mthdT-widen: G⊢mthdT≼resTy dynM
      by − (drule wf-mdecl-bodyD,
           auto simp add: callee-lcl-def
                cong add: lname.case-cong ename.case-cong)
    with dynM' iscls-invDeclC invDeclC'
    have
      (|prg=G,cls=invDeclC,lcl=L'|)
        ⊢(Methd invDeclC (|name = mn, parTs = pTs'|))::−mthdT
      by (auto intro: wt.Methd)
    moreover
    obtain M where
      (|prg=G,cls=invDeclC,lcl=L'|)
        ⊢ dom (locals (store s3'))
          »In1l (Methd invDeclC (|name = mn, parTs = pTs'|))» M
    proof −
      from wf-dynM
      obtain M' where
        da-body:
        (|prg=G, cls=invDeclC
         ,lcl=callee-lcl invDeclC (|name = mn, parTs = pTs'|) (mthd dynM)
         |) ⊢ parameters (mthd dynM) »⟨stmt (mbody (mthd dynM))⟩» M' and
        res: Result ∈ nrm M'
        by (rule wf-mdeclE) iprover
      from da-body is-static-eq L' have
        (|prg=G, cls=invDeclC,lcl=L'|)
          ⊢ parameters (mthd dynM) »⟨stmt (mbody (mthd dynM))⟩» M'
        by (simp add: callee-lcl-def
            cong add: lname.case-cong ename.case-cong)
      moreover have parameters (mthd dynM) ⊆  dom (locals (store s3'))
      proof −
        from is-static-eq
        have (invmode (mthd dynM) e) = (invmode statM e)
          by (simp add: invmode-def)
        moreover
        have length (pars (mthd dynM)) = length vs
        proof −
          from normal-s2 conf-args
          have length vs = length pTs
            by (simp add: list-all2-def)
          also from pTs-widen
          have ... = length pTs'
            by (simp add: widens-def list-all2-def)
          also from wf-dynM
          have ... = length (pars (mthd dynM))
            by (simp add: wf-mdecl-def wf-mhead-def)
          finally show ?thesis ..
        qed
        moreover note init-lvars dynM' is-static-eq normal-s2 mode
        ultimately
```

```
      have parameters (mthd dynM) = dom (locals (store s3))
        using dom-locals-init-lvars
          [of mthd dynM G invDeclC (|name=mn,parTs=pTs'|) vs e a s2]
        by simp
      also from check
      have dom (locals (store s3)) ⊆  dom (locals (store s3'))
        by (simp add:  eq-s3'-s3)
      finally show ?thesis .
    qed
    ultimately obtain M2 where
      da:
      (|prg=G, cls=invDeclC,lcl=L'|)
        ⊢ dom (locals (store s3')) »⟨stmt (mbody (mthd dynM))⟩» M2 and
      M2: nrm M' ⊆ nrm M2
      by (rule da-weakenE)
    from res M2 have Result ∈ nrm M2
      by blast
    moreover from wf-dynM
    have jumpNestingOkS {Ret} (stmt (mbody (mthd dynM)))
      by (rule wf-mdeclE)
    ultimately
    obtain M3 where
      (|prg=G, cls=invDeclC,lcl=L'|) ⊢ dom (locals (store s3'))
            »⟨Body (declclass dynM) (stmt (mbody (mthd dynM)))⟩» M3
      using da
      by (iprover intro: da.Body assigned.select-convs)
    from - this [simplified]
    show ?thesis
      by (rule da.Methd [simplified,elim-format]) (auto intro: dynM' that)
  qed
  ultimately obtain
    conf-s4: s4::≼(G, L') and
    conf-Res: normal s4 ⟶ G,store s4⊢v::≼mthdT and
    error-free-s4: error-free s4
    by (rule hyp-methd [elim-format])
       (simp add: error-free-s3 eq-s3'-s3)
  from init-lvars eval-methd eq-s3'-s3
  have store s2≤|store s4
    by (cases s2) (auto dest!: eval-gext simp add: init-lvars-def2 )
  moreover
  have abrupt s4 ≠ Some (Jump Ret)
  proof −
    from normal-s2 init-lvars
    have abrupt s3 ≠ Some (Jump Ret)
      by (cases s2) (simp add: init-lvars-def2 abrupt-if-def)
    with check
    have abrupt s3' ≠ Some (Jump Ret)
      by (cases s3) (auto simp add: check-method-access-def Let-def)
    with eval-methd
    show ?thesis
      by (rule Methd-no-jump)
  qed
  ultimately
  have (set-lvars (locals (store s2))) s4::≼(G, L)
    using conf-s2 conf-s4
    by (cases s2,cases s4) (auto intro: conforms-return)
  moreover
  from conf-Res mthdT-widen resTy-widen wf
  have normal s4
```

⟶ *G*,*store s4*⊢*v*::≼(*resTy statM*)
by (*auto dest*: *widen-trans*)
then
have *normal* ((*set-lvars* (*locals* (*store s2*))) *s4*)
⟶ *G*,*store*((*set-lvars* (*locals* (*store s2*))) *s4*) ⊢*v*::≼(*resTy statM*)
by (*cases s4*) *auto*
moreover note *error-free-s4 T*
ultimately
show *?thesis*
by *simp*
qed
qed
qed
next
case (*Methd s0 D sig v s1 L accC T A*)
note ‹*G*⊢*Norm s0* −*body G D sig*−≻*v*→ *s1*›
note *hyp* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l* (*body G D sig*)) (*In1 v*)›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*In1l* (*Methd D sig*)::*T*›
then obtain *m bodyT* **where**
D: *is-class G D* **and**
m: *methd G D sig* = *Some m* **and**
wt-body: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈
⊢*Body* (*declclass m*) (*stmt* (*mbody* (*mthd m*)))::−*bodyT* **and**
T: *T*=*Inl bodyT*
by (*rule wt-elim-cases*) *auto*
moreover
from *Methd.prems m* **have**
da-body: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))
»*In1l* (*Body* (*declclass m*) (*stmt* (*mbody* (*mthd m*))))» *A*
by − (*erule da-elim-cases*,*simp*)
ultimately
show *s1*::≼(*G*, *L*) ∧
(*normal s1* ⟶ *G*,*L*,*snd s1*⊢*In1l* (*Methd D sig*)≻*In1 v*::≼*T*) ∧
(*error-free* (*Norm s0*) = *error-free s1*)
using *hyp* [*of* - - (*Inl bodyT*)] *conf-s0*
by (*auto simp add*: *Let-def body-def*)
next
case (*Body s0 D s1 c s2 s3 L accC T A*)
note *eval-init* = ‹*G*⊢*Norm s0* −*Init D*→ *s1*›
note *eval-c* = ‹*G*⊢*s1* −*c*→ *s2*›
note *hyp-init* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1r* (*Init D*)) ♢›
note *hyp-c* = ‹*PROP ?TypeSafe s1 s2* (*In1r c*) ♢›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*In1l* (*Body D c*)::*T*›
then obtain *bodyT* **where**
iscls-D: *is-class G D* **and**
wt-c: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*c*::√ **and**
resultT: *L Result* = *Some bodyT* **and**
isty-bodyT: *is-type G bodyT* **and**
T: *T*=*Inl bodyT*
by (*rule wt-elim-cases*) *auto*
from *Body.prems* **obtain** *C* **where**
da-c: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))»*In1r c*» *C* **and**
jmpOk: *jumpNestingOkS* {*Ret*} *c* **and**
res: *Result* ∈ *nrm C*
by (*elim da-elim-cases*) *simp*

```
note conf-s0
moreover from iscls-D
have (|prg=G, cls=accC, lcl=L|)⊢Init D::√ by auto
moreover obtain I where
  (|prg=G,cls=accC,lcl=L|)
     ⊢ dom (locals (store ((Norm s0)::state))) »In1r (Init D)» I
  by (auto intro: da-Init [simplified] assigned.select-convs)
ultimately obtain
  conf-s1: s1::≼(G, L) and error-free-s1: error-free s1
   by (rule hyp-init [elim-format]) simp
obtain C' where da-C': (|prg=G,cls=accC,lcl=L|)
                          ⊢ (dom (locals (store s1)))»In1r c» C'
         and nrm-C': nrm C ⊆ nrm C'
proof −
  from eval-init
  have (dom (locals (store ((Norm s0)::state))))
                ⊆ (dom (locals (store s1)))
    by (rule dom-locals-eval-mono-elim)
  with da-c show thesis by (rule da-weakenE) (rule that)
qed
from conf-s1 wt-c da-C'
obtain conf-s2: s2::≼(G, L) and error-free-s2: error-free s2
  by (rule hyp-c [elim-format]) (simp add: error-free-s1)
from conf-s2
have abupd (absorb Ret) s2::≼(G, L)
  by (cases s2) (auto intro: conforms-absorb)
moreover
from error-free-s2
have error-free (abupd (absorb Ret) s2)
  by simp
moreover have abrupt (abupd (absorb Ret) s3) ≠ Some (Jump Ret)
  by (cases s3) (simp add: absorb-def)
moreover have s3=s2
proof −
  from iscls-D
  have wt-init: (|prg=G, cls=accC, lcl=L|)⊢(Init D)::√
    by auto
  from eval-init wf
  have s1-no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
    by − (rule eval-statement-no-jump [OF - - - wt-init],auto)
  from eval-c - wt-c wf
  have ⋀ j. abrupt s2 = Some (Jump j) ⟹ j=Ret
    by (rule jumpNestingOk-evalE) (auto intro: jmpOk simp add: s1-no-jmp)
  moreover
  note ‹s3 =
           (if ∃ l. abrupt s2 = Some (Jump (Break l)) ∨
                   abrupt s2 = Some (Jump (Cont l))
            then abupd (λx. Some (Error CrossMethodJump)) s2 else s2)›
  ultimately show ?thesis
    by force
qed
moreover
{
  assume normal-upd-s2: normal (abupd (absorb Ret) s2)
  have Result ∈ dom (locals (store s2))
  proof −
    from normal-upd-s2
    have normal s2 ∨ abrupt s2 = Some (Jump Ret)
      by (cases s2) (simp add: absorb-def)
```

```
        thus ?thesis
        proof
          assume normal s2
          with eval-c wt-c da-C′ wf res nrm-C′
          show ?thesis
            by (cases rule: da-good-approxE′) blast
        next
          assume abrupt s2 = Some (Jump Ret)
          with conf-s2 show ?thesis
            by (cases s2) (auto dest: conforms-RetD simp add: dom-def)
        qed
      qed
    }
    moreover note T resultT
    ultimately
    show abupd (absorb Ret) s3::≼(G, L) ∧
           (normal (abupd (absorb Ret) s3) ⟶
             G,L,store (abupd (absorb Ret) s3)
             ⊢In1l (Body D c)≻In1 (the (locals (store s2) Result))::≼T) ∧
          (error-free (Norm s0) = error-free (abupd (absorb Ret) s3))
      by (cases s2) (auto intro: conforms-locals)
  next
    case (LVar s vn L accC T)
    note conf-s = ‹Norm s::≼(G, L)› and
      wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In2 (LVar vn)::T›
    then obtain vnT where
      vnT: L vn = Some vnT and
        T: T=Inl vnT
      by (auto elim!: wt-elim-cases)
    from conf-s vnT
    have conf-fst: locals s vn ≠ None ⟶ G,s⊢fst (lvar vn s)::≼vnT
     by (auto elim: conforms-localD [THEN wlconfD]
             simp add: lvar-def)
    moreover
    from conf-s conf-fst vnT
    have s≤|snd (lvar vn s)≼vnT::≼(G, L)
      by (auto elim: conforms-lupd simp add: assign-conforms-def lvar-def)
    moreover note conf-s T
    ultimately
    show Norm s::≼(G, L) ∧
                 (normal (Norm s) ⟶
                    G,L,store (Norm s)⊢In2 (LVar vn)≻In2 (lvar vn s)::≼T) ∧
                 (error-free (Norm s) = error-free (Norm s))
      by (simp add: lvar-def)
  next
    case (FVar s0 statDeclC s1 e a s2 v s2′ stat fn s3 accC L accC′ T A)
    note eval-init = ‹G⊢Norm s0 −Init statDeclC→ s1›
    note eval-e = ‹G⊢s1 −e−≻a→ s2›
    note fvar = ‹(v, s2′) = fvar statDeclC stat fn a s2›
    note check = ‹s3 = check-field-access G accC statDeclC fn stat a s2′›
    note hyp-init = ‹PROP ?TypeSafe (Norm s0) s1 (In1r (Init statDeclC)) ♢›
    note hyp-e = ‹PROP ?TypeSafe s1 s2 (In1l e) (In1 a)›
    note conf-s0 = ‹Norm s0::≼(G, L)›
    note wt = ‹(|prg=G, cls=accC′, lcl=L|)⊢In2 ({accC,statDeclC,stat}e..fn)::T›
    then obtain statC f where
                wt-e: (|prg=G, cls=accC, lcl=L|)⊢e::−Class statC and
            accfield: accfield G accC statC fn = Some (statDeclC,f) and
       eq-accC-accC′: accC=accC′ and
                stat: stat=is-static f and
```

T: *T*=(*Inl* (*type f*))
by (*rule wt-elim-cases*) (*auto simp add*: *member-is-static-simp*)
from *FVar.prems eq-accC-accC'*
have *da-e*: (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))»*In1l e*» *A*
by (*elim da-elim-cases*) *simp*
note *conf-s0*
moreover
from *wf wt-e*
have *iscls-statC*: *is-class G statC*
by (*auto dest*: *ty-expr-is-type type-is-class*)
with *wf accfield*
have *iscls-statDeclC*: *is-class G statDeclC*
by (*auto dest*!: *accfield-fields dest*: *fields-declC*)
hence (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)⊢(*Init statDeclC*)::√
by *simp*
moreover obtain *I* **where**
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
⊢ *dom* (*locals* (*store* ((*Norm s0*)::*state*))) »*In1r* (*Init statDeclC*)» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)
ultimately
obtain *conf-s1*: *s1*::⪯(*G*, *L*) **and** *error-free-s1*: *error-free s1*
by (*rule hyp-init* [*elim-format*]) *simp*
obtain *A'* **where**
(|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|) ⊢ (*dom* (*locals* (*store s1*)))»*In1l e*» *A'*
proof −
from *eval-init*
have (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))
⊆ (*dom* (*locals* (*store s1*)))
by (*rule dom-locals-eval-mono-elim*)
with *da-e* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *conf-s1 wt-e*
obtain *conf-s2*: *s2*::⪯(*G*, *L*) **and**
conf-a: *normal s2* ⟶ *G*,*store s2*⊢*a*::⪯*Class statC* **and**
error-free-s2: *error-free s2*
by (*rule hyp-e* [*elim-format*]) (*simp add*: *error-free-s1*)
from *fvar*
have *store-s2'*: *store s2'*=*store s2*
by (*cases s2*) (*simp add*: *fvar-def2*)
with *fvar conf-s2*
have *conf-s2'*: *s2'*::⪯(*G*, *L*)
by (*cases s2*,*cases stat*) (*auto simp add*: *fvar-def2*)
from *eval-init*
have *initd-statDeclC-s1*: *initd statDeclC s1*
by (*rule init-yields-initd*)
from *accfield wt-e eval-init eval-e conf-s2 conf-a fvar stat check wf*
have *eq-s3-s2'*: *s3*=*s2'*
by (*auto dest*!: *error-free-field-access*)
have *conf-v*: *normal s2'* ⟹
G,*store s2'*⊢*fst v*::⪯*type f* ∧ *store s2'*≤|*snd v*⪯*type f*::⪯(*G*, *L*)
proof −
assume *normal*: *normal s2'*
obtain *vv vf x2 store2 store2'*
where *v*: *v*=(*vv*,*vf*) **and**
s2: *s2*=(*x2*,*store2*) **and**
store2': *store s2'* = *store2'*
by (*cases v*,*cases s2*,*cases s2'*) *blast*

from *iscls-statDeclC* **obtain** *c*
where *c*: *class G statDeclC* = *Some c*
by *auto*
have *G,store2′⊢vv::≼type f* ∧ *store2′≤|vf≼type f::≼(G, L)*
proof (*rule FVar-lemma* [*of vv vf store2′ statDeclC f fn a x2 store2*
statC G c L store s1])
from *v normal s2 fvar stat store2′*
show ((*vv*, *vf*), *Norm store2′*) =
fvar statDeclC (*static f*) *fn a* (*x2*, *store2*)
by (*auto simp add*: *member-is-static-simp*)
from *accfield iscls-statC wf*
show *G⊢statC*$\preceq_C$ *statDeclC*
by (*auto dest*!: *accfield-fields dest*: *fields-declC*)
from *accfield*
show *fld*: *table-of* (*DeclConcepts.fields G statC*) (*fn*, *statDeclC*) = *Some f*
by (*auto dest*!: *accfield-fields*)
from *wf* **show** *wf-prog G* **.**
from *conf-a s2* **show** *x2* = *None* ⟶ *G,store2⊢a::≼Class statC*
by *auto*
from *fld wf iscls-statC*
show *statDeclC* ≠ *Object*
by (*cases statDeclC=Object*) (*drule fields-declC,simp+*)+
from *c* **show** *class G statDeclC* = *Some c* **.**
from *conf-s2 s2* **show** (*x2*, *store2*)::≼(*G*, *L*) **by** *simp*
from *eval-e s2* **show** *snd s1≤|store2* **by** (*auto dest*: *eval-gext*)
from *initd-statDeclC-s1* **show** *inited statDeclC* (*globs* (*snd s1*))
by *simp*
qed
with *v s2 store2′*
show *?thesis*
by *simp*
qed
from *fvar error-free-s2*
have *error-free s2′*
by (*cases s2*)
(*auto simp add*: *fvar-def2 intro*!: *error-free-FVar-lemma*)
with *conf-v T conf-s2′ eq-s3-s2′*
show *s3*::≼(*G*, *L*) ∧
(*normal s3*
⟶ *G,L,store s3⊢In2* ({*accC,statDeclC,stat*}*e..fn*)≻*In2 v*::≼*T*) ∧
(*error-free* (*Norm s0*) = *error-free s3*)
by *auto*
next
case (*AVar s0 e1 a s1 e2 i s2 v s2′ L accC T A*)
note *eval-e1* = ‹*G⊢Norm s0* −*e1*−≻*a*→ *s1*›
note *eval-e2* = ‹*G⊢s1* −*e2*−≻*i*→ *s2*›
note *hyp-e1* = ‹*PROP ?TypeSafe* (*Norm s0*) *s1* (*In1l e1*) (*In1 a*)›
note *hyp-e2* = ‹*PROP ?TypeSafe s1 s2* (*In1l e2*) (*In1 i*)›
note *avar* = ‹(*v*, *s2′*) = *avar G i a s2*›
note *conf-s0* = ‹*Norm s0*::≼(*G*, *L*)›
note *wt* = ‹⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*In2* (*e1*.[*e2*])::*T*›
then obtain *elemT*
where *wt-e1*: ⦇*prg=G,cls=accC,lcl=L*⦈⊢*e1*::−*elemT*.[] **and**
wt-e2: ⦇*prg=G,cls=accC,lcl=L*⦈⊢*e2*::−*PrimT Integer* **and**
T: *T*= *Inl elemT*
by (*rule wt-elim-cases*) *auto*
from *AVar.prems* **obtain** *E1* **where**
da-e1: ⦇*prg=G,cls=accC,lcl=L*⦈
⊢ (*dom* (*locals* (*store* ((*Norm s0*)::*state*))))»*In1l e1*» *E1* **and**

```
  da-e2: ⦇prg=G,cls=accC,lcl=L⦈⊢ nrm E1 »In1l e2» A
  by (elim da-elim-cases) simp
from conf-s0 wt-e1 da-e1
obtain conf-s1: s1::≼(G, L) and
      conf-a: (normal s1 ⟶ G,store s1⊢a::≼elemT.[]) and
      error-free-s1: error-free s1
  by (rule hyp-e1 [elim-format]) simp
show s2'::≼(G, L) ∧
     (normal s2' ⟶ G,L,store s2'⊢In2 (e1.[e2])≻In2 v::≼T) ∧
     (error-free (Norm s0) = error-free s2')
proof (cases normal s1)
  case False
  moreover
  from False eval-e2 have eq-s2-s1: s2=s1 by auto
  moreover
  from eq-s2-s1 False have ¬ normal s2 by simp
  then have snd (avar G i a s2) = s2
    by (cases s2) (simp add: avar-def2)
  with avar have s2'=s2
    by (cases (avar G i a s2)) simp
  ultimately show ?thesis
    using conf-s1 error-free-s1
    by auto
next
  case True
  obtain A' where
    ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals (store s1)) »In1l e2» A'
  proof −
    from eval-e1 wt-e1 da-e1 wf True
    have nrm E1 ⊆ dom (locals (store s1))
      by (cases rule: da-good-approxE') iprover
    with da-e2 show thesis
      by (rule da-weakenE) (rule that)
  qed
  with conf-s1 wt-e2
  obtain conf-s2: s2::≼(G, L) and error-free-s2: error-free s2
    by (rule hyp-e2 [elim-format]) (simp add: error-free-s1)
  from avar
  have store s2'=store s2
    by (cases s2) (simp add: avar-def2)
  with avar conf-s2
  have conf-s2': s2'::≼(G, L)
    by (cases s2) (auto simp add: avar-def2)
  from avar error-free-s2
  have error-free-s2': error-free s2'
    by (cases s2) (auto simp add: avar-def2 )
  have normal s2' ⟹
    G,store s2'⊢fst v::≼elemT ∧ store s2'≤|snd v≼elemT::≼(G, L)
  proof −
    assume normal: normal s2'
    show ?thesis
    proof −
      obtain vv vf x1 store1 x2 store2 store2'
        where v: v=(vv,vf) and
              s1: s1=(x1,store1) and
              s2: s2=(x2,store2) and
         store2': store2'=store s2'
        by (cases v,cases s1, cases s2, cases s2') blast
      have G,store2'⊢vv::≼elemT ∧ store2'≤|vf≼elemT::≼(G, L)
```

```
        proof (rule AVar-lemma [of G x1 store1 e2 i x2 store2 vv vf store2' a,
                                 OF wf])
          from s1 s2 eval-e2 show G⊢(x1, store1) −e2−≻i→ (x2, store2)
            by simp
          from v normal s2 store2' avar
          show ((vv, vf), Norm store2') = avar G i a (x2, store2)
            by auto
          from s2 conf-s2 show (x2, store2)::≼(G, L) by simp
          from s1 conf-a show  x1 = None ⟶ G,store1⊢a::≼elemT.[] by simp
          from eval-e2 s1 s2 show store1≤|store2 by (auto dest: eval-gext)
        qed
        with v s1 s2 store2'
        show ?thesis
          by simp
      qed
    qed
    with conf-s2' error-free-s2' T
    show ?thesis
      by auto
  qed
next
  case (Nil s0 L accC T)
  then show ?case
    by (auto elim!: wt-elim-cases)
next
  case (Cons s0 e v s1 es vs s2 L accC T A)
  note eval-e = ‹G⊢Norm s0 −e−≻v→ s1›
  note eval-es = ‹G⊢s1 −es≐≻vs→ s2›
  note hyp-e = ‹PROP ?TypeSafe (Norm s0) s1 (In1l e) (In1 v)›
  note hyp-es = ‹PROP ?TypeSafe s1 s2 (In3 es) (In3 vs)›
  note conf-s0 = ‹Norm s0::≼(G, L)›
  note wt = ‹(|prg = G, cls = accC, lcl = L|)⊢In3 (e # es)::T›
  then obtain eT esT where
     wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT and
     wt-es: (|prg = G, cls = accC, lcl = L|)⊢es::≐esT and
     T: T=Inr (eT#esT)
    by (rule wt-elim-cases) blast
  from Cons.prems obtain E where
    da-e: (|prg=G,cls=accC,lcl=L|)
            ⊢ (dom (locals (store ((Norm s0)::state))))»In1l e» E and
    da-es: (|prg=G,cls=accC,lcl=L|)⊢ nrm E »In3 es» A
    by (elim da-elim-cases) simp
  from conf-s0 wt-e da-e
  obtain conf-s1: s1::≼(G, L) and error-free-s1: error-free s1 and
    conf-v: normal s1 ⟶ G,store s1⊢v::≼eT
    by (rule hyp-e [elim-format]) simp
  show
    s2::≼(G, L) ∧
    (normal s2 ⟶ G,L,store s2⊢In3 (e # es)≻In3 (v # vs)::≼T) ∧
    (error-free (Norm s0) = error-free s2)
  proof (cases normal s1)
    case False
    with eval-es have s2=s1 by auto
    with False conf-s1 error-free-s1
    show ?thesis
      by auto
  next
    case True
    obtain A' where
```

    $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$ *dom* (*locals* (*store s1*)) »*In3 es*» A'
  proof −
    from *eval-e wt-e da-e wf True*
    have *nrm* $E \subseteq$ *dom* (*locals* (*store s1*))
      by (*cases rule*: *da-good-approxE'*) *iprover*
    with *da-es* **show** *thesis*
      by (*rule da-weakenE*) (*rule that*)
  qed
  with *conf-s1 wt-es*
  obtain *conf-s2*: $s2::\preceq(G,\ L)$ **and**
      error-free-s2: *error-free s2* **and**
      conf-vs: *normal s2* $\longrightarrow$ *list-all2* (*conf G* (*store s2*)) *vs esT*
    by (*rule hyp-es* [*elim-format*]) (*simp add*: *error-free-s1*)
  moreover
  from *True eval-es conf-v*
  have *conf-v'*: G,*store s2*$\vdash v::\preceq eT$
    apply *clarify*
    apply (*rule conf-gext*)
    apply (*auto dest*: *eval-gext*)
    done
  ultimately show *?thesis* **using** T **by** *simp*
 qed
qed
from *this* **and** *conf-s0 wt da* **show** *?thesis* **.**
qed

corollary *eval-type-soundE* [*consumes 5*]:
assumes *eval*: $G\vdash s0\ -t\succ\rightarrow\ (v,\ s1)$
and *conf*: $s0::\preceq(G,\ L)$
and *wt*: $(\!|prg\ =\ G,\ cls\ =\ accC,\ lcl\ =\ L|\!)\vdash t::T$
and *da*: $(\!|prg\ =\ G,\ cls\ =\ accC,\ lcl\ =\ L|\!)\vdash$ *dom* (*locals* (*snd s0*)) »t» A
and *wf*: *wf-prog G*
and *elim*: $[\![s1::\preceq(G,\ L)$; *normal s1* $\Longrightarrow G,L$,*snd s1*$\vdash t\succ v::\preceq T$;
  error-free s0 = *error-free s1*$]\!] \Longrightarrow P$
shows P
using *eval wt da wf conf*
by (*rule eval-type-sound* [*elim-format*]) (*iprover intro*: *elim*)

corollary *eval-ts*:
$[\![G\vdash s\ -e-\succ v\ \rightarrow\ s'$; *wf-prog G*; $s::\preceq(G,L)$; $(\!|prg=G,cls=C,lcl=L|\!)\vdash e::-T$;
 $(\!|prg=G,cls=C,lcl=L|\!)\vdash$*dom* (*locals* (*store s*))»*In1l e*»$A]\!]$
$\Longrightarrow\ s'::\preceq(G,L) \wedge$ (*normal* $s' \longrightarrow G$,*store* $s'\vdash v::\preceq T) \wedge$
 (*error-free s* = *error-free* s')
apply (*drule* (*4*) *eval-type-sound*)
apply *clarsimp*
done

corollary *evals-ts*:
$[\![G\vdash s\ -es\doteq\succ vs\rightarrow\ s'$; *wf-prog G*; $s::\preceq(G,L)$; $(\!|prg=G,cls=C,lcl=L|\!)\vdash es::\doteq Ts$;
 $(\!|prg=G,cls=C,lcl=L|\!)\vdash$*dom* (*locals* (*store s*))»*In3 es*»$A]\!]$
$\Longrightarrow\ s'::\preceq(G,L) \wedge$ (*normal* $s' \longrightarrow$ *list-all2* (*conf G* (*store* s')) *vs Ts*) $\wedge$
 (*error-free s* = *error-free* s')
apply (*drule* (*4*) *eval-type-sound*)
apply *clarsimp*
done

corollary *evar-ts*:
$[\![G\vdash s\ -v=\succ vf\rightarrow\ s'$; *wf-prog G*; $s::\preceq(G,L)$; $(\!|prg=G,cls=C,lcl=L|\!)\vdash v::=T$;

(|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*dom* (*locals* (*store s*))»*In2 v*»*A*⟧ ⟹
s′::≼(*G*,*L*) ∧ (*normal s*′ ⟶ *G*,*L*,(*store s*′)⊢*In2 v*≻*In2 vf*::≼*Inl T*) ∧
(*error-free s* = *error-free s*′)
apply (*drule* (*4*) *eval-type-sound*)
apply *clarsimp*
done

theorem *exec-ts*:
⟦*G*⊢*s* −*c*→ *s*′; *wf-prog G*; *s*::≼(*G*,*L*); (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*c*::√;
(|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*dom* (*locals* (*store s*))»*In1r c*»*A*⟧
⟹ *s*′::≼(*G*,*L*) ∧ (*error-free s* ⟶ *error-free s*′)
apply (*drule* (*4*) *eval-type-sound*)
apply *clarsimp*
done

lemma *wf-eval-Fin*:
assumes *wf*: *wf-prog G*
and *wt-c1*: (|*prg* = *G*, *cls* = *C*, *lcl* = *L*|)⊢*In1r c1*::*Inl* (*PrimT Void*)
and *da-c1*: (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*dom* (*locals* (*store* (*Norm s0*)))»*In1r c1*»*A*
and *conf-s0*: *Norm s0*::≼(*G*, *L*)
and *eval-c1*: *G*⊢*Norm s0* −*c1*→ (*x1*,*s1*)
and *eval-c2*: *G*⊢*Norm s1* −*c2*→ *s2*
and *s3*: *s3*=*abupd* (*abrupt-if* (*x1*≠*None*) *x1*) *s2*
shows *G*⊢*Norm s0* −*c1 Finally c2*→ *s3*
proof −
from *eval-c1 wt-c1 da-c1 wf conf-s0*
have *error-free* (*x1*,*s1*)
by (*auto dest*: *eval-type-sound*)
with *eval-c1 eval-c2 s3*
show *?thesis*
by − (*rule eval.Fin*, *auto simp add*: *error-free-def*)
qed

48 Ideas for the future

In the type soundness proof and the correctness proof of definite assignment we perform induction on the evaluation relation with the further preconditions that the term is welltyped and definitely assigned. During the proofs we have to establish the welltypedness and definite assignment of the subterms to be able to apply the induction hypothesis. So large parts of both proofs are the same work in propagating welltypedness and definite assignment. So we can derive a new induction rule for induction on the evaluation of a wellformed term, were these propagations is already done, once and forever. Then we can do the proofs with this rule and can enjoy the time we have saved. Here is a first and incomplete sketch of such a rule.

theorem *wellformed-eval-induct* [*consumes 4*, *case-names Abrupt Skip Expr Lab*
Comp If]:
assumes *eval*: *G*⊢*s0* −*t*≻→ (*v*,*s1*)
and *wt*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*t*::*T*
and *da*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*dom* (*locals* (*store s0*))»*t*»*A*
and *wf*: *wf-prog G*
and *abrupt*: ⋀ *s t abr L accC T A*.
⟦(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*t*::*T*;
(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*dom* (*locals* (*store* (*Some abr*,*s*)))»*t*»*A*
⟧ ⟹ *P L accC* (*Some abr*, *s*) *t* (*arbitrary3 t*) (*Some abr*, *s*)
and *skip*: ⋀ *s L accC*. *P L accC* (*Norm s*) ⟨*Skip*⟩$_s$ ◇ (*Norm s*)
and *expr*: ⋀ *e s0 s1 v L accC eT E*.
⟦(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*e*::−*eT*;

$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\quad\vdash dom\ (locals\ (store\ ((Norm\ s0)::state)))\text{»}\langle e\rangle_e\text{»}E;$
$P\ L\ accC\ (Norm\ s0)\ \langle e\rangle_e\ \lfloor v\rfloor_e\ s1]\!]$
$\Longrightarrow\ P\ L\ accC\ (Norm\ s0)\ \langle Expr\ e\rangle_s\ \diamondsuit\ s1$

and *lab*: $\bigwedge\ c\ l\ s0\ s1\ L\ accC\ C.$
$[\![(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash c::\surd;$
$(\!|prg{=}G,cls{=}accC,\ lcl{=}L|\!)$
$\quad\vdash dom\ (locals\ (store\ ((Norm\ s0)::state)))\text{»}\langle c\rangle_s\text{»}C;$
$P\ L\ accC\ (Norm\ s0)\ \langle c\rangle_s\ \diamondsuit\ s1]\!]$
$\Longrightarrow P\ L\ accC\ (Norm\ s0)\ \langle l\bullet\ c\rangle_s\ \diamondsuit\ (abupd\ (absorb\ l)\ s1)$

and *comp*: $\bigwedge\ c1\ c2\ s0\ s1\ s2\ L\ accC\ C1.$
$[\![G\vdash Norm\ s0\ -c1\ \rightarrow\ s1;G\vdash s1\ -c2\ \rightarrow\ s2;$
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash c1::\surd;$
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash c2::\surd;$
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash$
$\quad dom\ (locals\ (store\ ((Norm\ s0)::state)))\ \text{»}\langle c1\rangle_s\text{»}\ C1;$
$P\ L\ accC\ (Norm\ s0)\ \langle c1\rangle_s\ \diamondsuit\ s1;$
$\bigwedge\ Q.\ [\![normal\ s1;$
$\quad\bigwedge\ C2.[\![(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\qquad\vdash dom\ (locals\ (store\ s1))\ \text{»}\langle c2\rangle_s\text{»}\ C2;$
$\qquad P\ L\ accC\ s1\ \langle c2\rangle_s\ \diamondsuit\ s2]\!]\Longrightarrow Q$
$\quad]\!]\Longrightarrow Q$
$]\!]\Longrightarrow P\ L\ accC\ (Norm\ s0)\ \langle c1;;\ c2\rangle_s\ \diamondsuit\ s2$

and *if*: $\bigwedge\ b\ c1\ c2\ e\ s0\ s1\ s2\ L\ accC\ E.$
$[\![G\vdash Norm\ s0\ -e{-}\succ b\rightarrow\ s1;$
$G\vdash s1\ -(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)\rightarrow\ s2;$
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash e::-PrimT\ Boolean;$
$(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)::\surd;$
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash$
$\quad dom\ (locals\ (store\ ((Norm\ s0)::state)))\ \text{»}\langle e\rangle_e\text{»}\ E;$
$P\ L\ accC\ (Norm\ s0)\ \langle e\rangle_e\ \lfloor b\rfloor_e\ s1;$
$\bigwedge\ Q.\ [\![normal\ s1;$
$\quad\bigwedge\ C.\ [\![(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash\ (dom\ (locals\ (store\ s1)))$
$\qquad\text{»}\langle if\ the\text{-}Bool\ b\ then\ c1\ else\ c2\rangle_s\text{»}\ C;$
$\qquad P\ L\ accC\ s1\ \langle if\ the\text{-}Bool\ b\ then\ c1\ else\ c2\rangle_s\ \diamondsuit\ s2$
$\qquad]\!]\Longrightarrow Q$
$\quad]\!]\Longrightarrow Q$
$]\!]\Longrightarrow\ P\ L\ accC\ (Norm\ s0)\ \langle If(e)\ c1\ Else\ c2\rangle_s\ \diamondsuit\ s2$

shows $P\ L\ accC\ s0\ t\ v\ s1$

proof –
note *inj-term-simps* [*simp*]
from *eval*
show $\bigwedge\ L\ accC\ T\ A.\ [\![(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash t::T;$
$\quad(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash dom\ (locals\ (store\ s0))\text{»}t\text{»}A]\!]$
$\Longrightarrow\ P\ L\ accC\ s0\ t\ v\ s1$ (**is** *PROP ?Hyp s0 t v s1*)
proof (*induct*)
case *Abrupt* **with** *abrupt* **show** *?case* **.**
next
case *Skip* **from** *skip* **show** *?case* **by** *simp*
next
case (*Expr s0 e v s1 L accC T A*)
from *Expr.prems* **obtain** *eT* **where**
$(\!|prg\ =\ G,\ cls\ =\ accC,\ lcl\ =\ L|\!)\vdash e::-eT$
by (*elim wt-elim-cases*)
moreover
from *Expr.prems* **obtain** *E* **where**
$(\!|prg{=}G,cls{=}accC,\ lcl{=}L|\!)\vdash dom\ (locals\ (store\ ((Norm\ s0)::state)))\text{»}\langle e\rangle_e\text{»}E$
by (*elim da-elim-cases*) *simp*
moreover from *calculation*

```
    have P L accC (Norm s0) ⟨e⟩ₑ ⌊v⌋ₑ s1
      by (rule Expr.hyps)
    ultimately show ?case
      by (rule expr)
  next
    case (Lab s0 c s1 l L accC T A)
    from Lab.prems
    have (|prg = G, cls = accC, lcl = L|)⊢c::√
      by (elim wt-elim-cases)
    moreover
    from Lab.prems obtain C where
      (|prg=G,cls=accC, lcl=L|)⊢dom (locals (store ((Norm s0)::state)))»⟨c⟩ₛ»C
      by (elim da-elim-cases) simp
    moreover from calculation
    have P L accC (Norm s0) ⟨c⟩ₛ ♢ s1
      by (rule  Lab.hyps)
    ultimately show ?case
      by (rule lab)
  next
    case (Comp s0 c1 s1 c2 s2 L accC T A)
    note eval-c1 = ‹G⊢Norm s0 −c1→ s1›
    note eval-c2 = ‹G⊢s1 −c2→ s2›
    from Comp.prems obtain
      wt-c1: (|prg = G, cls = accC, lcl = L|)⊢c1::√ and
      wt-c2: (|prg = G, cls = accC, lcl = L|)⊢c2::√
      by (elim wt-elim-cases)
    from Comp.prems
    obtain C1 C2
      where da-c1: (|prg=G, cls=accC, lcl=L|)⊢
                     dom (locals (store ((Norm s0)::state))) »⟨c1⟩ₛ» C1 and
            da-c2: (|prg=G, cls=accC, lcl=L|)⊢  nrm C1 »⟨c2⟩ₛ» C2
      by (elim da-elim-cases) simp
    from wt-c1 da-c1
    have P-c1: P L accC (Norm s0) ⟨c1⟩ₛ ♢ s1
      by (rule Comp.hyps)
    {
      fix Q
      assume normal-s1: normal s1
      assume elim: ⋀ C2'.
                  ⟦(|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s1))»⟨c2⟩ₛ»C2';
                    P L accC s1 ⟨c2⟩ₛ ♢ s2⟧ ⟹ Q
      have Q
      proof −
        obtain C2' where
          da: (|prg=G, cls=accC, lcl=L|)⊢ dom (locals (store s1)) »⟨c2⟩ₛ» C2'
        proof −
          from eval-c1 wt-c1 da-c1 wf normal-s1
          have nrm C1 ⊆ dom (locals (store s1))
            by (cases rule: da-good-approxE') iprover
          with da-c2 show thesis
            by (rule da-weakenE) (rule that)
        qed
        with wt-c2 have P L accC s1 ⟨c2⟩ₛ ♢ s2
          by (rule Comp.hyps)
        with da show ?thesis
          using elim by iprover
      qed
    }
    with eval-c1 eval-c2 wt-c1 wt-c2 da-c1 P-c1
```

```
    show ?case
      by (rule comp) iprover+
  next
    case (If s0 e b s1 c1 c2 s2 L accC T A)
    note eval-e = ‹G⊢Norm s0 −e−≻b→ s1›
    note eval-then-else = ‹G⊢s1 −(if the-Bool b then c1 else c2)→ s2›
    from If.prems
    obtain
             wt-e: (|prg=G, cls=accC, lcl=L|)⊢e::−PrimT Boolean and
      wt-then-else: (|prg=G, cls=accC, lcl=L|)⊢(if the-Bool b then c1 else c2)::√
      by (elim wt-elim-cases) (auto split add: split-if)
    from If.prems obtain E C where
      da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store ((Norm s0)::state)))
                                     »⟨e⟩e» E and
      da-then-else:
      (|prg=G,cls=accC,lcl=L|)⊢
         (dom (locals (store ((Norm s0)::state))) ∪ assigns-if (the-Bool b) e)
          »⟨if the-Bool b then c1 else c2⟩s» C
      by (elim da-elim-cases) (cases the-Bool b,auto)
    from wt-e da-e
    have P-e: P L accC (Norm s0) ⟨e⟩e ⌊b⌋e s1
      by (rule If.hyps)
    {
      fix Q
      assume normal-s1: normal s1
      assume elim: ⋀ C. ⟦(|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s1)))
                                »⟨if the-Bool b then c1 else c2⟩s» C;
                          P L accC s1 ⟨if the-Bool b then c1 else c2⟩s ♢ s2
                          ⟧ ⟹ Q
      have Q
      proof −
        obtain C' where
          da: (|prg=G,cls=accC,lcl=L|)⊢
               (dom (locals (store s1)))»⟨if the-Bool b then c1 else c2⟩s » C'
        proof −
          from eval-e have
            dom (locals (store ((Norm s0)::state))) ⊆ dom (locals (store s1))
            by (rule dom-locals-eval-mono-elim)
          moreover
          from eval-e normal-s1 wt-e
          have assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
            by (rule assigns-if-good-approx')
          ultimately
          have dom (locals (store ((Norm s0)::state)))
            ∪ assigns-if (the-Bool b) e ⊆ dom (locals (store s1))
            by (rule Un-least)
          with da-then-else show thesis
            by (rule da-weakenE) (rule that)
        qed
        with wt-then-else
        have P L accC s1 ⟨if the-Bool b then c1 else c2⟩s ♢ s2
          by (rule If.hyps)
        with da show ?thesis using elim by iprover
      qed
    }
    with eval-e eval-then-else wt-e wt-then-else da-e P-e
    show ?case
      by (rule if) iprover+
  next
```

oops

end

Chapter 20

Evaln

49 Operational evaluation (big-step) semantics of Java expressions and statements

theory *Evaln* **imports** *TypeSafe* **begin**

Variant of *eval* relation with counter for bounded recursive depth. In principal *evaln* could replace *eval*.

Validity of the axiomatic semantics builds on *evaln*. For recursive method calls the axiomatic semantics rule assumes the method ok to derive a proof for the body. To prove the method rule sound we need to perform induction on the recursion depth. For the completeness proof of the axiomatic semantics the notion of the most general formula is used. The most general formula right now builds on the ordinary evaluation relation *eval*. So sometimes we have to switch between *evaln* and *eval* and vice versa. To make this switch easy *evaln* also does all the technical accessibility tests *check-field-access* and *check-method-access* like *eval*. If it would omit them *evaln* and *eval* would only be equivalent for welltyped, and definitely assigned terms.

inductive
  evaln :: [*prog*, *state*, *term*, *nat*, *vals*, *state*] ⇒ *bool*
    (-⊢- −-≻−-→ ′(-, -′) [61,61,80,61,0,0] 60)
  and *evarn* :: [*prog*, *state*, *var*, *vvar*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-=≻-−-→ - [61,61,90,61,61,61] 60)
  and *eval-n*:: [*prog*, *state*, *expr*, *val*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-−≻-−-→ - [61,61,80,61,61,61] 60)
  and *evalsn* :: [*prog*, *state*, *expr list*, *val list*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-≐≻-−-→ - [61,61,61,61,61,61] 60)
  and *execn* :: [*prog*, *state*, *stmt*, *nat*, *state*] ⇒ *bool*
    (-⊢- −-−-→ - [61,61,65, 61,61] 60)
  for *G* :: *prog*
where

  $G\vdash s\ -c\ \ -n\rightarrow\ s' \equiv G\vdash s\ -In1r\ c\succ-n\rightarrow (\diamondsuit\ ,\ s')$
| $G\vdash s\ -e-\succ v\ -n\rightarrow\ s' \equiv G\vdash s\ -In1l\ e\succ-n\rightarrow (In1\ v\ ,\ s')$
| $G\vdash s\ -e{=}\succ vf\ -n\rightarrow\ s' \equiv G\vdash s\ -In2\ e\succ-n\rightarrow (In2\ vf,\ s')$
| $G\vdash s\ -e\doteq\succ v\ -n\rightarrow\ s' \equiv G\vdash s\ -In3\ e\succ-n\rightarrow (In3\ v\ ,\ s')$

— propagation of abrupt completion

| *Abrupt*: $G\vdash(Some\ xc,s)\ -t\succ-n\rightarrow (arbitrary3\ t,(Some\ xc,s))$

— evaluation of variables

| *LVar*: $G\vdash Norm\ s\ -LVar\ vn{=}\succ lvar\ vn\ s-n\rightarrow Norm\ s$

| *FVar*: $[\![G\vdash Norm\ s0\ -Init\ statDeclC-n\rightarrow s1;\ G\vdash s1\ -e-\succ a-n\rightarrow s2;$
  $(v,s2') = fvar\ statDeclC\ stat\ fn\ a\ s2;$
  $s3 = check\text{-}field\text{-}access\ G\ accC\ statDeclC\ fn\ stat\ a\ s2']\!] \Longrightarrow$
  $G\vdash Norm\ s0\ -\{accC,statDeclC,stat\}e..fn{=}\succ v-n\rightarrow s3$

| *AVar*: $[\![G\vdash Norm\ s0\ -e1-\succ a-n\rightarrow s1\ ;\ G\vdash s1\ -e2-\succ i-n\rightarrow s2;$
  $(v,s2') = avar\ G\ i\ a\ s2]\!] \Longrightarrow$
    $G\vdash Norm\ s0\ -e1.[e2]{=}\succ v-n\rightarrow s2'$

— evaluation of expressions

| *NewC*: $[\![G\vdash Norm\ s0\ -Init\ C-n\rightarrow s1;$

```
        G⊢      s1 −halloc (CInst C)≻a→ s2⟧ ⟹
                              G⊢Norm s0 −NewC C−≻Addr a−n→ s2

| NewA: ⟦G⊢Norm s0 −init-comp-ty T−n→ s1; G⊢s1 −e−≻i′−n→ s2;
        G⊢abupd (check-neg i′) s2 −halloc (Arr T (the-Intg i′))≻a→ s3⟧ ⟹
                            G⊢Norm s0 −New T[e]−≻Addr a−n→ s3

| Cast: ⟦G⊢Norm s0 −e−≻v−n→ s1;
        s2 = abupd (raise-if (¬G,snd s1⊢v fits T) ClassCast) s1⟧ ⟹
                            G⊢Norm s0 −Cast T e−≻v−n→ s2

| Inst: ⟦G⊢Norm s0 −e−≻v−n→ s1;
        b = (v≠Null ∧ G,store s1⊢v fits RefT T)⟧ ⟹
                          G⊢Norm s0 −e InstOf T−≻Bool b−n→ s1

| Lit:                  G⊢Norm s −Lit v−≻v−n→ Norm s

| UnOp: ⟦G⊢Norm s0 −e−≻v−n→ s1⟧
       ⟹ G⊢Norm s0 −UnOp unop e−≻(eval-unop unop v)−n→ s1

| BinOp: ⟦G⊢Norm s0 −e1−≻v1−n→ s1;
          G⊢s1 −(if need-second-arg binop v1 then (In1l e2) else (In1r Skip))
           ≻−n→ (In1 v2,s2)⟧
       ⟹ G⊢Norm s0 −BinOp binop e1 e2−≻(eval-binop binop v1 v2)−n→ s2

| Super:                 G⊢Norm s −Super−≻val-this s−n→ Norm s

| Acc:  ⟦G⊢Norm s0 −va=≻(v,f)−n→ s1⟧ ⟹
                              G⊢Norm s0 −Acc va−≻v−n→ s1

| Ass:  ⟦G⊢Norm s0 −va=≻(w,f)−n→ s1;
        G⊢      s1 −e−≻v      −n→ s2⟧ ⟹
                               G⊢Norm s0 −va:=e−≻v−n→ assign f v s2

| Cond: ⟦G⊢Norm s0 −e0−≻b−n→ s1;
        G⊢      s1 −(if the-Bool b then e1 else e2)−≻v−n→ s2⟧ ⟹
                         G⊢Norm s0 −e0 ? e1 : e2−≻v−n→ s2

| Call:
  ⟦G⊢Norm s0 −e−≻a′−n→ s1; G⊢s1 −args≐≻vs−n→ s2;
   D = invocation-declclass G mode (store s2) a′ statT (|name=mn,parTs=pTs|);
   s3=init-lvars G D (|name=mn,parTs=pTs|) mode a′ vs s2;
   s3′ = check-method-access G accC statT mode (|name=mn,parTs=pTs|) a′ s3;
   G⊢s3′−Methd D (|name=mn,parTs=pTs|)−≻v−n→ s4
  ⟧
  ⟹
   G⊢Norm s0 −{accC,statT,mode}e·mn({pTs}args)−≻v−n→ (restore-lvars s2 s4)

| Methd:⟦G⊢Norm s0 −body G D sig−≻v−n→ s1⟧ ⟹
                            G⊢Norm s0 −Methd D sig−≻v−Suc n→ s1

| Body: ⟦G⊢Norm s0−Init D−n→ s1; G⊢s1 −c−n→ s2;
        s3 = (if (∃ l. abrupt s2 = Some (Jump (Break l)) ∨
                       abrupt s2 = Some (Jump (Cont l)))
                then abupd (λ x. Some (Error CrossMethodJump)) s2
                else s2)⟧⟹
       G⊢Norm s0 −Body D c
        −≻the (locals (store s2) Result)−n→abupd (absorb Ret) s3
```

— evaluation of expression lists

| *Nil*:

$$G\vdash Norm\ s0\ -[]\doteq\succ[]-n\rightarrow\ Norm\ s0$$

| *Cons*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -e\ -\succ\ v\ -n\rightarrow\ s1;\\ G\vdash\quad s1\ -es\doteq\succ vs-n\rightarrow\ s2]\!]\Longrightarrow\\ \qquad G\vdash Norm\ s0\ -e\#es\doteq\succ v\#vs-n\rightarrow\ s2\end{array}$$

— execution of statements

| *Skip*:

$$G\vdash Norm\ s\ -Skip-n\rightarrow\ Norm\ s$$

| *Expr*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -e-\succ v-n\rightarrow\ s1]\!]\Longrightarrow\\ \qquad G\vdash Norm\ s0\ -Expr\ e-n\rightarrow\ s1\end{array}$$

| *Lab*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -c\ -n\rightarrow\ s1]\!]\Longrightarrow\\ \qquad G\vdash Norm\ s0\ -l\bullet\ c-n\rightarrow\ abupd\ (absorb\ l)\ s1\end{array}$$

| *Comp*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -c1\ -n\rightarrow\ s1;\\ G\vdash\quad s1\ -c2\ -n\rightarrow\ s2]\!]\Longrightarrow\\ \qquad G\vdash Norm\ s0\ -c1;;\ c2-n\rightarrow\ s2\end{array}$$

| *If*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -e-\succ b-n\rightarrow\ s1;\\ G\vdash\quad s1-(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)-n\rightarrow\ s2]\!]\Longrightarrow\\ \qquad G\vdash Norm\ s0\ -If(e)\ c1\ Else\ c2\ -n\rightarrow\ s2\end{array}$$

| *Loop*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -e-\succ b-n\rightarrow\ s1;\\ if\ the\text{-}Bool\ b\\ \quad then\ (G\vdash s1\ -c-n\rightarrow\ s2\ \wedge\\ \qquad G\vdash(abupd\ (absorb\ (Cont\ l))\ s2)\ -l\bullet\ While(e)\ c-n\rightarrow\ s3)\\ \quad else\ s3\ =\ s1]\!]\Longrightarrow\\ \qquad G\vdash Norm\ s0\ -l\bullet\ While(e)\ c-n\rightarrow\ s3\end{array}$$

| *Jmp*:

$$G\vdash Norm\ s\ -Jmp\ j-n\rightarrow\ (Some\ (Jump\ j),\ s)$$

| *Throw*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -e-\succ a'-n\rightarrow\ s1]\!]\Longrightarrow\\ \qquad G\vdash Norm\ s0\ -Throw\ e-n\rightarrow\ abupd\ (throw\ a')\ s1\end{array}$$

| *Try*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -c1-n\rightarrow\ s1;\ G\vdash s1\ -sxalloc\rightarrow\ s2;\\ if\ G,s2\vdash catch\ tn\ then\ G\vdash new\text{-}xcpt\text{-}var\ vn\ s2\ -c2-n\rightarrow\ s3\ else\ s3\ =\ s2]\!]\\ \Longrightarrow\\ \qquad G\vdash Norm\ s0\ -Try\ c1\ Catch(tn\ vn)\ c2-n\rightarrow\ s3\end{array}$$

| *Fin*:

$$\begin{array}{l}[\![G\vdash Norm\ s0\ -c1-n\rightarrow\ (x1,s1);\\ G\vdash Norm\ s1\ -c2-n\rightarrow\ s2;\\ s3=(if\ (\exists\ err.\ x1=Some\ (Error\ err))\\ \quad then\ (x1,s1)\\ \quad else\ abupd\ (abrupt\text{-}if\ (x1\neq None)\ x1)\ s2)]\!]\Longrightarrow\\ \quad G\vdash Norm\ s0\ -c1\ Finally\ c2-n\rightarrow\ s3\end{array}$$

| *Init*:

$$\begin{array}{l}[\![the\ (class\ G\ C)\ =\ c;\\ if\ inited\ C\ (globs\ s0)\ then\ s3\ =\ Norm\ s0\\ else\ (G\vdash Norm\ (init\text{-}class\text{-}obj\ G\ C\ s0)\\ \qquad -(if\ C\ =\ Object\ then\ Skip\ else\ Init\ (super\ c))-n\rightarrow\ s1\ \wedge\\ \quad G\vdash set\text{-}lvars\ empty\ s1\ -init\ c-n\rightarrow\ s2\ \wedge\\ \quad s3\ =\ restore\text{-}lvars\ s1\ s2)]\!]\\ \Longrightarrow\end{array}$$

$G\vdash Norm\ s0\ -Init\ C-n\rightarrow\ s3$

monos
if-bool-eq-conj

declare *split-if* [*split del*] *split-if-asm* [*split del*]
option.split [*split del*] *option.split-asm* [*split del*]
not-None-eq [*simp del*]
split-paired-All [*simp del*] *split-paired-Ex* [*simp del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss delloop split-all-tac*)) ⟫

inductive-cases *evaln-cases*: $G\vdash s\ -t\succ-n\rightarrow\ (v,\ s')$

inductive-cases *evaln-elim-cases*:
$G\vdash(Some\ xc,\ s)\ -t\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1r\ Skip\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (Jmp\ j)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (l\bullet\ c)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In3\ ([])\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In3\ (e\#es)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Lit\ w)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (UnOp\ unop\ e)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (BinOp\ binop\ e1\ e2)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In2\ (LVar\ vn)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Cast\ T\ e)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (e\ InstOf\ T)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Super)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Acc\ va)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Expr\ e)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (c1;;\ c2)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (Methd\ C\ sig)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (Body\ D\ c)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (e0\ ?\ e1\ :\ e2)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (If(e)\ c1\ Else\ c2)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (l\bullet\ While(e)\ c)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (c1\ Finally\ c2)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (Throw\ e)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1l\ (NewC\ C)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (New\ T[e])\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (Ass\ va\ e)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Try\ c1\ Catch(tn\ vn)\ c2)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In2\ (\{accC,statDeclC,stat\}e..fn)\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In2\ (e1.[e2])\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1l\ (\{accC,statT,mode\}e\cdot mn(\{pT\}p))\ \succ-n\rightarrow\ (v,\ s')$
$G\vdash Norm\ s\ -In1r\ (Init\ C)\ \succ-n\rightarrow\ (x,\ s')$
$G\vdash Norm\ s\ -In1r\ (Init\ C)\ \succ-n\rightarrow\ (x,\ s')$

declare *split-if* [*split*] *split-if-asm* [*split*]
option.split [*split*] *option.split-asm* [*split*]
not-None-eq [*simp*]
split-paired-All [*simp*] *split-paired-Ex* [*simp*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss addloop* (*split-all-tac*, *split-all-tac*))) ⟫

lemma *evaln-Inj-elim*: $G\vdash s\ -t\succ-n\rightarrow\ (w,s') \Longrightarrow case\ t\ of\ In1\ ec \Rightarrow$
$(case\ ec\ of\ Inl\ e \Rightarrow (\exists\, v.\ w = In1\ v)\ |\ Inr\ c \Rightarrow w = \diamondsuit)$
$|\ In2\ e \Rightarrow (\exists\, v.\ w = In2\ v)\ |\ In3\ e \Rightarrow (\exists\, v.\ w = In3\ v)$
apply (*erule evaln-cases* , *auto*)
apply (*induct-tac t*)

apply (*induct-tac a*)
apply *auto*
done

The following simplification procedures set up the proper injections of terms and their corresponding values in the evaluation relation: E.g. an expression (injection *In1l* into terms) always evaluates to ordinary values (injection *In1* into generalised values *vals*).

lemma *evaln-expr-eq*: $G\vdash s\ -In1l\ t\succ-n\rightarrow\ (w,\ s') = (\exists\, v.\ w{=}In1\ v \land G\vdash s\ -t-\succ v\ -n\rightarrow\ s')$
by (*auto*, *frule evaln-Inj-elim*, *auto*)

lemma *evaln-var-eq*: $G\vdash s\ -In2\ t\succ-n\rightarrow\ (w,\ s') = (\exists\, vf.\ w{=}In2\ vf \land G\vdash s\ -t{=}\succ vf{-}n\rightarrow\ s')$
by (*auto*, *frule evaln-Inj-elim*, *auto*)

lemma *evaln-exprs-eq*: $G\vdash s\ -In3\ t\succ-n\rightarrow\ (w,\ s') = (\exists\, vs.\ w{=}In3\ vs \land G\vdash s\ -t\doteq\succ vs{-}n\rightarrow\ s')$
by (*auto*, *frule evaln-Inj-elim*, *auto*)

lemma *evaln-stmt-eq*: $G\vdash s\ -In1r\ t\succ-n\rightarrow\ (w,\ s') = (w{=}\diamondsuit \land G\vdash s\ -t\ -n\rightarrow\ s')$
by (*auto*, *frule evaln-Inj-elim*, *auto*, *frule evaln-Inj-elim*, *auto*)

simproc-setup *evaln-expr* ($G\vdash s\ -In1l\ t\succ-n\rightarrow\ (w,\ s')$) = ⟪
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-expr-eq*})) ⟫

simproc-setup *evaln-var* ($G\vdash s\ -In2\ t\succ-n\rightarrow\ (w,\ s')$) = ⟪
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-var-eq*})) ⟫

simproc-setup *evaln-exprs* ($G\vdash s\ -In3\ t\succ-n\rightarrow\ (w,\ s')$) = ⟪
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-exprs-eq*})) ⟫

simproc-setup *evaln-stmt* ($G\vdash s\ -In1r\ t\succ-n\rightarrow\ (w,\ s')$) = ⟪
fn - => fn - => fn ct =>
(*case Thm.term-of ct of*
(- $ - $ - $ - $ - $ (*Const* - $ -) $ -) => *NONE*
| - => *SOME* (*mk-meta-eq* @{*thm evaln-stmt-eq*})) ⟫

ML-setup ⟪ *bind-thms* (*evaln-AbruptIs*, *sum3-instantiate* @{*thm evaln.Abrupt*}) ⟫
declare *evaln-AbruptIs* [*intro*!]

lemma *evaln-Callee*: $G\vdash Norm\ s{-}In1l\ (Callee\ l\ e)\succ-n\rightarrow\ (v,s') = False$
proof −
{ **fix** $s\ t\ v\ s'$
assume *eval*: $G\vdash s\ -t\succ-n\rightarrow\ (v,s')$ **and**
normal: *normal s* **and**
callee: $t{=}In1l\ (Callee\ l\ e)$
then have *False* **by** *induct auto*
}

then show *?thesis*
  by (*cases* s') *fastsimp*
qed

lemma *evaln-InsInitE*: $G \vdash Norm\ s -In1l\ (InsInitE\ c\ e) \succ -n \rightarrow (v,s') = False$
proof –
 { **fix** $s\ t\ v\ s'$
  assume *eval*: $G \vdash s\ -t \succ -n \rightarrow (v,s')$ **and**
   normal: *normal s* **and**
   callee: $t = In1l\ (InsInitE\ c\ e)$
  then have *False* **by** *induct auto*
 }
 then show *?thesis*
  by (*cases* s') *fastsimp*
qed

lemma *evaln-InsInitV*: $G \vdash Norm\ s -In2\ (InsInitV\ c\ w) \succ -n \rightarrow (v,s') = False$
proof –
 { **fix** $s\ t\ v\ s'$
  assume *eval*: $G \vdash s\ -t \succ -n \rightarrow (v,s')$ **and**
   normal: *normal s* **and**
   callee: $t = In2\ (InsInitV\ c\ w)$
  then have *False* **by** *induct auto*
 }
 then show *?thesis*
  by (*cases* s') *fastsimp*
qed

lemma *evaln-FinA*: $G \vdash Norm\ s -In1r\ (FinA\ a\ c) \succ -n \rightarrow (v,s') = False$
proof –
 { **fix** $s\ t\ v\ s'$
  assume *eval*: $G \vdash s\ -t \succ -n \rightarrow (v,s')$ **and**
   normal: *normal s* **and**
   callee: $t = In1r\ (FinA\ a\ c)$
  then have *False* **by** *induct auto*
 }
 then show *?thesis*
  by (*cases* s') *fastsimp*
qed

lemma *evaln-abrupt-lemma*: $G \vdash s\ -e \succ -n \rightarrow (v,s') \Longrightarrow$
$fst\ s = Some\ xc \longrightarrow s' = s \wedge v = arbitrary3\ e$
apply (*erule evaln-cases* , *auto*)
done

lemma *evaln-abrupt*:
$\bigwedge s'.\ G \vdash (Some\ xc,s)\ -e \succ -n \rightarrow (w,s') = (s' = (Some\ xc,s) \wedge$
 $w = arbitrary3\ e \wedge G \vdash (Some\ xc,s)\ -e \succ -n \rightarrow (arbitrary3\ e,(Some\ xc,s)))$
apply *auto*
apply (*frule evaln-abrupt-lemma*, *auto*)+
done

simproc-setup *evaln-abrupt* $(G \vdash (Some\ xc,s)\ -e \succ -n \rightarrow (w,s')) = \langle\langle$
 fn - => fn - => fn ct =>

```
  (case Thm.term-of ct of
    (- $ - $ - $ - $ - $ - $ (Const (@{const-name Pair}, -) $ (Const (@{const-name Some},-) $ -)$ -))
      => NONE
  | - => SOME (mk-meta-eq @{thm evaln-abrupt}))
⟫
```

lemma *evaln-LitI*: $G \vdash s\ -Lit\ v -\succ (if\ normal\ s\ then\ v\ else\ arbitrary) -n\rightarrow\ s$
apply (*case-tac s*, *case-tac a = None*)
by (*auto intro*!: *evaln.Lit*)

lemma *CondI*:
$\bigwedge s1.\ [\![G \vdash s\ -e -\succ b -n\rightarrow\ s1;\ G \vdash s1\ -(if\ the\text{-}Bool\ b\ then\ e1\ else\ e2) -\succ v -n\rightarrow\ s2]\!] \Longrightarrow$
$G \vdash s\ -e\ ?\ e1 : e2 -\succ (if\ normal\ s1\ then\ v\ else\ arbitrary) -n\rightarrow\ s2$
apply (*case-tac s*, *case-tac a = None*)
by (*auto intro*!: *evaln.Cond*)

lemma *evaln-SkipI* [*intro*!]: $G \vdash s\ -Skip -n\rightarrow\ s$
apply (*case-tac s*, *case-tac a = None*)
by (*auto intro*!: *evaln.Skip*)

lemma *evaln-ExprI*: $G \vdash s\ -e -\succ v -n\rightarrow\ s' \Longrightarrow G \vdash s\ -Expr\ e -n\rightarrow\ s'$
apply (*case-tac s*, *case-tac a = None*)
by (*auto intro*!: *evaln.Expr*)

lemma *evaln-CompI*: $[\![G \vdash s\ -c1 -n\rightarrow\ s1;\ G \vdash s1\ -c2 -n\rightarrow\ s2]\!] \Longrightarrow G \vdash s\ -c1;;\ c2 -n\rightarrow\ s2$
apply (*case-tac s*, *case-tac a = None*)
by (*auto intro*!: *evaln.Comp*)

lemma *evaln-IfI*:
$[\![G \vdash s\ -e -\succ v -n\rightarrow\ s1;\ G \vdash s1\ -(if\ the\text{-}Bool\ v\ then\ c1\ else\ c2) -n\rightarrow\ s2]\!] \Longrightarrow$
$G \vdash s\ -If(e)\ c1\ Else\ c2 -n\rightarrow\ s2$
apply (*case-tac s*, *case-tac a = None*)
by (*auto intro*!: *evaln.If*)

lemma *evaln-SkipD* [*dest*!]: $G \vdash s\ -Skip -n\rightarrow\ s' \Longrightarrow s' = s$
by (*erule evaln-cases*, *auto*)

lemma *evaln-Skip-eq* [*simp*]: $G \vdash s\ -Skip -n\rightarrow\ s' = (s = s')$
apply *auto*
done

evaln implies eval

lemma *evaln-eval*:
  assumes *evaln*: $G \vdash s0\ -t \succ -n\rightarrow\ (v,s1)$
  shows $G \vdash s0\ -t \succ\rightarrow\ (v,s1)$
using *evaln*
proof (*induct*)
  case (*Loop s0 e b n s1 c s2 l s3*)
  note ⟨$G \vdash Norm\ s0\ -e -\succ b\rightarrow\ s1$⟩
  moreover

have *if the-Bool b*
then $(G\vdash s1\ -c\rightarrow\ s2) \wedge$
$G\vdash abupd\ (absorb\ (Cont\ l))\ s2\ -l\bullet\ While(e)\ c\rightarrow\ s3$
else $s3 = s1$
using *Loop.hyps* **by** *simp*
ultimately show *?case* **by** (*rule eval.Loop*)
next
case (*Try s0 c1 n s1 s2 C vn c2 s3*)
note ‹$G\vdash Norm\ s0\ -c1\rightarrow\ s1$›
moreover
note ‹$G\vdash s1\ -sxalloc\rightarrow\ s2$›
moreover
have *if* $G,s2\vdash catch\ C$ *then* $G\vdash new\text{-}xcpt\text{-}var\ vn\ s2\ -c2\rightarrow\ s3$ *else* $s3 = s2$
using *Try.hyps* **by** *simp*
ultimately show *?case* **by** (*rule eval.Try*)
next
case (*Init C c s0 s3 n s1 s2*)
note ‹*the* (*class G C*) = *c*›
moreover
have *if inited C* (*globs s0*)
then $s3 = Norm\ s0$
else $G\vdash Norm\ ((init\text{-}class\text{-}obj\ G\ C)\ s0)$
$-(if\ C = Object\ then\ Skip\ else\ Init\ (super\ c))\rightarrow\ s1 \wedge$
$G\vdash(set\text{-}lvars\ empty)\ s1\ -init\ c\rightarrow\ s2 \wedge$
$s3 = (set\text{-}lvars\ (locals\ (store\ s1)))\ s2$
using *Init.hyps* **by** *simp*
ultimately show *?case* **by** (*rule eval.Init*)
qed (*rule eval.intros*,(*assumption+* | *assumption?*))+

lemma *Suc-le-D-lemma*: $[\![Suc\ n <= m';\ (\bigwedge m.\ n <= m \Longrightarrow P\ (Suc\ m))\ ]\!] \Longrightarrow P\ m'$
apply (*frule Suc-le-D*)
apply *fast*
done

lemma *evaln-nonstrict* [*rule-format* (*no-asm*), *elim*]:
$G\vdash s\ -t\succ-n\rightarrow\ (w,\ s') \Longrightarrow \forall m.\ n\le m \longrightarrow G\vdash s\ -t\succ-m\rightarrow\ (w,\ s')$
apply (*erule evaln.induct*)
apply (*tactic* ⟪ *ALLGOALS* (*EVERY'*[*strip-tac*, *TRY o etac* (*thm Suc-le-D-lemma*),
REPEAT o smp-tac 1,
resolve-tac (*thms evaln.intros*) *THEN-ALL-NEW TRY o atac*]) ⟫)

apply (*auto split del*: *split-if*)
done

lemmas *evaln-nonstrict-Suc* = *evaln-nonstrict* [*OF - le-refl* [*THEN le-SucI*]]

lemma *evaln-max2*: $[\![G\vdash s1\ -t1\succ-n1\rightarrow\ (w1,\ s1');\ G\vdash s2\ -t2\succ-n2\rightarrow\ (w2,\ s2')]\!] \Longrightarrow$
$G\vdash s1\ -t1\succ-max\ n1\ n2\rightarrow\ (w1,\ s1') \wedge G\vdash s2\ -t2\succ-max\ n1\ n2\rightarrow\ (w2,\ s2')$
by (*fast intro*: *le-maxI1 le-maxI2*)

corollary *evaln-max2E* [*consumes 2*]:
$[\![G\vdash s1\ -t1\succ-n1\rightarrow\ (w1,\ s1');\ G\vdash s2\ -t2\succ-n2\rightarrow\ (w2,\ s2');$
$[\![G\vdash s1\ -t1\succ-max\ n1\ n2\rightarrow\ (w1,\ s1');G\vdash s2\ -t2\succ-max\ n1\ n2\rightarrow\ (w2,\ s2')\ ]\!] \Longrightarrow P\ ]\!] \Longrightarrow P$
by (*drule* (*1*) *evaln-max2*) *simp*

lemma *evaln-max3*:
$[\![G\vdash s1 \ -t1\succ-n1\to (w1,\ s1'); G\vdash s2 \ -t2\succ-n2\to (w2,\ s2'); G\vdash s3 \ -t3\succ-n3\to (w3,\ s3')]\!] \Longrightarrow$
$G\vdash s1 \ -t1\succ-max\ (max\ n1\ n2)\ n3\to (w1,\ s1') \wedge$
$G\vdash s2 \ -t2\succ-max\ (max\ n1\ n2)\ n3\to (w2,\ s2') \wedge$
$G\vdash s3 \ -t3\succ-max\ (max\ n1\ n2)\ n3\to (w3,\ s3')$
apply (*drule* (*1*) *evaln-max2*, *erule thin-rl*)
apply (*fast intro*!: *le-maxI1 le-maxI2*)
done

corollary *evaln-max3E*:
$[\![G\vdash s1 \ -t1\succ-n1\to (w1,\ s1'); G\vdash s2 \ -t2\succ-n2\to (w2,\ s2'); G\vdash s3 \ -t3\succ-n3\to (w3,\ s3');$
$[\![G\vdash s1 \ -t1\succ-max\ (max\ n1\ n2)\ n3\to (w1,\ s1');$
$G\vdash s2 \ -t2\succ-max\ (max\ n1\ n2)\ n3\to (w2,\ s2');$
$G\vdash s3 \ -t3\succ-max\ (max\ n1\ n2)\ n3\to (w3,\ s3')$
$]\!] \Longrightarrow P$
$]\!] \Longrightarrow P$
by (*drule* (*2*) *evaln-max3*) *simp*

lemma *le-max3I1*: $(n2::nat) \le max\ n1\ (max\ n2\ n3)$
proof −
  have $n2 \le max\ n2\ n3$
    by (*rule le-maxI1*)
  also
  have $max\ n2\ n3 \le max\ n1\ (max\ n2\ n3)$
    by (*rule le-maxI2*)
  finally
  show *?thesis* **.**
qed

lemma *le-max3I2*: $(n3::nat) \le max\ n1\ (max\ n2\ n3)$
proof −
  have $n3 \le max\ n2\ n3$
    by (*rule le-maxI2*)
  also
  have $max\ n2\ n3 \le max\ n1\ (max\ n2\ n3)$
    by (*rule le-maxI2*)
  finally
  show *?thesis* **.**
qed

declare [[*simproc del*: *wt-expr wt-var wt-exprs wt-stmt*]]

eval implies evaln

lemma *eval-evaln*:
  assumes *eval*: $G\vdash s0 \ -t\succ\to (v,s1)$
  shows $\exists n.\ G\vdash s0 \ -t\succ-n\to (v,s1)$
using *eval*
proof (*induct*)
  case (*Abrupt xc s t*)
  obtain *n* **where**
    $G\vdash(Some\ xc,\ s) \ -t\succ-n\to (arbitrary3\ t,\ (Some\ xc,\ s))$
    by (*iprover intro*: *evaln.Abrupt*)
  then show *?case* **..**
next

```
  case Skip
  show ?case by (blast intro: evaln.Skip)
next
  case (Expr s0 e v s1)
  then obtain n where
    G⊢Norm s0 −e−≻v−n→ s1
    by (iprover)
  then have G⊢Norm s0 −Expr e−n→ s1
    by (rule evaln.Expr)
  then show ?case ..
next
  case (Lab s0 c s1 l)
  then obtain n where
    G⊢Norm s0 −c−n→ s1
    by (iprover)
  then have G⊢Norm s0 −l· c−n→ abupd (absorb l) s1
    by (rule evaln.Lab)
  then show ?case ..
next
  case (Comp s0 c1 s1 c2 s2)
  then obtain n1 n2 where
    G⊢Norm s0 −c1−n1→ s1
    G⊢s1 −c2−n2→ s2
    by (iprover)
  then have G⊢Norm s0 −c1;; c2−max n1 n2→ s2
    by (blast intro: evaln.Comp dest: evaln-max2 )
  then show ?case ..
next
  case (If s0 e b s1 c1 c2 s2)
  then obtain n1 n2 where
    G⊢Norm s0 −e−≻b−n1→ s1
    G⊢s1 −(if the-Bool b then c1 else c2)−n2→ s2
    by (iprover)
  then have G⊢Norm s0 −If(e) c1 Else c2−max n1 n2→ s2
    by (blast intro: evaln.If dest: evaln-max2)
  then show ?case ..
next
  case (Loop s0 e b s1 c s2 l s3)
  from Loop.hyps obtain n1 where
    G⊢Norm s0 −e−≻b−n1→ s1
    by (iprover)
  moreover from Loop.hyps obtain n2 where
    if the-Bool b
        then (G⊢s1 −c−n2→ s2 ∧
              G⊢(abupd (absorb (Cont l)) s2)−l· While(e) c−n2→ s3)
        else s3 = s1
    by simp (iprover intro: evaln-nonstrict le-maxI1 le-maxI2)
  ultimately
  have G⊢Norm s0 −l· While(e) c−max n1 n2→ s3
    apply −
    apply (rule evaln.Loop)
    apply   (iprover intro: evaln-nonstrict intro: le-maxI1)

    apply   (auto intro: evaln-nonstrict intro: le-maxI2)
    done
  then show ?case ..
next
  case (Jmp s j)
  have G⊢Norm s −Jmp j−n→ (Some (Jump j), s)
```

```
    by (rule evaln.Jmp)
  then show ?case ..
next
  case (Throw s0 e a s1)
  then obtain n where
    G⊢Norm s0 −e−≻a−n→ s1
    by (iprover)
  then have G⊢Norm s0 −Throw e−n→ abupd (throw a) s1
    by (rule evaln.Throw)
  then show ?case ..
next
  case (Try s0 c1 s1 s2 catchC vn c2 s3)
  from Try.hyps obtain n1 where
    G⊢Norm s0 −c1−n1→ s1
    by (iprover)
  moreover
  note sxalloc = ‹G⊢s1 −sxalloc→ s2›
  moreover
  from Try.hyps obtain n2 where
    if G,s2⊢catch catchC then G⊢new-xcpt-var vn s2 −c2−n2→ s3 else s3 = s2
    by fastsimp
  ultimately
  have G⊢Norm s0 −Try c1 Catch(catchC vn) c2−max n1 n2→ s3
    by (auto intro!: evaln.Try le-maxI1 le-maxI2)
  then show ?case ..
next
  case (Fin s0 c1 x1 s1 c2 s2 s3)
  from Fin obtain n1 n2 where
    G⊢Norm s0 −c1−n1→ (x1, s1)
    G⊢Norm s1 −c2−n2→ s2
    by iprover
  moreover
  note s3 = ‹s3 = (if ∃ err. x1 = Some (Error err)
                   then (x1, s1)
                   else abupd (abrupt-if (x1 ≠ None) x1) s2)›
  ultimately
  have
    G⊢Norm s0 −c1 Finally c2−max n1 n2→ s3
    by (blast intro: evaln.Fin dest: evaln-max2)
  then show ?case ..
next
  case (Init C c s0 s3 s1 s2)
  note cls = ‹the (class G C) = c›
  moreover from Init.hyps obtain n where
      if inited C (globs s0) then s3 = Norm s0
       else (G⊢Norm (init-class-obj G C s0)
             −(if C = Object then Skip else Init (super c))−n→ s1 ∧
                  G⊢set-lvars empty s1 −init c−n→ s2 ∧
                  s3 = restore-lvars s1 s2)
    by (auto intro: evaln-nonstrict le-maxI1 le-maxI2)
  ultimately have G⊢Norm s0 −Init C−n→ s3
    by (rule evaln.Init)
  then show ?case ..
next
  case (NewC s0 C s1 a s2)
  then obtain n where
    G⊢Norm s0 −Init C−n→ s1
    by (iprover)
  with NewC
```

have $G\vdash Norm\ s0\ -NewC\ C-\succ Addr\ a-n\rightarrow\ s2$
  by (*iprover intro*: *evaln.NewC*)
then show *?case* **..**
next
case (*NewA s0 T s1 e i s2 a s3*)
then obtain *n1 n2* **where**
  $G\vdash Norm\ s0\ -init\text{-}comp\text{-}ty\ T-n1\rightarrow\ s1$
  $G\vdash s1\ -e-\succ i-n2\rightarrow\ s2$
  by (*iprover*)
moreover
note ‹$G\vdash abupd\ (check\text{-}neg\ i)\ s2\ -halloc\ Arr\ T\ (the\text{-}Intg\ i)\succ a\rightarrow\ s3$›
ultimately
have $G\vdash Norm\ s0\ -New\ T[e]-\succ Addr\ a-max\ n1\ n2\rightarrow\ s3$
  by (*blast intro*: *evaln.NewA dest*: *evaln-max2*)
then show *?case* **..**
next
case (*Cast s0 e v s1 s2 castT*)
then obtain *n* **where**
  $G\vdash Norm\ s0\ -e-\succ v-n\rightarrow\ s1$
  by (*iprover*)
moreover
note ‹$s2 = abupd\ (raise\text{-}if\ (\neg\ G,snd\ s1\vdash v\ fits\ castT)\ ClassCast)\ s1$›
ultimately
have $G\vdash Norm\ s0\ -Cast\ castT\ e-\succ v-n\rightarrow\ s2$
  by (*rule evaln.Cast*)
then show *?case* **..**
next
case (*Inst s0 e v s1 b T*)
then obtain *n* **where**
  $G\vdash Norm\ s0\ -e-\succ v-n\rightarrow\ s1$
  by (*iprover*)
moreover
note ‹$b = (v \neq Null \wedge G,snd\ s1\vdash v\ fits\ RefT\ T)$›
ultimately
have $G\vdash Norm\ s0\ -e\ InstOf\ T-\succ Bool\ b-n\rightarrow\ s1$
  by (*rule evaln.Inst*)
then show *?case* **..**
next
case (*Lit s v*)
have $G\vdash Norm\ s\ -Lit\ v-\succ v-n\rightarrow\ Norm\ s$
  by (*rule evaln.Lit*)
then show *?case* **..**
next
case (*UnOp s0 e v s1 unop*)
then obtain *n* **where**
  $G\vdash Norm\ s0\ -e-\succ v-n\rightarrow\ s1$
  by (*iprover*)
hence $G\vdash Norm\ s0\ -UnOp\ unop\ e-\succ eval\text{-}unop\ unop\ v-n\rightarrow\ s1$
  by (*rule evaln.UnOp*)
then show *?case* **..**
next
case (*BinOp s0 e1 v1 s1 binop e2 v2 s2*)
then obtain *n1 n2* **where**
  $G\vdash Norm\ s0\ -e1-\succ v1-n1\rightarrow\ s1$
  $G\vdash s1\ -(if\ need\text{-}second\text{-}arg\ binop\ v1\ then\ In1l\ e2$
    $else\ In1r\ Skip)\succ-n2\rightarrow\ (In1\ v2,\ s2)$
  by (*iprover*)
hence $G\vdash Norm\ s0\ -BinOp\ binop\ e1\ e2-\succ(eval\text{-}binop\ binop\ v1\ v2)-max\ n1\ n2$
  $\rightarrow\ s2$

```
    by (blast intro!: evaln.BinOp dest: evaln-max2)
  then show ?case ..
next
  case (Super s )
  have G⊢Norm s −Super−≻val-this s−n→ Norm s
    by (rule evaln.Super)
  then show ?case ..
next
  case (Acc s0 va v f s1)
  then obtain n where
    G⊢Norm s0 −va=≻(v, f)−n→ s1
    by (iprover)
  then
  have G⊢Norm s0 −Acc va−≻v−n→ s1
    by (rule evaln.Acc)
  then show ?case ..
next
  case (Ass s0 var w f s1 e v s2)
  then obtain n1 n2 where
    G⊢Norm s0 −var=≻(w, f)−n1→ s1
    G⊢s1 −e−≻v−n2→ s2
    by (iprover)
  then
  have G⊢Norm s0 −var:=e−≻v−max n1 n2→ assign f v s2
    by (blast intro: evaln.Ass dest: evaln-max2)
  then show ?case ..
next
  case (Cond s0 e0 b s1 e1 e2 v s2)
  then obtain n1 n2 where
    G⊢Norm s0 −e0−≻b−n1→ s1
    G⊢s1 −(if the-Bool b then e1 else e2)−≻v−n2→ s2
    by (iprover)
  then
  have G⊢Norm s0 −e0 ? e1 : e2−≻v−max n1 n2→ s2
    by (blast intro: evaln.Cond dest: evaln-max2)
  then show ?case ..
next
  case (Call s0 e a' s1 args vs s2 invDeclC mode statT mn pTs' s3 s3' accC' v s4)
  then obtain n1 n2 where
    G⊢Norm s0 −e−≻a'−n1→ s1
    G⊢s1 −args≐≻vs−n2→ s2
    by iprover
  moreover
  note ‹invDeclC = invocation-declclass G mode (store s2) a' statT
                  (|name=mn,parTs=pTs'|)›
  moreover
  note ‹s3 = init-lvars G invDeclC (|name=mn,parTs=pTs'|) mode a' vs s2›
  moreover
  note ‹s3'=check-method-access G accC' statT mode (|name=mn,parTs=pTs'|) a' s3›
  moreover
  from Call.hyps
  obtain m where
    G⊢s3' −Methd invDeclC (|name=mn, parTs=pTs'|)−≻v−m→ s4
    by iprover
  ultimately
  have G⊢Norm s0 −{accC',statT,mode}e·mn( {pTs'}args)−≻v−max n1 (max n2 m)→
          (set-lvars (locals (store s2))) s4
    by (auto intro!: evaln.Call le-maxI1 le-max3I1 le-max3I2)
  thus ?case ..
```

```
next
  case (Methd s0 D sig v s1)
  then obtain n where
    G⊢Norm s0 −body G D sig−≻v−n→ s1
    by iprover
  then have G⊢Norm s0 −Methd D sig−≻v−Suc n→ s1
    by (rule evaln.Methd)
  then show ?case ..
next
  case (Body s0 D s1 c s2 s3)
  from Body.hyps obtain n1 n2 where
    evaln-init: G⊢Norm s0 −Init D−n1→ s1 and
    evaln-c: G⊢s1 −c−n2→ s2
    by (iprover)
  moreover
  note ‹s3 = (if ∃ l. fst s2 = Some (Jump (Break l)) ∨
                    fst s2 = Some (Jump (Cont l))
              then abupd (λx. Some (Error CrossMethodJump)) s2
              else s2)›
  ultimately
  have
     G⊢Norm s0 −Body D c−≻the (locals (store s2) Result)−max n1 n2
       → abupd (absorb Ret) s3
    by (iprover intro: evaln.Body dest: evaln-max2)
  then show ?case ..
next
  case (LVar s vn )
  obtain n where
    G⊢Norm s −LVar vn=≻lvar vn s−n→ Norm s
    by (iprover intro: evaln.LVar)
  then show ?case ..
next
  case (FVar s0 statDeclC s1 e a s2 v s2' stat fn s3 accC)
  then obtain n1 n2 where
    G⊢Norm s0 −Init statDeclC−n1→ s1
    G⊢s1 −e−≻a−n2→ s2
    by iprover
  moreover
  note ‹s3 = check-field-access G accC statDeclC fn stat a s2'›
    and ‹(v, s2') = fvar statDeclC stat fn a s2›
  ultimately
  have G⊢Norm s0 −{accC,statDeclC,stat}e..fn=≻v−max n1 n2→ s3
    by (iprover intro: evaln.FVar dest: evaln-max2)
  then show ?case ..
next
  case (AVar s0 e1 a s1 e2 i s2 v s2')
  then obtain n1 n2 where
    G⊢Norm s0 −e1−≻a−n1→ s1
    G⊢s1 −e2−≻i−n2→ s2
    by iprover
  moreover
  note ‹(v, s2') = avar G i a s2›
  ultimately
  have G⊢Norm s0 −e1.[e2]=≻v−max n1 n2→ s2'
    by (blast intro!: evaln.AVar dest: evaln-max2)
  then show ?case ..
next
  case (Nil s0)
  show ?case by (iprover intro: evaln.Nil)
```

next
  case (*Cons s0 e v s1 es vs s2*)
  then obtain *n1 n2* **where**
    $G \vdash Norm\ s0\ -e-\succ v-n1\rightarrow\ s1$
    $G \vdash s1\ -es\dot{=}\succ vs-n2\rightarrow\ s2$
    by *iprover*
  then
  have $G \vdash Norm\ s0\ -e\ \#\ es\dot{=}\succ v\ \#\ vs-max\ n1\ n2\rightarrow\ s2$
    by (*blast intro*!: *evaln.Cons dest*: *evaln-max2*)
  then show *?case* **..**
qed

end

Chapter 21

Trans

```
theory Trans imports Evaln begin

constdefs groundVar:: var ⇒ bool
groundVar v ≡ (case v of
                  LVar ln ⇒ True
                | {accC,statDeclC,stat}e..fn ⇒ ∃ a. e=Lit a
                | e1.[e2] ⇒ ∃ a i. e1= Lit a ∧ e2 = Lit i
                | InsInitV c v ⇒ False)


lemma groundVar-cases [consumes 1, case-names LVar FVar AVar]:
  assumes ground: groundVar v and
          LVar: ⋀ ln. ⟦v=LVar ln⟧ ⟹ P and
          FVar: ⋀ accC statDeclC stat a fn.
                  ⟦v={accC,statDeclC,stat}(Lit a)..fn⟧ ⟹ P and
          AVar: ⋀ a i. ⟦v=(Lit a).[Lit i]⟧ ⟹ P
  shows P
proof −
  from ground LVar FVar AVar
  show ?thesis
    apply (cases v)
    apply (simp add: groundVar-def)
    apply (simp add: groundVar-def,blast)
    apply (simp add: groundVar-def,blast)
    apply (simp add: groundVar-def)
    done
qed

constdefs groundExprs:: expr list ⇒ bool
groundExprs es ≡ list-all (λ e. ∃ v. e=Lit v) es

consts the-val:: expr ⇒ val
primrec
the-val (Lit v) = v

consts the-var:: prog ⇒ state ⇒ var ⇒ (vvar × state)
primrec
the-var G s (LVar ln)                    =(lvar ln (store s),s)
the-var-FVar-def:
the-var G s ({accC,statDeclC,stat}a..fn) =fvar statDeclC stat fn (the-val a) s
the-var-AVar-def:
the-var G s(a.[i])                       =avar G (the-val i) (the-val a) s
```

lemma *the-var-FVar-simp*[*simp*]:
the-var G s ({*accC*,*statDeclC*,*stat*}(*Lit a*)..*fn*) = *fvar statDeclC stat fn a s*
by (*simp*)
declare *the-var-FVar-def* [*simp del*]

lemma *the-var-AVar-simp*:
the-var G s ((*Lit a*).[*Lit i*]) = *avar G i a s*
by (*simp*)
declare *the-var-AVar-def* [*simp del*]

syntax (*xsymbols*)
  Ref :: *loc* ⇒ *expr*
  SKIP :: *expr*

translations
  Ref a == *Lit* (*Addr a*)
  SKIP == *Lit Unit*

inductive
  step :: [*prog*,*term* × *state*,*term* × *state*] ⇒ *bool* (-⊢- ↦1 -[*61*,*82*,*82*] *81*)
  for *G* :: *prog*
where

Abrupt: ⟦∀ *v*. *t* ≠ ⟨*Lit v*⟩;
∀ *t*. *t* ≠ ⟨*l*• *Skip*⟩;
∀ *C vn c*. *t* ≠ ⟨*Try Skip Catch*(*C vn*) *c*⟩;
∀ *x c*. *t* ≠ ⟨*Skip Finally c*⟩ ∧ *xc* ≠ *Xcpt x*;
∀ *a c*. *t* ≠ ⟨*FinA a c*⟩⟧
⟹
G⊢(*t*,*Some xc*,*s*) ↦*1* (⟨*Lit arbitrary*⟩,*Some xc*,*s*)

| *InsInitE*: ⟦*G*⊢(⟨*c*⟩,*Norm s*) ↦*1* (⟨*c'*⟩, *s'*)⟧
⟹
G⊢(⟨*InsInitE c e*⟩,*Norm s*) ↦*1* (⟨*InsInitE c' e*⟩, *s'*)

| *NewC*: *G*⊢(⟨*NewC C*⟩,*Norm s*) ↦*1* (⟨*InsInitE* (*Init C*) (*NewC C*)⟩, *Norm s*)
| *NewCInited*: ⟦*G*⊢ *Norm s* −*halloc* (*CInst C*)≻*a*→ *s'*⟧
⟹
G⊢(⟨*InsInitE Skip* (*NewC C*)⟩,*Norm s*) ↦*1* (⟨*Ref a*⟩, *s'*)

| *NewA*:
G⊢(⟨*New T*[*e*]⟩,*Norm s*) ↦*1* (⟨*InsInitE* (*init-comp-ty T*) (*New T*[*e*])⟩,*Norm s*)
| *InsInitNewAIdx*:
⟦*G*⊢(⟨*e*⟩,*Norm s*) ↦*1* (⟨*e'*⟩, *s'*)⟧
⟹
G⊢(⟨*InsInitE Skip* (*New T*[*e*])⟩,*Norm s*) ↦*1* (⟨*InsInitE Skip* (*New T*[*e'*])⟩,*s'*)

| *InsInitNewA*:
  $[\![G\vdash abupd\ (check\text{-}neg\ i)\ (Norm\ s)\ -halloc\ (Arr\ T\ (the\text{-}Intg\ i))\succ a\rightarrow\ s'\ ]\!]$
  $\Longrightarrow$
  $G\vdash(\langle InsInitE\ Skip\ (New\ T[Lit\ i])\rangle,Norm\ s)\ \mapsto 1\ (\langle Ref\ a\rangle,s')$

| *CastE*:
  $[\![G\vdash(\langle e\rangle,Norm\ s)\ \mapsto 1\ (\langle e'\rangle,s')]\!]$
  $\Longrightarrow$
  $G\vdash(\langle Cast\ T\ e\rangle,None,s)\ \mapsto 1\ (\langle Cast\ T\ e'\rangle,s')$
| *Cast*:
  $[\![s' = abupd\ (raise\text{-}if\ (\neg G,s\vdash v\ fits\ T)\ \ ClassCast)\ (Norm\ s)]\!]$
  $\Longrightarrow$
  $G\vdash(\langle Cast\ T\ (Lit\ v)\rangle,Norm\ s)\ \mapsto 1\ (\langle Lit\ v\rangle,s')$

| *InstE*: $[\![G\vdash(\langle e\rangle,Norm\ s)\ \mapsto 1\ (\langle e'::expr\rangle,s')]\!]$
  $\Longrightarrow$
  $G\vdash(\langle e\ InstOf\ T\rangle,Norm\ s)\ \mapsto 1\ (\langle e'\rangle,s')$
| *Inst*: $[\![b = (v\neq Null\ \wedge\ G,s\vdash v\ fits\ RefT\ T)]\!]$
  $\Longrightarrow$
  $G\vdash(\langle (Lit\ v)\ InstOf\ T\rangle,Norm\ s)\ \mapsto 1\ (\langle Lit\ (Bool\ b)\rangle,s')$

| *UnOpE*: $[\![G\vdash(\langle e\rangle,Norm\ s)\ \mapsto 1\ (\langle e'\rangle,s')\ ]\!]$
  $\Longrightarrow$
  $G\vdash(\langle UnOp\ unop\ e\rangle,Norm\ s)\ \mapsto 1\ (\langle UnOp\ unop\ e'\rangle,s')$
| *UnOp*: $G\vdash(\langle UnOp\ unop\ (Lit\ v)\rangle,Norm\ s)\ \mapsto 1\ (\langle Lit\ (eval\text{-}unop\ unop\ v)\rangle,Norm\ s)$

| *BinOpE1*: $[\![G\vdash(\langle e1\rangle,Norm\ s)\ \mapsto 1\ (\langle e1'\rangle,s')\ ]\!]$
  $\Longrightarrow$
  $G\vdash(\langle BinOp\ binop\ e1\ e2\rangle,Norm\ s)\ \mapsto 1\ (\langle BinOp\ binop\ e1'\ e2\rangle,s')$
| *BinOpE2*: $[\![need\text{-}second\text{-}arg\ binop\ v1;\ G\vdash(\langle e2\rangle,Norm\ s)\ \mapsto 1\ (\langle e2'\rangle,s')\ ]\!]$
  $\Longrightarrow$
  $G\vdash(\langle BinOp\ binop\ (Lit\ v1)\ e2\rangle,Norm\ s)$
  $\mapsto 1\ (\langle BinOp\ binop\ (Lit\ v1)\ e2'\rangle,s')$
| *BinOpTerm*: $[\![\neg\ need\text{-}second\text{-}arg\ binop\ v1]\!]$
  $\Longrightarrow$
  $G\vdash(\langle BinOp\ binop\ (Lit\ v1)\ e2\rangle,Norm\ s)$
  $\mapsto 1\ (\langle Lit\ v1\rangle,Norm\ s)$
| *BinOp*: $G\vdash(\langle BinOp\ binop\ (Lit\ v1)\ (Lit\ v2)\rangle,Norm\ s)$
  $\mapsto 1\ (\langle Lit\ (eval\text{-}binop\ binop\ v1\ v2)\rangle,Norm\ s)$

| *Super*: $G\vdash(\langle Super\rangle,Norm\ s)\ \mapsto 1\ (\langle Lit\ (val\text{-}this\ s)\rangle,Norm\ s)$

| *AccVA*: $[\![G\vdash(\langle va\rangle,Norm\ s)\ \mapsto 1\ (\langle va'\rangle,s')\ ]\!]$
  $\Longrightarrow$
  $G\vdash(\langle Acc\ va\rangle,Norm\ s)\ \mapsto 1\ (\langle Acc\ va'\rangle,s')$
| *Acc*: $[\![groundVar\ va;\ ((v,vf),s') = the\text{-}var\ G\ (Norm\ s)\ va]\!]$
  $\Longrightarrow$
  $G\vdash(\langle Acc\ va\rangle,Norm\ s)\ \mapsto 1\ (\langle Lit\ v\rangle,s')$

| *AssVA*: $[\![G\vdash(\langle va\rangle,Norm\ s)\ \mapsto 1\ (\langle va'\rangle,s')]\!]$
  $\Longrightarrow$

$$
\begin{array}{ll}
 & G\vdash(\langle va{:=}e\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle va'{:=}e\rangle,s') \\
\mid \mathit{AssE}\colon & [\![\mathit{groundVar}\ va;\ G\vdash(\langle e\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle e'\rangle,s')]\!] \\
 & \Longrightarrow \\
 & G\vdash(\langle va{:=}e\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle va{:=}e'\rangle,s') \\
\mid \mathit{Ass}\colon & [\![\mathit{groundVar}\ va;\ ((w,f),s') = \mathit{the\text{-}var}\ G\ (\mathit{Norm}\ s)\ va]\!] \\
 & \Longrightarrow \\
 & G\vdash(\langle va{:=}(\mathit{Lit}\ v)\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle \mathit{Lit}\ v\rangle,\mathit{assign}\ f\ v\ s')
\end{array}
$$

$$
\begin{array}{ll}
\mid \mathit{CondC}\colon & [\![G\vdash(\langle e0\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle e0'\rangle,s')]\!] \\
 & \Longrightarrow \\
 & G\vdash(\langle e0?\ e1{:}e2\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle e0'?\ e1{:}e2\rangle,s') \\
\mid \mathit{Cond}\colon & G\vdash(\langle \mathit{Lit}\ b?\ e1{:}e2\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle \mathit{if}\ \mathit{the\text{-}Bool}\ b\ \mathit{then}\ e1\ \mathit{else}\ e2\rangle,\mathit{Norm}\ s)
\end{array}
$$

$$
\begin{array}{ll}
\mid \mathit{CallTarget}\colon & [\![G\vdash(\langle e\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle e'\rangle,s')]\!] \\
 & \Longrightarrow \\
 & G\vdash(\langle\{accC,statT,mode\}e{\cdot}mn(\{pTs\}args)\rangle,\mathit{Norm}\ s) \\
 & \ \mapsto 1\ (\langle\{accC,statT,mode\}e'{\cdot}mn(\{pTs\}args)\rangle,s') \\
\mid \mathit{CallArgs}\colon & [\![G\vdash(\langle args\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle args'\rangle,s')]\!] \\
 & \Longrightarrow \\
 & G\vdash(\langle\{accC,statT,mode\}\mathit{Lit}\ a{\cdot}mn(\{pTs\}args)\rangle,\mathit{Norm}\ s) \\
 & \ \mapsto 1\ (\langle\{accC,statT,mode\}\mathit{Lit}\ a{\cdot}mn(\{pTs\}args')\rangle,s') \\
\mid \mathit{Call}\colon & [\![\mathit{groundExprs}\ args;\ vs = \mathit{map}\ \mathit{the\text{-}val}\ args; \\
 & \ D = \mathit{invocation\text{-}declclass}\ G\ mode\ s\ a\ statT\ (\!|name{=}mn,parTs{=}pTs|\!); \\
 & \ s'{=}\mathit{init\text{-}lvars}\ G\ D\ (\!|name{=}mn,parTs{=}pTs|\!)\ mode\ a'\ vs\ (\mathit{Norm}\ s)]\!] \\
 & \Longrightarrow \\
 & G\vdash(\langle\{accC,statT,mode\}\mathit{Lit}\ a{\cdot}mn(\{pTs\}args)\rangle,\mathit{Norm}\ s) \\
 & \ \mapsto 1\ (\langle \mathit{Callee}\ (\mathit{locals}\ s)\ (\mathit{Methd}\ D\ (\!|name{=}mn,parTs{=}pTs|\!))\rangle,s')
\end{array}
$$

$$
\begin{array}{ll}
\mid \mathit{Callee}\colon & [\![G\vdash(\langle e\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle e'{::}expr\rangle,s')]\!] \\
 & \Longrightarrow \\
 & G\vdash(\langle \mathit{Callee}\ \mathit{lcls\text{-}caller}\ e\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle e'\rangle,s')
\end{array}
$$

$$
\begin{array}{ll}
\mid \mathit{CalleeRet}\colon & G\vdash(\langle \mathit{Callee}\ \mathit{lcls\text{-}caller}\ (\mathit{Lit}\ v)\rangle,\mathit{Norm}\ s) \\
 & \ \mapsto 1\ (\langle \mathit{Lit}\ v\rangle,(\mathit{set\text{-}lvars}\ \mathit{lcls\text{-}caller}\ (\mathit{Norm}\ s)))
\end{array}
$$

$$
\mid \mathit{Methd}\colon G\vdash(\langle \mathit{Methd}\ D\ sig\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle \mathit{body}\ G\ D\ sig\rangle,\mathit{Norm}\ s)
$$

$$
\mid \mathit{Body}\colon G\vdash(\langle \mathit{Body}\ D\ c\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle \mathit{InsInitE}\ (\mathit{Init}\ D)\ (\mathit{Body}\ D\ c)\rangle,\mathit{Norm}\ s)
$$

$$
\begin{array}{l}
\mid \mathit{InsInitBody}\colon \\
\quad [\![G\vdash(\langle c\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle c'\rangle,s')]\!] \\
\quad \Longrightarrow \\
\quad G\vdash(\langle \mathit{InsInitE}\ \mathit{Skip}\ (\mathit{Body}\ D\ c)\rangle,\mathit{Norm}\ s) \mapsto 1(\langle \mathit{InsInitE}\ \mathit{Skip}\ (\mathit{Body}\ D\ c')\rangle,s') \\
\mid \mathit{InsInitBodyRet}\colon \\
\quad G\vdash(\langle \mathit{InsInitE}\ \mathit{Skip}\ (\mathit{Body}\ D\ \mathit{Skip})\rangle,\mathit{Norm}\ s) \\
\quad\ \mapsto 1\ (\langle \mathit{Lit}\ (\mathit{the}\ ((\mathit{locals}\ s)\ \mathit{Result}))\rangle,\mathit{abupd}\ (\mathit{absorb}\ \mathit{Ret})\ (\mathit{Norm}\ s))
\end{array}
$$

$$
\begin{array}{l}
\mid \mathit{FVar}\colon [\![\neg\ \mathit{inited}\ \mathit{statDeclC}\ (\mathit{globs}\ s)]\!] \\
\quad \Longrightarrow \\
\quad G\vdash(\langle\{accC,statDeclC,stat\}e..fn\rangle,\mathit{Norm}\ s) \\
\quad\ \mapsto 1\ (\langle \mathit{InsInitV}\ (\mathit{Init}\ statDeclC)\ (\{accC,statDeclC,stat\}e..fn)\rangle,\mathit{Norm}\ s) \\
\mid \mathit{InsInitFVarE}\colon \\
\quad [\![G\vdash(\langle e\rangle,\mathit{Norm}\ s) \mapsto 1\ (\langle e'\rangle,s')]\!] \\
\quad \Longrightarrow \\
\quad G\vdash(\langle \mathit{InsInitV}\ \mathit{Skip}\ (\{accC,statDeclC,stat\}e..fn)\rangle,\mathit{Norm}\ s) \\
\quad\ \mapsto 1\ (\langle \mathit{InsInitV}\ \mathit{Skip}\ (\{accC,statDeclC,stat\}e'..fn)\rangle,s')
\end{array}
$$

| *InsInitFVar*:
$G\vdash(\langle InsInitV\ Skip\ (\{accC,statDeclC,stat\}Lit\ a..fn)\rangle,Norm\ s)$
$\mapsto 1\ (\langle\{accC,statDeclC,stat\}Lit\ a..fn\rangle,Norm\ s)$
— Notice, that we do not have literal values for *vars*. The rules for accessing variables (*Acc*) and assigning to variables (*Ass*), test this with the predicate *groundVar*. After initialisation is done and the *FVar* is evaluated, we can't just throw away the *InsInitFVar* term and return a literal value, as in the cases of *New* or *NewC*. Instead we just return the evaluated *FVar* and test for initialisation in the rule *FVar*.

| *AVarE1*: $[\![G\vdash(\langle e1\rangle,Norm\ s)\mapsto 1\ (\langle e1'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle e1.[e2]\rangle,Norm\ s)\mapsto 1\ (\langle e1'.[e2]\rangle,s')$

| *AVarE2*: $G\vdash(\langle e2\rangle,Norm\ s)\mapsto 1\ (\langle e2'\rangle,s')$
$\Longrightarrow$
$G\vdash(\langle Lit\ a.[e2]\rangle,Norm\ s)\mapsto 1\ (\langle Lit\ a.[e2']\rangle,s')$

— *Nil* is fully evaluated

| *ConsHd*: $[\![G\vdash(\langle e{::}expr\rangle,Norm\ s)\mapsto 1\ (\langle e'{::}expr\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle e\#es\rangle,Norm\ s)\mapsto 1\ (\langle e'\#es\rangle,s')$

| *ConsTl*: $[\![G\vdash(\langle es\rangle,Norm\ s)\mapsto 1\ (\langle es'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle(Lit\ v)\#es\rangle,Norm\ s)\mapsto 1\ (\langle(Lit\ v)\#es'\rangle,s')$

| *Skip*: $G\vdash(\langle Skip\rangle,Norm\ s)\mapsto 1\ (\langle SKIP\rangle,Norm\ s)$

| *ExprE*: $[\![G\vdash(\langle e\rangle,Norm\ s)\mapsto 1\ (\langle e'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle Expr\ e\rangle,Norm\ s)\mapsto 1\ (\langle Expr\ e'\rangle,s')$
| *Expr*: $G\vdash(\langle Expr\ (Lit\ v)\rangle,Norm\ s)\mapsto 1\ (\langle Skip\rangle,Norm\ s)$

| *LabC*: $[\![G\vdash(\langle c\rangle,Norm\ s)\mapsto 1\ (\langle c'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle l\cdot\ c\rangle,Norm\ s)\mapsto 1\ (\langle l\cdot\ c'\rangle,s')$
| *Lab*: $G\vdash(\langle l\cdot\ Skip\rangle,s)\mapsto 1\ (\langle Skip\rangle,\ abupd\ (absorb\ l)\ s)$

| *CompC1*: $[\![G\vdash(\langle c1\rangle,Norm\ s)\mapsto 1\ (\langle c1'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle c1;;\ c2\rangle,Norm\ s)\mapsto 1\ (\langle c1';;\ c2\rangle,s')$

| *Comp*: $G\vdash(\langle Skip;;\ c2\rangle,Norm\ s)\mapsto 1\ (\langle c2\rangle,Norm\ s)$

| *IfE*: $[\![G\vdash(\langle e\rangle\ ,Norm\ s)\mapsto 1\ (\langle e'\rangle,s')]\!]$
$\Longrightarrow$
$G\vdash(\langle If(e)\ s1\ Else\ s2\rangle,Norm\ s)\mapsto 1\ (\langle If(e')\ s1\ Else\ s2\rangle,s')$
| *If*: $G\vdash(\langle If(Lit\ v)\ s1\ Else\ s2\rangle,Norm\ s)$

$\mapsto 1$ (⟨*if the-Bool v then s1 else s2*⟩,*Norm s*)

| *Loop*: *G*⊢(⟨*l*· *While*(*e*) *c*⟩,*Norm s*)
$\mapsto 1$ (⟨*If*(*e*) (*Cont l*·*c*;; *l*· *While*(*e*) *c*) *Else Skip*⟩,*Norm s*)

| *Jmp*: *G*⊢(⟨*Jmp j*⟩,*Norm s*) $\mapsto 1$ (⟨*Skip*⟩,(*Some* (*Jump j*), *s*))

| *ThrowE*: ⟦*G*⊢(⟨*e*⟩,*Norm s*) $\mapsto 1$ (⟨*e'*⟩,*s'*)⟧
$\Longrightarrow$
G⊢(⟨*Throw e*⟩,*Norm s*) $\mapsto 1$ (⟨*Throw e'*⟩,*s'*)
| *Throw*: *G*⊢(⟨*Throw* (*Lit a*)⟩,*Norm s*) $\mapsto 1$ (⟨*Skip*⟩,*abupd* (*throw a*) (*Norm s*))

| *TryC1*: ⟦*G*⊢(⟨*c1*⟩,*Norm s*) $\mapsto 1$ (⟨*c1'*⟩,*s'*)⟧
$\Longrightarrow$
G⊢(⟨*Try c1 Catch*(*C vn*) *c2*⟩, *Norm s*) $\mapsto 1$ (⟨*Try c1' Catch*(*C vn*) *c2*⟩,*s'*)
| *Try*: ⟦*G*⊢*s* −*sxalloc*→ *s'*⟧
$\Longrightarrow$
G⊢(⟨*Try Skip Catch*(*C vn*) *c2*⟩, *s*)
$\mapsto 1$ (*if G*,*s'*⊢*catch C then* (⟨*c2*⟩,*new-xcpt-var vn s'*)
else (⟨*Skip*⟩,*s'*))

| *FinC1*: ⟦*G*⊢(⟨*c1*⟩,*Norm s*) $\mapsto 1$ (⟨*c1'*⟩,*s'*)⟧
$\Longrightarrow$
G⊢(⟨*c1 Finally c2*⟩,*Norm s*) $\mapsto 1$ (⟨*c1' Finally c2*⟩,*s'*)

| *Fin*: *G*⊢(⟨*Skip Finally c2*⟩,(*a*,*s*)) $\mapsto 1$ (⟨*FinA a c2*⟩,*Norm s*)

| *FinAC*: ⟦*G*⊢(⟨*c*⟩,*s*) $\mapsto 1$ (⟨*c'*⟩,*s'*)⟧
$\Longrightarrow$
G⊢(⟨*FinA a c*⟩,*s*) $\mapsto 1$ (⟨*FinA a c'*⟩,*s'*)
| *FinA*: *G*⊢(⟨*FinA a Skip*⟩,*s*) $\mapsto 1$ (⟨*Skip*⟩,*abupd* (*abrupt-if* (*a*≠*None*) *a*) *s*)

| *Init1*: ⟦*inited C* (*globs s*)⟧
$\Longrightarrow$
G⊢(⟨*Init C*⟩,*Norm s*) $\mapsto 1$ (⟨*Skip*⟩,*Norm s*)
| *Init*: ⟦*the* (*class G C*)=*c*; ¬ *inited C* (*globs s*)⟧
$\Longrightarrow$
G⊢(⟨*Init C*⟩,*Norm s*)
$\mapsto 1$ (⟨(*if C = Object then Skip else* (*Init* (*super c*)));;
Expr (*Callee* (*locals s*) (*InsInitE* (*init c*) *SKIP*))⟩
,*Norm* (*init-class-obj G C s*))
— *InsInitE* is just used as trick to embed the statement *init c* into an expression
| *InsInitESKIP*:
G⊢(⟨*InsInitE Skip SKIP*⟩,*Norm s*) $\mapsto 1$ (⟨*SKIP*⟩,*Norm s*)

abbreviation
stepn:: [*prog*, *term* × *state*,*nat*,*term* × *state*] ⇒ *bool* (-⊢- ↦- -[*61*,*82*,*82*] *81*)
where *G*⊢*p* ↦*n p'* ≡ (*p*,*p'*) ∈ {(*x*, *y*). *step G x y*}ˆ*n*

abbreviation
steptr:: [*prog*,*term* × *state*,*term* × *state*] ⇒ *bool* (-⊢- ↦∗ -[*61*,*82*,*82*] *81*)
where *G*⊢*p* ↦∗ *p'* ≡ (*p*,*p'*) ∈ {(*x*, *y*). *step G x y*}*

lemma *rtrancl-imp-rel-pow*: *p* ∈ *R*ˆ∗ ⟹ ∃ *n*. *p* ∈ *R*ˆ*n*

proof $-$
  assume $p \in R^*$
  moreover obtain $x\ y$ **where** p: $p = (x,y)$ **by** (*cases* p)
  ultimately have $(x,y) \in R^*$ **by** *hypsubst*
  hence $\exists n.\ (x,y) \in R\hat{}n$
  proof *induct*
    fix a **have** $(a,a) \in R\hat{}0$ **by** *simp*
    thus $\exists n.\ (a,a) \in R\ \hat{}\ n$ **..**
  next
    fix $a\ b\ c$ **assume** $\exists n.\ (a,b) \in R\ \hat{}\ n$
    then obtain n **where** $(a,b) \in R\hat{}n$ **..**
    moreover assume $(b,c) \in R$
    ultimately have $(a,c) \in R\hat{}(Suc\ n)$ **by** *auto*
    thus $\exists n.\ (a,c) \in R\hat{}n$ **..**
  qed
  with p **show** *?thesis* **by** *hypsubst*
qed

end

Chapter 22

AxSem

50 Axiomatic semantics of Java expressions and statements (see also Eval.thy)

theory *AxSem* **imports** *Evaln TypeSafe* **begin**

design issues:

- a strong version of validity for triples with premises, namely one that takes the recursive depth needed to complete execution, enables correctness proof
- auxiliary variables are handled first-class (-¿ Thomas Kleymann)
- expressions not flattened to elementary assignments (as usual for axiomatic semantics) but treated first-class =¿ explicit result value handling
- intermediate values not on triple, but on assertion level (with result entry)
- multiple results with semantical substitution mechnism not requiring a stack
- because of dynamic method binding, terms need to be dependent on state. this is also useful for conditional expressions and statements
- result values in triples exactly as in eval relation (also for xcpt states)
- validity: additional assumption of state conformance and well-typedness, which is required for soundness and thus rule hazard required of completeness

restrictions:

- all triples in a derivation are of the same type (due to weak polymorphism)

types *res* = *vals* — result entry
syntax
  Val :: *val* ⇒ *res*
  Var :: *var* ⇒ *res*
  Vals :: *val list* ⇒ *res*
translations
  Val x => (*In1 x*)
  Var x => (*In2 x*)
  Vals x => (*In3 x*)

syntax
  -Val :: [*pttrn*] => *pttrn* (*Val*:- [*951*] *950*)
  -Var :: [*pttrn*] => *pttrn* (*Var*:- [*951*] *950*)
  -Vals :: [*pttrn*] => *pttrn* (*Vals*:- [*951*] *950*)

translations
  λ*Val*:*v* . *b* == (λ*v*. *b*) ∘ *the-In1*
  λ*Var*:*v* . *b* == (λ*v*. *b*) ∘ *the-In2*
  λ*Vals*:*v*. *b* == (λ*v*. *b*) ∘ *the-In3*

— relation on result values, state and auxiliary variables
types *'a assn* = *res* ⇒ *state* ⇒ *'a* ⇒ *bool*
translations
  res <= (*type*) *AxSem.res*
  a assn <= (*type*) *vals* ⇒ *state* ⇒ *a* ⇒ *bool*

constdefs
  assn-imp :: *'a assn* ⇒ *'a assn* ⇒ *bool* (**infixr** ⇒ *25*)
  $P \Rightarrow Q \equiv \forall\, Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow Q\ Y\ s\ Z$

lemma *assn-imp-def2* [*iff*]: $(P \Rightarrow Q) = (\forall Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow Q\ Y\ s\ Z)$
apply (*unfold assn-imp-def*)
apply (*rule HOL.refl*)
done

assertion transformers

51 peek-and

constdefs
peek-and :: $'a\ assn \Rightarrow (state \Rightarrow\ bool) \Rightarrow 'a\ assn$ (**infixl** $\wedge.$ *13*)
$P \wedge.\ p \equiv \lambda Y\ s\ Z.\ P\ Y\ s\ Z \wedge p\ s$

lemma *peek-and-def2* [*simp*]: *peek-and* $P\ p\ Y\ s = (\lambda Z.\ (P\ Y\ s\ Z \wedge p\ s))$
apply (*unfold peek-and-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-Not* [*simp*]: $(P \wedge.\ (\lambda s.\ \neg f\ s)) = (P \wedge.\ Not \circ f)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-and* [*simp*]: *peek-and* (*peek-and* $P\ p$) p = *peek-and* $P\ p$
apply (*unfold peek-and-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-commut*: $(P \wedge.\ p \wedge.\ q) = (P \wedge.\ q \wedge.\ p)$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply *auto*
done

syntax
Normal :: $'a\ assn \Rightarrow 'a\ assn$
translations
Normal P == $P \wedge.\ normal$

lemma *peek-and-Normal* [*simp*]: *peek-and* (*Normal* P) p = *Normal* (*peek-and* $P\ p$)
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply *auto*
done

52 assn-supd

constdefs
assn-supd :: $'a\ assn \Rightarrow (state \Rightarrow state) \Rightarrow 'a\ assn$ (**infixl** $;.$ *13*)
$P\ ;.\ f \equiv \lambda Y\ s'\ Z.\ \exists s.\ P\ Y\ s\ Z \wedge s' = f\ s$

lemma *assn-supd-def2* [*simp*]: *assn-supd* $P\ f\ Y\ s'\ Z = (\exists\, s.\ P\ Y\ s\ Z \wedge s' = f\ s)$
apply (*unfold assn-supd-def*)
apply (*simp* (*no-asm*))
done

53 supd-assn

constdefs
supd-assn :: (*state* ⇒ *state*) ⇒ 'a *assn* ⇒ 'a *assn* (**infixr** .; *13*)
f .; $P \equiv \lambda Y\ s.\ P\ Y\ (f\ s)$

lemma *supd-assn-def2* [*simp*]: $(f$.; $P)\ Y\ s = P\ Y\ (f\ s)$
apply (*unfold supd-assn-def*)
apply (*simp* (*no-asm*))
done

lemma *supd-assn-supdD* [*elim*]: $((f$.; $Q)$;. $f)\ Y\ s\ Z \Longrightarrow Q\ Y\ s\ Z$
apply *auto*
done

lemma *supd-assn-supdI* [*elim*]: $Q\ Y\ s\ Z \Longrightarrow (f$.; $(Q$;. $f))\ Y\ s\ Z$
apply (*auto simp del*: *split-paired-Ex*)
done

54 subst-res

constdefs
subst-res :: 'a *assn* ⇒ *res* ⇒ 'a *assn* (-←- [*60*,*61*] *60*)
$P{\leftarrow}w \equiv \lambda Y.\ P\ w$

lemma *subst-res-def2* [*simp*]: $(P{\leftarrow}w)\ Y = P\ w$
apply (*unfold subst-res-def*)
apply (*simp* (*no-asm*))
done

lemma *subst-subst-res* [*simp*]: $P{\leftarrow}w{\leftarrow}v = P{\leftarrow}w$
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-subst-res* [*simp*]: $(P \wedge.\ p){\leftarrow}w = (P{\leftarrow}w \wedge.\ p)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

55 subst-Bool

constdefs
subst-Bool :: 'a *assn* ⇒ *bool* ⇒ 'a *assn* (-←=- [*60*,*61*] *60*)

$P \leftarrow = b \equiv \lambda Y\ s\ Z.\ \exists v.\ P\ (Val\ v)\ s\ Z \wedge (normal\ s \longrightarrow the\text{-}Bool\ v{=}b)$

lemma *subst-Bool-def2* [*simp*]:
$(P \leftarrow = b)\ Y\ s\ Z = (\exists v.\ P\ (Val\ v)\ s\ Z \wedge (normal\ s \longrightarrow the\text{-}Bool\ v{=}b))$
apply (*unfold subst-Bool-def*)
apply (*simp* (*no-asm*))
done

lemma *subst-Bool-the-BoolI*: $P\ (Val\ b)\ s\ Z \Longrightarrow (P \leftarrow = the\text{-}Bool\ b)\ Y\ s\ Z$
apply *auto*
done

56 peek-res

constdefs
peek-res :: $(res \Rightarrow 'a\ assn) \Rightarrow 'a\ assn$
$peek\text{-}res\ Pf \equiv \lambda Y.\ Pf\ Y\ Y$

syntax
@peek-res :: $pttrn \Rightarrow 'a\ assn \Rightarrow 'a\ assn$ (λ-:. - [*0*,*3*] *3*)
translations
$\lambda w{:}.\ P$ == $peek\text{-}res\ (\lambda w.\ P)$

lemma *peek-res-def2* [*simp*]: $peek\text{-}res\ P\ Y = P\ Y\ Y$
apply (*unfold peek-res-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-res-subst-res* [*simp*]: $peek\text{-}res\ P \leftarrow w = P\ w \leftarrow w$
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-subst-res-allI*:
$(\bigwedge a.\ T\ a\ (P\ (f\ a) \leftarrow f\ a)) \Longrightarrow \forall a.\ T\ a\ (peek\text{-}res\ P \leftarrow f\ a)$
apply (*rule allI*)
apply (*simp* (*no-asm*))
apply *fast*
done

57 ign-res

constdefs
ign-res :: $'a\ assn \Rightarrow 'a\ assn$ (-$\downarrow$ [*1000*] *1000*)
$P\downarrow \equiv \lambda Y\ s\ Z.\ \exists Y.\ P\ Y\ s\ Z$

lemma *ign-res-def2* [*simp*]: $P\downarrow\ Y\ s\ Z = (\exists Y.\ P\ Y\ s\ Z)$
apply (*unfold ign-res-def*)
apply (*simp* (*no-asm*))
done

lemma *ign-ign-res* [*simp*]: $P{\downarrow}{\downarrow} = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *ign-subst-res* [*simp*]: $P{\downarrow}{\leftarrow}w = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-ign-res* [*simp*]: $(P \wedge.\ p){\downarrow} = (P{\downarrow} \wedge.\ p)$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

58 peek-st

constdefs
  peek-st :: $(st \Rightarrow {}'a\ assn) \Rightarrow {}'a\ assn$
 peek-st $P \equiv \lambda Y\ s.\ P\ (store\ s)\ Y\ s$

syntax
@*peek-st* :: $pttrn \Rightarrow {}'a\ assn \Rightarrow {}'a\ assn$ (λ-.. - [*0*,*3*] *3*)
translations
  $\lambda s..\ P$ == *peek-st* $(\lambda s.\ P)$

lemma *peek-st-def2* [*simp*]: $(\lambda s..\ Pf\ s)\ Y\ s = Pf\ (store\ s)\ Y\ s$
apply (*unfold peek-st-def*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-triv* [*simp*]: $(\lambda s..\ P) = P$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-st* [*simp*]: $(\lambda s..\ \lambda s'..\ P\ s\ s') = (\lambda s..\ P\ s\ s)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-split* [*simp*]: $(\lambda s..\ \lambda Y\ s'.\ P\ s\ Y\ s') = (\lambda Y\ s.\ P\ (store\ s)\ Y\ s)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))

done

lemma *peek-st-subst-res* [*simp*]: $(\lambda s.. P\ s)\leftarrow w = (\lambda s.. P\ s\leftarrow w)$
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *peek-st-Normal* [*simp*]: $(\lambda s..(Normal\ (P\ s))) = Normal\ (\lambda s.. P\ s)$
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

59 ign-res-eq

constdefs
 $ign\text{-}res\text{-}eq :: 'a\ assn \Rightarrow res \Rightarrow 'a\ assn$ $(\text{-}{\downarrow}{=}\text{-}\ [60,61]\ 60)$
 $P{\downarrow}{=}w \quad \equiv \lambda Y{:}.\ P{\downarrow} \wedge.\ (\lambda s.\ Y{=}w)$

lemma *ign-res-eq-def2* [*simp*]: $(P{\downarrow}{=}w)\ Y\ s\ Z = ((\exists\, Y.\ P\ Y\ s\ Z) \wedge Y{=}w)$
apply (*unfold ign-res-eq-def*)
apply *auto*
done

lemma *ign-ign-res-eq* [*simp*]: $(P{\downarrow}{=}w){\downarrow} = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *ign-res-eq-subst-res*: $P{\downarrow}{=}w{\leftarrow}w = P{\downarrow}$
apply (*rule ext*)
apply (*rule ext*)
apply (*rule ext*)
apply (*simp* (*no-asm*))
done

lemma *subst-Bool-ign-res-eq*: $((P{\leftarrow}{=}b){\downarrow}{=}x)\ Y\ s\ Z = ((P{\leftarrow}{=}b)\ Y\ s\ Z \ \wedge\ Y{=}x)$
apply (*simp* (*no-asm*))
done

60 RefVar

constdefs
 $RefVar \quad :: (state \Rightarrow vvar \times state) \Rightarrow 'a\ assn \Rightarrow 'a\ assn$(**infixr** $..;\ 13$)
 $vf\ ..;\ P \equiv \lambda Y\ s.\ let\ (v,s') = vf\ s\ in\ P\ (Var\ v)\ s'$

lemma *RefVar-def2* [*simp*]: $(vf\ ..;\ P)\ Y\ s =$
 $P\ (Var\ (fst\ (vf\ s)))\ (snd\ (vf\ s))$

apply (*unfold RefVar-def Let-def*)
apply (*simp* (*no-asm*) *add*: *split-beta*)
done

61 allocation

constdefs
Alloc :: *prog* ⇒ *obj-tag* ⇒ *'a assn* ⇒ *'a assn*
Alloc G otag P ≡ λ*Y s Z*.
∀ *s' a*. *G*⊢*s* −*halloc otag*≻*a*→ *s'*⟶ *P* (*Val* (*Addr a*)) *s' Z*

SXAlloc :: *prog* ⇒ *'a assn* ⇒ *'a assn*
SXAlloc G P ≡ λ*Y s Z*. ∀ *s'*. *G*⊢*s* −*sxalloc*→ *s'* ⟶ *P Y s' Z*

lemma *Alloc-def2* [*simp*]: *Alloc G otag P Y s Z* =
(∀ *s' a*. *G*⊢*s* −*halloc otag*≻*a*→ *s'*⟶ *P* (*Val* (*Addr a*)) *s' Z*)
apply (*unfold Alloc-def*)
apply (*simp* (*no-asm*))
done

lemma *SXAlloc-def2* [*simp*]:
SXAlloc G P Y s Z = (∀ *s'*. *G*⊢*s* −*sxalloc*→ *s'* ⟶ *P Y s' Z*)
apply (*unfold SXAlloc-def*)
apply (*simp* (*no-asm*))
done

validity

constdefs
type-ok :: *prog* ⇒ *term* ⇒ *state* ⇒ *bool*
type-ok G t s ≡
∃ *L T C A*. (*normal s* ⟶ (|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*t*::*T* ∧
(|*prg*=*G*,*cls*=*C*,*lcl*=*L*|)⊢*dom* (*locals* (*store s*))»*t*»*A*)
∧ *s*::≼(*G*,*L*)

datatype *'a triple* = *triple* (*'a assn*) *term* (*'a assn*)
({(1-)}/ ->/ {(1-)} [3,65,3]75)
types *'a triples* = *'a triple set*

syntax

var-triple :: [*'a assn*, *var* ,*'a assn*] ⇒ *'a triple*
({(1-)}/ -=>/ {(1-)} [3,80,3] 75)
expr-triple :: [*'a assn*, *expr* ,*'a assn*] ⇒ *'a triple*
({(1-)}/ -->/ {(1-)} [3,80,3] 75)
exprs-triple :: [*'a assn*, *expr list* ,*'a assn*] ⇒ *'a triple*
({(1-)}/ -#>/ {(1-)} [3,65,3] 75)
stmt-triple :: [*'a assn*, *stmt*, *'a assn*] ⇒ *'a triple*
({(1-)}/ .-./ {(1-)} [3,65,3] 75)

syntax (*xsymbols*)

triple :: [*'a assn*, *term* ,*'a assn*] ⇒ *'a triple*
({(1-)}/ -≻/ {(1-)} [3,65,3] 75)
var-triple :: [*'a assn*, *var* ,*'a assn*] ⇒ *'a triple*
({(1-)}/ -=≻/ {(1-)} [3,80,3] 75)

$expr\text{-}triple$ :: $['a\ assn,\ expr\ \ \ ,'a\ assn] \Rightarrow 'a\ triple$
$(\{(1\text{-})\}/\ \text{-}{-}{\succ}/\ \{(1\text{-})\}\quad [3,80,3]\ 75)$
$exprs\text{-}triple$ :: $['a\ assn,\ expr\ list\ \ ,'a\ assn] \Rightarrow 'a\ triple$
$(\{(1\text{-})\}/\ \text{-}\dot{=}{\succ}/\ \{(1\text{-})\}\quad [3,65,3]\ 75)$

translations
$\{P\}\ e{-}{\succ}\ \{Q\} == \{P\}\ In1l\ e{\succ}\ \{Q\}$
$\{P\}\ e{=}{\succ}\ \{Q\} == \{P\}\ In2\ \ e{\succ}\ \{Q\}$
$\{P\}\ e\dot{=}{\succ}\ \{Q\} == \{P\}\ In3\ \ e{\succ}\ \{Q\}$
$\{P\}\ .c.\ \{Q\} == \{P\}\ In1r\ c{\succ}\ \{Q\}$

lemma *inj-triple*: $inj\ (\lambda(P,t,Q).\ \{P\}\ t{\succ}\ \{Q\})$
apply (*rule inj-onI*)
apply *auto*
done

lemma *triple-inj-eq*: $(\{P\}\ t{\succ}\ \{Q\} = \{P'\}\ t'{\succ}\ \{Q'\}\ ) = (P{=}P' \wedge t{=}t' \wedge Q{=}Q')$
apply *auto*
done

constdefs
$mtriples$ :: $('c \Rightarrow 'sig \Rightarrow 'a\ assn) \Rightarrow ('c \Rightarrow 'sig \Rightarrow expr) \Rightarrow$
$('c \Rightarrow 'sig \Rightarrow 'a\ assn) \Rightarrow ('c \times\ 'sig)\ set \Rightarrow 'a\ triples$
$(\{\{(1\text{-})\}/\ \text{-}{-}{\succ}/\ \{(1\text{-})\}\ |\ \text{-}\}[3,65,3,65]75)$
$\{\{P\}\ tf{-}{\succ}\ \{Q\}\ |\ ms\} \equiv (\lambda(C,sig).\ \{Normal(P\ C\ sig)\}\ tf\ C\ sig{-}{\succ}\ \{Q\ C\ sig\})`ms$

consts

$triple\text{-}valid$ :: $prog \Rightarrow nat \Rightarrow\ \ \ \ 'a\ triple\ \ \Rightarrow bool$
$(\ \ \ \text{-}{\models}\text{-}{:}\text{-}\ [61,0,\ 58]\ 57)$
$ax\text{-}valids$ :: $prog \Rightarrow 'b\ triples \Rightarrow 'a\ triples \Rightarrow bool$
$(\text{-},\text{-}|{\models}\text{-}\ \ \ [61,58,58]\ 57)$

syntax

$triples\text{-}valid$:: $prog \Rightarrow nat \Rightarrow\ \ \ \ 'a\ triples \Rightarrow bool$
$(\ \ \text{-}||{=}\text{-}{:}\text{-}\ [61,0,\ 58]\ 57)$
$ax\text{-}valid$ :: $prog \Rightarrow\ \ 'b\ triples \Rightarrow 'a\ triple\ \ \Rightarrow bool$
$(\ \text{-},\text{-}|{=}\text{-}\ \ \ [61,58,58]\ 57)$

syntax (*xsymbols*)

$triples\text{-}valid$:: $prog \Rightarrow nat \Rightarrow\ \ \ \ 'a\ triples \Rightarrow bool$
$(\ \ \text{-}|{\models}\text{-}{:}\text{-}\ [61,0,\ 58]\ 57)$
$ax\text{-}valid$ :: $prog \Rightarrow\ \ 'b\ triples \Rightarrow 'a\ triple\ \ \Rightarrow bool$
$(\ \text{-},\text{-}{\models}\text{-}\ \ \ [61,58,58]\ 57)$

defs *triple-valid-def*: $G{\models}n{:}t\ \equiv case\ t\ of\ \{P\}\ t{\succ}\ \{Q\} \Rightarrow$
$\forall\,Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow type\text{-}ok\ G\ t\ s \longrightarrow$
$(\forall\,Y'\ s'.\ G{\vdash}s\ {-}t{\succ}{-}n{\rightarrow}\ (Y',s') \longrightarrow Q\ Y'\ s'\ Z)$
translations $G|{\models}n{:}ts == Ball\ ts\ (triple\text{-}valid\ G\ n)$
defs *ax-valids-def*: $G,A|{\models}ts\ \equiv\ \forall\,n.\ G|{\models}n{:}A \longrightarrow G|{\models}n{:}ts$
translations $G,A\ {\models}t\ == G,A|{\models}\{t\}$

lemma *triple-valid-def2*: $G{\models}n{:}\{P\}\ t{\succ}\ \{Q\} =$
$(\forall\,Y\ s\ Z.\ P\ Y\ s\ Z$

$\longrightarrow (\exists\, L.\ (normal\ s \longrightarrow (\exists\ \ C\ T\ A.\ (\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash t{::}T\ \wedge$
$\quad (\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash dom\ (locals\ (store\ s))\text{»}t\text{»}A))\ \wedge$
$\quad s{::}\preceq(G,L))$
$\longrightarrow (\forall\, Y'\ s'.\ G\vdash s\ -t\succ-n\rightarrow\ (Y',s')\longrightarrow\ Q\ Y'\ s'\ Z))$

apply (*unfold triple-valid-def type-ok-def*)
apply (*simp* (*no-asm*))
done

declare *split-paired-All* [*simp del*] *split-paired-Ex* [*simp del*]
declare *split-if* [*split del*] *split-if-asm* [*split del*]
option.split [*split del*] *option.split-asm* [*split del*]
declaration ⟪ *K* (*Simplifier.map-ss* (*fn ss* => *ss delloop split-all-tac*)) ⟫
declaration ⟪ *K* (*Classical.map-cs* (*fn cs* => *cs delSWrapper split-all-tac*)) ⟫

inductive
ax-derivs :: *prog* ⇒ *'a triples* ⇒ *'a triples* ⇒ *bool* (-,-|⊢- [*61*,*58*,*58*] *57*)
and *ax-deriv* :: *prog* ⇒ *'a triples* ⇒ *'a triple* ⇒ *bool* (-,-⊢- [*61*,*58*,*58*] *57*)
for *G* :: *prog*
where

$G,A\ \vdash t\ \equiv\ G,A|\!\vdash\{t\}$

| *empty*: $G,A|\!\vdash\{\}$
| *insert*: $[\![G,A\vdash t;\ G,A|\!\vdash ts]\!] \Longrightarrow$
$G,A|\!\vdash insert\ t\ ts$

| *asm*: $ts\subseteq A \Longrightarrow G,A|\!\vdash ts$

| *weaken*: $[\![G,A|\!\vdash ts';\ ts\subseteq ts']\!] \Longrightarrow G,A|\!\vdash ts$

| *conseq*: $\forall\ Y\ s\ Z\ .\ P\ \ Y\ s\ Z\ \longrightarrow (\exists\, P'\ Q'.\ G,A\vdash\{P'\}\ t\succ\ \{Q'\}\ \wedge (\forall\ Y'\ s'.$
$(\forall\ Y\ \ \ Z'.\ P'\ Y\ s\ Z' \longrightarrow Q'\ Y'\ s'\ Z') \longrightarrow$
$Q\ \ Y'\ s'\ Z\ ))$
$\Longrightarrow G,A\vdash\{P\ \}\ t\succ\ \{Q\ \}$

| *hazard*: $G,A\vdash\{P\ \wedge.\ Not\circ type\text{-}ok\ G\ t\}\ t\succ\ \{Q\}$

| *Abrupt*: $G,A\vdash\{P\leftarrow(arbitrary3\ t)\ \wedge.\ Not\circ normal\}\ t\succ\ \{P\}$

— variables
| *LVar*: $G,A\vdash\{Normal\ (\lambda s..\ P\leftarrow Var\ (lvar\ vn\ s))\}\ LVar\ vn{=}\succ\ \{P\}$

| *FVar*: $[\![G,A\vdash\{Normal\ P\}\ .Init\ C.\ \{Q\};$
$G,A\vdash\{Q\}\ e{-}\succ\ \{\lambda Val{:}a{:}.\ fvar\ C\ stat\ fn\ a\ ..;\ R\}]\!] \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ \{accC,C,stat\}e..fn{=}\succ\ \{R\}$

| *AVar*: $[\![G,A\vdash\{Normal\ P\}\ e1{-}\succ\ \{Q\};$
$\forall\, a.\ G,A\vdash\{Q\leftarrow Val\ a\}\ e2{-}\succ\ \{\lambda Val{:}i{:}.\ avar\ G\ i\ a\ ..;\ R\}]\!] \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ e1.[e2]{=}\succ\ \{R\}$
— expressions

| *NewC*: $[\![G,A\vdash\{Normal\ P\}\ .Init\ C.\ \{Alloc\ G\ (CInst\ C)\ Q\}]\!] \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ NewC\ C{-}\succ\ \{Q\}$

| *NewA*: $[\![G,A\vdash\{Normal\ P\}\ .init\text{-}comp\text{-}ty\ T.\ \{Q\};\ \ G,A\vdash\{Q\}\ e{-}\succ$
$\{\lambda Val{:}i{:}.\ abupd\ (check\text{-}neg\ i)\ .;\ Alloc\ G\ (Arr\ T\ (the\text{-}Intg\ i))\ R\}]\!] \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ New\ T[e]{-}\succ\ \{R\}$

$| \; Cast\colon [\![G,A \vdash \{Normal\ P\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ \lambda s..$
$\quad abupd\ (raise\text{-}if\ (\neg G,s \vdash v\ fits\ T)\ ClassCast)\ .;\ Q \leftarrow Val\ v\}]\!] \Longrightarrow$
$\qquad G,A \vdash \{Normal\ P\}\ Cast\ T\ e{-}\!\succ\ \{Q\}$

$| \; Inst\colon [\![G,A \vdash \{Normal\ P\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ \lambda s..$
$\quad Q \leftarrow Val\ (Bool\ (v \neq Null \wedge G,s \vdash v\ fits\ RefT\ T))\}]\!] \Longrightarrow$
$\qquad G,A \vdash \{Normal\ P\}\ e\ InstOf\ T{-}\!\succ\ \{Q\}$

$| \; Lit\colon \qquad G,A \vdash \{Normal\ (P \leftarrow Val\ v)\}\ Lit\ v{-}\!\succ\ \{P\}$

$| \; UnOp\colon [\![G,A \vdash \{Normal\ P\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ Q \leftarrow Val\ (eval\text{-}unop\ unop\ v)\}]\!]$
$\quad \Longrightarrow$
$\quad G,A \vdash \{Normal\ P\}\ UnOp\ unop\ e{-}\!\succ\ \{Q\}$

$| \; BinOp\colon$
$[\![G,A \vdash \{Normal\ P\}\ e1{-}\!\succ\ \{Q\};$
$\forall v1.\ G,A \vdash \{Q \leftarrow Val\ v1\}$
$\quad (if\ need\text{-}second\text{-}arg\ binop\ v1\ then\ (In1l\ e2)\ else\ (In1r\ Skip)){\succ}$
$\quad \{\lambda Val{:}v2{:}.\ R \leftarrow Val\ (eval\text{-}binop\ binop\ v1\ v2)\}]\!]$
$\Longrightarrow$
$G,A \vdash \{Normal\ P\}\ BinOp\ binop\ e1\ e2{-}\!\succ\ \{R\}$

$| \; Super\colon G,A \vdash \{Normal\ (\lambda s..\ P \leftarrow Val\ (val\text{-}this\ s))\}\ Super{-}\!\succ\ \{P\}$

$| \; Acc\colon [\![G,A \vdash \{Normal\ P\}\ va{=}\!\succ\ \{\lambda Var{:}(v,f){:}.\ Q \leftarrow Val\ v\}]\!] \Longrightarrow$
$\qquad G,A \vdash \{Normal\ P\}\ Acc\ va{-}\!\succ\ \{Q\}$

$| \; Ass\colon [\![G,A \vdash \{Normal\ P\}\ va{=}\!\succ\ \{Q\};$
$\quad \forall vf.\ G,A \vdash \{Q \leftarrow Var\ vf\}\ e{-}\!\succ\ \{\lambda Val{:}v{:}.\ assign\ (snd\ vf)\ v\ .;\ R\}]\!] \Longrightarrow$
$\qquad G,A \vdash \{Normal\ P\}\ va{:=}e{-}\!\succ\ \{R\}$

$| \; Cond\colon [\![G,A \vdash \{Normal\ P\}\ e0{-}\!\succ\ \{P'\};$
$\quad \forall b.\ G,A \vdash \{P' \leftarrow = b\}\ (if\ b\ then\ e1\ else\ e2){-}\!\succ\ \{Q\}]\!] \Longrightarrow$
$\qquad G,A \vdash \{Normal\ P\}\ e0\ ?\ e1\ :\ e2{-}\!\succ\ \{Q\}$

$| \; Call\colon$
$[\![G,A \vdash \{Normal\ P\}\ e{-}\!\succ\ \{Q\};\ \forall a.\ G,A \vdash \{Q \leftarrow Val\ a\}\ args{\doteq}\!\succ\ \{R\ a\};$
$\forall a\ vs\ invC\ declC\ l.\ G,A \vdash \{(R\ a \leftarrow Vals\ vs \wedge.$
$(\lambda s.\ declC = invocation\text{-}declclass\ G\ mode\ (store\ s)\ a\ statT\ (\!|name{=}mn,parTs{=}pTs|\!) \wedge$
$\quad invC = invocation\text{-}class\ mode\ (store\ s)\ a\ statT \wedge$
$\quad\quad l = locals\ (store\ s))\ ;.$
$\quad init\text{-}lvars\ G\ declC\ (\!|name{=}mn,parTs{=}pTs|\!)\ mode\ a\ vs) \wedge.$
$\quad (\lambda s.\ normal\ s \longrightarrow G \vdash mode \rightarrow invC \preceq statT)\}$
$Methd\ declC\ (\!|name{=}mn,parTs{=}pTs|\!){-}\!\succ\ \{set\text{-}lvars\ l\ .;\ S\}]\!] \Longrightarrow$
$\quad G,A \vdash \{Normal\ P\}\ \{accC,statT,mode\}e{\cdot}mn(\{pTs\}args){-}\!\succ\ \{S\}$

$| \; Methd\colon [\![G,A \cup\ \{\{P\}\ Methd{-}\!\succ\ \{Q\}\ |\ ms\}\ |\!\vdash\ \{\{P\}\ body\ G{-}\!\succ\ \{Q\}\ |\ ms\}]\!] \Longrightarrow$
$\qquad G,A |\!\vdash \{\{P\}\ Methd{-}\!\succ\ \{Q\}\ |\ ms\}$

$| \; Body\colon [\![G,A \vdash \{Normal\ P\}\ .Init\ D.\ \{Q\};$
$G,A \vdash \{Q\}\ .c.\ \{\lambda s..\ abupd\ (absorb\ Ret)\ .;\ R \leftarrow (In1\ (the\ (locals\ s\ Result)))\}]\!]$
$\Longrightarrow$
$\qquad G,A \vdash \{Normal\ P\}\ Body\ D\ c{-}\!\succ\ \{R\}$

— expression lists

$| \; Nil\colon \qquad G,A \vdash \{Normal\ (P \leftarrow Vals\ [])\}\ []{\doteq}\!\succ\ \{P\}$

| *Cons*: $[\![G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{Q\};$
$\quad\forall v.\ G,A\vdash\{Q\leftarrow Val\ v\}\ es\dot{=}\!\succ\ \{\lambda Vals{:}vs{:}.\ R\leftarrow Vals\ (v\#vs)\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ e\#es\dot{=}\!\succ\ \{R\}$

— statements

| *Skip*: $\qquad G,A\vdash\{Normal\ (P\leftarrow\Diamond)\}\ .Skip.\ \{P\}$

| *Expr*: $[\![G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{Q\leftarrow\Diamond\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ .Expr\ e.\ \{Q\}$

| *Lab*: $[\![G,A\vdash\{Normal\ P\}\ .c.\ \{abupd\ (absorb\ l)\ .;\ Q\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ .l\cdot\ c.\ \{Q\}$

| *Comp*: $[\![G,A\vdash\{Normal\ P\}\ .c1.\ \{Q\};$
$\quad G,A\vdash\{Q\}\ .c2.\ \{R\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ .c1;;c2.\ \{R\}$

| *If*: $[\![G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{P'\};$
$\quad\forall b.\ G,A\vdash\{P'\leftarrow=b\}\ .(if\ b\ then\ c1\ else\ c2).\ \{Q\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ .If(e)\ c1\ Else\ c2.\ \{Q\}$

| *Loop*: $[\![G,A\vdash\{P\}\ e{-}\!\succ\ \{P'\};$
$\quad G,A\vdash\{Normal\ (P'\leftarrow=True)\}\ .c.\ \{abupd\ (absorb\ (Cont\ l))\ .;\ P\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{P\}\ .l\cdot\ While(e)\ c.\ \{(P'\leftarrow=False)\downarrow=\Diamond\}$

| *Jmp*: $G,A\vdash\{Normal\ (abupd\ (\lambda a.\ (Some\ (Jump\ j)))\ .;\ P\leftarrow\Diamond)\}\ .Jmp\ j.\ \{P\}$

| *Throw*: $[\![G,A\vdash\{Normal\ P\}\ e{-}\!\succ\ \{\lambda Val{:}a{:}.\ abupd\ (throw\ a)\ .;\ Q\leftarrow\Diamond\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ .Throw\ e.\ \{Q\}$

| *Try*: $[\![G,A\vdash\{Normal\ P\}\ .c1.\ \{SXAlloc\ G\ Q\};$
$\quad G,A\vdash\{Q\ \wedge.\ (\lambda s.\ \ G,s\vdash catch\ C)\ ;.\ new\text{-}xcpt\text{-}var\ vn\}\ .c2.\ \{R\};$
$\quad\ (Q\ \wedge.\ (\lambda s.\ \neg G,s\vdash catch\ C))\Rightarrow R]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ .Try\ c1\ Catch(C\ vn)\ c2.\ \{R\}$

| *Fin*: $[\![G,A\vdash\{Normal\ P\}\ .c1.\ \{Q\};$
$\quad\forall x.\ G,A\vdash\{Q\ \wedge.\ (\lambda s.\ x=fst\ s)\ ;.\ abupd\ (\lambda x.\ None)\}$
$\quad\ .c2.\ \{abupd\ (abrupt\text{-}if\ (x\neq None)\ x)\ .;\ R\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ P\}\ .c1\ Finally\ c2.\ \{R\}$

| *Done*: $\qquad G,A\vdash\{Normal\ (P\leftarrow\Diamond\ \wedge.\ initd\ C)\}\ .Init\ C.\ \{P\}$

| *Init*: $[\![the\ (class\ G\ C)=c;$
$\quad G,A\vdash\{Normal\ ((P\ \wedge.\ Not\circ initd\ C)\ ;.\ supd\ (init\text{-}class\text{-}obj\ G\ C))\}$
$\quad\ .(if\ C=Object\ then\ Skip\ else\ Init\ (super\ c)).\ \{Q\};$
$\quad\forall l.\ G,A\vdash\{Q\ \wedge.\ (\lambda s.\ l=locals\ (store\ s))\ ;.\ set\text{-}lvars\ empty\}$
$\quad\ .init\ c.\ \{set\text{-}lvars\ l\ .;\ R\}]\!]\Longrightarrow$
$\qquad\qquad G,A\vdash\{Normal\ (P\ \wedge.\ Not\circ initd\ C)\}\ .Init\ C.\ \{R\}$

— Some dummy rules for the intermediate terms *Callee*, *InsInitE*, *InsInitV*, *FinA* only used by the smallstep semantics.
| *InsInitV*: $G,A\vdash\{Normal\ P\}\ InsInitV\ c\ v{=}\!\succ\ \{Q\}$
| *InsInitE*: $G,A\vdash\{Normal\ P\}\ InsInitE\ c\ e{-}\!\succ\ \{Q\}$
| *Callee*: $G,A\vdash\{Normal\ P\}\ Callee\ l\ e{-}\!\succ\ \{Q\}$
| *FinA*: $G,A\vdash\{Normal\ P\}\ .FinA\ a\ c.\ \{Q\}$

constdefs

adapt-pre :: $'a$ *assn* $\Rightarrow$ $'a$ *assn* $\Rightarrow$ $'a$ *assn* $\Rightarrow$ $'a$ *assn*
adapt-pre $P\ Q\ Q' \equiv \lambda Y\ s\ Z.\ \forall Y'\ s'.\ \exists Z'.\ P\ Y\ s\ Z' \wedge (Q\ Y'\ s'\ Z' \longrightarrow Q'\ Y'\ s'\ Z)$

rules derived by induction

lemma *cut-valid*: $[\![G,A'|\models ts;\ G,A|\models A']\!] \Longrightarrow G,A|\models ts$
apply (*unfold ax-valids-def*)
apply *fast*
done

lemma *ax-thin* [*rule-format* (*no-asm*)]:
$G,(A'::'a\ triple\ set)|\vdash(ts::'a\ triple\ set) \Longrightarrow \forall A.\ A' \subseteq A \longrightarrow G,A|\vdash ts$
apply (*erule ax-derivs.induct*)
apply (*tactic ALLGOALS*(*EVERY'*[*clarify-tac* @{*claset*}, *REPEAT o smp-tac 1*]))
apply (*rule ax-derivs.empty*)
apply (*erule* (*1*) *ax-derivs.insert*)
apply (*fast intro*: *ax-derivs.asm*)

apply (*fast intro*: *ax-derivs.weaken*)
apply (*rule ax-derivs.conseq*, *intro strip*, *tactic smp-tac 3 1*,*clarify*, *tactic smp-tac 1 1*,*rule exI*, *rule exI*, *erule* (*1*) *conjI*)

prefer *18*
apply (*rule ax-derivs.Methd*, *drule spec*, *erule mp*, *fast*)
apply (*tactic* ⟪ *TRYALL* (*resolve-tac* ((*funpow 5 tl*) (*thms ax-derivs.intros*))) ⟫)
apply *auto*
done

lemma *ax-thin-insert*: $G,(A::'a\ triple\ set)\vdash(t::'a\ triple) \Longrightarrow G,insert\ x\ A\vdash t$
apply (*erule ax-thin*)
apply *fast*
done

lemma *subset-mtriples-iff*:
$ts \subseteq \{\{P\}\ mb{-}\!\succ\ \{Q\}\ |\ ms\} = (\exists ms'.\ ms' \subseteq ms \wedge\ ts = \{\{P\}\ mb{-}\!\succ\ \{Q\}\ |\ ms'\})$
apply (*unfold mtriples-def*)
apply (*rule subset-image-iff*)
done

lemma *weaken*:
$G,(A::'a\ triple\ set)|\vdash(ts'::'a\ triple\ set) \Longrightarrow\ !ts.\ ts \subseteq ts' \longrightarrow G,A|\vdash ts$
apply (*erule ax-derivs.induct*)

apply (*tactic ALLGOALS strip-tac*)
apply (*tactic* ⟪ *ALLGOALS*(*REPEAT o* (*EVERY'*[*dtac* (*thm subset-singletonD*), *etac disjE*, *fast-tac* (*claset*() *addSIs* [*thm ax-derivs.empty*])]))⟫)
apply (*tactic TRYALL hyp-subst-tac*)
apply (*simp*, *rule ax-derivs.empty*)
apply (*drule subset-insertD*)
apply (*blast intro*: *ax-derivs.insert*)
apply (*fast intro*: *ax-derivs.asm*)

apply (*fast intro*: *ax-derivs.weaken*)
apply (*rule ax-derivs.conseq*, *clarify*, *tactic smp-tac 3 1*, *blast*)

apply (*tactic* ⟪ *TRYALL* (*resolve-tac* ((*funpow 5 tl*) (*thms ax-derivs.intros*))
THEN-ALL-NEW fast-tac @{*claset*}) ⟫)

apply (*clarsimp simp add*: *subset-mtriples-iff*)
apply (*rule ax-derivs.Methd*)
apply (*drule spec*)
apply (*erule impE*)
apply (*rule exI*)
apply (*erule conjI*)
apply (*rule HOL.refl*)
oops

rules derived from conseq

In the following rules we often have to give some type annotations like: $G,A\vdash\{P\}\ t\succ\ \{Q\}$. Given only the term above without annotations, Isabelle would infer a more general type were we could have different types of auxiliary variables in the assumption set (A) and in the triple itself (P and Q). But *ax-derivs.Methd* enforces the same type in the inductive definition of the derivation. So we have to restrict the types to be able to apply the rules.

lemma *conseq12*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q'\};$
$\forall\ Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow (\forall\ Y'\ s'.\ (\forall\ Y\ Z'.\ P'\ Y\ s\ Z' \longrightarrow Q'\ Y'\ s'\ Z') \longrightarrow$
$Q\ Y'\ s'\ Z)]\!]$
$\Longrightarrow\ G,A\vdash\{P\ ::'a\ assn\}\ t\succ\ \{Q\ \}$
apply (*rule ax-derivs.conseq*)
apply *clarsimp*
apply *blast*
done

— Nice variant, since it is so symmetric we might be able to memorise it.

lemma *conseq12'*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q'\};\ \forall\ s\ Y'\ s'.$
$(\forall\ Y\ Z.\ P'\ Y\ s\ Z \longrightarrow Q'\ Y'\ s'\ Z) \longrightarrow$
$(\forall\ Y\ Z.\ P\ \ Y\ s\ Z \longrightarrow Q\ \ Y'\ s'\ Z)]\!]$
$\Longrightarrow\ G,A\vdash\{P::'a\ assn\ \}\ t\succ\ \{Q\ \}$
apply (*erule conseq12*)
apply *fast*
done

lemma *conseq12-from-conseq12'*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q'\};$
$\forall\ Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow (\forall\ Y'\ s'.\ (\forall\ Y\ Z'.\ P'\ Y\ s\ Z' \longrightarrow Q'\ Y'\ s'\ Z') \longrightarrow$
$Q\ Y'\ s'\ Z)]\!]$
$\Longrightarrow\ G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q\ \}$
apply (*erule conseq12'*)
apply *blast*
done

lemma *conseq1*: $[\![G,(A::'a\ triple\ set)\vdash\{P'::'a\ assn\}\ t\succ\ \{Q\};\ P \Rightarrow P']\!]$
$\Longrightarrow G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}$
apply (*erule conseq12*)
apply *blast*
done

lemma *conseq2*: $[\![G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q'\};\ Q' \Rightarrow Q]\!]$
$\Longrightarrow G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}$

apply (*erule conseq12*)
apply *blast*
done

lemma *ax-escape*:
$[\![\forall\, Y\ s\ Z.\ P\ Y\ s\ Z$
$\quad\longrightarrow G,(A::'a\ triple\ set)\vdash\{\lambda Y'\ s'\ (Z'::'a).\ (Y',s') = (Y,s)\}$
$\qquad\qquad t\succ$
$\qquad\qquad \{\lambda Y\ s\ Z'.\ Q\ Y\ s\ Z\}$
$]\!]\Longrightarrow\ G,A\vdash\{P::'a\ assn\}\ t\succ\ \{Q::'a\ assn\}$
apply (*rule ax-derivs.conseq*)
apply *force*
done

lemma *ax-constant*: $[\![\ C \Longrightarrow G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}]\!]$
$\Longrightarrow G,A\vdash\{\lambda Y\ s\ Z.\ C \wedge P\ Y\ s\ Z\}\ t\succ\ \{Q\}$
apply (*rule ax-escape*)
apply *clarify*
apply (*rule conseq12*)
apply *fast*
apply *auto*
done

lemma *ax-impossible* [*intro*]:
$G,(A::'a\ triple\ set)\vdash\{\lambda Y\ s\ Z.\ False\}\ t\succ\ \{Q::'a\ assn\}$
apply (*rule ax-escape*)
apply *clarify*
done

lemma *ax-nochange-lemma*: $[\![P\ \ Y\ s;\ All\ (op = w)]\!] \Longrightarrow P\ w\ s$
apply *auto*
done

lemma *ax-nochange*:
$G,(A::(res \times state)\ triple\ set)\vdash\{\lambda Y\ s\ Z.\ (Y,s){=}Z\}\ t\succ\ \{\lambda Y\ s\ Z.\ (Y,s){=}Z\}$
$\Longrightarrow G,A\vdash\{P::(res \times state)\ assn\}\ t\succ\ \{P\}$
apply (*erule conseq12*)
apply *auto*
apply (*erule* (*1*) *ax-nochange-lemma*)
done

lemma *ax-trivial*: $G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ \ t\succ\ \{\lambda Y\ s\ Z.\ True\}$
apply (*rule ax-derivs.conseq*)
apply *auto*
done

lemma *ax-disj*:
$[\![G,(A::'a\ triple\ set)\vdash\{P1::'a\ assn\}\ t\succ\ \{Q1\};\ G,A\vdash\{P2::'a\ assn\}\ t\succ\ \{Q2\}]\!]$
$\Longrightarrow\ G,A\vdash\{\lambda Y\ s\ Z.\ P1\ Y\ s\ Z\ \vee\ P2\ Y\ s\ Z\}\ t\succ\ \{\lambda Y\ s\ Z.\ Q1\ Y\ s\ Z\ \vee\ Q2\ Y\ s\ Z\}$
apply (*rule ax-escape*)
apply *safe*
apply (*erule conseq12*, *fast*)+
done

lemma *ax-supd-shuffle*:
$(\exists\,Q.\ G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ .c1.\ \{Q\}\ \wedge\ G,A\vdash\{Q\ ;.\ f\}\ .c2.\ \{R\}) =$
$(\exists\,Q'.\ G,A\vdash\{P\}\ .c1.\ \{f\ .;\ Q'\}\ \wedge\ G,A\vdash\{Q'\}\ .c2.\ \{R\})$
apply (*best elim*!: *conseq1 conseq2*)
done

lemma *ax-cases*:
$[\![G,(A::'a\ triple\ set)\vdash\{P\ \wedge.\qquad C\}\ t\succ\ \{Q::'a\ assn\};$
$G,A\vdash\{P\ \wedge.\ Not\ \circ\ C\}\ t\succ\ \{Q\}]\!]\Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Q\}$
apply (*unfold peek-and-def*)
apply (*rule ax-escape*)
apply *clarify*
apply (*case-tac C s*)
apply (*erule conseq12*, *force*)+
done

lemma *ax-adapt*: $G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}$
$\Longrightarrow G,A\vdash\{adapt\text{-}pre\ P\ Q\ Q'\}\ t\succ\ \{Q'\}$
apply (*unfold adapt-pre-def*)
apply (*erule conseq12*)
apply *fast*
done

lemma *adapt-pre-adapts*: $G,(A::'a\ triple\ set)\models\{P::'a\ assn\}\ t\succ\ \{Q\}$
$\longrightarrow G,A\models\{adapt\text{-}pre\ P\ Q\ Q'\}\ t\succ\ \{Q'\}$
apply (*unfold adapt-pre-def*)
apply (*simp add*: *ax-valids-def triple-valid-def2*)
apply *fast*
done

lemma *adapt-pre-weakest*:
$\forall\,G\ (A::'a\ triple\ set)\ t.\ G,A\models\{P\}\ t\succ\ \{Q\}\longrightarrow G,A\models\{P'\}\ t\succ\ \{Q'\}\Longrightarrow$
$P'\Rightarrow adapt\text{-}pre\ P\ Q\ (Q'::'a\ assn)$
apply (*unfold adapt-pre-def*)
apply (*drule spec*)
apply (*drule-tac* $x = \{\}$ **in** *spec*)
apply (*drule-tac* $x =$ *In1r Skip* **in** *spec*)
apply (*simp add*: *ax-valids-def triple-valid-def2*)
oops

lemma *peek-and-forget1-Normal*:
$G,(A::'a\ triple\ set)\vdash\{Normal\ P\}\ t\succ\ \{Q::'a\ assn\}$

$\Longrightarrow G,A\vdash\{Normal\ (P \wedge.\ p)\}\ t\succ\ \{Q\}$
apply (*erule conseq1*)
apply (*simp* (*no-asm*))
done

lemma *peek-and-forget1*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q\}$
$\Longrightarrow G,A\vdash\{P \wedge.\ p\}\ t\succ\ \{Q\}$
apply (*erule conseq1*)
apply (*simp* (*no-asm*))
done

lemmas *ax-NormalD* = *peek-and-forget1* [*of - - - - - normal*]

lemma *peek-and-forget2*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ\ \{Q \wedge.\ p\}$
$\Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Q\}$
apply (*erule conseq2*)
apply (*simp* (*no-asm*))
done

lemma *ax-subst-Val-allI*:
$\forall v.\ G,(A::'a\ triple\ set)\vdash\{(P'\quad v\ )\leftarrow Val\ v\}\ t\succ\ \{(Q\ v)::'a\ assn\}$
$\Longrightarrow\ \forall v.\ G,A\vdash\{(\lambda w{:}.\ P'\ (the\text{-}In1\ w))\leftarrow Val\ v\}\ t\succ\ \{Q\ v\}$
apply (*force elim*!: *conseq1*)
done

lemma *ax-subst-Var-allI*:
$\forall v.\ G,(A::'a\ triple\ set)\vdash\{(P'\quad v\ )\leftarrow Var\ v\}\ t\succ\ \{(Q\ v)::'a\ assn\}$
$\Longrightarrow\ \forall v.\ G,A\vdash\{(\lambda w{:}.\ P'\ (the\text{-}In2\ w))\leftarrow Var\ v\}\ t\succ\ \{Q\ v\}$
apply (*force elim*!: *conseq1*)
done

lemma *ax-subst-Vals-allI*:
$(\forall v.\ G,(A::'a\ triple\ set)\vdash\{(\quad P'\quad v\ )\leftarrow Vals\ v\}\ t\succ\ \{(Q\ v)::'a\ assn\})$
$\Longrightarrow\ \forall v.\ G,A\vdash\{(\lambda w{:}.\ P'\ (the\text{-}In3\ w))\leftarrow Vals\ v\}\ t\succ\ \{Q\ v\}$
apply (*force elim*!: *conseq1*)
done

alternative axioms

lemma *ax-Lit2*:
$G,(A::'a\ triple\ set)\vdash\{Normal\ P::'a\ assn\}\ Lit\ v{-}\succ\ \{Normal\ (P{\downarrow}{=}Val\ v)\}$
apply (*rule ax-derivs.Lit* [*THEN conseq1*])
apply *force*
done

lemma *ax-Lit2-test-complete*:
$G,(A::'a\ triple\ set)\vdash\{Normal\ (P\leftarrow Val\ v)::'a\ assn\}\ Lit\ v{-}\succ\ \{P\}$
apply (*rule ax-Lit2* [*THEN conseq2*])
apply *force*
done

lemma *ax-LVar2*: G,(A::'a triple set)⊢{Normal P::'a assn} LVar vn=≻ {Normal (λs.. P↓=Var (lvar vn s))}
apply (rule ax-derivs.LVar [THEN conseq1])
apply force
done

lemma *ax-Super2*: G,(A::'a triple set)⊢
{Normal P::'a assn} Super−≻ {Normal (λs.. P↓=Val (val-this s))}
apply (rule ax-derivs.Super [THEN conseq1])
apply force
done

lemma *ax-Nil2*:
G,(A::'a triple set)⊢{Normal P::'a assn} []≐≻ {Normal (P↓=Vals [])}
apply (rule ax-derivs.Nil [THEN conseq1])
apply force
done

misc derived structural rules

lemma *ax-finite-mtriples-lemma*: ⟦F ⊆ ms; finite ms; ∀(C,sig)∈ms.
G,(A::'a triple set)⊢{Normal (P C sig)::'a assn} mb C sig−≻ {Q C sig}⟧ ⟹
G,A|⊢{{P} mb−≻ {Q} | F}
apply (frule (1) finite-subset)
apply (erule rev-mp)
apply (erule thin-rl)
apply (erule finite-induct)
apply (unfold mtriples-def)
apply (clarsimp intro!: ax-derivs.empty ax-derivs.insert)+
apply force
done
lemmas *ax-finite-mtriples* = ax-finite-mtriples-lemma [OF subset-refl]

lemma *ax-derivs-insertD*:
G,(A::'a triple set)|⊢insert (t::'a triple) ts ⟹ G,A⊢t ∧ G,A|⊢ts
apply (fast intro: ax-derivs.weaken)
done

lemma *ax-methods-spec*:
⟦G,(A::'a triple set)|⊢split f ‘ ms; (C,sig) ∈ ms⟧⟹ G,A⊢((f C sig)::'a triple)
apply (erule ax-derivs.weaken)
apply (force del: image-eqI intro: rev-image-eqI)
done

lemma *ax-finite-pointwise-lemma* [rule-format]: ⟦F ⊆ ms; finite ms⟧ ⟹
((∀(C,sig)∈F. G,(A::'a triple set)⊢(f C sig::'a triple)) ⟶ (∀(C,sig)∈ms. G,A⊢(g C sig::'a triple))) ⟶
G,A|⊢split f ‘ F ⟶ G,A|⊢split g ‘ F
apply (frule (1) finite-subset)
apply (erule rev-mp)
apply (erule thin-rl)
apply (erule finite-induct)
apply clarsimp+
apply (drule ax-derivs-insertD)

apply (*rule ax-derivs.insert*)
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*auto elim*: *ax-methods-spec*)
done
lemmas *ax-finite-pointwise* = *ax-finite-pointwise-lemma* [*OF subset-refl*]

lemma *ax-no-hazard*:
$G,(A::'a\ triple\ set)\vdash\{P\ \wedge.\ type\text{-}ok\ G\ t\}\ t\succ\ \{Q::'a\ assn\} \Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Q\}$
apply (*erule ax-cases*)
apply (*rule ax-derivs.hazard* [*THEN conseq1*])
apply *force*
done

lemma *ax-free-wt*:
$(\exists\ T\ L\ C.\ (\!|prg=G,cls=C,lcl=L|\!)\vdash t::T)$
$\longrightarrow G,(A::'a\ triple\ set)\vdash\{Normal\ P\}\ t\succ\ \{Q::'a\ assn\} \Longrightarrow$
$G,A\vdash\{Normal\ P\}\ t\succ\ \{Q\}$
apply (*rule ax-no-hazard*)
apply (*rule ax-escape*)
apply *clarify*
apply (*erule mp* [*THEN conseq12*])
apply (*auto simp add*: *type-ok-def*)
done

ML-setup ⟨⟨ *bind-thms* (*ax-Abrupts*, *sum3-instantiate* @{*thm ax-derivs.Abrupt*}) ⟩⟩
declare *ax-Abrupts* [*intro*!]

lemmas *ax-Normal-cases* = *ax-cases* [*of - - - normal*]

lemma *ax-Skip* [*intro*!]: $G,(A::'a\ triple\ set)\vdash\{P\leftarrow\Diamond\}\ .Skip.\ \{P::'a\ assn\}$
apply (*rule ax-Normal-cases*)
apply (*rule ax-derivs.Skip*)
apply *fast*
done
lemmas *ax-SkipI* = *ax-Skip* [*THEN conseq1*, *standard*]

derived rules for methd call

lemma *ax-Call-known-DynT*:
$[\![G\vdash IntVir\rightarrow C\preceq statT;$
$\forall\, a\ vs\ l.\ G,A\vdash\{(R\ a\leftarrow Vals\ vs\ \wedge.\ (\lambda s.\ l = locals\ (store\ s))\ ;.$
$init\text{-}lvars\ G\ C\ (\!|name=mn,parTs=pTs|\!)\ IntVir\ a\ vs)\}$
$Methd\ C\ (\!|name=mn,parTs=pTs|\!)\!-\!\succ\ \{set\text{-}lvars\ l\ .;\ S\};$
$\forall\, a.\ G,A\vdash\{Q\leftarrow Val\ a\}\ args\doteq\succ$
$\{R\ a\ \wedge.\ (\lambda s.\ C = obj\text{-}class\ (the\ (heap\ (store\ s)\ (the\text{-}Addr\ a)))\ \wedge$
$C = invocation\text{-}declclass$
$G\ IntVir\ (store\ s)\ a\ statT\ (\!|name=mn,parTs=pTs|\!)\ )\};$
$G,(A::'a\ triple\ set)\vdash\{Normal\ P\}\ e\!-\!\succ\ \{Q::'a\ assn\}]\!]$
$\Longrightarrow G,A\vdash\{Normal\ P\}\ \{accC,statT,IntVir\}e\cdot mn(\{pTs\}args)\!-\!\succ\ \{S\}$
apply (*erule ax-derivs.Call*)
apply *safe*
apply (*erule spec*)
apply (*rule ax-escape*, *clarsimp*)
apply (*drule spec*, *drule spec*, *drule spec*,*erule conseq12*)
apply *force*
done

lemma *ax-Call-Static*:
$[\![\forall\, a\ vs\ l.\ G{,}A{\vdash}\{R\ a{\leftarrow}Vals\ vs \wedge.\ (\lambda s.\ l = locals\ (store\ s))\ ;.$
$\quad init\text{-}lvars\ G\ C\ (\![name{=}mn{,}parTs{=}pTs]\!)\ Static\ any\text{-}Addr\ vs\}$
$\quad Methd\ C\ (\![name{=}mn{,}parTs{=}pTs]\!){-}{\succ}\ \{set\text{-}lvars\ l\ .;\ S\};$
$G{,}A{\vdash}\{Normal\ P\}\ e{-}{\succ}\ \{Q\};$
$\forall\ a.\ G{,}(A{::}'a\ triple\ set){\vdash}\{Q{\leftarrow}Val\ a\}\ args{\doteq}{\succ}\ \{(R{::}val \Rightarrow 'a\ assn)\ \ a$
$\wedge.\ (\lambda\ s.\ C{=}invocation\text{-}declclass$
$\quad G\ Static\ (store\ s)\ a\ statT\ (\![name{=}mn{,}parTs{=}pTs]\!))\}$
$]\!]\ \Longrightarrow\ \ G{,}A{\vdash}\{Normal\ P\}\ \{accC{,}statT{,}Static\}e{\cdot}mn(\{pTs\}args){-}{\succ}\ \{S\}$
apply (*erule ax-derivs.Call*)
apply *safe*
apply (*erule spec*)
apply (*rule ax-escape*, *clarsimp*)
apply (*erule-tac V = ?P ⟶ ?Q* **in** *thin-rl*)
apply (*drule spec*,*drule spec*,*drule spec*, *erule conseq12*)
apply (*force simp add*: *init-lvars-def Let-def*)
done

lemma *ax-Methd1*:
$[\![G{,}A{\cup}\{\{P\}\ Methd{-}{\succ}\ \{Q\}\ |\ ms\}{|\!\vdash}\ \{\{P\}\ body\ G{-}{\succ}\ \{Q\}\ |\ ms\};\ (C{,}sig){\in}\ ms]\!]\ \Longrightarrow$
$\quad G{,}A{\vdash}\{Normal\ (P\ C\ sig)\}\ Methd\ C\ sig{-}{\succ}\ \{Q\ C\ sig\}$
apply (*drule ax-derivs.Methd*)
apply (*unfold mtriples-def*)
apply (*erule* (*1*) *ax-methods-spec*)
done

lemma *ax-MethdN*:
$G{,}insert(\{Normal\ P\}\ Methd\ \ C\ sig{-}{\succ}\ \{Q\})\ A{\vdash}$
$\quad \{Normal\ P\}\ body\ G\ C\ sig{-}{\succ}\ \{Q\}\ \Longrightarrow$
$\quad G{,}A{\vdash}\{Normal\ P\}\ Methd\ \ \ C\ sig{-}{\succ}\ \{Q\}$
apply (*rule ax-Methd1*)
apply (*rule-tac* [*2*] *singletonI*)
apply (*unfold mtriples-def*)
apply *clarsimp*
done

lemma *ax-StatRef*:
$G{,}(A{::}'a\ triple\ set){\vdash}\{Normal\ (P{\leftarrow}Val\ Null)\}\ StatRef\ rt{-}{\succ}\ \{P{::}'a\ assn\}$
apply (*rule ax-derivs.Cast*)
apply (*rule ax-Lit2* [*THEN conseq2*])
apply *clarsimp*
done

rules derived from Init and Done

lemma *ax-InitS*: $[\![the\ (class\ G\ C) = c;\ C \neq Object;$
$\forall\, l.\ G{,}A{\vdash}\{Q \wedge.\ (\lambda s.\ l = locals\ (store\ s))\ ;.\ set\text{-}lvars\ empty\}$
$\quad .init\ c.\ \{set\text{-}lvars\ l\ .;\ R\};$
$\quad G{,}A{\vdash}\{Normal\ ((P \wedge.\ Not \circ initd\ C)\ ;.\ supd\ (init\text{-}class\text{-}obj\ G\ C))\}$
$.Init\ (super\ c).\ \{Q\}]\!]\ \Longrightarrow$
$G{,}(A{::}'a\ triple\ set){\vdash}\{Normal\ (P \wedge.\ Not \circ initd\ C)\}\ .Init\ C.\ \{R{::}'a\ assn\}$
apply (*erule ax-derivs.Init*)

apply (*simp* (*no-asm-simp*))
apply *assumption*
done

lemma *ax-Init-Skip-lemma*:
$\forall l.\ G,(A::'a\ triple\ set)\vdash\{P\leftarrow\Diamond \wedge.\ (\lambda s.\ l = locals\ (store\ s))\ ;.\ set\text{-}lvars\ l'\}$
$.Skip.\ \{(set\text{-}lvars\ l\ .;\ P)::'a\ assn\}$
apply (*rule allI*)
apply (*rule ax-SkipI*)
apply *clarsimp*
done

lemma *ax-triv-InitS*: $[\![the\ (class\ G\ C) = c;init\ c = Skip;\ C \neq Object;$
$P\leftarrow\Diamond \Rightarrow (supd\ (init\text{-}class\text{-}obj\ G\ C)\ .;\ P);$
$G,A\vdash\{Normal\ (P \wedge.\ initd\ C)\}\ .Init\ (super\ c).\ \{(P \wedge.\ initd\ C)\leftarrow\Diamond\}]\!] \Longrightarrow$
$G,(A::'a\ triple\ set)\vdash\{Normal\ P\leftarrow\Diamond\}\ .Init\ C.\ \{(P \wedge.\ initd\ C)::'a\ assn\}$
apply (*rule-tac C = initd C* **in** *ax-cases*)
apply (*rule conseq1*, *rule ax-derivs.Done*, *clarsimp*)
apply (*simp* (*no-asm*))
apply (*erule* (*1*) *ax-InitS*)
apply *simp*
apply (*rule ax-Init-Skip-lemma*)
apply (*erule conseq1*)
apply *force*
done

lemma *ax-Init-Object*: $wf\text{-}prog\ G \Longrightarrow G,(A::'a\ triple\ set)\vdash$
$\{Normal\ ((supd\ (init\text{-}class\text{-}obj\ G\ Object)\ .;\ P\leftarrow\Diamond) \wedge.\ Not \circ initd\ Object)\}$
$.Init\ Object.\ \{(P \wedge.\ initd\ Object)::'a\ assn\}$
apply (*rule ax-derivs.Init*)
apply (*drule class-Object*, *force*)
apply (*simp-all* (*no-asm*))
apply (*rule-tac* [*2*] *ax-Init-Skip-lemma*)
apply (*rule ax-SkipI*, *force*)
done

lemma *ax-triv-Init-Object*: $[\![wf\text{-}prog\ G;$
$(P::'a\ assn) \Rightarrow (supd\ (init\text{-}class\text{-}obj\ G\ Object)\ .;\ P)]\!] \Longrightarrow$
$G,(A::'a\ triple\ set)\vdash\{Normal\ P\leftarrow\Diamond\}\ .Init\ Object.\ \{P \wedge.\ initd\ Object\}$
apply (*rule-tac C = initd Object* **in** *ax-cases*)
apply (*rule conseq1*, *rule ax-derivs.Done*, *clarsimp*)
apply (*erule ax-Init-Object* [*THEN conseq1*])
apply *force*
done

introduction rules for Alloc and SXAlloc

lemma *ax-SXAlloc-Normal*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ .c.\ \{Normal\ Q\}$
$\Longrightarrow G,A\vdash\{P\}\ .c.\ \{SXAlloc\ G\ Q\}$
apply (*erule conseq2*)
apply (*clarsimp elim*!: *sxalloc-elim-cases simp add*: *split-tupled-all*)
done

lemma *ax-Alloc*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ$
$\{Normal\ (\lambda Y\ (x,s)\ Z.\ (\forall a.\ new\text{-}Addr\ (heap\ s) = Some\ a \longrightarrow$
$Q\ (Val\ (Addr\ a))\ (Norm(init\text{-}obj\ G\ (CInst\ C)\ (Heap\ a)\ s))\ Z)) \wedge.$
$heap\text{-}free\ (Suc\ (Suc\ 0))\}$
$\Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Alloc\ G\ (CInst\ C)\ Q\}$
apply (*erule conseq2*)
apply (*auto elim*!: *halloc-elim-cases*)
done

lemma *ax-Alloc-Arr*:
$G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ$
$\{\lambda Val{:}i{:}.\ Normal\ (\lambda Y\ (x,s)\ Z.\ \neg the\text{-}Intg\ i{<}0\ \wedge$
$(\forall a.\ new\text{-}Addr\ (heap\ s) = Some\ a \longrightarrow$
$Q\ (Val\ (Addr\ a))\ (Norm\ (init\text{-}obj\ G\ (Arr\ T\ (the\text{-}Intg\ i))\ (Heap\ a)\ s))\ Z)) \wedge.$
$heap\text{-}free\ (Suc\ (Suc\ 0))\}$
$\Longrightarrow$
$G,A\vdash\{P\}\ t\succ\ \{\lambda Val{:}i{:}.\ abupd\ (check\text{-}neg\ i)\ .;\ Alloc\ G\ (Arr\ T(the\text{-}Intg\ i))\ Q\}$
apply (*erule conseq2*)
apply (*auto elim*!: *halloc-elim-cases*)
done

lemma *ax-SXAlloc-catch-SXcpt*:
$[\![G,(A::'a\ triple\ set)\vdash\{P::'a\ assn\}\ t\succ$
$\{(\lambda Y\ (x,s)\ Z.\ x{=}Some\ (Xcpt\ (Std\ xn)) \wedge$
$(\forall a.\ new\text{-}Addr\ (heap\ s) = Some\ a \longrightarrow$
$Q\ Y\ (Some\ (Xcpt\ (Loc\ a)),init\text{-}obj\ G\ (CInst\ (SXcpt\ xn))\ (Heap\ a)\ s)\ Z))$
$\wedge.\ heap\text{-}free\ (Suc\ (Suc\ 0))\}]\!]$
$\Longrightarrow$
$G,A\vdash\{P\}\ t\succ\ \{SXAlloc\ G\ (\lambda Y\ s\ Z.\ Q\ Y\ s\ Z \wedge G,s\vdash catch\ SXcpt\ xn)\}$
apply (*erule conseq2*)
apply (*auto elim*!: *sxalloc-elim-cases halloc-elim-cases*)
done

end

Chapter 23

AxSound

62 Soundness proof for Axiomatic semantics of Java expressions and statements

theory *AxSound* **imports** *AxSem* **begin**

validity

consts

triple-valid2:: *prog* ⇒ *nat* ⇒ *'a triple* ⇒ *bool* (-⊨-::-[61,0, 58] 57)
ax-valids2:: *prog* ⇒ *'a triples* ⇒ *'a triples* ⇒ *bool* (-,-|⊨::- [61,58,58] 57)

defs *triple-valid2-def*: $G \models n{::}t \equiv case\ t\ of\ \{P\}\ t\succ\ \{Q\} \Rightarrow$
$\forall Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow (\forall L.\ s{::}\preceq(G,L)$
$\longrightarrow (\forall T\ C\ A.\ (normal\ s \longrightarrow ((\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash t{::}T \wedge$
$(\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash dom\ (locals\ (store\ s))\text{»}t\text{»}A)) \longrightarrow$
$(\forall Y'\ s'.\ G\vdash s\ {-}t\succ{-}n\rightarrow\ (Y',s') \longrightarrow Q\ Y'\ s'\ Z \wedge s'{::}\preceq(G,L))))$

This definition differs from the ordinary *triple-valid-def* manly in the conclusion: We also ensures conformance of the result state. So we don't have to apply the type soundness lemma all the time during induction. This definition is only introduced for the soundness proof of the axiomatic semantics, in the end we will conclude to the ordinary definition.

defs *ax-valids2-def*: $G,A|\models{::}ts \equiv\ \forall n.\ (\forall t{\in}A.\ G\models n{::}t) \longrightarrow (\forall t{\in}ts.\ G\models n{::}t)$

lemma *triple-valid2-def2*: $G\models n{::}\{P\}\ t\succ\ \{Q\} =$
$(\forall Y\ s\ Z.\ P\ Y\ s\ Z \longrightarrow (\forall Y'\ s'.\ G\vdash s\ {-}t\succ{-}n\rightarrow\ (Y',s')\longrightarrow$
$(\forall L.\ s{::}\preceq(G,L) \longrightarrow (\forall T\ C\ A.\ (normal\ s \longrightarrow ((\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash t{::}T \wedge$
$(\!|prg{=}G,cls{=}C,lcl{=}L|\!)\vdash dom\ (locals\ (store\ s))\text{»}t\text{»}A)) \longrightarrow$
$Q\ Y'\ s'\ Z \wedge s'{::}\preceq(G,L))))))$
apply (*unfold triple-valid2-def*)
apply (*simp* (*no-asm*) *add*: *split-paired-All*)
apply *blast*
done

lemma *triple-valid2-eq* [*rule-format* (*no-asm*)]:
wf-prog G ==> triple-valid2 G = triple-valid G
apply (*rule ext*)
apply (*rule ext*)
apply (*rule triple.induct*)
apply (*simp* (*no-asm*) *add*: *triple-valid-def2 triple-valid2-def2*)
apply (*rule iffI*)
apply *fast*
apply *clarify*
apply (*tactic smp-tac 3 1*)
apply (*case-tac normal s*)
apply *clarsimp*
apply (*elim conjE impE*)
apply *blast*

apply (*tactic smp-tac 2 1*)
apply (*drule evaln-eval*)
apply (*drule* (*1*) *eval-type-sound* [*THEN conjunct1*],*simp*, *assumption+*)
apply *simp*

apply *clarsimp*
done

lemma *ax-valids2-eq*: *wf-prog G* $\Longrightarrow$ *G*,*A*$|\models$::*ts* = *G*,*A*$|\models$*ts*
apply (*unfold ax-valids-def ax-valids2-def*)
apply (*force simp add*: *triple-valid2-eq*)
done

lemma *triple-valid2-Suc* [*rule-format* (*no-asm*)]: *G*$\models$*Suc n*::*t* $\longrightarrow$ *G*$\models$*n*::*t*
apply (*induct-tac t*)
apply (*subst triple-valid2-def2*)
apply (*subst triple-valid2-def2*)
apply (*fast intro*: *evaln-nonstrict-Suc*)
done

lemma *Methd-triple-valid2-0*: *G*$\models$*0*::{*Normal P*} *Methd C sig*$-\succ$ {*Q*}
apply (*clarsimp elim*!: *evaln-elim-cases simp add*: *triple-valid2-def2*)
done

lemma *Methd-triple-valid2-SucI*:
$[\![$*G*$\models$*n*::{*Normal P*} *body G C sig*$-\succ${*Q*}$]\!]$
  $\Longrightarrow$ *G*$\models$*Suc n*::{*Normal P*} *Methd C sig*$-\succ$ {*Q*}
apply (*simp* (*no-asm-use*) *add*: *triple-valid2-def2*)
apply (*intro strip*, *tactic smp-tac 3 1*, *clarify*)
apply (*erule wt-elim-cases*, *erule da-elim-cases*, *erule evaln-elim-cases*)
apply (*unfold body-def Let-def*)
apply (*clarsimp simp add*: *inj-term-simps*)
apply *blast*
done

lemma *triples-valid2-Suc*:
 Ball ts (*triple-valid2 G* (*Suc n*)) $\Longrightarrow$ *Ball ts* (*triple-valid2 G n*)
apply (*fast intro*: *triple-valid2-Suc*)
done

lemma *G*$|\models$*n*:*insert t A* = (*G*$\models$*n*:*t* $\wedge$ *G*$|\models$*n*:*A*)
oops

soundness

lemma *Methd-sound*:
  assumes *recursive*: *G*,*A*$\cup$ {{*P*} *Methd*$-\succ$ {*Q*} | *ms*}$|\models$::{{*P*} *body G*$-\succ$ {*Q*} | *ms*}
  shows *G*,*A*$|\models$::{{*P*} *Methd*$-\succ$ {*Q*} | *ms*}
proof −
  {
    fix *n*
    assume *recursive*: $\bigwedge$ *n*. $\forall$ *t*$\in$(*A* $\cup$ {{*P*} *Methd*$-\succ$ {*Q*} | *ms*}). *G*$\models$*n*::*t*
                $\Longrightarrow$ $\forall$ *t*$\in${{*P*} *body G*$-\succ$ {*Q*} | *ms*}. *G*$\models$*n*::*t*
    have $\forall$ *t*$\in$*A*. *G*$\models$*n*::*t* $\Longrightarrow$ $\forall$ *t*$\in${{*P*} *Methd*$-\succ$ {*Q*} | *ms*}. *G*$\models$*n*::*t*
    proof (*induct n*)
      case *0*
      show $\forall$ *t*$\in${{*P*} *Methd*$-\succ$ {*Q*} | *ms*}. *G*$\models$*0*::*t*
      proof −
        {

```
      fix C sig
      assume (C,sig) ∈ ms
      have G⊨0::{Normal (P C sig)} Methd C sig−≻ {Q C sig}
        by (rule Methd-triple-valid2-0)
    }
    thus ?thesis
      by (simp add: mtriples-def split-def)
  qed
next
  case (Suc m)
  note hyp = ‹∀ t∈A. G⊨m::t ⟹ ∀ t∈{{P} Methd−≻ {Q} | ms}. G⊨m::t›
  note prem = ‹∀ t∈A. G⊨Suc m::t›
  show ∀ t∈{{P} Methd−≻ {Q} | ms}. G⊨Suc m::t
  proof −
    {
      fix C sig
      assume m: (C,sig) ∈ ms
      have G⊨Suc m::{Normal (P C sig)} Methd C sig−≻ {Q C sig}
      proof −
        from prem have prem-m: ∀ t∈A. G⊨m::t
          by (rule triples-valid2-Suc)
        hence ∀ t∈{{P} Methd−≻ {Q} | ms}. G⊨m::t
          by (rule hyp)
        with prem-m
        have ∀ t∈(A ∪ {{P} Methd−≻ {Q} | ms}). G⊨m::t
          by (simp add: ball-Un)
        hence ∀ t∈{{P} body G−≻ {Q} | ms}. G⊨m::t
          by (rule recursive)
        with m have G⊨m::{Normal (P C sig)} body G C sig−≻ {Q C sig}
          by (auto simp add: mtriples-def split-def)
        thus ?thesis
          by (rule Methd-triple-valid2-SucI)
      qed
    }
    thus ?thesis
      by (simp add: mtriples-def split-def)
  qed
qed
}
with recursive show ?thesis
  by (unfold ax-valids2-def) blast
qed
```

```
lemma valids2-inductI: ∀ s t n Y' s'. G⊢s−t≻−n→ (Y',s') ⟶ t = c ⟶
  Ball A (triple-valid2 G n) ⟶ (∀ Y Z. P Y s Z ⟶
  (∀ L. s::≼(G,L) ⟶
    (∀ T C A. (normal s ⟶ (⦇prg=G,cls=C,lcl=L⦈⊢t::T) ∧
                          ⦇prg=G,cls=C,lcl=L⦈⊢dom (locals (store s))»t»A) ⟶
    Q Y' s' Z ∧ s'::≼(G, L)))) ⟹
  G,A|⊨::{ {P} c≻ {Q}}
apply (simp (no-asm) add: ax-valids2-def triple-valid2-def2)
apply clarsimp
done
```

```
lemma da-good-approx-evalnE [consumes 4]:
  assumes evaln: G⊢s0 −t≻−n→ (v, s1)
```

```
    and      wt: (|prg=G,cls=C,lcl=L|)⊢t::T
    and      da: (|prg=G,cls=C,lcl=L|)⊢ dom (locals (store s0)) »t» A
    and      wf: wf-prog G
    and    elim: [[normal s1 ⟹ nrm A ⊆ dom (locals (store s1));
                  ⋀ l. [[abrupt s1 = Some (Jump (Break l)); normal s0]]
                       ⟹ brk A l ⊆ dom (locals (store s1));
                  [[abrupt s1 = Some (Jump Ret);normal s0]]
                  ⟹Result ∈ dom (locals (store s1))
                  ]] ⟹ P
  shows P
proof −
  from evaln have G⊢s0 −t≻→ (v, s1)
    by (rule evaln-eval)
  from this wt da wf elim show P
    by (rule da-good-approxE') iprover+
qed


lemma validI:
   assumes I: ⋀ n s0 L accC T C v s1 Y Z.
              [[∀ t∈A. G⊨n::t; s0::≼(G,L);
              normal s0 ⟹ (|prg=G,cls=accC,lcl=L|)⊢t::T;
              normal s0 ⟹ (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»t»C;
              G⊢s0 −t≻−n→ (v,s1); P Y s0 Z]] ⟹ Q v s1 Z ∧ s1::≼(G,L)
  shows G,A|⊨::{ {P} t≻ {Q} }
apply (simp add: ax-valids2-def triple-valid2-def2)
apply (intro allI impI)
apply (case-tac normal s)
apply   clarsimp
apply   (rule I,(assumption|simp)+)

apply   (rule I,auto)
done


declare [[simproc add: wt-expr wt-var wt-exprs wt-stmt]]


lemma valid-stmtI:
   assumes I: ⋀ n s0 L accC C s1 Y Z.
             [[∀ t∈A. G⊨n::t; s0::≼(G,L);
              normal s0⟹ (|prg=G,cls=accC,lcl=L|)⊢c::√;
              normal s0⟹(|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨c⟩ₛ»C;
              G⊢s0 −c−n→ s1; P Y s0 Z]] ⟹ Q ◇ s1 Z ∧ s1::≼(G,L)
  shows G,A|⊨::{ {P} ⟨c⟩ₛ≻ {Q} }
apply (simp add: ax-valids2-def triple-valid2-def2)
apply (intro allI impI)
apply (case-tac normal s)
apply   clarsimp
apply   (rule I,(assumption|simp)+)

apply   (rule I,auto)
done


lemma valid-stmt-NormalI:
   assumes I: ⋀ n s0 L accC C s1 Y Z.
              [[∀ t∈A. G⊨n::t; s0::≼(G,L); normal s0; (|prg=G,cls=accC,lcl=L|)⊢c::√;
              (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨c⟩ₛ»C;
```

$G\vdash s0\ -c-n\rightarrow\ s1$; $(Normal\ P)\ Y\ s0\ Z]\!] \Longrightarrow Q \diamondsuit s1\ Z \wedge s1::\preceq(G,L)$
shows $G,A|\models::\{\ \{Normal\ P\}\ \langle c\rangle_s\succ\ \{Q\}\ \}$
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*elim exE conjE*)
apply (*rule I*)
by *auto*

lemma *valid-var-NormalI*:
assumes I: $\bigwedge\ n\ s0\ L\ accC\ T\ C\ vf\ s1\ Y\ Z.$
$[\![\forall t\in A.\ G\models n::t;\ s0::\preceq(G,L);\ normal\ s0;$
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash t::=T;$
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash dom\ (locals\ (store\ s0))\gg\langle t\rangle_v\gg C;$
$G\vdash s0\ -t=\succ vf-n\rightarrow\ s1;\ (Normal\ P)\ Y\ s0\ Z]\!]$
$\Longrightarrow Q\ (In2\ vf)\ s1\ Z \wedge s1::\preceq(G,L)$
shows $G,A|\models::\{\ \{Normal\ P\}\ \langle t\rangle_v\succ\ \{Q\}\ \}$
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*elim exE conjE*)
apply *simp*
apply (*rule I*)
by *auto*

lemma *valid-expr-NormalI*:
assumes I: $\bigwedge\ n\ s0\ L\ accC\ T\ C\ v\ s1\ Y\ Z.$
$[\![\forall t\in A.\ G\models n::t;\ s0::\preceq(G,L);\ normal\ s0;$
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash t::-T;$
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash dom\ (locals\ (store\ s0))\gg\langle t\rangle_e\gg C;$
$G\vdash s0\ -t-\succ v-n\rightarrow\ s1;\ (Normal\ P)\ Y\ s0\ Z]\!]$
$\Longrightarrow Q\ (In1\ v)\ s1\ Z \wedge s1::\preceq(G,L)$
shows $G,A|\models::\{\ \{Normal\ P\}\ \langle t\rangle_e\succ\ \{Q\}\ \}$
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*elim exE conjE*)
apply *simp*
apply (*rule I*)
by *auto*

lemma *valid-expr-list-NormalI*:
assumes I: $\bigwedge\ n\ s0\ L\ accC\ T\ C\ vs\ s1\ Y\ Z.$
$[\![\forall t\in A.\ G\models n::t;\ s0::\preceq(G,L);\ normal\ s0;$
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash t::\doteq T;$
$(\!|prg=G,cls=accC,lcl=L|\!)\vdash dom\ (locals\ (store\ s0))\gg\langle t\rangle_l\gg C;$
$G\vdash s0\ -t\doteq\succ vs-n\rightarrow\ s1;\ (Normal\ P)\ Y\ s0\ Z]\!]$
$\Longrightarrow Q\ (In3\ vs)\ s1\ Z \wedge s1::\preceq(G,L)$
shows $G,A|\models::\{\ \{Normal\ P\}\ \langle t\rangle_l\succ\ \{Q\}\ \}$
apply (*simp add*: *ax-valids2-def triple-valid2-def2*)
apply (*intro allI impI*)
apply (*elim exE conjE*)
apply *simp*
apply (*rule I*)
by *auto*

lemma *validE* [*consumes 5*]:
assumes *valid*: $G,A|\models::\{\ \{P\}\ t\succ\ \{Q\}\ \}$

and *P*: *P Y s0 Z*
and *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
and *conf*: *s0*::≼(*G*,*L*)
and *eval*: *G*⊢*s0* −*t*≻−*n*→ (*v*,*s1*)
and *wt*: *normal s0* ⟹ ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*t*::*T*
and *da*: *normal s0* ⟹ ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»*t*»*C*
and *elim*: ⟦*Q v s1 Z*; *s1*::≼(*G*,*L*)⟧ ⟹ *concl*
shows *concl*
using *prems*
by (*simp add*: *ax-valids2-def triple-valid2-def2*) *fast*

lemma *all-empty*: (!*x*. *P*) = *P*
by *simp*

corollary *evaln-type-sound*:
assumes *evaln*: *G*⊢*s0* −*t*≻−*n*→ (*v*,*s1*) **and**
wt: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*t*::*T* **and**
da: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*)) »*t*» *A* **and**
conf-s0: *s0*::≼(*G*,*L*) **and**
wf: *wf-prog G*
shows *s1*::≼(*G*,*L*) ∧ (*normal s1* ⟶ *G*,*L*,*store s1*⊢*t*≻*v*::≼*T*) ∧
(*error-free s0* = *error-free s1*)
proof −
from *evaln* **have** *G*⊢*s0* −*t*≻→ (*v*,*s1*)
by (*rule evaln-eval*)
from *this wt da wf conf-s0* **show** *?thesis*
by (*rule eval-type-sound*)
qed

corollary *dom-locals-evaln-mono-elim* [*consumes 1*]:
assumes
evaln: *G*⊢ *s0* −*t*≻−*n*→ (*v*,*s1*) **and**
hyps: ⟦*dom* (*locals* (*store s0*)) ⊆ *dom* (*locals* (*store s1*));
⋀ *vv s val*. ⟦*v*=*In2 vv*; *normal s1*⟧
⟹ *dom* (*locals* (*store s*))
⊆ *dom* (*locals* (*store* ((*snd vv*) *val s*)))⟧ ⟹ *P*
shows *P*
proof −
from *evaln* **have** *G*⊢ *s0* −*t*≻→ (*v*,*s1*) **by** (*rule evaln-eval*)
from *this hyps* **show** *?thesis*
by (*rule dom-locals-eval-mono-elim*) *iprover*+
qed

lemma *evaln-no-abrupt*:
⋀*s s*′. ⟦*G*⊢*s* −*t*≻−*n*→ (*w*,*s*′); *normal s*′⟧ ⟹ *normal s*
by (*erule evaln-cases*,*auto*)

declare *inj-term-simps* [*simp*]

lemma *ax-sound2*:
assumes *wf*: *wf-prog G*
and *deriv*: *G*,*A*|⊢*ts*
shows *G*,*A*|⊨::*ts*
using *deriv*

```
proof (induct)
  case (empty A)
  show ?case
    by (simp add: ax-valids2-def triple-valid2-def2)
next
  case (insert A t ts)
  note valid-t = ‹G,A|⊨::{t}›
  moreover
  note valid-ts = ‹G,A|⊨::ts›
  {
    fix n assume valid-A: ∀ t∈A. G⊨n::t
    have G⊨n::t and ∀ t∈ts. G⊨n::t
    proof −
      from valid-A valid-t show G⊨n::t
        by (simp add: ax-valids2-def)
    next
      from valid-A valid-ts show ∀ t∈ts. G⊨n::t
        by (unfold ax-valids2-def) blast
    qed
    hence ∀ t'∈insert t ts. G⊨n::t'
      by simp
  }
  thus ?case
    by (unfold ax-valids2-def) blast
next
  case (asm ts A)
  from ‹ts ⊆ A›
  show G,A|⊨::ts
    by (auto simp add: ax-valids2-def triple-valid2-def)
next
  case (weaken A ts' ts)
  note ‹G,A|⊨::ts'›
  moreover note ‹ts ⊆ ts'›
  ultimately show G,A|⊨::ts
    by (unfold ax-valids2-def triple-valid2-def) blast
next
  case (conseq P A t Q)
  note con = ‹∀ Y s Z. P Y s Z ⟶
            (∃ P' Q'.
                (G,A⊢{P'} t≻ {Q'} ∧ G,A|⊨::{ {P'} t≻ {Q'} }) ∧
                (∀ Y' s'. (∀ Y Z'. P' Y s Z' ⟶ Q' Y' s' Z') ⟶ Q Y' s' Z))›
  show G,A|⊨::{ {P} t≻ {Q} }
  proof (rule validI)
    fix n s0 L accC T C v s1 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf: s0::≼(G,L)
    assume wt: normal s0 ⟹ ⦇prg=G,cls=accC,lcl=L⦈⊢t::T
    assume da: normal s0
                ⟹ ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0)) »t» C
    assume eval: G⊢s0 −t≻−n→ (v, s1)
    assume P: P Y s0 Z
    show Q v s1 Z ∧ s1::≼(G, L)
    proof −
      from valid-A conf wt da eval P con
      have Q v s1 Z
        apply (simp add: ax-valids2-def triple-valid2-def2)
        apply (tactic smp-tac 3 1)
        apply clarify
        apply (tactic smp-tac 1 1)
```

```
      apply (erule allE,erule allE, erule mp)
      apply (intro strip)
      apply (tactic smp-tac 3 1)
      apply (tactic smp-tac 2 1)
      apply (tactic smp-tac 1 1)
      by blast
    moreover have s1::≼(G, L)
    proof (cases normal s0)
      case True
      from eval wt [OF True] da [OF True] conf wf
      show ?thesis
        by (rule evaln-type-sound [elim-format]) simp
    next
      case False
      with eval have s1=s0
        by auto
      with conf show ?thesis by simp
    qed
    ultimately show ?thesis ..
  qed
 qed
next
 case (hazard A P t Q)
 show G,A|⊨::{ {P ∧. Not ∘ type-ok G t} t≻ {Q} }
  by (simp add: ax-valids2-def triple-valid2-def2 type-ok-def) fast
next
 case (Abrupt A P t)
 show G,A|⊨::{ {P←arbitrary3 t ∧. Not ∘ normal} t≻ {P} }
 proof (rule validI)
  fix n s0 L accC T C v s1 Y Z
  assume conf-s0: s0::≼(G, L)
  assume eval: G⊢s0 −t≻−n→ (v, s1)
  assume (P←arbitrary3 t ∧. Not ∘ normal) Y s0 Z
  then obtain P: P (arbitrary3 t) s0 Z and abrupt-s0: ¬ normal s0
    by simp
  from eval abrupt-s0 obtain s1=s0 and v=arbitrary3 t
    by auto
  with P conf-s0
  show P v s1 Z ∧ s1::≼(G, L)
    by simp
 qed
next
 case (LVar A P vn)
 show G,A|⊨::{ {Normal (λs.. P←In2 (lvar vn s))} LVar vn=≻ {P} }
 proof (rule valid-var-NormalI)
  fix n s0 L accC T C vf s1 Y Z
  assume conf-s0: s0::≼(G, L)
  assume normal-s0: normal s0
  assume wt: (|prg = G, cls = accC, lcl = L|)⊢LVar vn::=T
  assume da: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨LVar vn⟩_v» C
  assume eval: G⊢s0 −LVar vn=≻vf−n→ s1
  assume P: (Normal (λs.. P←In2 (lvar vn s))) Y s0 Z
  show P (In2 vf) s1 Z ∧ s1::≼(G, L)
  proof
    from eval normal-s0 obtain s1=s0 vf=lvar vn (store s0)
      by (fastsimp elim: evaln-elim-cases)
    with P show P (In2 vf) s1 Z
      by simp
  next
```

from *eval wt da conf-s0 wf*
show $s1::\preceq(G,\ L)$
by (*rule evaln-type-sound* [*elim-format*]) *simp*
qed
qed
next
case (*FVar A P statDeclC Q e stat fn R accC*)
note *valid-init* = ‹$G,A|\models::\{$ {*Normal P*} .*Init statDeclC*. {*Q*} }›
note *valid-e* = ‹$G,A|\models::\{$ {*Q*} $e{-}\succ$ {λ*Val:a:. fvar statDeclC stat fn a* ..; *R*} }›
show $G,A|\models::\{$ {*Normal P*} {*accC,statDeclC,stat*}*e..fn*$=\succ$ {*R*} }
proof (*rule valid-var-NormalI*)
fix *n s0 L accC′ T V vf s3 Y Z*
assume *valid-A*: $\forall\, t{\in}A.\ G\models n::t$
assume *conf-s0*: $s0::\preceq(G,L)$
assume *normal-s0*: *normal s0*
assume *wt*: (|*prg=G,cls=accC′,lcl=L*|)$\vdash${*accC,statDeclC,stat*}*e..fn*$::=T$
assume *da*: (|*prg=G,cls=accC′,lcl=L*|)
$\vdash$ *dom* (*locals* (*store s0*)) »⟨{*accC,statDeclC,stat*}*e..fn*⟩$_v$» *V*
assume *eval*: $G\vdash s0$ −{*accC,statDeclC,stat*}*e..fn*$=\succ$*vf*$-n\rightarrow$ *s3*
assume *P*: (*Normal P*) *Y s0 Z*
show *R* $\lfloor vf\rfloor_v$ *s3 Z* $\wedge$ $s3::\preceq(G,\ L)$
proof −
from *wt* **obtain** *statC f* **where**
wt-e: (|*prg=G, cls=accC, lcl=L*|)$\vdash e::-$*Class statC* **and**
accfield: *accfield G accC statC fn = Some* (*statDeclC,f*) **and**
eq-accC: *accC=accC′* **and**
stat: *stat=is-static f* **and**
T: *T=*(*type f*)
by (*cases*) (*auto simp add*: *member-is-static-simp*)
from *da eq-accC*
have *da-e*: (|*prg=G, cls=accC, lcl=L*|)$\vdash$*dom* (*locals* (*store s0*))»⟨*e*⟩$_e$» *V*
by *cases simp*
from *eval* **obtain** *a s1 s2 s2′* **where**
eval-init: $G\vdash s0$ −*Init statDeclC*$-n\rightarrow$ *s1* **and**
eval-e: $G\vdash s1\ {-}e{-}\succ a{-}n\rightarrow$ *s2* **and**
fvar: (*vf,s2′*)=*fvar statDeclC stat fn a s2* **and**
s3: *s3 = check-field-access G accC statDeclC fn stat a s2′*
using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
have *wt-init*: (|*prg=G, cls=accC, lcl=L*|)$\vdash$(*Init statDeclC*)$::\surd$
proof −
from *wf wt-e*
have *iscls-statC*: *is-class G statC*
by (*auto dest*: *ty-expr-is-type type-is-class*)
with *wf accfield*
have *iscls-statDeclC*: *is-class G statDeclC*
by (*auto dest!*: *accfield-fields dest*: *fields-declC*)
thus *?thesis* **by** *simp*
qed
obtain *I* **where**
da-init: (|*prg=G,cls=accC,lcl=L*|)
$\vdash$ *dom* (*locals* (*store s0*)) »⟨*Init statDeclC*⟩$_s$» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)
from *valid-init P valid-A conf-s0 eval-init wt-init da-init*
obtain *Q*: *Q* $\diamondsuit$ *s1 Z* **and** *conf-s1*: $s1::\preceq(G,\ L)$
by (*rule validE*)
obtain
R: *R* $\lfloor vf\rfloor_v$ *s2′ Z* **and**
conf-s2: $s2::\preceq(G,\ L)$ **and**
conf-a: *normal s2* $\longrightarrow$ *G,store s2*$\vdash a::\preceq$*Class statC*

proof (*cases normal s1*)
case *True*
obtain V' **where**
da-e':
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!) \vdash dom\ (locals\ (store\ s1)) \text{»}\langle e\rangle_e\text{»}\ V'$
proof −
from *eval-init*
have $(dom\ (locals\ (store\ s0))) \subseteq (dom\ (locals\ (store\ s1)))$
by (*rule dom-locals-evaln-mono-elim*)
with *da-e* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *valid-e Q valid-A conf-s1 eval-e wt-e*
obtain $R\ \lfloor vf\rfloor_v\ s2'\ Z$ **and** $s2{::}\preceq(G,\ L)$
by (*rule validE*) (*simp add*: *fvar* [*symmetric*])
moreover
from *eval-e wt-e da-e' conf-s1 wf*
have $normal\ s2 \longrightarrow G{,}store\ s2 \vdash a{::}\preceq Class\ statC$
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
next
case *False*
with *valid-e Q valid-A conf-s1 eval-e*
obtain $R\ \lfloor vf\rfloor_v\ s2'\ Z$ **and** $s2{::}\preceq(G,\ L)$
by (*cases rule*: *validE*) (*simp add*: *fvar* [*symmetric*])+
moreover from *False eval-e* **have** $\neg$ *normal s2*
by *auto*
hence $normal\ s2 \longrightarrow G{,}store\ s2 \vdash a{::}\preceq Class\ statC$
by *auto*
ultimately show *?thesis* **..**
qed
from *accfield wt-e eval-init eval-e conf-s2 conf-a fvar stat s3 wf*
have *eq-s3-s2'*: $s3{=}s2'$
using *normal-s0* **by** (*auto dest!*: *error-free-field-access evaln-eval*)
moreover
from *eval wt da conf-s0 wf*
have $s3{::}\preceq(G,\ L)$
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **using** *Q R* **by** *simp*
qed
qed
next
case (*AVar A P e1 Q e2 R*)
note *valid-e1* = $\langle G{,}A|\!\models{::}\{\ \{Normal\ P\}\ e1{-}\!\succ\ \{Q\}\ \}\rangle$
have *valid-e2*: $\bigwedge\ a.\ G{,}A|\!\models{::}\{\ \{Q{\leftarrow}In1\ a\}\ e2{-}\!\succ\ \{\lambda Val{:}i{:}.\ avar\ G\ i\ a\ ..;\ R\}\ \}$
using *AVar.hyps* **by** *simp*
show $G{,}A|\!\models{::}\{\ \{Normal\ P\}\ e1.[e2]{=}\!\succ\ \{R\}\ \}$
proof (*rule valid-var-NormalI*)
fix $n\ s0\ L\ accC\ T\ V\ vf\ s2'\ Y\ Z$
assume *valid-A*: $\forall t{\in}A.\ G|\!\models n{::}t$
assume *conf-s0*: $s0{::}\preceq(G{,}L)$
assume *normal-s0*: *normal s0*
assume *wt*: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!) \vdash e1.[e2]{::}{=}T$
assume *da*: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash\ dom\ (locals\ (store\ s0))\ \text{»}\langle e1.[e2]\rangle_v\text{»}\ V$
assume *eval*: $G\vdash s0\ {-}e1.[e2]{=}\!\succ vf{-}n{\rightarrow}\ s2'$
assume *P*: $(Normal\ P)\ Y\ s0\ Z$
show $R\ \lfloor vf\rfloor_v\ s2'\ Z \wedge s2'{::}\preceq(G,\ L)$
proof −

```
      from wt obtain
        wt-e1: (|prg=G,cls=accC,lcl=L|)⊢e1::−T.[] and
        wt-e2: (|prg=G,cls=accC,lcl=L|)⊢e2::−PrimT Integer
        by (rule wt-elim-cases) simp
      from da obtain E1 where
        da-e1: (|prg=G,cls=accC,lcl=L|) ⊢dom (locals (store s0))»⟨e1⟩e» E1 and
        da-e2: (|prg=G,cls=accC,lcl=L|)⊢ nrm E1 »⟨e2⟩e» V
        by (rule da-elim-cases) simp
      from eval obtain s1 a i s2 where
        eval-e1: G⊢s0 −e1−≻a−n→ s1 and
        eval-e2: G⊢s1 −e2−≻i−n→ s2 and
        avar: avar G i a s2 =(vf, s2')
        using normal-s0 by (fastsimp elim: evaln-elim-cases)
      from valid-e1 P valid-A conf-s0 eval-e1 wt-e1 da-e1
      obtain Q: Q ⌊a⌋e s1 Z and conf-s1: s1::≼(G, L)
        by (rule validE)
      from Q have Q': ⋀ v. (Q←In1 a) v s1 Z
        by simp
      have R ⌊vf⌋v s2' Z
      proof (cases normal s1)
        case True
        obtain V' where
          (|prg=G,cls=accC,lcl=L|) ⊢dom (locals (store s1))»⟨e2⟩e» V'
        proof −
          from eval-e1  wt-e1 da-e1 wf True
          have nrm E1 ⊆ dom (locals (store s1))
            by (cases rule: da-good-approx-evalnE) iprover
          with da-e2 show thesis
            by (rule da-weakenE) (rule that)
        qed
        with valid-e2 Q' valid-A conf-s1 eval-e2 wt-e2
        show ?thesis
          by (rule validE) (simp add: avar)
      next
        case False
        with valid-e2 Q' valid-A conf-s1 eval-e2
        show ?thesis
          by (cases rule: validE) (simp add: avar)+
      qed
      moreover
      from eval wt da conf-s0 wf
      have s2'::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (NewC A P C Q)
  note valid-init = ‹G,A|⊨::{ {Normal P} .Init C. {Alloc G (CInst C) Q} }›
  show G,A|⊨::{ {Normal P} NewC C−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s2 Y Z
    assume valid-A: ∀ t∈A. G|⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢NewC C::−T
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢ dom (locals (store s0)) »⟨NewC C⟩e» E
    assume eval: G⊢s0 −NewC C−≻v−n→ s2
```

assume *P*: (*Normal P*) *Y s0 Z*
show *Q* $\lfloor v \rfloor_e$ *s2 Z* ∧ *s2*::≼(*G*, *L*)
proof −
from *wt* **obtain** *is-cls-C*: *is-class G C*
by (*rule wt-elim-cases*) (*auto dest*: *is-acc-classD*)
hence *wt-init*: (|*prg*=*G*, *cls*=*accC*, *lcl*=*L*|)⊢*Init C*::√
by *auto*
obtain *I* **where**
da-init: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢ *dom* (*locals* (*store s0*)) »⟨*Init C*⟩$_s$» *I*
by (*auto intro*: *da-Init* [*simplified*] *assigned.select-convs*)
from *eval* **obtain** *s1 a* **where**
eval-init: *G*⊢*s0* −*Init C*−*n*→ *s1* **and**
alloc: *G*⊢*s1* −*halloc CInst C*≻*a*→ *s2* **and**
v: *v*=*Addr a*
using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
from *valid-init P valid-A conf-s0 eval-init wt-init da-init*
obtain (*Alloc G* (*CInst C*) *Q*) ♢ *s1 Z*
by (*rule validE*)
with *alloc v* **have** *Q* $\lfloor v \rfloor_e$ *s2 Z*
by *simp*
moreover
from *eval wt da conf-s0 wf*
have *s2*::≼(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*NewA A P T Q e R*)
note *valid-init* = ‹*G*,*A*|⊨::{ {*Normal P*} *.init-comp-ty T*. {*Q*} }›
note *valid-e* = ‹*G*,*A*|⊨::{ {*Q*} *e*−≻ {λ*Val*:*i*:. *abupd* (*check-neg i*) .;
Alloc G (*Arr T* (*the-Intg i*)) *R*}}›
show *G*,*A*|⊨::{ {*Normal P*} *New T*[*e*]−≻ {*R*} }
proof (*rule valid-expr-NormalI*)
fix *n s0 L accC arrT E v s3 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*New T*[*e*]::−*arrT*
assume *da*: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*dom* (*locals* (*store s0*)) »⟨*New T*[*e*]⟩$_e$» *E*
assume *eval*: *G*⊢*s0* −*New T*[*e*]−≻*v*−*n*→ *s3*
assume *P*: (*Normal P*) *Y s0 Z*
show *R* $\lfloor v \rfloor_e$ *s3 Z* ∧ *s3*::≼(*G*, *L*)
proof −
from *wt* **obtain**
wt-init: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*init-comp-ty T*::√ **and**
wt-e: (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢*e*::−*PrimT Integer*
by (*rule wt-elim-cases*) (*auto intro*: *wt-init-comp-ty*)
from *da* **obtain**
da-e:(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)⊢ *dom* (*locals* (*store s0*)) »⟨*e*⟩$_e$» *E*
by *cases simp*
from *eval* **obtain** *s1 i s2 a* **where**
eval-init: *G*⊢*s0* −*init-comp-ty T*−*n*→ *s1* **and**
eval-e: *G*⊢*s1* −*e*−≻*i*−*n*→ *s2* **and**
alloc: *G*⊢*abupd* (*check-neg i*) *s2* −*halloc Arr T* (*the-Intg i*)≻*a*→ *s3* **and**
v: *v*=*Addr a*
using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
obtain *I* **where**
da-init:

```
      (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0)) »⟨init-comp-ty T⟩s» I
    proof (cases ∃ C. T = Class C)
      case True
      thus ?thesis
        by − (rule that, (auto intro: da-Init [simplified]
                                   assigned.select-convs
                          simp add: init-comp-ty-def))

    next
      case False
      thus ?thesis
        by − (rule that, (auto intro: da-Skip [simplified]
                                   assigned.select-convs
                          simp add: init-comp-ty-def))

    qed
    with valid-init P valid-A conf-s0 eval-init wt-init
    obtain Q: Q ♢ s1 Z and conf-s1: s1::≼(G, L)
      by (rule validE)
    obtain E' where
     (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨e⟩e» E'
    proof −
      from eval-init
      have dom (locals (store s0)) ⊆ dom (locals (store s1))
        by (rule dom-locals-evaln-mono-elim)
      with da-e show thesis
        by (rule da-weakenE) (rule that)
    qed
    with valid-e Q valid-A conf-s1 eval-e wt-e
    have (λVal:i:. abupd (check-neg i) .;
                  Alloc G (Arr T (the-Intg i)) R) ⌊i⌋e s2 Z
      by (rule validE)
    with alloc v have R ⌊v⌋e s3 Z
      by simp
    moreover
    from eval wt da conf-s0 wf
    have s3::≼(G, L)
      by (rule evaln-type-sound [elim-format]) simp
    ultimately show ?thesis ..
  qed
 qed
next
 case (Cast A P e T Q)
 note valid-e = ‹G,A|⊨::{ {Normal P} e−≻
              {λVal:v:. λs.. abupd (raise-if (¬ G,s⊢v fits T) ClassCast) .;
               Q←In1 v} }›
 show G,A|⊨::{ {Normal P} Cast T e−≻ {Q} }
 proof (rule valid-expr-NormalI)
  fix n s0 L accC castT E v s2 Y Z
  assume valid-A: ∀ t∈A. G⊨n::t
  assume conf-s0: s0::≼(G,L)
  assume normal-s0: normal s0
  assume wt: (|prg=G,cls=accC,lcl=L|)⊢Cast T e::−castT
  assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0)) »⟨Cast T e⟩e» E
  assume eval: G⊢s0 −Cast T e−≻v−n→ s2
  assume P: (Normal P) Y s0 Z
  show Q ⌊v⌋e s2 Z ∧ s2::≼(G, L)
  proof −
    from wt obtain eT where
```

```
      wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT
      by cases simp
    from da obtain
      da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
      by cases simp
    from eval obtain s1 where
      eval-e: G⊢s0 −e−≻v−n→ s1 and
      s2: s2 = abupd (raise-if (¬ G,snd s1⊢v fits T) ClassCast) s1
      using normal-s0 by (fastsimp elim: evaln-elim-cases)
    from valid-e P valid-A conf-s0 eval-e wt-e da-e
    have (λVal:v:. λs.. abupd (raise-if (¬ G,s⊢v fits T) ClassCast) .;
              Q←In1 v) ⌊v⌋e s1 Z
      by (rule validE)
    with s2 have Q ⌊v⌋e s2 Z
      by simp
    moreover
    from eval wt da conf-s0 wf
    have s2::≼(G, L)
      by (rule evaln-type-sound [elim-format]) simp
    ultimately show ?thesis ..
  qed
 qed
next
 case (Inst A P e Q T)
 assume valid-e: G,A|⊨::{ {Normal P} e−≻
            {λVal:v:. λs.. Q←In1 (Bool (v ≠ Null ∧ G,s⊢v fits RefT T))} }
 show G,A|⊨::{ {Normal P} e InstOf T−≻ {Q} }
 proof (rule valid-expr-NormalI)
  fix n s0 L accC instT E v s1 Y Z
  assume valid-A: ∀ t∈A. G⊨n::t
  assume conf-s0: s0::≼(G,L)
  assume normal-s0: normal s0
  assume wt: (|prg=G,cls=accC,lcl=L|)⊢e InstOf T::−instT
  assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨e InstOf T⟩e» E
  assume eval: G⊢s0 −e InstOf T−≻v−n→ s1
  assume P: (Normal P) Y s0 Z
  show Q ⌊v⌋e s1 Z ∧ s1::≼(G, L)
  proof −
    from wt obtain eT where
      wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT
      by cases simp
    from da obtain
      da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
      by cases simp
    from eval obtain a where
      eval-e: G⊢s0 −e−≻a−n→ s1 and
      v: v = Bool (a ≠ Null ∧ G,store s1⊢a fits RefT T)
      using normal-s0 by (fastsimp elim: evaln-elim-cases)
    from valid-e P valid-A conf-s0 eval-e wt-e da-e
    have (λVal:v:. λs.. Q←In1 (Bool (v ≠ Null ∧ G,s⊢v fits RefT T)))
          ⌊a⌋e s1 Z
      by (rule validE)
    with v have Q ⌊v⌋e s1 Z
      by simp
    moreover
    from eval wt da conf-s0 wf
    have s1::≼(G, L)
      by (rule evaln-type-sound [elim-format]) simp
    ultimately show ?thesis ..
```

```
    qed
  qed
next
  case (Lit A P v)
  show G,A|⊨::{ {Normal (P←In1 v)} Lit v−≻ {P} }
  proof (rule valid-expr-NormalI)
    fix n L s0 s1 v'  Y Z
    assume conf-s0: s0::≼(G, L)
    assume normal-s0:  normal s0
    assume eval: G⊢s0 −Lit v−≻v'−n→ s1
    assume P: (Normal (P←In1 v)) Y s0 Z
    show P ⌊v'⌋e s1 Z ∧ s1::≼(G, L)
    proof −
      from eval have s1=s0 and  v'=v
        using normal-s0 by (auto elim: evaln-elim-cases)
      with P conf-s0 show ?thesis by simp
    qed
  qed
next
  case (UnOp A P e Q unop)
  assume valid-e: G,A|⊨::{ {Normal P}e−≻{λVal:v:. Q←In1 (eval-unop unop v)} }
  show G,A|⊨::{ {Normal P} UnOp unop e−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s1 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢UnOp unop e::−T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨UnOp unop e⟩e»E
    assume eval: G⊢s0 −UnOp unop e−≻v−n→ s1
    assume P: (Normal P) Y s0 Z
    show Q ⌊v⌋e s1 Z ∧ s1::≼(G, L)
    proof −
      from wt obtain eT where
        wt-e: (|prg = G, cls = accC, lcl = L|)⊢e::−eT
        by cases simp
      from da obtain
        da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
        by cases simp
      from eval obtain ve where
        eval-e: G⊢s0 −e−≻ve−n→ s1 and
        v: v = eval-unop unop ve
        using normal-s0 by (fastsimp elim: evaln-elim-cases)
      from valid-e P valid-A conf-s0 eval-e wt-e da-e
      have (λVal:v:. Q←In1 (eval-unop unop v)) ⌊ve⌋e s1 Z
        by (rule validE)
      with v have Q ⌊v⌋e s1 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s1::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (BinOp A P e1 Q binop e2 R)
  assume valid-e1: G,A|⊨::{ {Normal P} e1−≻ {Q} }
  have valid-e2: ⋀ v1.  G,A|⊨::{ {Q←In1 v1}
```

```
          (if need-second-arg binop v1 then In1l e2 else In1r Skip)≻
          {λVal:v2:. R←In1 (eval-binop binop v1 v2)} }
    using BinOp.hyps by simp
  show G,A|⊨::{ {Normal P} BinOp binop e1 e2−≻ {R} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s2 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢BinOp binop e1 e2::−T
    assume da: (|prg=G,cls=accC,lcl=L|)
                ⊢dom (locals (store s0)) »⟨BinOp binop e1 e2⟩e» E
    assume eval: G⊢s0 −BinOp binop e1 e2−≻v−n→ s2
    assume P: (Normal P) Y s0 Z
    show R ⌊v⌋e s2 Z ∧ s2::≼(G, L)
    proof −
      from wt obtain e1T e2T where
        wt-e1: (|prg=G,cls=accC,lcl=L|)⊢e1::−e1T and
        wt-e2: (|prg=G,cls=accC,lcl=L|)⊢e2::−e2T and
        wt-binop: wt-binop G binop e1T e2T
        by cases simp
      have wt-Skip: (|prg = G, cls = accC, lcl = L|)⊢Skip::√
        by simp

      from da obtain E1 where
        da-e1: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨e1⟩e» E1
        by cases simp+
      from eval obtain v1 s1 v2 where
        eval-e1: G⊢s0 −e1−≻v1−n→ s1 and
        eval-e2: G⊢s1 −(if need-second-arg binop v1 then ⟨e2⟩e else ⟨Skip⟩s)
                   ≻−n→ (⌊v2⌋e, s2) and
        v: v=eval-binop binop v1 v2
        using normal-s0 by (fastsimp elim: evaln-elim-cases)
      from valid-e1 P valid-A conf-s0 eval-e1 wt-e1 da-e1
      obtain Q: Q ⌊v1⌋e s1 Z and conf-s1: s1::≼(G,L)
        by (rule validE)
      from Q have Q': ⋀ v. (Q←In1 v1) v s1 Z
        by simp
      have (λVal:v2:. R←In1 (eval-binop binop v1 v2)) ⌊v2⌋e s2 Z
      proof (cases normal s1)
        case True
        from eval-e1 wt-e1 da-e1 conf-s0 wf
        have conf-v1: G,store s1⊢v1::≼e1T
          by (rule evaln-type-sound [elim-format]) (insert True,simp)
        from eval-e1
        have G⊢s0 −e1−≻v1→ s1
          by (rule evaln-eval)
        from da wt-e1 wt-e2 wt-binop conf-s0 True this conf-v1 wf
        obtain E2 where
          da-e2: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1))
                  »(if need-second-arg binop v1 then ⟨e2⟩e else ⟨Skip⟩s)» E2
          by (rule da-e2-BinOp [elim-format]) iprover
        from wt-e2 wt-Skip obtain T2
          where (|prg=G,cls=accC,lcl=L|)
                ⊢(if need-second-arg binop v1 then ⟨e2⟩e else ⟨Skip⟩s)::T2
          by (cases need-second-arg binop v1) auto
        note ve=validE [OF valid-e2,OF Q' valid-A conf-s1 eval-e2 this da-e2]

        thus ?thesis
```

```
        by (rule ve)
      next
        case False
        note ve=validE [OF valid-e2,OF Q' valid-A conf-s1 eval-e2]
        with False show ?thesis
          by iprover
      qed
      with v have R ⌊v⌋e s2 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s2::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Super A P)
  show G,A|⊨::{ {Normal (λs.. P←In1 (val-this s))} Super−≻ {P} }
  proof (rule valid-expr-NormalI)
    fix n L s0 s1 v  Y Z
    assume conf-s0: s0::≼(G, L)
    assume normal-s0:  normal s0
    assume eval: G⊢s0 −Super−≻v−n→ s1
    assume P: (Normal (λs.. P←In1 (val-this s))) Y s0 Z
    show P ⌊v⌋e s1 Z ∧ s1::≼(G, L)
    proof −
      from eval have s1=s0 and  v=val-this (store s0)
        using normal-s0 by (auto elim: evaln-elim-cases)
      with P conf-s0 show ?thesis by simp
    qed
  qed
next
  case (Acc A P var Q)
  note valid-var = ‹G,A|⊨::{ {Normal P} var=≻ {λVar:(v, f):. Q←In1 v} }›
  show G,A|⊨::{ {Normal P} Acc var−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s1 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Acc var::−T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨Acc var⟩e»E
    assume eval: G⊢s0 −Acc var−≻v−n→ s1
    assume P: (Normal P) Y s0 Z
    show Q ⌊v⌋e s1 Z ∧ s1::≼(G, L)
    proof −
      from wt obtain
        wt-var: (|prg=G,cls=accC,lcl=L|)⊢var::=T
        by cases simp
      from da obtain V where
        da-var: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨var⟩v» V
        by (cases ∃ n. var=LVar n) (insert da.LVar,auto elim!: da-elim-cases)
      from eval obtain w upd where
        eval-var: G⊢s0 −var=≻(v, upd)−n→ s1
        using normal-s0 by (fastsimp elim: evaln-elim-cases)
      from valid-var P valid-A conf-s0 eval-var wt-var da-var
      have (λVar:(v, f):. Q←In1 v) ⌊(v, upd)⌋v s1 Z
        by (rule validE)
```

```
      then have Q ⌊v⌋e s1 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s1::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Ass A P var Q e R)
  note valid-var = ‹G,A|⊨::{ {Normal P} var=≻ {Q} }›
  have valid-e: ⋀ vf.
                 G,A|⊨::{ {Q←In2 vf} e−≻ {λVal:v:. assign (snd vf) v .; R} }
    using Ass.hyps by simp
  show G,A|⊨::{ {Normal P} var:=e−≻ {R} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s3 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢var:=e::−T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨var:=e⟩e»E
    assume eval: G⊢s0 −var:=e−≻v−n→ s3
    assume P: (Normal P) Y s0 Z
    show R ⌊v⌋e s3 Z ∧ s3::≼(G, L)
    proof −
      from wt obtain varT  where
        wt-var: (|prg=G,cls=accC,lcl=L|)⊢var::=varT and
        wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−T
        by cases simp
      from eval obtain w upd s1 s2 where
        eval-var: G⊢s0 −var=≻(w, upd)−n→ s1 and
        eval-e: G⊢s1 −e−≻v−n→ s2 and
        s3: s3=assign upd v s2
        using normal-s0 by (auto elim: evaln-elim-cases)
      have R ⌊v⌋e s3 Z
      proof (cases ∃ vn. var = LVar vn)
        case False
        with da obtain V where
          da-var: (|prg=G,cls=accC,lcl=L|)
                    ⊢ dom (locals (store s0)) »⟨var⟩v» V and
          da-e:   (|prg=G,cls=accC,lcl=L|) ⊢ nrm V »⟨e⟩e» E
          by cases simp+
        from valid-var P valid-A conf-s0 eval-var wt-var da-var
        obtain Q: Q ⌊(w,upd)⌋v s1 Z and conf-s1: s1::≼(G,L)
          by (rule validE)
        hence Q': ⋀ v. (Q←In2 (w,upd)) v s1 Z
          by simp
        have (λVal:v:. assign (snd (w,upd)) v .; R) ⌊v⌋e s2 Z
        proof (cases normal s1)
          case True
          obtain E' where
            da-e': (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨e⟩e» E'
          proof −
            from eval-var wt-var da-var wf True
            have nrm V ⊆  dom (locals (store s1))
              by (cases rule: da-good-approx-evalnE) iprover
            with da-e show thesis
```

```
        by (rule da-weakenE) (rule that)
    qed
    note ve=validE [OF valid-e,OF Q' valid-A conf-s1 eval-e wt-e da-e']
    show ?thesis
      by (rule ve)
  next
    case False
    note ve=validE [OF valid-e,OF Q' valid-A conf-s1 eval-e]
    with False show ?thesis
      by iprover
  qed
  with s3 show R ⌊v⌋e s3 Z
    by simp
next
  case True
  then obtain vn where
    vn: var = LVar vn
    by auto
  with da obtain E where
      da-e:  (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨e⟩e» E
    by cases simp+
  from da.LVar vn obtain  V where
    da-var: (|prg=G,cls=accC,lcl=L|)
              ⊢ dom (locals (store s0)) »⟨var⟩v» V
    by auto
  from valid-var P valid-A conf-s0 eval-var wt-var da-var
  obtain Q: Q ⌊(w,upd)⌋v s1 Z and conf-s1: s1::≼(G,L)
    by (rule validE)
  hence Q': ⋀ v. (Q←In2 (w,upd)) v s1 Z
    by simp
  have (λVal:v:. assign (snd (w,upd)) v .; R) ⌊v⌋e s2 Z
  proof (cases normal s1)
    case True
    obtain E' where
      da-e': (|prg=G,cls=accC,lcl=L|)
               ⊢ dom (locals (store s1)) »⟨e⟩e» E'
    proof −
      from eval-var
      have dom (locals (store s0)) ⊆ dom (locals (store (s1)))
        by (rule dom-locals-evaln-mono-elim)
      with da-e show thesis
        by (rule da-weakenE) (rule that)
    qed
    note ve=validE [OF valid-e,OF Q' valid-A conf-s1 eval-e wt-e da-e']
    show ?thesis
      by (rule ve)
  next
    case False
    note ve=validE [OF valid-e,OF Q' valid-A conf-s1 eval-e]
    with False show ?thesis
      by iprover
  qed
  with s3 show R ⌊v⌋e s3 Z
    by simp
qed
moreover
from eval wt da conf-s0 wf
have s3::≼(G, L)
  by (rule evaln-type-sound [elim-format]) simp
```

```
      ultimately show ?thesis ..
    qed
  qed
next
  case (Cond A P e0 P' e1 e2 Q)
  note valid-e0 = ‹G,A|⊨::{ {Normal P} e0−≻ {P'} }›
  have valid-then-else:⋀ b.  G,A|⊨::{ {P'←=b} (if b then e1 else e2)−≻ {Q} }
    using Cond.hyps by simp
  show G,A|⊨::{ {Normal P} e0 ? e1 : e2−≻ {Q} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s2 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: ⦇prg=G,cls=accC,lcl=L⦈⊢e0 ? e1 : e2::−T
    assume da: ⦇prg=G,cls=accC,lcl=L⦈⊢dom (locals (store s0))»⟨e0 ? e1:e2⟩ₑ»E
    assume eval: G⊢s0 −e0 ? e1 : e2−≻v−n→ s2
    assume P: (Normal P) Y s0 Z
    show Q ⌊v⌋ₑ s2 Z ∧ s2::≼(G, L)
    proof −
      from wt obtain T1 T2 where
        wt-e0: ⦇prg=G,cls=accC,lcl=L⦈⊢e0::−PrimT Boolean and
        wt-e1: ⦇prg=G,cls=accC,lcl=L⦈⊢e1::−T1 and
        wt-e2: ⦇prg=G,cls=accC,lcl=L⦈⊢e2::−T2
        by cases simp
      from da obtain E0 E1 E2 where
        da-e0: ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals (store s0)) »⟨e0⟩ₑ» E0 and
        da-e1: ⦇prg=G,cls=accC,lcl=L⦈
                ⊢(dom (locals (store s0)) ∪ assigns-if True e0)»⟨e1⟩ₑ» E1 and
        da-e2: ⦇prg=G,cls=accC,lcl=L⦈
                ⊢(dom (locals (store s0)) ∪ assigns-if False e0)»⟨e2⟩ₑ» E2
        by cases simp+
      from eval obtain b s1 where
        eval-e0: G⊢s0 −e0−≻b−n→ s1 and
        eval-then-else: G⊢s1 −(if the-Bool b then e1 else e2)−≻v−n→ s2
        using normal-s0 by (fastsimp elim: evaln-elim-cases)
      from valid-e0 P valid-A conf-s0 eval-e0 wt-e0 da-e0
      obtain P' ⌊b⌋ₑ s1 Z and conf-s1: s1::≼(G,L)
        by (rule validE)
      hence P': ⋀ v. (P'←=(the-Bool b)) v s1 Z
        by (cases normal s1) auto
      have Q ⌊v⌋ₑ s2 Z
      proof (cases normal s1)
        case True
        note normal-s1=this
        from wt-e1 wt-e2 obtain T' where
          wt-then-else:
          ⦇prg=G,cls=accC,lcl=L⦈⊢(if the-Bool b then e1 else e2)::−T'
          by (cases the-Bool b) simp+
        have s0-s1: dom (locals (store s0))
                     ∪ assigns-if (the-Bool b) e0 ⊆ dom (locals (store s1))
        proof −
          from eval-e0
          have eval-e0': G⊢s0 −e0−≻b→ s1
            by (rule evaln-eval)
          hence
            dom (locals (store s0)) ⊆ dom (locals (store s1))
            by (rule dom-locals-eval-mono-elim)
          moreover
```

```
          from eval-e0′ True wt-e0
          have assigns-if (the-Bool b) e0 ⊆ dom (locals (store s1))
            by (rule assigns-if-good-approx′)
          ultimately show ?thesis by (rule Un-least)
        qed
        obtain E′ where
          da-then-else:
          (|prg=G,cls=accC,lcl=L|)
             ⊢dom (locals (store s1))»⟨if the-Bool b then e1 else e2⟩e» E′
        proof (cases the-Bool b)
          case True
          with that da-e1 s0-s1 show ?thesis
            by simp (erule da-weakenE,auto)
        next
          case False
          with that da-e2 s0-s1 show ?thesis
            by simp (erule da-weakenE,auto)
        qed
        with valid-then-else P′ valid-A conf-s1 eval-then-else wt-then-else
        show ?thesis
          by (rule validE)
      next
        case False
        with valid-then-else P′ valid-A conf-s1 eval-then-else
        show ?thesis
          by (cases rule: validE) iprover+
      qed
      moreover
      from eval wt da conf-s0 wf
      have s2::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Call A P e Q args R mode statT mn pTs′ S accC′)
  note valid-e = ‹G,A|⊨::{ {Normal P} e−≻ {Q} }›
  have valid-args: ⋀ a. G,A|⊨::{ {Q←In1 a} args≐≻ {R a} }
    using Call.hyps by simp
  have valid-methd: ⋀ a vs invC declC l.
        G,A|⊨::{ {R a←In3 vs ∧.
                 (λs. declC =
                    invocation-declclass G mode (store s) a statT
                     (|name = mn, parTs = pTs′|) ∧
                    invC = invocation-class mode (store s) a statT ∧
                    l = locals (store s)) ;.
                 init-lvars G declC (|name = mn, parTs = pTs′|) mode a vs ∧.
                 (λs. normal s ⟶ G⊢mode→invC≼statT)}
             Methd declC (|name=mn,parTs=pTs′|)−≻ {set-lvars l .; S} }
    using Call.hyps by simp
  show G,A|⊨::{ {Normal P} {accC′,statT,mode}e·mn( {pTs′}args)−≻ {S} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s5 Y Z
    assume valid-A: ∀ t∈A. G|⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢{accC′,statT,mode}e·mn( {pTs′}args)::−T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))
                 »⟨{accC′,statT,mode}e·mn( {pTs′}args)⟩e» E
```

```
assume eval: G⊢s0 −{accC′,statT,mode}e·mn( {pTs′}args)−≻v−n→ s5
assume P: (Normal P) Y s0 Z
show S ⌊v⌋e s5 Z ∧ s5::≼(G, L)
proof −
  from wt obtain pTs statDeclT statM where
             wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−RefT statT and
          wt-args: (|prg=G,cls=accC,lcl=L|)⊢args::≐pTs and
            statM: max-spec G accC statT (|name=mn,parTs=pTs|)
                     = {((statDeclT,statM),pTs′)} and
             mode: mode = invmode statM e and
                T: T =(resTy statM) and
    eq-accC-accC′: accC=accC′
    by cases fastsimp+
  from da obtain C where
    da-e: (|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s0)))»⟨e⟩e» C and
    da-args: (|prg=G,cls=accC,lcl=L|)⊢ nrm C »⟨args⟩l» E
    by cases simp
  from eval eq-accC-accC′ obtain a s1 vs s2 s3 s3′ s4 invDeclC where
    evaln-e: G⊢s0 −e−≻a−n→ s1 and
    evaln-args: G⊢s1 −args≐≻vs−n→ s2 and
    invDeclC: invDeclC = invocation-declclass
          G mode (store s2) a statT (|name=mn,parTs=pTs′|) and
    s3: s3 = init-lvars G invDeclC (|name=mn,parTs=pTs′|) mode a vs s2 and
    check: s3′ = check-method-access G
                       accC′ statT mode (|name = mn, parTs = pTs′|) a s3 and
    evaln-methd:
       G⊢s3′ −Methd invDeclC (|name=mn,parTs=pTs′|)−≻v−n→ s4 and
    s5: s5=(set-lvars (locals (store s2))) s4
    using normal-s0 by (auto elim: evaln-elim-cases)

  from evaln-e
  have eval-e: G⊢s0 −e−≻a→ s1
    by (rule evaln-eval)

  from eval-e - wt-e wf
  have s1-no-return: abrupt s1 ≠ Some (Jump Ret)
    by (rule eval-expression-no-jump
             [where ?Env=(|prg=G,cls=accC,lcl=L|),simplified])
       (insert normal-s0,auto)

  from valid-e P valid-A conf-s0 evaln-e wt-e da-e
  obtain Q ⌊a⌋e s1 Z and conf-s1: s1::≼(G,L)
    by (rule validE)
  hence Q: ⋀ v. (Q←In1 a) v s1 Z
    by simp
  obtain
    R: (R a) ⌊vs⌋l s2 Z and
    conf-s2: s2::≼(G,L) and
    s2-no-return: abrupt s2 ≠ Some (Jump Ret)
  proof (cases normal s1)
    case True
    obtain E′ where
      da-args′:
      (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨args⟩l» E′
    proof −
      from evaln-e wt-e da-e wf True
      have nrm C ⊆  dom (locals (store s1))
        by (cases rule: da-good-approx-evalnE) iprover
      with da-args show thesis
```

by (*rule da-weakenE*) (*rule that*)
qed
with *valid-args Q valid-A conf-s1 evaln-args wt-args*
obtain (*R a*) $\lfloor vs \rfloor_l$ *s2 Z s2*::⪯(*G*,*L*)
by (*rule validE*)
moreover
from *evaln-args*
have *e*: *G*⊢*s1* −*args*≐≻*vs*→ *s2*
by (*rule evaln-eval*)
from *this s1-no-return wt-args wf*
have *abrupt s2* ≠ *Some* (*Jump Ret*)
by (*rule eval-expression-list-no-jump*
[**where** *?Env*=(|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|),*simplified*])
ultimately show *?thesis* **..**
next
case *False*
with *valid-args Q valid-A conf-s1 evaln-args*
obtain (*R a*) $\lfloor vs \rfloor_l$ *s2 Z s2*::⪯(*G*,*L*)
by (*cases rule*: *validE*) *iprover*+
moreover
from *False evaln-args* **have** *s2*=*s1*
by *auto*
with *s1-no-return* **have** *abrupt s2* ≠ *Some* (*Jump Ret*)
by *simp*
ultimately show *?thesis* **..**
qed

obtain *invC* **where**
invC: *invC* = *invocation-class mode* (*store s2*) *a statT*
by *simp*
with *s3*
have *invC′*: *invC* = (*invocation-class mode* (*store s3*) *a statT*)
by (*cases s2*,*cases mode*) (*auto simp add*: *init-lvars-def2*)
obtain *l* **where**
l: *l* = *locals* (*store s2*)
by *simp*

from *eval wt da conf-s0 wf*
have *conf-s5*: *s5*::⪯(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
let *PROP ?R* = ⋀ *v*.
(*R a*←*In3 vs* ∧.
(λ*s*. *invDeclC* = *invocation-declclass G mode* (*store s*) *a statT*
(|*name* = *mn*, *parTs* = *pTs′*|) ∧
invC = *invocation-class mode* (*store s*) *a statT* ∧
l = *locals* (*store s*)) ;.
init-lvars G invDeclC (|*name* = *mn*, *parTs* = *pTs′*|) *mode a vs* ∧.
(λ*s*. *normal s* ⟶ *G*⊢*mode*→*invC*⪯*statT*)
) *v s3′ Z*
{
assume *abrupt-s3*: ¬ *normal s3*
have *S* $\lfloor v \rfloor_e$ *s5 Z*
proof −
from *abrupt-s3 check* **have** *eq-s3′-s3*: *s3′*=*s3*
by (*auto simp add*: *check-method-access-def Let-def*)
with *R s3 invDeclC invC l abrupt-s3*
have *R′*: *PROP ?R*
by *auto*
have *conf-s3′*: *s3′*::⪯(*G*, *empty*)

```
      proof −
        from s2-no-return s3
        have abrupt s3 ≠ Some (Jump Ret)
          by (cases s2) (auto simp add: init-lvars-def2 split: split-if-asm)
        moreover
        obtain abr2 str2 where s2: s2=(abr2,str2)
          by (cases s2)
        from s3 s2 conf-s2 have (abrupt s3,str2)::≼(G, L)
          by (auto simp add: init-lvars-def2 split: split-if-asm)
        ultimately show ?thesis
          using s3 s2 eq-s3'-s3
          apply (simp add: init-lvars-def2)
          apply (rule conforms-set-locals [OF - wlconf-empty])
          by auto
      qed
      from valid-methd R' valid-A conf-s3' evaln-methd abrupt-s3 eq-s3'-s3
      have (set-lvars l .; S) ⌊v⌋e s4 Z
        by (cases rule: validE) simp+
      with s5 l show ?thesis
        by simp
    qed
  } note abrupt-s3-lemma = this

  have S ⌊v⌋e s5 Z
  proof (cases normal s2)
    case False
    with s3 have abrupt-s3: ¬ normal s3
      by (cases s2) (simp add: init-lvars-def2)
    thus ?thesis
      by (rule abrupt-s3-lemma)
  next
    case True
    note normal-s2 = this
    with evaln-args
    have normal-s1: normal s1
      by (rule evaln-no-abrupt)
    obtain E' where
      da-args':
      (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨args⟩l» E'
    proof −
      from evaln-e wt-e da-e wf normal-s1
      have nrm C ⊆  dom (locals (store s1))
        by (cases rule: da-good-approx-evalnE) iprover
      with da-args show thesis
        by (rule da-weakenE) (rule that)
    qed
    from evaln-args
    have eval-args: G⊢s1 −args≐≻vs→ s2
      by (rule evaln-eval)
    from evaln-e wt-e da-e conf-s0 wf
    have conf-a: G, store s1⊢a::≼RefT statT
      by (rule evaln-type-sound [elim-format]) (insert normal-s1,simp)
    with normal-s1 normal-s2 eval-args
    have conf-a-s2: G, store s2⊢a::≼RefT statT
      by (auto dest: eval-gext intro: conf-gext)
    from evaln-args wt-args da-args' conf-s1 wf
    have conf-args: list-all2 (conf G (store s2)) vs pTs
      by (rule evaln-type-sound [elim-format]) (insert normal-s2,simp)
```

```
from statM
obtain
  statM': (statDeclT,statM)∈mheads G accC statT ⦇name=mn,parTs=pTs'⦈
  and
  pTs-widen: G⊢pTs[≼]pTs'
  by (blast dest: max-spec2mheads)
show ?thesis
proof (cases normal s3)
  case False
  thus ?thesis
    by (rule abrupt-s3-lemma)
next
  case True
  note normal-s3 = this
  with s3 have notNull: mode = IntVir ⟶ a ≠ Null
    by (cases s2) (auto simp add: init-lvars-def2)
  from conf-s2 conf-a-s2 wf notNull invC
  have dynT-prop: G⊢mode→invC≼statT
    by (cases s2) (auto intro: DynT-propI)

  with wt-e statM' invC mode wf
  obtain dynM where
    dynM: dynlookup G statT invC  ⦇name=mn,parTs=pTs'⦈ = Some dynM and
    acc-dynM: G ⊢Methd  ⦇name=mn,parTs=pTs'⦈ dynM
                in invC dyn-accessible-from accC
    by (force dest!: call-access-ok)
  with invC' check eq-accC-accC'
  have eq-s3'-s3: s3'=s3
    by (auto simp add: check-method-access-def Let-def)

  with dynT-prop R s3 invDeclC invC l
  have R': PROP ?R
    by auto

  from dynT-prop wf wt-e statM' mode invC invDeclC dynM
  obtain
    dynM: dynlookup G statT invC  ⦇name=mn,parTs=pTs'⦈ = Some dynM and
    wf-dynM: wf-mdecl G invDeclC (⦇name=mn,parTs=pTs'⦈,mthd dynM) and
      dynM': methd G invDeclC ⦇name=mn,parTs=pTs'⦈ = Some dynM and
    iscls-invDeclC: is-class G invDeclC and
         invDeclC': invDeclC = declclass dynM and
      invC-widen: G⊢invC≼C invDeclC and
     resTy-widen: G⊢resTy dynM≼resTy statM and
    is-static-eq: is-static dynM = is-static statM and
    involved-classes-prop:
     (if invmode statM e = IntVir
       then ∀ statC. statT = ClassT statC ⟶ G⊢invC≼C statC
       else ((∃ statC. statT = ClassT statC ∧ G⊢statC≼C invDeclC) ∨
             (∀ statC. statT ≠ ClassT statC ∧ invDeclC = Object)) ∧
              statDeclT = ClassT invDeclC)
    by (cases rule: DynT-mheadsE) simp
  obtain L' where
    L':L'=(λ k.
           (case k of
              EName e
              ⇒ (case e of
                    VNam v
                    ⇒(table-of (lcls (mbody (mthd dynM)))
                       (pars (mthd dynM)[↦]pTs')) v
```

```
                | Res ⇒ Some (resTy dynM))
            | This ⇒ if is-static statM
                    then None else Some (Class invDeclC)))
    by simp
  from wf-dynM [THEN wf-mdeclD1, THEN conjunct1] normal-s2 conf-s2 wt-e
    wf eval-args conf-a mode notNull wf-dynM involved-classes-prop
  have conf-s3: s3::≼(G,L')
    apply −

    apply (drule conforms-init-lvars [of G invDeclC
            (|name=mn,parTs=pTs'|) dynM store s2 vs pTs abrupt s2
            L statT invC a (statDeclT,statM) e])
    apply (rule wf)
    apply (rule conf-args)
    apply (simp add: pTs-widen)
    apply (cases s2,simp)
    apply (rule dynM')
    apply (force dest: ty-expr-is-type)
    apply (rule invC-widen)
    apply (force intro: conf-gext dest: eval-gext)
    apply simp
    apply simp
    apply (simp add: invC)
    apply (simp add: invDeclC)
    apply (simp add: normal-s2)
    apply (cases s2, simp add: L' init-lvars-def2 s3
                cong add: lname.case-cong ename.case-cong)
    done
  with eq-s3'-s3 have conf-s3': s3'::≼(G,L') by simp
  from is-static-eq wf-dynM L'
  obtain mthdT where
    (|prg=G,cls=invDeclC,lcl=L'|)
      ⊢Body invDeclC (stmt (mbody (mthd dynM)))::−mthdT and
    mthdT-widen: G⊢mthdT≼resTy dynM
    by − (drule wf-mdecl-bodyD,
          auto simp add: callee-lcl-def
              cong add: lname.case-cong ename.case-cong)
  with dynM' iscls-invDeclC invDeclC'
  have
    wt-methd:
    (|prg=G,cls=invDeclC,lcl=L'|)
      ⊢(Methd invDeclC (|name = mn, parTs = pTs'|))::−mthdT
    by (auto intro: wt.Methd)
  obtain M where
    da-methd:
    (|prg=G,cls=invDeclC,lcl=L'|)
      ⊢ dom (locals (store s3'))
          »⟨Methd invDeclC (|name=mn,parTs=pTs'|)⟩ₑ» M
  proof −
    from wf-dynM
    obtain M' where
      da-body:
      (|prg=G, cls=invDeclC
       ,lcl=callee-lcl invDeclC (|name = mn, parTs = pTs'|) (mthd dynM)
       |) ⊢ parameters (mthd dynM) »⟨stmt (mbody (mthd dynM))⟩» M' and
      res: Result ∈ nrm M'
      by (rule wf-mdeclE) iprover
    from da-body is-static-eq L' have
      (|prg=G, cls=invDeclC,lcl=L'|)
```

```
        ⊢ parameters (mthd dynM) »⟨stmt (mbody (mthd dynM))⟩» M′
      by (simp add: callee-lcl-def
          cong add: lname.case-cong ename.case-cong)
    moreover have parameters (mthd dynM) ⊆ dom (locals (store s3′))
    proof −
      from is-static-eq
      have (invmode (mthd dynM) e) = (invmode statM e)
        by (simp add: invmode-def)
      moreover
      have length (pars (mthd dynM)) = length vs
      proof −
        from normal-s2 conf-args
        have length vs = length pTs
          by (simp add: list-all2-def)
        also from pTs-widen
        have ... = length pTs′
          by (simp add: widens-def list-all2-def)
        also from wf-dynM
        have ... = length (pars (mthd dynM))
          by (simp add: wf-mdecl-def wf-mhead-def)
        finally show ?thesis ..
      qed
      moreover note s3 dynM′ is-static-eq normal-s2 mode
      ultimately
      have parameters (mthd dynM) = dom (locals (store s3))
        using dom-locals-init-lvars
          [of mthd dynM G invDeclC (|name=mn,parTs=pTs′|) vs e a s2]
        by simp
      thus ?thesis using eq-s3′-s3 by simp
    qed
    ultimately obtain M2 where
      da:
      (|prg=G, cls=invDeclC,lcl=L′|)
        ⊢ dom (locals (store s3′)) »⟨stmt (mbody (mthd dynM))⟩» M2 and
      M2: nrm M′ ⊆ nrm M2
      by (rule da-weakenE)
    from res M2 have Result ∈ nrm M2
      by blast
    moreover from wf-dynM
    have jumpNestingOkS {Ret} (stmt (mbody (mthd dynM)))
      by (rule wf-mdeclE)
    ultimately
    obtain M3 where
      (|prg=G, cls=invDeclC,lcl=L′|) ⊢ dom (locals (store s3′))
           »⟨Body (declclass dynM) (stmt (mbody (mthd dynM)))⟩» M3
      using da
      by (iprover intro: da.Body assigned.select-convs)
    from - this [simplified]
    show thesis
      by (rule da.Methd [simplified,elim-format])
         (auto intro: dynM′ that)
  qed
  from valid-methd R′ valid-A conf-s3′ evaln-methd wt-methd da-methd
  have (set-lvars l .; S) ⌊v⌋ₑ s4 Z
    by (cases rule: validE) iprover+
  with s5 l show ?thesis
    by simp
 qed
qed
```

```
      with conf-s5 show ?thesis by iprover
    qed
  qed
next
  case (Methd A P Q ms)
  note valid-body = ‹G,A ∪ {{P} Methd−≻ {Q} | ms}|⊨::{{P} body G−≻ {Q} | ms}›
  show G,A|⊨::{{P} Methd−≻ {Q} | ms}
    by (rule Methd-sound) (rule Methd.hyps)
next
  case (Body A P D Q c R)
  note valid-init = ‹G,A|⊨::{ {Normal P} .Init D. {Q} }›
  note valid-c = ‹G,A|⊨::{ {Q} .c.
             {λs.. abupd (absorb Ret) .; R←In1 (the (locals s Result))} }›
  show G,A|⊨::{ {Normal P} Body D c−≻ {R} }
  proof (rule valid-expr-NormalI)
    fix n s0 L accC T E v s4 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Body D c::−T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0))»⟨Body D c⟩ₑ»E
    assume eval: G⊢s0 −Body D c−≻v−n→ s4
    assume P: (Normal P) Y s0 Z
    show R ⌊v⌋ₑ s4 Z ∧ s4::≼(G, L)
    proof −
      from wt obtain
        iscls-D: is-class G D and
        wt-init: (|prg=G,cls=accC,lcl=L|)⊢Init D::√ and
        wt-c: (|prg=G,cls=accC,lcl=L|)⊢c::√
        by cases auto
      obtain I where
        da-init:(|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨Init D⟩ₛ» I
        by (auto intro: da-Init [simplified] assigned.select-convs)
      from da obtain C where
        da-c: (|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s0)))»⟨c⟩ₛ» C and
        jmpOk: jumpNestingOkS {Ret} c
        by cases simp
      from eval obtain s1 s2 s3 where
        eval-init: G⊢s0 −Init D−n→ s1 and
        eval-c: G⊢s1 −c−n→ s2 and
        v: v = the (locals (store s2) Result) and
        s3: s3 =(if ∃ l. abrupt s2 = Some (Jump (Break l)) ∨
                         abrupt s2 = Some (Jump (Cont l))
                  then abupd (λx. Some (Error CrossMethodJump)) s2 else s2)and
        s4: s4 = abupd (absorb Ret) s3
        using normal-s0 by (fastsimp elim: evaln-elim-cases)
      obtain C' where
        da-c': (|prg=G,cls=accC,lcl=L|)⊢ (dom (locals (store s1)))»⟨c⟩ₛ» C'
      proof −
        from eval-init
        have (dom (locals (store s0))) ⊆ (dom (locals (store s1)))
          by (rule dom-locals-evaln-mono-elim)
        with da-c show thesis by (rule da-weakenE) (rule that)
      qed
      from valid-init P valid-A conf-s0 eval-init wt-init da-init
      obtain Q: Q ♢ s1 Z and conf-s1: s1::≼(G,L)
        by (rule validE)
      from valid-c Q valid-A conf-s1 eval-c wt-c da-c'
      have R: (λs.. abupd (absorb Ret) .; R←In1 (the (locals s Result)))
```

```
              ♢ s2 Z
        by (rule validE)
      have s3=s2
      proof −
        from eval-init [THEN evaln-eval] wf
        have s1-no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
          by − (rule eval-statement-no-jump [OF - - - wt-init],
                insert normal-s0,auto)
        from eval-c [THEN evaln-eval] - wt-c wf
        have ⋀ j. abrupt s2 = Some (Jump j) ⟹ j=Ret
          by (rule jumpNestingOk-evalE) (auto intro: jmpOk simp add: s1-no-jmp)
        moreover note s3
        ultimately show ?thesis
          by (force split: split-if)
      qed
      with R v s4
      have R ⌊v⌋e s4 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s4::≼(G, L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Nil A P)
  show G,A|⊨::{ {Normal (P←⌊[]⌋l)} []≐≻ {P} }
  proof (rule valid-expr-list-NormalI)
    fix s0 s1 vs n L Y Z
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume eval: G⊢s0 −[]≐≻vs−n→ s1
    assume P: (Normal (P←⌊[]⌋l)) Y s0 Z
    show P ⌊vs⌋l s1 Z ∧ s1::≼(G, L)
    proof −
      from eval obtain vs=[] s1=s0
        using normal-s0 by (auto elim: evaln-elim-cases)
      with P conf-s0 show ?thesis
        by simp
    qed
  qed
next
  case (Cons A P e Q es R)
  note valid-e = ‹G,A|⊨::{ {Normal P} e−≻ {Q} }›
  have valid-es: ⋀ v. G,A|⊨::{ {Q←⌊v⌋e} es≐≻ {λVals:vs:. R←⌊(v # vs)⌋l} }
    using Cons.hyps by simp
  show G,A|⊨::{ {Normal P} e # es≐≻ {R} }
  proof (rule valid-expr-list-NormalI)
    fix n s0 L accC T E v s2 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0: s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢e # es::≐T
    assume da: (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s0)) »⟨e # es⟩l» E
    assume eval: G⊢s0 −e # es≐≻v−n→ s2
    assume P: (Normal P) Y s0 Z
    show R ⌊v⌋l s2 Z ∧ s2::≼(G, L)
    proof −
```

from *wt* **obtain** *eT esT* **where**
  wt-e: $(\!|prg=G,cls=accC,lcl=L|\!)\vdash e{::}-eT$ **and**
  wt-es: $(\!|prg=G,cls=accC,lcl=L|\!)\vdash es{::}\doteq esT$
  by *cases simp*
from *da* **obtain** *E1* **where**
  da-e: $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$ $(dom\ (locals\ (store\ s0)))$»$\langle e\rangle_e$» *E1* **and**
  da-es: $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$ *nrm E1* »$\langle es\rangle_l$» *E*
  by *cases simp*
from *eval* **obtain** *s1 ve vs* **where**
  eval-e: $G\vdash s0\ -e\!-\!\succ ve\!-\!n\!\rightarrow\ s1$ **and**
  eval-es: $G\vdash s1\ -es\doteq\!\succ vs\!-\!n\!\rightarrow\ s2$ **and**
  v: $v{=}ve\#vs$
  using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
from *valid-e P valid-A conf-s0 eval-e wt-e da-e*
obtain *Q*: $Q\ \lfloor ve\rfloor_e\ s1\ Z$ **and** *conf-s1*: $s1{::}\preceq(G,L)$
  by (*rule validE*)
from *Q* **have** *Q'*: $\bigwedge\ v.\ (Q{\leftarrow}\lfloor ve\rfloor_e)\ v\ s1\ Z$
  by *simp*
have $(\lambda\, Vals{:}vs{:}.\ R{\leftarrow}\lfloor(ve\ \#\ vs)\rfloor_l)\ \lfloor vs\rfloor_l\ s2\ Z$
proof (*cases normal s1*)
  case *True*
  obtain *E'* **where**
    da-es': $(\!|prg=G,cls=accC,lcl=L|\!)\vdash$ *dom* (*locals* (*store s1*)) »$\langle es\rangle_l$» *E'*
  proof −
    from *eval-e wt-e da-e wf True*
    have *nrm E1* $\subseteq$ *dom* (*locals* (*store s1*))
      by (*cases rule*: *da-good-approx-evalnE*) *iprover*
    with *da-es* **show** *thesis*
      by (*rule da-weakenE*) (*rule that*)
  qed
  from *valid-es Q' valid-A conf-s1 eval-es wt-es da-es'*
  show *?thesis*
    by (*rule validE*)
next
  case *False*
  with *valid-es Q' valid-A conf-s1 eval-es*
  show *?thesis*
    by (*cases rule*: *validE*) *iprover+*
qed
with *v* **have** $R\ \lfloor v\rfloor_l\ s2\ Z$
  by *simp*
moreover
from *eval wt da conf-s0 wf*
have $s2{::}\preceq(G,\ L)$
  by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Skip A P*)
show $G,A|\!\!\models{::}\{\ \{Normal\ (P{\leftarrow}\Diamond)\}\ .Skip.\ \{P\}\ \}$
proof (*rule valid-stmt-NormalI*)
  fix *s0 s1 n L Y Z*
  assume *conf-s0*: $s0{::}\preceq(G,L)$
  assume *normal-s0*: *normal s0*
  assume *eval*: $G\vdash s0\ -Skip\!-\!n\!\rightarrow\ s1$
  assume *P*: $(Normal\ (P{\leftarrow}\Diamond))\ Y\ s0\ Z$
  show $P\ \Diamond\ s1\ Z\ \wedge\ s1{::}\preceq(G,\ L)$
  proof −

from *eval* **obtain** *s1=s0*
using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
with *P conf-s0* **show** *?thesis*
by *simp*
qed
qed
next
case (*Expr A P e Q*)
note *valid-e* = ‹*G*,*A*|⊨::{ {*Normal P*} *e*−≻ {*Q*←◇} }›
show *G*,*A*|⊨::{ {*Normal P*} .*Expr e*. {*Q*} }
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s1 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*Expr e*::√
assume *da*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*)) »⟨*Expr e*⟩$_s$» *C*
assume *eval*: *G*⊢*s0* −*Expr e*−*n*→ *s1*
assume *P*: (*Normal P*) *Y s0 Z*
show *Q* ◇ *s1 Z* ∧ *s1*::≼(*G*, *L*)
proof −
from *wt* **obtain** *eT* **where**
wt-e: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*e*::−*eT*
by *cases simp*
from *da* **obtain** *E* **where**
da-e: ⦇*prg*=*G*,*cls*=*accC*, *lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*e*⟩$_e$»*E*
by *cases simp*
from *eval* **obtain** *v* **where**
eval-e: *G*⊢*s0* −*e*−≻*v*−*n*→ *s1*
using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
from *valid-e P valid-A conf-s0 eval-e wt-e da-e*
obtain *Q*: (*Q*←◇) ⌊*v*⌋$_e$ *s1 Z* **and** *s1*::≼(*G*,*L*)
by (*rule validE*)
thus *?thesis* **by** *simp*
qed
qed
next
case (*Lab A P c l Q*)
note *valid-c* = ‹*G*,*A*|⊨::{ {*Normal P*} .*c*. {*abupd* (*absorb l*) .; *Q*} }›
show *G*,*A*|⊨::{ {*Normal P*} .*l*• *c*. {*Q*} }
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s2 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*l*• *c*::√
assume *da*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*)) »⟨*l*• *c*⟩$_s$» *C*
assume *eval*: *G*⊢*s0* −*l*• *c*−*n*→ *s2*
assume *P*: (*Normal P*) *Y s0 Z*
show *Q* ◇ *s2 Z* ∧ *s2*::≼(*G*, *L*)
proof −
from *wt* **obtain**
wt-c: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*c*::√
by *cases simp*
from *da* **obtain** *E* **where**
da-c: ⦇*prg*=*G*,*cls*=*accC*, *lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*c*⟩$_s$»*E*
by *cases simp*
from *eval* **obtain** *s1* **where**
eval-c: *G*⊢*s0* −*c*−*n*→ *s1* **and**

s2: *s2* = *abupd* (*absorb l*) *s1*
using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
from *valid-c P valid-A conf-s0 eval-c wt-c da-c*
obtain *Q*: (*abupd* (*absorb l*) .; *Q*) ♢ *s1 Z*
by (*rule validE*)
with *s2* **have** *Q* ♢ *s2 Z*
by *simp*
moreover
from *eval wt da conf-s0 wf*
have *s2*::≼(*G*, *L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Comp A P c1 Q c2 R*)
note *valid-c1* = ‹*G*,*A*|⊨::{ {*Normal P*} .*c1*. {*Q*} }›
note *valid-c2* = ‹*G*,*A*|⊨::{ {*Q*} .*c2*. {*R*} }›
show *G*,*A*|⊨::{ {*Normal P*} .*c1*;; *c2*. {*R*} }
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s2 Y Z*
assume *valid-A*: ∀ *t*∈*A*. *G*⊨*n*::*t*
assume *conf-s0*: *s0*::≼(*G*,*L*)
assume *normal-s0*: *normal s0*
assume *wt*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢(*c1*;; *c2*)::√
assume *da*: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*dom* (*locals* (*store s0*))»⟨*c1*;;*c2*⟩$_s$»*C*
assume *eval*: *G*⊢*s0* −*c1*;; *c2*−*n*→ *s2*
assume *P*: (*Normal P*) *Y s0 Z*
show *R* ♢ *s2 Z* ∧ *s2*::≼(*G*,*L*)
proof −
from *eval* **obtain** *s1* **where**
eval-c1: *G*⊢*s1* −*c1* −*n*→ *s1* **and**
eval-c2: *G*⊢*s1* −*c2* −*n*→ *s2*
using *normal-s0* **by** (*fastsimp elim*: *evaln-elim-cases*)
from *wt* **obtain**
wt-c1: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*c1*::√ **and**
wt-c2: ⦇*prg* = *G*, *cls* = *accC*, *lcl* = *L*⦈⊢*c2*::√
by *cases simp*
from *da* **obtain** *C1 C2* **where**
da-c1: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *dom* (*locals* (*store s0*)) »⟨*c1*⟩$_s$» *C1* **and**
da-c2: ⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢*nrm C1* »⟨*c2*⟩$_s$» *C2*
by *cases simp*
from *valid-c1 P valid-A conf-s0 eval-c1 wt-c1 da-c1*
obtain *Q*: *Q* ♢ *s1 Z* **and** *conf-s1*: *s1*::≼(*G*,*L*)
by (*rule validE*)
have *R* ♢ *s2 Z*
proof (*cases normal s1*)
case *True*
obtain *C2'* **where**
⦇*prg*=*G*,*cls*=*accC*,*lcl*=*L*⦈⊢ *dom* (*locals* (*store s1*)) »⟨*c2*⟩$_s$» *C2'*
proof −
from *eval-c1 wt-c1 da-c1 wf True*
have *nrm C1* ⊆ *dom* (*locals* (*store s1*))
by (*cases rule*: *da-good-approx-evalnE*) *iprover*
with *da-c2* **show** *thesis*
by (*rule da-weakenE*) (*rule that*)
qed
with *valid-c2 Q valid-A conf-s1 eval-c2 wt-c2*
show *?thesis*

by (*rule validE*)
next
case *False*
from *valid-c2 Q valid-A conf-s1 eval-c2 False*
show *?thesis*
by (*cases rule*: *validE*) *iprover+*
qed
moreover
from *eval wt da conf-s0 wf*
have $s2::\preceq(G, L)$
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*If A P e P′ c1 c2 Q*)
note *valid-e* = ‹$G,A|\models::\{ \{Normal\ P\}\ e{-}{\succ}\ \{P'\} \}$›
have *valid-then-else*: $\bigwedge$ *b*. $G,A|\models::\{ \{P'{\leftarrow}{=}b\}\ .(if\ b\ then\ c1\ else\ c2).\ \{Q\} \}$
using *If.hyps* **by** *simp*
show $G,A|\models::\{ \{Normal\ P\}\ .If(e)\ c1\ Else\ c2.\ \{Q\} \}$
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s2 Y Z*
assume *valid-A*: $\forall t\in A.\ G\models n::t$
assume *conf-s0*: $s0::\preceq(G,L)$
assume *normal-s0*: *normal s0*
assume *wt*: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash If(e)\ c1\ Else\ c2::\surd$
assume *da*: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash dom\ (locals\ (store\ s0))»\langle If(e)\ c1\ Else\ c2\rangle_s»C$
assume *eval*: $G\vdash s0\ {-}If(e)\ c1\ Else\ c2{-}n{\rightarrow}\ s2$
assume *P*: (*Normal P*) *Y s0 Z*
show $Q\ \diamondsuit\ s2\ Z \wedge s2::\preceq(G,L)$
proof −
from *eval* **obtain** *b s1* **where**
eval-e: $G\vdash s0\ {-}e{-}{\succ}b{-}n{\rightarrow}\ s1$ **and**
eval-then-else: $G\vdash s1\ {-}(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2){-}n{\rightarrow}\ s2$
using *normal-s0* **by** (*auto elim*: *evaln-elim-cases*)
from *wt* **obtain**
wt-e: $(\!|prg{=}G,\ cls{=}accC,\ lcl{=}L|\!)\vdash e::{-}PrimT\ Boolean$ **and**
wt-then-else: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash(if\ the\text{-}Bool\ b\ then\ c1\ else\ c2)::\surd$
by *cases* (*simp split*: *split-if*)
from *da* **obtain** *E S* **where**
da-e: $(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash\ dom\ (locals\ (store\ s0))\ »\langle e\rangle_e»\ E$ **and**
da-then-else:
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)\vdash$
$(dom\ (locals\ (store\ s0)) \cup assigns\text{-}if\ (the\text{-}Bool\ b)\ e)$
$»\langle if\ the\text{-}Bool\ b\ then\ c1\ else\ c2\rangle_s»\ S$
by *cases* (*cases the-Bool b*,*auto*)
from *valid-e P valid-A conf-s0 eval-e wt-e da-e*
obtain $P'\ \lfloor b\rfloor_e\ s1\ Z$ **and** *conf-s1*: $s1::\preceq(G,L)$
by (*rule validE*)
hence *P′*: $\bigwedge v.\ (P'{\leftarrow}{=}the\text{-}Bool\ b)\ v\ s1\ Z$
by (*cases normal s1*) *auto*
have $Q\ \diamondsuit\ s2\ Z$
proof (*cases normal s1*)
case *True*
have *s0-s1*: $dom\ (locals\ (store\ s0))$
$\cup\ assigns\text{-}if\ (the\text{-}Bool\ b)\ e \subseteq dom\ (locals\ (store\ s1))$
proof −
from *eval-e*

have *eval-e'*: $G\vdash s0\ -e\!-\!\succ b\!\rightarrow\ s1$
  by (*rule evaln-eval*)
hence
  dom (*locals* (*store s0*)) $\subseteq$ *dom* (*locals* (*store s1*))
  by (*rule dom-locals-eval-mono-elim*)
moreover
from *eval-e' True wt-e*
have *assigns-if* (*the-Bool b*) *e* $\subseteq$ *dom* (*locals* (*store s1*))
  by (*rule assigns-if-good-approx'*)
ultimately show *?thesis* **by** (*rule Un-least*)
qed
with *da-then-else*
obtain S' **where**
  (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
    $\vdash$*dom* (*locals* (*store s1*))»⟨*if the-Bool b then c1 else c2*⟩$_s$» S'
  by (*rule da-weakenE*)
with *valid-then-else P' valid-A conf-s1 eval-then-else wt-then-else*
show *?thesis*
  by (*rule validE*)
next
  case *False*
  with *valid-then-else P' valid-A conf-s1 eval-then-else*
  show *?thesis*
    by (*cases rule*: *validE*) *iprover+*
qed
moreover
from *eval wt da conf-s0 wf*
have *s2*::$\preceq$(*G*, *L*)
  by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Loop A P e P' c l*)
note *valid-e* = ‹*G*,*A*|$\models$::{ {*P*} *e*$-\succ$ {*P'*} }›
note *valid-c* = ‹*G*,*A*|$\models$::{ {*Normal* (*P'*$\leftarrow$=*True*)}
  .*c*.
  {*abupd* (*absorb* (*Cont l*)) .; *P*} }›
show *G*,*A*|$\models$::{ {*P*} .*l*• *While*(*e*) *c*. {*P'*$\leftarrow$=*False*$\downarrow$=$\diamondsuit$} }
proof (*rule valid-stmtI*)
  fix *n s0 L accC C s3 Y Z*
  assume *valid-A*: $\forall$ *t*$\in$*A*. *G*$\models$*n*::*t*
  assume *conf-s0*: *s0*::$\preceq$(*G*,*L*)
  assume *wt*: *normal s0* $\Longrightarrow$ (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)$\vdash$*l*• *While*(*e*) *c*::$\surd$
  assume *da*: *normal s0* $\Longrightarrow$ (|*prg*=*G*,*cls*=*accC*,*lcl*=*L*|)
    $\vdash$ *dom* (*locals* (*store s0*)) »⟨*l*• *While*(*e*) *c*⟩$_s$» *C*
  assume *eval*: $G\vdash s0\ -l\bullet\ While(e)\ c\!-\!n\!\rightarrow\ s3$
  assume *P*: *P Y s0 Z*
  show (*P'*$\leftarrow$=*False*$\downarrow$=$\diamondsuit$) $\diamondsuit$ *s3 Z* $\wedge$ *s3*::$\preceq$(*G*,*L*)
  proof –

— From the given hypothesises *valid-e* and *valid-c* we can only reach the state after unfolding the loop once, i.e. *P* $\diamondsuit$ *s2 Z*, where *s2* is the state after executing *c*. To gain validity of the further execution of while, to finally get (*P'*$\leftarrow$=*False*$\downarrow$=$\diamondsuit$) $\diamondsuit$ *s3 Z* we have to get a hypothesis about the subsequent unfoldings (the whole loop again), too. We can achieve this, by performing induction on the evaluation relation, with all the necessary preconditions to apply *valid-e* and *valid-c* in the goal.

{
  fix *t s s' v*
  assume $G\vdash s\ -t\succ\!-n\!\rightarrow\ (v,\ s')$
  hence $\bigwedge$ *Y' T E*.

```
      ⟦t = ⟨l• While(e) c⟩s; ∀ t∈A. G⊨n::t; P Y' s Z; s::≼(G, L);
       normal s ⟹ (|prg=G,cls=accC,lcl=L|)⊢t::T;
       normal s ⟹ (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s))»t»E
      ⟧⟹ (P'←=False↓=◇) v s' Z
  (is PROP ?Hyp n t s v s')
proof (induct)
  case (Loop s0' e' b n' s1' c' s2' l' s3' Y' T E)
  note while = ‹(⟨l'• While(e') c'⟩s::term) = ⟨l• While(e) c⟩s›
  hence eqs: l'=l e'=e c'=c by simp-all
  note valid-A = ‹∀ t∈A. G⊨n'::t›
  note P = ‹P Y' (Norm s0') Z›
  note conf-s0' = ‹Norm s0'::≼(G, L)›
  have wt: (|prg=G,cls=accC,lcl=L|)⊢⟨l• While(e) c⟩s::T
    using Loop.prems eqs by simp
  have da: (|prg=G,cls=accC,lcl=L|)⊢
          dom (locals (store ((Norm s0')::state)))»⟨l• While(e) c⟩s»E
    using Loop.prems eqs by simp
  have evaln-e: G⊢Norm s0' −e−≻b−n'→ s1'
    using Loop.hyps eqs by simp
  show (P'←=False↓=◇) ◇ s3' Z
  proof −
    from wt obtain
      wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−PrimT Boolean and
      wt-c: (|prg=G,cls=accC,lcl=L|)⊢c::√
      by cases (simp add: eqs)
    from da obtain E S where
      da-e: (|prg=G,cls=accC,lcl=L|)
             ⊢ dom (locals (store ((Norm s0')::state))) »⟨e⟩e» E and
      da-c: (|prg=G,cls=accC,lcl=L|)
             ⊢ (dom (locals (store ((Norm s0')::state)))
                  ∪ assigns-if True e) »⟨c⟩s» S
      by cases (simp add: eqs)
    from evaln-e
    have eval-e: G⊢Norm s0' −e−≻b→ s1'
      by (rule evaln-eval)
    from valid-e P valid-A conf-s0' evaln-e wt-e da-e
    obtain P': P' ⌊b⌋e s1' Z and conf-s1': s1'::≼(G,L)
      by (rule validE)
    show (P'←=False↓=◇) ◇ s3' Z
    proof (cases normal s1')
      case True
      note normal-s1'=this
      show ?thesis
      proof (cases the-Bool b)
        case True
        with P' normal-s1' have P'': (Normal (P'←=True)) ⌊b⌋e s1' Z
          by auto
        from True Loop.hyps obtain
          eval-c: G⊢s1' −c−n'→ s2' and
          eval-while:
             G⊢abupd (absorb (Cont l)) s2' −l• While(e) c−n'→ s3'
          by (simp add: eqs)
        from True Loop.hyps have
          hyp: PROP ?Hyp n' ⟨l• While(e) c⟩s
                 (abupd (absorb (Cont l')) s2') ◇ s3'
          apply (simp only: True if-True eqs)
          apply (elim conjE)
          apply (tactic smp-tac 3 1)
          apply fast
```

```
    done
  from eval-e
  have s0'-s1': dom (locals (store ((Norm s0')::state)))
                  ⊆ dom (locals (store s1'))
    by (rule dom-locals-eval-mono-elim)
  obtain S' where
    da-c':
     (|prg=G,cls=accC,lcl=L|)⊢(dom (locals (store s1')))»⟨c⟩s» S'
  proof −
    note s0'-s1'
    moreover
    from eval-e normal-s1' wt-e
    have assigns-if True e ⊆ dom (locals (store s1'))
      by (rule assigns-if-good-approx' [elim-format])
         (simp add: True)
    ultimately
    have dom (locals (store ((Norm s0')::state)))
           ∪ assigns-if True e ⊆ dom (locals (store s1'))
      by (rule Un-least)
    with da-c show thesis
      by (rule da-weakenE) (rule that)
  qed
  with valid-c P'' valid-A conf-s1' eval-c wt-c
  obtain (abupd (absorb (Cont l)) .; P) ♢ s2' Z and
    conf-s2': s2'::≼(G,L)
    by (rule validE)
  hence P-s2': P ♢ (abupd (absorb (Cont l)) s2') Z
    by simp
  from conf-s2'
  have conf-absorb: abupd (absorb (Cont l)) s2' ::≼(G, L)
    by (cases s2') (auto intro: conforms-absorb)
  moreover
  obtain E' where
    da-while':
     (|prg=G,cls=accC,lcl=L|)⊢
       dom (locals(store (abupd (absorb (Cont l)) s2')))
        »⟨l• While(e) c⟩s» E'
  proof −
    note s0'-s1'
    also
    from eval-c
    have G⊢s1' −c→ s2'
      by (rule evaln-eval)
    hence dom (locals (store s1')) ⊆ dom (locals (store s2'))
      by (rule dom-locals-eval-mono-elim)
    also
    have . . .⊆dom (locals (store (abupd (absorb (Cont l)) s2')))
      by simp
    finally
    have dom (locals (store ((Norm s0')::state))) ⊆ . . . .
    with da show thesis
      by (rule da-weakenE) (rule that)
  qed
  from valid-A P-s2' conf-absorb wt da-while'
  show (P'←=False↓=♢) ♢ s3' Z
    using hyp by (simp add: eqs)
next
  case False
  with Loop.hyps obtain s3'=s1'
```

```
          by simp
        with P' False show ?thesis
          by auto
      qed
    next
      case False
      note abnormal-s1'=this
      have s3'=s1'
      proof −
        from False obtain abr where abr: abrupt s1' = Some abr
          by (cases s1') auto
        from eval-e - wt-e wf
        have no-jmp: ⋀ j. abrupt s1' ≠ Some (Jump j)
          by (rule eval-expression-no-jump
                [where ?Env=(|prg=G,cls=accC,lcl=L|),simplified])
              simp
        show ?thesis
        proof (cases the-Bool b)
          case True
          with Loop.hyps obtain
            eval-c: G⊢s1' −c−n'→ s2' and
            eval-while:
             G⊢abupd (absorb (Cont l)) s2' −l• While(e) c−n'→ s3'
            by (simp add: eqs)
          from eval-c abr have s2'=s1' by auto
          moreover from calculation no-jmp
          have abupd (absorb (Cont l)) s2'=s2'
            by (cases s1') (simp add: absorb-def)
          ultimately show ?thesis
            using eval-while abr
            by auto
        next
          case False
          with Loop.hyps show ?thesis by simp
        qed
      qed
      with P' False show ?thesis
        by auto
    qed
  qed
next
  case (Abrupt abr s t' n' Y' T E)
  note t' = ‹t' = ⟨l• While(e) c⟩ₛ›
  note conf = ‹(Some abr, s)::≼(G, L)›
  note P = ‹P Y' (Some abr, s) Z›
  note valid-A = ‹∀ t∈A. G⊨n'::t›
  show (P'←=False↓=◇) (arbitrary3 t') (Some abr, s) Z
  proof −
    have eval-e:
      G⊢(Some abr,s) −⟨e⟩ₑ≻−n'→ (arbitrary3 ⟨e⟩ₑ,(Some abr,s))
      by auto
    from valid-e P valid-A conf eval-e
    have P' (arbitrary3 ⟨e⟩ₑ) (Some abr,s) Z
      by (cases rule: validE [where ?P=P]) simp+
    with t' show ?thesis
      by auto
  qed
qed simp-all
} note generalized=this
```

from *eval - valid-A P conf-s0 wt da*
have $(P' \leftarrow = False \downarrow = \diamondsuit)\ \diamondsuit\ s3\ Z$
by (*rule generalized*) *simp-all*
moreover
have $s3::\preceq(G,\ L)$
proof (*cases normal s0*)
case *True*
from *eval wt* [*OF True*] *da* [*OF True*] *conf-s0 wf*
show *?thesis*
by (*rule evaln-type-sound* [*elim-format*]) *simp*
next
case *False*
with *eval* **have** $s3=s0$
by *auto*
with *conf-s0* **show** *?thesis*
by *simp*
qed
ultimately show *?thesis* **..**
qed
qed
next
case (*Jmp A j P*)
show $G,A|\models::\{\ \{Normal\ (abupd\ (\lambda a.\ Some\ (Jump\ j))\ .;\ P \leftarrow \diamondsuit)\}\ .Jmp\ j.\ \{P\}\ \}$
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s1 Y Z*
assume *valid-A*: $\forall t \in A.\ G \models n::t$
assume *conf-s0*: $s0::\preceq(G,L)$
assume *normal-s0*: *normal s0*
assume *wt*: $(|prg=G,cls=accC,lcl=L|) \vdash Jmp\ j::\surd$
assume *da*: $(|prg=G,cls=accC,lcl=L|)$
$\vdash dom\ (locals\ (store\ s0))\gg\langle Jmp\ j\rangle_s\gg C$
assume *eval*: $G \vdash s0\ -Jmp\ j-n\rightarrow\ s1$
assume *P*: $(Normal\ (abupd\ (\lambda a.\ Some\ (Jump\ j))\ .;\ P \leftarrow \diamondsuit))\ Y\ s0\ Z$
show $P\ \diamondsuit\ s1\ Z \wedge s1::\preceq(G,L)$
proof −
from *eval* **obtain** *s* **where**
s: $s0=Norm\ s\ s1=(Some\ (Jump\ j),\ s)$
using *normal-s0* **by** (*auto elim*: *evaln-elim-cases*)
with *P* **have** $P\ \diamondsuit\ s1\ Z$
by *simp*
moreover
from *eval wt da conf-s0 wf*
have $s1::\preceq(G,L)$
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*Throw A P e Q*)
note *valid-e* = $\langle G,A|\models::\{\ \{Normal\ P\}\ e-\succ\ \{\lambda Val:a:.\ abupd\ (throw\ a)\ .;\ Q \leftarrow \diamondsuit\}\ \}\rangle$
show $G,A|\models::\{\ \{Normal\ P\}\ .Throw\ e.\ \{Q\}\ \}$
proof (*rule valid-stmt-NormalI*)
fix *n s0 L accC C s2 Y Z*
assume *valid-A*: $\forall t \in A.\ G \models n::t$
assume *conf-s0*: $s0::\preceq(G,L)$
assume *normal-s0*: *normal s0*
assume *wt*: $(|prg=G,cls=accC,lcl=L|) \vdash Throw\ e::\surd$
assume *da*: $(|prg=G,cls=accC,lcl=L|)$
$\vdash dom\ (locals\ (store\ s0))\gg\langle Throw\ e\rangle_s\gg C$

```
      assume eval: G⊢s0 −Throw e−n→ s2
      assume P: (Normal P) Y s0 Z
      show Q ◇ s2 Z ∧ s2::≼(G,L)
      proof −
        from eval obtain s1 a where
          eval-e: G⊢s0 −e−≻a−n→ s1 and
          s2: s2 = abupd (throw a) s1
          using normal-s0 by (auto elim: evaln-elim-cases)
        from wt obtain T where
          wt-e: (|prg=G,cls=accC,lcl=L|)⊢e::−T
          by cases simp
        from da obtain E where
          da-e: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨e⟩e» E
          by cases simp
        from valid-e P valid-A conf-s0 eval-e wt-e da-e
        obtain (λVal:a:. abupd (throw a) .; Q←◇) ⌊a⌋e s1 Z
          by (rule validE)
        with s2 have Q ◇ s2 Z
          by simp
        moreover
        from eval wt da conf-s0 wf
        have s2::≼(G,L)
          by (rule evaln-type-sound [elim-format]) simp
        ultimately show ?thesis ..
      qed
    qed
  next
    case (Try A P c1 Q C vn c2 R)
    note valid-c1 = ‹G,A|⊨::{ {Normal P} .c1. {SXAlloc G Q} }›
    note valid-c2 = ‹G,A|⊨::{ {Q ∧. (λs. G,s⊢catch C) ;. new-xcpt-var vn}
                              .c2.
                             {R} }›
    note Q-R = ‹(Q ∧. (λs. ¬ G,s⊢catch C)) ⇒ R›
    show G,A|⊨::{ {Normal P} .Try c1 Catch(C vn) c2. {R} }
    proof (rule valid-stmt-NormalI)
      fix n s0 L accC E s3 Y Z
      assume valid-A: ∀ t∈A. G⊨n::t
      assume conf-s0:  s0::≼(G,L)
      assume normal-s0: normal s0
      assume wt: (|prg=G,cls=accC,lcl=L|)⊢Try c1 Catch(C vn) c2::√
      assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0)) »⟨Try c1 Catch(C vn) c2⟩s» E
      assume eval: G⊢s0 −Try c1 Catch(C vn) c2−n→ s3
      assume P: (Normal P) Y s0 Z
      show R ◇ s3 Z ∧ s3::≼(G,L)
      proof −
        from eval obtain s1 s2 where
          eval-c1: G⊢s0 −c1−n→ s1 and
          sxalloc: G⊢s1 −sxalloc→ s2 and
          s3: if G,s2⊢catch C
                then G⊢new-xcpt-var vn s2 −c2−n→ s3
                else s3 = s2
          using normal-s0 by (fastsimp elim: evaln-elim-cases)
        from wt obtain
          wt-c1: (|prg=G,cls=accC,lcl=L|)⊢c1::√ and
          wt-c2: (|prg=G,cls=accC,lcl=L(VName vn↦Class C)|)⊢c2::√
          by cases simp
        from da obtain C1 C2 where
          da-c1: (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s0)) »⟨c1⟩s» C1 and
```

```
    da-c2: (|prg=G,cls=accC,lcl=L(VName vn↦Class C)|)
             ⊢ (dom (locals (store s0)) ∪ {VName vn}) »⟨c2⟩ₛ» C2
    by cases simp
  from valid-c1 P valid-A conf-s0 eval-c1 wt-c1 da-c1
  obtain sxQ: (SXAlloc G Q) ♢ s1 Z and conf-s1: s1::≼(G,L)
    by (rule validE)
  from sxalloc sxQ
  have Q: Q ♢ s2 Z
    by auto
  have R ♢ s3 Z
  proof (cases ∃ x. abrupt s1 = Some (Xcpt x))
    case False
    from sxalloc wf
    have s2=s1
      by (rule sxalloc-type-sound [elim-format])
         (insert False, auto split: option.splits abrupt.splits )
    with False
    have no-catch: ¬  G,s2⊢catch C
      by (simp add: catch-def)
    moreover
    from no-catch s3
    have s3=s2
      by simp
    ultimately show ?thesis
      using Q Q-R by simp
  next
    case True
    note exception-s1 = this
    show ?thesis
    proof (cases G,s2⊢catch C)
      case False
      with s3
      have s3=s2
        by simp
      with False Q Q-R show ?thesis
        by simp
    next
      case True
      with s3 have eval-c2: G⊢new-xcpt-var vn s2 −c2−n→ s3
        by simp
      from conf-s1 sxalloc wf
      have conf-s2: s2::≼(G, L)
        by (auto dest: sxalloc-type-sound
               split: option.splits abrupt.splits)
      from exception-s1 sxalloc wf
      obtain a
        where xcpt-s2: abrupt s2 = Some (Xcpt (Loc a))
        by (auto dest!: sxalloc-type-sound
                     split: option.splits abrupt.splits)
      with True
      have G⊢obj-ty (the (globs (store s2) (Heap a)))≼Class C
        by (cases s2) simp
      with xcpt-s2 conf-s2 wf
      have conf-new-xcpt: new-xcpt-var vn s2 ::≼(G, L(VName vn↦Class C))
        by (auto dest: Try-lemma)
      obtain C2' where
        da-c2':
        (|prg=G,cls=accC,lcl=L(VName vn↦Class C)|)
         ⊢ (dom (locals (store (new-xcpt-var vn s2)))) »⟨c2⟩ₛ» C2'
```

```
        proof -
          have (dom (locals (store s0)) ∪ {VName vn})
                ⊆ dom (locals (store (new-xcpt-var vn s2)))
          proof -
            from eval-c1
            have dom (locals (store s0))
                  ⊆ dom (locals (store s1))
              by (rule dom-locals-evaln-mono-elim)
            also
            from sxalloc
            have ... ⊆ dom (locals (store s2))
              by (rule dom-locals-sxalloc-mono)
            also
            have ... ⊆ dom (locals (store (new-xcpt-var vn s2)))
              by (cases s2) (simp add: new-xcpt-var-def, blast)
            also
            have {VName vn} ⊆ ...
              by (cases s2) simp
            ultimately show ?thesis
              by (rule Un-least)
          qed
          with da-c2 show thesis
            by (rule da-weakenE) (rule that)
        qed
        from Q eval-c2 True
        have (Q ∧. (λs. G,s⊢catch C) ;. new-xcpt-var vn)
               ◇ (new-xcpt-var vn s2) Z
          by auto
        from valid-c2 this valid-A conf-new-xcpt eval-c2 wt-c2 da-c2'
        show R ◇ s3 Z
          by (rule validE)
      qed
    qed
    moreover
    from eval wt da conf-s0 wf
    have s3::≼(G,L)
      by (rule evaln-type-sound [elim-format]) simp
    ultimately show ?thesis ..
  qed
 qed
next
 case (Fin A P c1 Q c2 R)
 note valid-c1 = ‹G,A|⊨::{ {Normal P} .c1. {Q} }›
 have valid-c2: ⋀ abr. G,A|⊨::{ {Q ∧. (λs. abr = fst s) ;. abupd (λx. None)}
                                .c2.
                                {abupd (abrupt-if (abr ≠ None) abr) .; R} }
   using Fin.hyps by simp
 show G,A|⊨::{ {Normal P} .c1 Finally c2. {R} }
 proof (rule valid-stmt-NormalI)
   fix n s0 L accC E s3 Y Z
   assume valid-A: ∀ t∈A. G⊨n::t
   assume conf-s0:  s0::≼(G,L)
   assume normal-s0: normal s0
   assume wt: ⦇prg=G,cls=accC,lcl=L⦈⊢c1 Finally c2::√
   assume da: ⦇prg=G,cls=accC,lcl=L⦈
              ⊢dom (locals (store s0)) »⟨c1 Finally c2⟩ₛ» E
   assume eval: G⊢s0 −c1 Finally c2−n→ s3
   assume P: (Normal P) Y s0 Z
   show R ◇ s3 Z ∧ s3::≼(G,L)
```

```
    proof −
      from eval obtain s1 abr1 s2 where
        eval-c1: G⊢s0 −c1−n→ (abr1, s1) and
        eval-c2: G⊢Norm s1 −c2−n→ s2 and
        s3: s3 = (if ∃ err. abr1 = Some (Error err)
                      then (abr1, s1)
                      else abupd (abrupt-if (abr1 ≠ None) abr1) s2)
        using normal-s0 by (fastsimp elim: evaln-elim-cases)
      from wt obtain
        wt-c1: (|prg=G,cls=accC,lcl=L|)⊢c1::√ and
        wt-c2: (|prg=G,cls=accC,lcl=L|)⊢c2::√
        by cases simp
      from da obtain C1 C2 where
        da-c1: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨c1⟩ₛ» C1 and
        da-c2: (|prg=G,cls=accC,lcl=L|) ⊢ dom (locals (store s0)) »⟨c2⟩ₛ» C2
        by cases simp
      from valid-c1 P valid-A conf-s0 eval-c1 wt-c1 da-c1
      obtain Q: Q ♢ (abr1,s1) Z and conf-s1: (abr1,s1)::≼(G,L)
        by (rule validE)
      from Q
      have Q′: (Q ∧. (λs. abr1 = fst s) ;. abupd (λx. None)) ♢ (Norm s1) Z
        by auto
      from eval-c1 wt-c1 da-c1 conf-s0 wf
      have  error-free (abr1,s1)
        by (rule evaln-type-sound  [elim-format]) (insert normal-s0,simp)
      with s3 have s3′: s3 = abupd (abrupt-if (abr1 ≠ None) abr1) s2
        by (simp add: error-free-def)
      from conf-s1
      have conf-Norm-s1: Norm s1::≼(G,L)
        by (rule conforms-NormI)
      obtain C2′ where
        da-c2′: (|prg=G,cls=accC,lcl=L|)
                 ⊢ dom (locals (store ((Norm s1)::state))) »⟨c2⟩ₛ» C2′
      proof −
        from eval-c1
        have dom (locals (store s0)) ⊆ dom (locals (store (abr1,s1)))
          by (rule dom-locals-evaln-mono-elim)
        hence dom (locals (store s0))
                ⊆ dom (locals (store ((Norm s1)::state)))
          by simp
        with da-c2 show thesis
          by (rule da-weakenE) (rule that)
      qed
      from valid-c2 Q′ valid-A conf-Norm-s1 eval-c2 wt-c2 da-c2′
      have (abupd (abrupt-if (abr1 ≠ None) abr1) .; R) ♢ s2 Z
        by (rule validE)
      with s3′ have R ♢ s3 Z
        by simp
      moreover
      from eval wt da conf-s0 wf
      have s3::≼(G,L)
        by (rule evaln-type-sound [elim-format]) simp
      ultimately show ?thesis ..
    qed
  qed
next
  case (Done A P C)
  show G,A|⊨::{ {Normal (P←♢ ∧. initd C)} .Init C. {P} }
  proof (rule valid-stmt-NormalI)
```

```
    fix n s0 L accC E s3 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0:  s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Init C::√
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0)) »⟨Init C⟩s» E
    assume eval: G⊢s0 −Init C−n→ s3
    assume P: (Normal (P←♢ ∧. initd C)) Y s0 Z
    show P ♢ s3 Z ∧ s3::≼(G,L)
    proof −
      from P have inited: inited C (globs (store s0))
        by simp
      with eval have s3=s0
        using normal-s0 by (auto elim: evaln-elim-cases)
      with P conf-s0 show ?thesis
        by simp
    qed
  qed
next
  case (Init C c A P Q R)
  note c = ‹the (class G C) = c›
  note valid-super =
       ‹G,A|⊨::{ {Normal (P ∧. Not ∘ initd C ;. supd (init-class-obj G C))}
               .(if C = Object then Skip else Init (super c)).
               {Q} }›
  have valid-init:
       ⋀ l.  G,A|⊨::{ {Q ∧. (λs. l = locals (snd s)) ;. set-lvars empty}
                      .init c.
                      {set-lvars l .; R} }
    using Init.hyps by simp
  show G,A|⊨::{ {Normal (P ∧. Not ∘ initd C)} .Init C. {R} }
  proof (rule valid-stmt-NormalI)
    fix n s0 L accC E s3 Y Z
    assume valid-A: ∀ t∈A. G⊨n::t
    assume conf-s0:  s0::≼(G,L)
    assume normal-s0: normal s0
    assume wt: (|prg=G,cls=accC,lcl=L|)⊢Init C::√
    assume da: (|prg=G,cls=accC,lcl=L|)
                 ⊢dom (locals (store s0)) »⟨Init C⟩s» E
    assume eval: G⊢s0 −Init C−n→ s3
    assume P: (Normal (P ∧. Not ∘ initd C)) Y s0 Z
    show R ♢ s3 Z ∧ s3::≼(G,L)
    proof −
      from P have not-inited: ¬ inited C (globs (store s0)) by simp
      with eval c obtain s1 s2 where
        eval-super:
        G⊢Norm ((init-class-obj G C) (store s0))
           −(if C = Object then Skip else Init (super c))−n→ s1 and
        eval-init: G⊢(set-lvars empty) s1 −init c−n→ s2 and
        s3: s3 = (set-lvars (locals (store s1))) s2
        using normal-s0 by (auto elim!: evaln-elim-cases)
      from wt c have
        cls-C: class G C = Some c
        by cases auto
      from wf cls-C have
        wt-super: (|prg=G,cls=accC,lcl=L|)
                     ⊢(if C = Object then Skip else Init (super c))::√
        by (cases C=Object)
```

(*auto dest*: *wf-prog-cdecl wf-cdecl-supD is-acc-classD*)
obtain S **where**
da-super:
$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$
$\vdash$ *dom* (*locals* (*store* ((*Norm*
((*init-class-obj G C*) (*store s0*)))::*state*)))
»⟨*if C = Object then Skip else Init* (*super c*)⟩$_s$» S
proof (*cases C=Object*)
case *True*
with *da-Skip* **show** *?thesis*
using *that* **by** (*auto intro*: *assigned.select-convs*)
next
case *False*
with *da-Init* **show** *?thesis*
by − (*rule that*, *auto intro*: *assigned.select-convs*)
qed
from *normal-s0 conf-s0 wf cls-C not-inited*
have *conf-init-cls*: (*Norm* ((*init-class-obj G C*) (*store s0*)))::$\preceq$(G, L)
by (*auto intro*: *conforms-init-class-obj*)
from P
have P': (*Normal* (P ∧. *Not* ∘ *initd C* ;. *supd* (*init-class-obj G C*)))
Y (*Norm* ((*init-class-obj G C*) (*store s0*))) Z
by *auto*

from *valid-super* P' *valid-A conf-init-cls eval-super wt-super da-super*
obtain Q: Q ◇ *s1* Z **and** *conf-s1*: *s1*::$\preceq$(G,L)
by (*rule validE*)

from *cls-C wf* **have** *wt-init*: $(\!|prg{=}G,\ cls{=}C,lcl{=}empty|\!)\vdash$(*init c*)::$\surd$
by (*rule wf-prog-cdecl* [*THEN wf-cdecl-wt-init*])
from *cls-C wf* **obtain** I **where**
$(\!|prg{=}G,cls{=}C,lcl{=}empty|\!)\vdash$ {} »⟨*init c*⟩$_s$» I
by (*rule wf-prog-cdecl* [*THEN wf-cdeclE*,*simplified*]) *blast*

then obtain I' **where**
da-init:
$(\!|prg{=}G,cls{=}C,lcl{=}empty|\!)\vdash$*dom* (*locals* (*store* ((*set-lvars empty*) *s1*)))
»⟨*init c*⟩$_s$» I'
by (*rule da-weakenE*) *simp*
have *conf-s1-empty*: (*set-lvars empty*) *s1*::$\preceq$(G, *empty*)
proof −
from *eval-super* **have**
$G\vdash$*Norm* ((*init-class-obj G C*) (*store s0*))
−(*if C = Object then Skip else Init* (*super c*))→ *s1*
by (*rule evaln-eval*)
from *this wt-super wf*
have *s1-no-ret*: $\bigwedge$ *j*. *abrupt s1* ≠ *Some* (*Jump j*)
by − (*rule eval-statement-no-jump*
[**where** *?Env*=$(\!|prg{=}G,cls{=}accC,lcl{=}L|\!)$], *auto split*: *split-if*)
with *conf-s1*
show *?thesis*
by (*cases s1*) (*auto intro*: *conforms-set-locals*)
qed

obtain l **where** l: l = *locals* (*store s1*)
by *simp*
with Q
have Q': (Q ∧. (λ*s*. l = *locals* (*snd s*)) ;. *set-lvars empty*)
◇ ((*set-lvars empty*) *s1*) Z

by *auto*
from *valid-init Q′ valid-A conf-s1-empty eval-init wt-init da-init*
have (*set-lvars l* .; *R*) $\Diamond$ *s2 Z*
by (*rule validE*)
with *s3 l* **have** *R* $\Diamond$ *s3 Z*
by *simp*
moreover
from *eval wt da conf-s0 wf*
have *s3*::$\preceq$(*G*,*L*)
by (*rule evaln-type-sound* [*elim-format*]) *simp*
ultimately show *?thesis* **..**
qed
qed
next
case (*InsInitV A P c v Q*)
show *G*,*A*|$\models$::{ {*Normal P*} *InsInitV c v*=$\succ$ {*Q*} }
proof (*rule valid-var-NormalI*)
fix *s0 vf n s1 L Z*
assume *normal s0*
moreover
assume *G*$\vdash$*s0* $-$*InsInitV c v*=$\succ$*vf*$-$*n*$\rightarrow$ *s1*
ultimately have *False*
by (*cases s0*) (*simp add*: *evaln-InsInitV*)
thus *Q* $\lfloor vf \rfloor_v$ *s1 Z* $\wedge$ *s1*::$\preceq$(*G*, *L*)**..**
qed
next
case (*InsInitE A P c e Q*)
show *G*,*A*|$\models$::{ {*Normal P*} *InsInitE c e*$-\succ$ {*Q*} }
proof (*rule valid-expr-NormalI*)
fix *s0 v n s1 L Z*
assume *normal s0*
moreover
assume *G*$\vdash$*s0* $-$*InsInitE c e*$-\succ$*v*$-$*n*$\rightarrow$ *s1*
ultimately have *False*
by (*cases s0*) (*simp add*: *evaln-InsInitE*)
thus *Q* $\lfloor v \rfloor_e$ *s1 Z* $\wedge$ *s1*::$\preceq$(*G*, *L*)**..**
qed
next
case (*Callee A P l e Q*)
show *G*,*A*|$\models$::{ {*Normal P*} *Callee l e*$-\succ$ {*Q*} }
proof (*rule valid-expr-NormalI*)
fix *s0 v n s1 L Z*
assume *normal s0*
moreover
assume *G*$\vdash$*s0* $-$*Callee l e*$-\succ$*v*$-$*n*$\rightarrow$ *s1*
ultimately have *False*
by (*cases s0*) (*simp add*: *evaln-Callee*)
thus *Q* $\lfloor v \rfloor_e$ *s1 Z* $\wedge$ *s1*::$\preceq$(*G*, *L*)**..**
qed
next
case (*FinA A P a c Q*)
show *G*,*A*|$\models$::{ {*Normal P*} .*FinA a c*. {*Q*} }
proof (*rule valid-stmt-NormalI*)
fix *s0 v n s1 L Z*
assume *normal s0*
moreover
assume *G*$\vdash$*s0* $-$*FinA a c*$-$*n*$\rightarrow$ *s1*
ultimately have *False*
by (*cases s0*) (*simp add*: *evaln-FinA*)

  thus $Q \diamondsuit\ s1\ Z \wedge s1{::}\preceq(G,\ L)$**..**
 qed
qed
declare *inj-term-simps* [*simp del*]

theorem *ax-sound*:
 $wf\text{-}prog\ G \Longrightarrow G,(A{::}'a\ triple\ set)|\!\vdash(ts{::}'a\ triple\ set) \Longrightarrow G,A|\!\models ts$
apply (*subst ax-valids2-eq* [*symmetric*])
apply *assumption*
apply (*erule* (*1*) *ax-sound2*)
done

lemma *sound-valid2-lemma*:
$[\![\forall v\ n.\ Ball\ A\ (triple\text{-}valid2\ G\ n) \longrightarrow P\ v\ n;\ Ball\ A\ (triple\text{-}valid2\ G\ n)]\!]$
 $\Longrightarrow P\ v\ n$
by *blast*

end

Chapter 24

AxCompl

63 Completeness proof for Axiomatic semantics of Java expressions and statements

theory *AxCompl* **imports** *AxSem* **begin**

design issues:

- proof structured by Most General Formulas (-¿ Thomas Kleymann)

set of not yet initialzed classes

constdefs

nyinitcls :: *prog* ⇒ *state* ⇒ *qtname set*
nyinitcls G s ≡ {*C*. *is-class G C* ∧ ¬ *initd C s*}

lemma *nyinitcls-subset-class*: *nyinitcls G s* ⊆ {*C*. *is-class G C*}
apply (*unfold nyinitcls-def*)
apply *fast*
done

lemmas *finite-nyinitcls* [*simp*] =
finite-is-class [*THEN nyinitcls-subset-class* [*THEN finite-subset*], *standard*]

lemma *card-nyinitcls-bound*: *card* (*nyinitcls G s*) ≤ *card* {*C*. *is-class G C*}
apply (*rule nyinitcls-subset-class* [*THEN finite-is-class* [*THEN card-mono*]])
done

lemma *nyinitcls-set-locals-cong* [*simp*]:
nyinitcls G (*x*,*set-locals l s*) = *nyinitcls G* (*x*,*s*)
apply (*unfold nyinitcls-def*)
apply (*simp* (*no-asm*))
done

lemma *nyinitcls-abrupt-cong* [*simp*]: *nyinitcls G* (*f x*, *y*) = *nyinitcls G* (*x*, *y*)
apply (*unfold nyinitcls-def*)
apply (*simp* (*no-asm*))
done

lemma *nyinitcls-abupd-cong* [*simp*]:!!*s*. *nyinitcls G* (*abupd f s*) = *nyinitcls G s*
apply (*unfold nyinitcls-def*)
apply (*simp* (*no-asm-simp*) *only*: *split-tupled-all*)
apply (*simp* (*no-asm*))
done

lemma *card-nyinitcls-abrupt-congE* [*elim*!]:
card (*nyinitcls G* (*x*, *s*)) ≤ *n* ⟹ *card* (*nyinitcls G* (*y*, *s*)) ≤ *n*
apply (*unfold nyinitcls-def*)
apply *auto*
done

lemma *nyinitcls-new-xcpt-var* [*simp*]:

nyinitcls G (new-xcpt-var vn s) = nyinitcls G s
apply (*unfold nyinitcls-def*)
apply (*induct-tac s*)
apply (*simp* (*no-asm*))
done

lemma *nyinitcls-init-lvars* [*simp*]:
nyinitcls G ((init-lvars G C sig mode a′ pvs) s) = nyinitcls G s
apply (*induct-tac s*)
apply (*simp* (*no-asm*) *add*: *init-lvars-def2 split add*: *split-if*)
done

lemma *nyinitcls-emptyD*: $\llbracket$*nyinitcls G s* = {}; *is-class G C*$\rrbracket \Longrightarrow$ *initd C s*
apply (*unfold nyinitcls-def*)
apply *fast*
done

lemma *card-Suc-lemma*:
$\llbracket$*card* (*insert a A*) $\leq$ *Suc n*; *a*$\notin$*A*; *finite A*$\rrbracket \Longrightarrow$ *card A* $\leq$ *n*
apply *clarsimp*
done

lemma *nyinitcls-le-SucD*:
$\llbracket$*card* (*nyinitcls G* (*x*,*s*)) $\leq$ *Suc n*; $\neg$*inited C* (*globs s*); *class G C*=*Some y*$\rrbracket \Longrightarrow$
card (*nyinitcls G* (*x*,*init-class-obj G C s*)) $\leq$ *n*
apply (*subgoal-tac*
nyinitcls G (*x*,*s*) = *insert C* (*nyinitcls G* (*x*,*init-class-obj G C s*)))
apply *clarsimp*
apply (*erule-tac V*=*nyinitcls G* (*x*, *s*) = *?rhs* **in** *thin-rl*)
apply (*rule card-Suc-lemma* [*OF - - finite-nyinitcls*])
apply (*auto dest*!: *not-initedD elim*!:
simp add: *nyinitcls-def inited-def split add*: *split-if-asm*)
done

lemma *inited-gext′*: $\llbracket$*s*$\leq$|*s′*;*inited C* (*globs s*)$\rrbracket \Longrightarrow$ *inited C* (*globs s′*)
by (*rule inited-gext*)

lemma *nyinitcls-gext*: *snd s*$\leq$|*snd s′* $\Longrightarrow$ *nyinitcls G s′* $\subseteq$ *nyinitcls G s*
apply (*unfold nyinitcls-def*)
apply (*force dest*!: *inited-gext′*)
done

lemma *card-nyinitcls-gext*:
$\llbracket$*snd s*$\leq$|*snd s′*; *card* (*nyinitcls G s*) $\leq$ *n*$\rrbracket \Longrightarrow$ *card* (*nyinitcls G s′*) $\leq$ *n*
apply (*rule le-trans*)
apply (*rule card-mono*)
apply (*rule finite-nyinitcls*)
apply (*erule nyinitcls-gext*)
apply *assumption*
done

init-le

constdefs
$init\text{-}le :: prog \Rightarrow nat \Rightarrow state \Rightarrow bool$ $\quad\quad (\text{-}\vdash init\leq\text{-}\ [51,51]\ 50)$
$G\vdash init\leq n \equiv \lambda s.\ card\ (nyinitcls\ G\ s) \leq n$

lemma *init-le-def2* [*simp*]: $(G\vdash init\leq n)\ s = (card\ (nyinitcls\ G\ s)\leq n)$
apply (*unfold init-le-def*)
apply *auto*
done

lemma *All-init-leD*:
$\forall n::nat.\ G,(A::'a\ triple\ set)\vdash\{P \wedge.\ G\vdash init\leq n\}\ t\succ\ \{Q::'a\ assn\}$
$\Longrightarrow G,A\vdash\{P\}\ t\succ\ \{Q\}$
apply (*drule spec*)
apply (*erule conseq1*)
apply *clarsimp*
apply (*rule card-nyinitcls-bound*)
done

Most General Triples and Formulas

constdefs

$remember\text{-}init\text{-}state :: state\ assn$ $\quad\quad (\doteq)$
$\doteq\ \equiv \lambda Y\ s\ Z.\ s = Z$

lemma *remember-init-state-def2* [*simp*]: $\doteq\ Y = op =$
apply (*unfold remember-init-state-def*)
apply (*simp* (*no-asm*))
done

consts

$MGF\ ::[state\ assn,\ term,\ prog] \Rightarrow state\ triple \quad (\{\text{-}\}\ \text{-}\succ\ \{\text{-}\rightarrow\}[3,65,3]62)$
$MGFn::[nat\quad\quad,\ term,\ prog] \Rightarrow state\ triple\ (\{=:\text{-}\}\ \text{-}\succ\ \{\text{-}\rightarrow\}[3,65,3]62)$

defs

MGF-def:
$\{P\}\ t\succ\ \{G\rightarrow\} \equiv \{P\}\ t\succ\ \{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\}$

MGFn-def:
$\{=:n\}\ t\succ\ \{G\rightarrow\} \equiv \{\doteq\ \wedge.\ G\vdash init\leq n\}\ t\succ\ \{G\rightarrow\}$

lemma *MGF-valid*: $wf\text{-}prog\ G \Longrightarrow G,\{\}\models\{\doteq\}\ t\succ\ \{G\rightarrow\}$
apply (*unfold MGF-def*)
apply (*simp add*: *ax-valids-def triple-valid-def2*)
apply (*auto elim*: *evaln-eval*)
done

lemma *MGF-res-eq-lemma* [*simp*]:
$(\forall\, Y'\ Y\ s.\ Y = Y' \wedge P\ s \longrightarrow Q\ s) = (\forall\, s.\ P\ s \longrightarrow Q\ s)$
apply *auto*
done

lemma *MGFn-def2*:
$G,A\vdash\{=:n\}\ t\succ\ \{G\rightarrow\} = G,A\vdash\{\doteq \wedge.\ G\vdash init\leq n\}$
$t\succ\ \{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\}$
apply (*unfold MGFn-def MGF-def*)
apply *fast*
done

lemma *MGF-MGFn-iff*:
$G,(A{::}state\ triple\ set)\vdash\{\doteq\}\ t\succ\ \{G\rightarrow\} = (\forall\, n.\ G,A\vdash\{=:n\}\ t\succ\ \{G\rightarrow\})$
apply (*simp* (*no-asm-use*) *add*: *MGFn-def2 MGF-def*)
apply *safe*
apply (*erule-tac* [*2*] *All-init-leD*)
apply (*erule conseq1*)
apply *clarsimp*
done

lemma *MGFnD*:
$G,(A{::}state\ triple\ set)\vdash\{=:n\}\ t\succ\ \{G\rightarrow\} \Longrightarrow$
$G,A\vdash\{(\lambda Y'\ s'\ s.\ s' = s \qquad \wedge\ P\ s)\ \wedge.\ G\vdash init\leq n\}$
$t\succ\ \{(\lambda Y'\ s'\ s.\ G\vdash s-t\succ\rightarrow(Y',s')\ \wedge\ P\ s)\ \wedge.\ G\vdash init\leq n\}$
apply (*unfold init-le-def*)
apply (*simp* (*no-asm-use*) *add*: *MGFn-def2*)
apply (*erule conseq12*)
apply *clarsimp*
apply (*erule* (*1*) *eval-gext* [*THEN card-nyinitcls-gext*])
done
lemmas *MGFnD'* = *MGFnD* [*of - - - -* $\lambda x.$ *True*]

To derive the most general formula, we can always assume a normal state in the precondition, since abrupt cases can be handled uniformally by the abrupt rule.

lemma *MGFNormalI*: $G,A\vdash\{Normal\ \doteq\}\ t\succ\ \{G\rightarrow\} \Longrightarrow$
$G,(A{::}state\ triple\ set)\vdash\{\doteq{::}state\ assn\}\ t\succ\ \{G\rightarrow\}$
apply (*unfold MGF-def*)
apply (*rule ax-Normal-cases*)
apply (*erule conseq1*)
apply *clarsimp*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*])
apply (*clarsimp simp add*: *Let-def*)
done

lemma *MGFNormalD*:
$G,(A{::}state\ triple\ set)\vdash\{\doteq\}\ t\succ\ \{G\rightarrow\} \Longrightarrow G,A\vdash\{Normal\ \doteq\}\ t\succ\ \{G\rightarrow\}$
apply (*unfold MGF-def*)
apply (*erule conseq1*)
apply *clarsimp*
done

Additionally to *MGFNormalI*, we also expand the definition of the most general formula here

lemma *MGFn-NormalI*:

$G,(A::state\ triple\ set)\vdash\{Normal((\lambda Y'\ s'\ s.\ s'=s \wedge normal\ s) \wedge.\ G\vdash init\le n)\}t\succ$
$\{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\} \Longrightarrow G,A\vdash\{=:n\}t\succ\{G\rightarrow\}$
apply (*simp* (*no-asm-use*) *add*: *MGFn-def2*)
apply (*rule ax-Normal-cases*)
apply (*erule conseq1*)
apply *clarsimp*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*])
apply (*clarsimp simp add*: *Let-def*)
done

To derive the most general formula, we can restrict ourselves to welltyped terms, since all others can be uniformally handled by the hazard rule.

lemma *MGFn-free-wt*:
$(\exists\ T\ L\ C.\ (\!|prg=G,cls=C,lcl=L|\!)\vdash t::T)$
$\longrightarrow G,(A::state\ triple\ set)\vdash\{=:n\}\ t\succ\ \{G\rightarrow\}$
$\Longrightarrow G,A\vdash\{=:n\}\ t\succ\ \{G\rightarrow\}$
apply (*rule MGFn-NormalI*)
apply (*rule ax-free-wt*)
apply (*auto elim*: *conseq12 simp add*: *MGFn-def MGF-def*)
done

To derive the most general formula, we can restrict ourselves to welltyped terms and assume that the state in the precondition conforms to the environment. All type violations can be uniformally handled by the hazard rule.

lemma *MGFn-free-wt-NormalConformI*:
$(\forall\ T\ L\ C\ .\ (\!|prg=G,cls=C,lcl=L|\!)\vdash t::T$
$\longrightarrow G,(A::state\ triple\ set)$
$\vdash\{Normal((\lambda Y'\ s'\ s.\ s'=s \wedge normal\ s) \wedge.\ G\vdash init\le n) \wedge.\ (\lambda\ s.\ s::\preceq(G,\ L))\}$
$t\succ$
$\{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\})$
$\Longrightarrow G,A\vdash\{=:n\}t\succ\{G\rightarrow\}$
apply (*rule MGFn-NormalI*)
apply (*rule ax-no-hazard*)
apply (*rule ax-escape*)
apply (*intro strip*)
apply (*simp only*: *type-ok-def peek-and-def*)
apply (*erule conjE*)+
apply (*erule exE*,*erule exE*, *erule exE*, *erule exE*,*erule conjE*,*drule* (*1*) *mp*,
erule conjE)
apply (*drule spec*,*drule spec*, *drule spec*, *drule* (*1*) *mp*)
apply (*erule conseq12*)
apply *blast*
done

To derive the most general formula, we can restrict ourselves to welltyped terms and assume that the state in the precondition conforms to the environment and that the term is definetly assigned with respect to this state. All type violations can be uniformally handled by the hazard rule.

lemma *MGFn-free-wt-da-NormalConformI*:
$(\forall\ T\ L\ C\ B.\ (\!|prg=G,cls=C,lcl=L|\!)\vdash t::T$
$\longrightarrow G,(A::state\ triple\ set)$
$\vdash\{Normal((\lambda Y'\ s'\ s.\ s'=s \wedge normal\ s) \wedge.\ G\vdash init\le n) \wedge.\ (\lambda\ s.\ s::\preceq(G,\ L))$
$\wedge.\ (\lambda\ s.\ (\!|prg=G,cls=C,lcl=L|\!)\vdash dom\ (locals\ (store\ s))\gg t\gg B)\}$
$t\succ$
$\{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,s')\})$
$\Longrightarrow G,A\vdash\{=:n\}t\succ\{G\rightarrow\}$
apply (*rule MGFn-NormalI*)
apply (*rule ax-no-hazard*)
apply (*rule ax-escape*)

```
apply (intro strip)
apply (simp only: type-ok-def peek-and-def)
apply (erule conjE)+
apply (erule exE,erule exE, erule exE, erule exE,erule conjE,drule (1) mp,
    erule conjE)
apply (drule spec,drule spec, drule spec,drule spec, drule (1) mp)
apply (erule conseq12)
apply blast
done
```

main lemmas

```
lemma MGFn-Init:
 assumes mgf-hyp: ∀ m. Suc m≤n ⟶ (∀ t. G,A⊢{=:m} t≻ {G→})
 shows G,(A::state triple set)⊢{=:n} ⟨Init C⟩ₛ≻ {G→}
proof (rule MGFn-free-wt [rule-format],elim exE,rule MGFn-NormalI)
  fix T L accC
  assume ⦇prg=G, cls=accC, lcl= L⦈⊢⟨Init C⟩ₛ::T
  hence is-cls: is-class G C
    by cases simp
  show G,A⊢{Normal ((λY' s' s. s' = s ∧ normal s) ∧. G⊢init≤n)}
            .Init C.
            {λY s' s. G⊢s −⟨Init C⟩ₛ≻→ (Y, s')}
       (is G,A⊢{Normal ?P} .Init C. {?R})
  proof (rule ax-cases [where ?C=initd C])
    show G,A⊢{Normal ?P  ∧. initd C} .Init C. {?R}
      by (rule ax-derivs.Done [THEN conseq1]) (fastsimp intro: init-done)
  next
    have G,A⊢{Normal (?P  ∧. Not ∘ initd C)} .Init C. {?R}
    proof (cases n)
      case 0
      with is-cls
      show ?thesis
        by − (rule ax-impossible [THEN conseq1],fastsimp dest: nyinitcls-emptyD)
    next
      case (Suc m)
      with mgf-hyp have mgf-hyp': ⋀ t. G,A⊢{=:m} t≻ {G→}
        by simp
      from is-cls obtain c where c: the (class G C) = c
        by auto
      let ?Q= (λY s' (x,s) .
          G⊢ (x,init-class-obj G C s)
             − (if C=Object then Skip else Init (super (the (class G C))))→ s'
          ∧ x=None ∧ ¬inited C (globs s)) ∧. G⊢init≤m
      from c
      show ?thesis
      proof (rule ax-derivs.Init [where ?Q=?Q])
        let ?P' = Normal ((λY s' s. s' = supd (init-class-obj G C) s
                           ∧ normal s ∧ ¬ initd C s) ∧. G⊢init≤m)
        show G,A⊢{Normal (?P ∧. Not ∘ initd C ;. supd (init-class-obj G C))}
                  .(if C = Object then Skip else Init (super c)).
                  {?Q}
        proof (rule conseq1 [where ?P'=?P'])
          show G,A⊢{?P'} .(if C = Object then Skip else Init (super c)). {?Q}
          proof (cases C=Object)
            case True
            have G,A⊢{?P'} .Skip. {?Q}
              by (rule ax-derivs.Skip [THEN conseq1])
                 (auto simp add: True intro: eval.Skip)
```

```
          with True show ?thesis
            by simp
        next
          case False
          from mgf-hyp'
          have G,A⊢{?P'} .Init (super c). {?Q}
            by (rule MGFnD' [THEN conseq12]) (fastsimp simp add: False c)
          with False show ?thesis
            by simp
        qed
      next
        from Suc is-cls
        show Normal (?P ∧. Not ∘ initd C ;. supd (init-class-obj G C))
              ⇒ ?P'
          by (fastsimp elim: nyinitcls-le-SucD)
      qed
    next
      from mgf-hyp'
      show ∀ l. G,A⊢{?Q ∧. (λs. l = locals (snd s)) ;. set-lvars empty}
                    .init c.
                    {set-lvars l .; ?R}
        apply (rule MGFnD' [THEN conseq12, THEN allI])
        apply (clarsimp simp add: split-paired-all)
        apply (rule eval.Init [OF c])
        apply (insert c)
        apply auto
        done
    qed
  qed
  thus G,A⊢{Normal ?P ∧. Not ∘ initd C} .Init C. {?R}
    by clarsimp
 qed
qed
lemmas MGFn-InitD = MGFn-Init [THEN MGFnD, THEN ax-NormalD]


lemma MGFn-Call:
 assumes mgf-methds:
         ∀ C sig. G,(A::state triple set)⊢{=:n} ⟨(Methd C sig)⟩ₑ≻ {G→}
 and mgf-e: G,A⊢{=:n} ⟨e⟩ₑ≻ {G→}
 and mgf-ps: G,A⊢{=:n} ⟨ps⟩ₗ≻ {G→}
 and wf: wf-prog G
 shows G,A⊢{=:n} ⟨{accC,statT,mode}e·mn({pTs'}ps)⟩ₑ≻ {G→}
proof (rule MGFn-free-wt-da-NormalConformI [rule-format],clarsimp)
 note inj-term-simps [simp]
 fix T L accC' E
 assume wt: ⦇prg=G,cls=accC',lcl = L⦈⊢⟨({accC,statT,mode}e·mn( {pTs'}ps))⟩ₑ::T
 then obtain pTs statDeclT statM where
               wt-e: ⦇prg=G,cls=accC,lcl=L⦈⊢e::−RefT statT and
            wt-args: ⦇prg=G,cls=accC,lcl=L⦈⊢ps::≐pTs and
              statM: max-spec G accC statT ⦇name=mn,parTs=pTs⦈
                       = {((statDeclT,statM),pTs')} and
               mode: mode = invmode statM e and
                  T: T =Inl (resTy statM) and
       eq-accC-accC': accC=accC'
       by cases fastsimp+
 let ?Q=(λY s1 (x,s) . x = None ∧
           (∃ a. G⊢Norm s −e−≻a→ s1 ∧
                (normal s1 ⟶ G, store s1⊢a::≼RefT statT)
```

```
              ∧ Y = In1 a) ∧
          (∃ P. normal s1
             ⟶ (|prg=G,cls=accC',lcl=L|)⊢dom (locals (store s1))»⟨ps⟩ₗ»P))
       ∧. G⊢init≤n ∧. (λ s. s::≼(G, L))::state assn
let ?R=λa. ((λY (x2,s2) (x,s) . x = None ∧
            (∃s1 pvs. G⊢Norm s −e−≻a→ s1 ∧
                     (normal s1 ⟶ G, store s1⊢a::≼RefT statT)∧
                     Y = ⌊pvs⌋ₗ ∧ G⊢s1 −ps≐≻pvs→ (x2,s2)))
          ∧. G⊢init≤n ∧. (λ s. s::≼(G, L)))::state assn

show G,A⊢{Normal ((λY' s' s. s' = s ∧ abrupt s = None) ∧. G⊢init≤n ∧.
                (λs. s::≼(G, L)) ∧.
                (λs. (|prg=G, cls=accC',lcl=L|) ⊢ dom (locals (store s))
                     » ⟨{accC,statT,mode}e·mn( {pTs'}ps)⟩ₑ» E))}
         {accC,statT,mode}e·mn( {pTs'}ps)−≻
         {λY s' s. ∃v. Y = ⌊v⌋ₑ ∧
                     G⊢s −{accC,statT,mode}e·mn( {pTs'}ps)−≻v→ s'}
  (is G,A⊢{Normal ?P} {accC,statT,mode}e·mn( {pTs'}ps)−≻ {?S})
proof (rule ax-derivs.Call [where ?Q=?Q and ?R=?R])
  from mgf-e
  show G,A⊢{Normal ?P} e−≻ {?Q}
  proof (rule MGFnD' [THEN conseq12],clarsimp)
    fix s0 s1 a
    assume conf-s0: Norm s0::≼(G, L)
    assume da: (|prg=G,cls=accC',lcl=L|)⊢
                dom (locals s0) »⟨{accC,statT,mode}e·mn( {pTs'}ps)⟩ₑ» E
    assume eval-e: G⊢Norm s0 −e−≻a→ s1
    show (abrupt s1 = None ⟶ G,store s1⊢a::≼RefT statT) ∧
          (abrupt s1 = None ⟶
            (∃P. (|prg=G,cls=accC',lcl=L|)⊢ dom (locals (store s1)) »⟨ps⟩ₗ» P))
          ∧ s1::≼(G, L)
    proof −
      from da obtain C where
        da-e: (|prg=G,cls=accC,lcl=L|)⊢
                dom (locals (store ((Norm s0)::state)))»⟨e⟩ₑ» C and
        da-ps: (|prg=G,cls=accC,lcl=L|)⊢ nrm C »⟨ps⟩ₗ» E
        by cases (simp add: eq-accC-accC')
      from eval-e conf-s0 wt-e da-e wf
      obtain (abrupt s1 = None ⟶ G,store s1⊢a::≼RefT statT)
        and  s1::≼(G, L)
        by (rule eval-type-soundE) simp
      moreover
      {
        assume normal-s1: normal s1
        have ∃P. (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨ps⟩ₗ» P
        proof −
          from eval-e wt-e da-e wf normal-s1
          have nrm C ⊆  dom (locals (store s1))
            by (cases rule: da-good-approxE') iprover
          with da-ps show ?thesis
            by (rule da-weakenE) iprover
        qed
      }
      ultimately show ?thesis
        using eq-accC-accC' by simp
    qed
  qed
next
  show ∀a. G,A⊢{?Q←In1 a} ps≐≻ {?R a} (is ∀ a. ?PS a)
```

```
  proof
    fix a
    show ?PS a
    proof (rule MGFnD' [OF mgf-ps, THEN conseq12],
          clarsimp simp add: eq-accC-accC' [symmetric])
      fix s0 s1 s2 vs
      assume conf-s1: s1::⪯(G, L)
      assume eval-e: G⊢Norm s0 −e−≻a→ s1
      assume conf-a: abrupt s1 = None ⟶ G,store s1⊢a::⪯RefT statT
      assume eval-ps: G⊢s1 −ps≐≻vs→ s2
      assume da-ps: abrupt s1 = None ⟶
                    (∃ P. (|prg=G,cls=accC,lcl=L|)⊢
                          dom (locals (store s1)) »⟨ps⟩l» P)
      show (∃ s1. G⊢Norm s0 −e−≻a→ s1 ∧
              (abrupt s1 = None ⟶ G,store s1⊢a::⪯RefT statT) ∧
              G⊢s1 −ps≐≻vs→ s2) ∧
            s2::⪯(G, L)
      proof (cases normal s1)
        case True
        with da-ps obtain P where
         (|prg=G,cls=accC,lcl=L|)⊢ dom (locals (store s1)) »⟨ps⟩l» P
          by auto
        from eval-ps conf-s1 wt-args this wf
        have s2::⪯(G, L)
          by (rule eval-type-soundE)
        with eval-e conf-a eval-ps
        show ?thesis
          by auto
      next
        case False
        with eval-ps have s2=s1 by auto
        with eval-e conf-a eval-ps conf-s1
        show ?thesis
          by auto
      qed
    qed
  qed
next
  show ∀ a vs invC declC l.
    G,A⊢{?R a←⌊vs⌋l ∧.
          (λs. declC =
               invocation-declclass G mode (store s) a statT
                  (|name=mn, parTs=pTs'|) ∧
               invC = invocation-class mode (store s) a statT ∧
               l = locals (store s)) ;.
          init-lvars G declC (|name=mn, parTs=pTs'|) mode a vs ∧.
          (λs. normal s ⟶ G⊢mode→invC⪯statT)}
        Methd declC (|name=mn,parTs=pTs'|)−≻
        {set-lvars l .; ?S}
    (is ∀  a vs invC declC l. ?METHD a vs invC declC l)
  proof (intro allI)
    fix a vs invC declC l
    from mgf-methds [rule-format]
    show ?METHD a vs invC declC l
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s4 s2 s1::state
      fix s0 v
      let ?D= invocation-declclass G mode (store s2) a statT
                (|name=mn,parTs=pTs'|)
```

```
        let ?s3= init-lvars G ?D (|name=mn, parTs=pTs'|) mode a vs s2
        assume inv-prop: abrupt ?s3=None
             ⟶ G⊢mode→invocation-class mode (store s2) a statT≼statT
        assume conf-s2: s2::≼(G, L)
        assume conf-a: abrupt s1 = None ⟶ G,store s1⊢a::≼RefT statT
        assume eval-e: G⊢Norm s0 −e−≻a→ s1
        assume eval-ps: G⊢s1 −ps≐≻vs→ s2
        assume eval-mthd: G⊢?s3 −Methd ?D (|name=mn,parTs=pTs'|)−≻v→ s4
        show G⊢Norm s0 −{accC,statT,mode}e·mn( {pTs'}ps)−≻v
                    → (set-lvars (locals (store s2))) s4
        proof −
          obtain D where D: D=?D by simp
          obtain s3 where s3: s3=?s3 by simp
          obtain s3' where
            s3': s3' = check-method-access G accC statT mode
                        (|name=mn,parTs=pTs'|) a s3
            by simp
          have eq-s3'-s3: s3'=s3
          proof −
            from inv-prop s3 mode
            have normal s3 ⟹
             G⊢invmode statM e→invocation-class mode (store s2) a statT≼statT
              by auto
            with eval-ps wt-e statM conf-s2 conf-a [rule-format]
            have check-method-access G accC statT (invmode statM e)
                     (|name=mn,parTs=pTs'|) a s3 = s3
              by (rule error-free-call-access) (auto simp add: s3 mode wf)
            thus ?thesis
              by (simp add: s3' mode)
          qed
          with eval-mthd D s3
          have G⊢s3' −Methd D (|name=mn,parTs=pTs'|)−≻v→ s4
            by simp
          with eval-e eval-ps D - s3'
          show ?thesis
            by (rule eval-Call) (auto simp add: s3 mode D)
        qed
      qed
    qed
  qed
qed


lemma eval-expression-no-jump':
  assumes eval: G⊢s0 −e−≻v→ s1
  and   no-jmp: abrupt s0 ≠ Some (Jump j)
  and       wt: (|prg=G, cls=C,lcl=L|)⊢e::−T
  and       wf: wf-prog G
shows abrupt s1 ≠ Some (Jump j)
using eval no-jmp wt wf
by − (rule eval-expression-no-jump
            [where ?Env=(|prg=G, cls=C,lcl=L|),simplified],auto)
```

To derive the most general formula for the loop statement, we need to come up with a proper loop invariant, which intuitively states that we are currently inside the evaluation of the loop. To define such an invariant, we unroll the loop in iterated evaluations of the expression and evaluations of the loop body.

constdefs

```
unroll:: prog ⇒ label ⇒ expr ⇒ stmt ⇒ (state × state) set

unroll G l e c ≡ {(s,t). ∃ v s1 s2.
                        G⊢s −e−≻v→ s1 ∧ the-Bool v ∧ normal s1 ∧
                        G⊢s1 −c→ s2 ∧ t=(abupd (absorb (Cont l)) s2)}
```

```
lemma unroll-while:
  assumes unroll: (s, t) ∈ (unroll G l e c)*
  and     eval-e: G⊢t −e−≻v→ s′
  and     normal-termination: normal s′ ⟶ ¬ the-Bool v
  and     wt: ⦇prg=G,cls=C,lcl=L⦈⊢e::−T
  and     wf: wf-prog G
  shows G⊢s −l· While(e) c→ s′
using unroll
proof (induct rule: converse-rtrancl-induct)
  show G⊢t −l· While(e) c→ s′
  proof (cases normal t)
    case False
    with eval-e have s′=t by auto
    with False show ?thesis by auto
  next
    case True
    note normal-t = this
    show ?thesis
    proof (cases normal s′)
      case True
      with normal-t eval-e normal-termination
      show ?thesis
        by (auto intro: eval.Loop)
    next
      case False
      note abrupt-s′ = this
      from eval-e - wt wf
      have no-cont: abrupt s′ ≠ Some (Jump (Cont l))
        by (rule eval-expression-no-jump′) (insert normal-t,simp)
      have
        if the-Bool v
            then (G⊢s′ −c→ s′ ∧
                  G⊢(abupd (absorb (Cont l)) s′) −l· While(e) c→ s′)
            else s′ = s′
      proof (cases the-Bool v)
        case False thus ?thesis by simp
      next
        case True
        with abrupt-s′ have G⊢s′ −c→ s′ by auto
        moreover from abrupt-s′ no-cont
        have no-absorb: (abupd (absorb (Cont l)) s′)=s′
          by (cases s′) (simp add: absorb-def split: split-if)
        moreover
        from no-absorb abrupt-s′
        have G⊢(abupd (absorb (Cont l)) s′) −l· While(e) c→ s′
          by auto
        ultimately show ?thesis
          using True by simp
      qed
      with eval-e
      show ?thesis
```

```
        using normal-t by (auto intro: eval.Loop)
      qed
    qed
next
  fix s s3
  assume unroll: (s,s3) ∈ unroll G l e c
  assume while: G⊢s3 −l· While(e) c→ s′
  show G⊢s −l· While(e) c→ s′
  proof −
    from unroll obtain v s1 s2 where
      normal-s1: normal s1 and
      eval-e: G⊢s −e−≻v→ s1 and
      continue: the-Bool v and
      eval-c: G⊢s1 −c→ s2 and
      s3: s3=(abupd (absorb (Cont l)) s2)
      by (unfold unroll-def) fast
    from eval-e normal-s1 have
      normal s
      by (rule eval-no-abrupt-lemma [rule-format])
    with while eval-e continue eval-c s3 show ?thesis
      by (auto intro!: eval.Loop)
  qed
qed


lemma MGFn-Loop:
  assumes mfg-e: G,(A::state triple set)⊢{=:n} ⟨e⟩_e≻ {G→}
  and     mfg-c: G,A⊢{=:n} ⟨c⟩_s≻ {G→}
  and     wf: wf-prog G
shows G,A⊢{=:n} ⟨l· While(e) c⟩_s≻ {G→}
proof (rule MGFn-free-wt [rule-format],elim exE)
  fix T L C
  assume wt: (|prg = G, cls = C, lcl = L|)⊢⟨l· While(e) c⟩_s::T
  then obtain eT where
    wt-e: (|prg = G, cls = C, lcl = L|)⊢e::−eT
    by cases simp
  show ?thesis
  proof (rule MGFn-NormalI)
    show G,A⊢{Normal ((λY′ s′ s. s′ = s ∧ normal s) ∧. G⊢init≤n)}
              .l· While(e) c.
              {λY s′ s. G⊢s −In1r (l· While(e) c)≻→ (Y, s′)}
    proof (rule conseq12
           [where ?P′=(λ Y s′ s. (s,s′) ∈ (unroll G l e c)* ) ∧. G⊢init≤n
             and  ?Q′=((λ Y s′ s. (∃ t b. (s,t) ∈ (unroll G l e c)* ∧
                          Y=⌊b⌋_e ∧ G⊢t −e−≻b→ s′))
                        ∧. G⊢init≤n)←=False↓=◇])
      show  G,A⊢{(λY s′ s. (s, s′) ∈ (unroll G l e c)*) ∧. G⊢init≤n}
                .l· While(e) c.
                {((λY s′ s. (∃t b. (s, t) ∈ (unroll G l e c)* ∧
                                   Y = In1 b ∧ G⊢t −e−≻b→ s′))
                             ∧. G⊢init≤n)←=False↓=◇}
      proof (rule ax-derivs.Loop)
        from mfg-e
        show G,A⊢{(λY s′ s. (s, s′) ∈ (unroll G l e c)*) ∧. G⊢init≤n}
                 e−≻
                {(λY s′ s. (∃t b. (s, t) ∈ (unroll G l e c)* ∧
                                     Y = In1 b ∧ G⊢t −e−≻b→ s′))
                 ∧. G⊢init≤n}
        proof (rule MGFnD′ [THEN conseq12],clarsimp)
```

fix $s\ Z\ s'\ v$
assume $(Z,\ s) \in (\mathit{unroll}\ G\ l\ e\ c)^*$
moreover
assume $G\vdash s\ -e\!-\!\succ v\!\rightarrow\ s'$
ultimately
show $\exists\, t.\ (Z,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \wedge G\vdash t\ -e\!-\!\succ v\!\rightarrow\ s'$
by *blast*
qed
next
from *mfg-c*
show $G,A\vdash\{\mathit{Normal}\ (((\lambda Y\ s'\ s.\ \exists\, t\ b.\ (s,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \wedge$
$Y = \lfloor b\rfloor_e \wedge G\vdash t\ -e\!-\!\succ b\!\rightarrow\ s')$
$\wedge.\ G\vdash \mathit{init}\!\le\! n)\!\leftarrow\!=\mathit{True})\}$
$.c.$
$\{\mathit{abupd}\ (\mathit{absorb}\ (\mathit{Cont}\ l))\ .;$
$((\lambda Y\ s'\ s.\ (s,\ s') \in (\mathit{unroll}\ G\ l\ e\ c)^*) \wedge.\ G\vdash \mathit{init}\!\le\! n)\}$
proof (*rule MGFnD′* [*THEN conseq12*],*clarsimp*)
fix $Z\ s'\ s\ v\ t$
assume *unroll*: $(Z,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^*$
assume *eval-e*: $G\vdash t\ -e\!-\!\succ v\!\rightarrow\ \mathit{Norm}\ s$
assume *true*: *the-Bool v*
assume *eval-c*: $G\vdash \mathit{Norm}\ s\ -c\!\rightarrow\ s'$
show $(Z,\ \mathit{abupd}\ (\mathit{absorb}\ (\mathit{Cont}\ l))\ s') \in (\mathit{unroll}\ G\ l\ e\ c)^*$
proof −
note *unroll*
also
from *eval-e true eval-c*
have $(t,\mathit{abupd}\ (\mathit{absorb}\ (\mathit{Cont}\ l))\ s') \in \mathit{unroll}\ G\ l\ e\ c$
by (*unfold unroll-def*) *force*
ultimately show *?thesis* **..**
qed
qed
qed
next
show
$\forall\, Y\ s\ Z.$
$(\mathit{Normal}\ ((\lambda Y'\ s'\ s.\ s' = s \wedge \mathit{normal}\ s) \wedge.\ G\vdash \mathit{init}\!\le\! n))\ Y\ s\ Z$
$\longrightarrow (\forall\, Y'\ s'.$
$(\forall\, Y\ Z'.$
$((\lambda Y\ s'\ s.\ (s,\ s') \in (\mathit{unroll}\ G\ l\ e\ c)^*) \wedge.\ G\vdash \mathit{init}\!\le\! n)\ Y\ s\ Z'$
$\longrightarrow (((\lambda Y\ s'\ s.\ \exists\, t\ b.\ (s,t) \in (\mathit{unroll}\ G\ l\ e\ c)^*$
$\wedge\ Y{=}\lfloor b\rfloor_e \wedge G\vdash t\ -e\!-\!\succ b\!\rightarrow\ s')$
$\wedge.\ G\vdash \mathit{init}\!\le\! n)\!\leftarrow\!=\mathit{False}\!\downarrow\!=\!\Diamond)\ Y'\ s'\ Z')$
$\longrightarrow G\vdash Z\ -\langle l\bullet\ \mathit{While}(e)\ c\rangle_s\!\succ\!\rightarrow (Y',\ s'))$
proof (*clarsimp*)
fix $Y'\ s'\ s$
assume *asm*:
$\forall\, Z'.\ (Z',\ \mathit{Norm}\ s) \in (\mathit{unroll}\ G\ l\ e\ c)^*$
$\longrightarrow \mathit{card}\ (\mathit{nyinitcls}\ G\ s') \le n\ \wedge$
$(\exists\, v.\ (\exists\, t.\ (Z',\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^* \wedge G\vdash t\ -e\!-\!\succ v\!\rightarrow\ s') \wedge$
$(\mathit{fst}\ s' = \mathit{None} \longrightarrow \neg\ \mathit{the\text{-}Bool}\ v)) \wedge Y' = \Diamond$
show $Y' = \Diamond \wedge G\vdash \mathit{Norm}\ s\ -l\bullet\ \mathit{While}(e)\ c\!\rightarrow\ s'$
proof −
from *asm* **obtain** $v\ t$ **where**
— Z' gets instantiated with *Norm s*
unroll: $(\mathit{Norm}\ s,\ t) \in (\mathit{unroll}\ G\ l\ e\ c)^*$ **and**
eval-e: $G\vdash t\ -e\!-\!\succ v\!\rightarrow\ s'$ **and**
normal-termination: $\mathit{normal}\ s' \longrightarrow \neg\ \mathit{the\text{-}Bool}\ v$ **and**
Y': $Y' = \Diamond$

by *auto*
from *unroll eval-e normal-termination wt-e wf*
have $G\vdash Norm\ s\ -l\cdot\ While(e)\ c\rightarrow\ s'$
by (*rule unroll-while*)
with Y'
show *?thesis*
by *simp*
qed
qed
qed
qed
qed

lemma *MGFn-FVar*:
fixes A :: *state triple set*
assumes *mgf-init*: $G,A\vdash\{=:n\}\ \langle Init\ statDeclC\rangle_s\succ\ \{G\rightarrow\}$
and *mgf-e*: $G,A\vdash\{=:n\}\ \langle e\rangle_e\succ\ \{G\rightarrow\}$
and *wf*: *wf-prog G*
shows $G,A\vdash\{=:n\}\ \langle\{accC,statDeclC,stat\}e..fn\rangle_v\succ\ \{G\rightarrow\}$
proof (*rule MGFn-free-wt-da-NormalConformI* [*rule-format*],*clarsimp*)
note *inj-term-simps* [*simp*]
fix $T\ L\ accC'\ V$
assume *wt*: $(\!|prg = G,\ cls = accC',\ lcl = L|\!)\vdash\langle\{accC,statDeclC,stat\}e..fn\rangle_v{::}T$
then obtain *statC f* **where**
wt-e: $(\!|prg{=}G,\ cls{=}accC',\ lcl{=}L|\!)\vdash e{::}-Class\ statC$ **and**
accfield: $accfield\ G\ accC'\ statC\ fn = Some\ (statDeclC,f\ )$ **and**
eq-accC: $accC{=}accC'$ **and**
stat: $stat{=}is\text{-}static\ \ f$
by (*cases*) (*auto simp add*: *member-is-static-simp*)
let $?Q{=}(\lambda Y\ s1\ (x,s)\ .\ x = None\ \wedge$
$(G\vdash Norm\ s\ -Init\ statDeclC\rightarrow\ s1)\ \wedge$
$(\exists\ E.\ (\!|prg{=}G,cls{=}accC',lcl{=}L|\!)\vdash dom\ (locals\ (store\ s1))\ »\langle e\rangle_e»\ E))$
$\wedge.\ G\vdash init{\le}n\ \wedge.\ (\lambda\ s.\ s{::}{\preceq}(G,\ L))$
show $G,A\vdash\{Normal$
$((\lambda Y'\ s'\ s.\ s' = s\ \wedge\ abrupt\ s = None)\ \wedge.\ G\vdash init{\le}n\ \wedge.$
$(\lambda s.\ s{::}{\preceq}(G,\ L))\ \wedge.$
$(\lambda s.\ (\!|prg{=}G,cls{=}accC',lcl{=}L|\!)$
$\vdash\ dom\ (locals\ (store\ s))\ »\ \langle\{accC,statDeclC,stat\}e..fn\rangle_v»\ V))$
$\}\ \{accC,statDeclC,stat\}e..fn{=}\!\succ$
$\{\lambda Y\ s'\ s.\ \exists vf.\ Y = \lfloor vf\rfloor_v\ \wedge$
$G\vdash s\ -\{accC,statDeclC,stat\}e..fn{=}\!\succ vf\rightarrow\ s'\}$
(**is** $G,A\vdash\{Normal\ ?P\}\ \{accC,statDeclC,stat\}e..fn{=}\!\succ\ \{?R\}$)
proof (*rule ax-derivs.FVar* [**where** $?Q{=}?Q$])
from *mgf-init*
show $G,A\vdash\{Normal\ ?P\}\ .Init\ statDeclC.\ \{?Q\}$
proof (*rule MGFnD′* [*THEN conseq12*],*clarsimp*)
fix $s\ s'$
assume *conf-s*: $Norm\ s{::}{\preceq}(G,\ L)$
assume *da*: $(\!|prg{=}G,cls{=}accC',lcl{=}L|\!)$
$\vdash\ dom\ (locals\ s)\ »\langle\{accC,statDeclC,stat\}e..fn\rangle_v»\ V$
assume *eval-init*: $G\vdash Norm\ s\ -Init\ statDeclC\rightarrow\ s'$
show $(\exists E.\ (\!|prg{=}G,\ cls{=}accC',\ lcl{=}L|\!)\vdash\ dom\ (locals\ (store\ s'))\ »\langle e\rangle_e»\ E)\ \wedge$
$s'{::}{\preceq}(G,\ L))$
proof −
from *da*
obtain E **where**
$(\!|prg{=}G,\ cls{=}accC',\ lcl{=}L|\!)\vdash\ dom\ (locals\ s)\ »\langle e\rangle_e»\ E$
by *cases simp*

```
      moreover
      from eval-init
      have dom (locals s) ⊆ dom (locals (store s′))
        by (rule dom-locals-eval-mono [elim-format]) simp
      ultimately obtain E′ where
        (|prg=G, cls=accC′, lcl=L|)⊢ dom (locals (store s′)) »⟨e⟩ₑ» E′
        by (rule da-weakenE)
      moreover
      have s′::≼(G, L)
      proof −
        have wt-init: (|prg=G, cls=accC, lcl=L|)⊢(Init statDeclC)::√
        proof −
          from wf wt-e
          have iscls-statC: is-class G statC
            by (auto dest: ty-expr-is-type type-is-class)
          with wf accfield
          have iscls-statDeclC: is-class G statDeclC
            by (auto dest!: accfield-fields dest: fields-declC)
          thus ?thesis by simp
        qed
        obtain I where
          da-init: (|prg=G,cls=accC,lcl=L|)
            ⊢ dom (locals (store ((Norm s)::state))) »⟨Init statDeclC⟩ₛ» I
          by (auto intro: da-Init [simplified] assigned.select-convs)
        from eval-init conf-s wt-init da-init  wf
        show ?thesis
          by (rule eval-type-soundE)
      qed
      ultimately show ?thesis by iprover
    qed
  qed
next
  from mgf-e
  show G,A⊢{?Q} e−≻ {λVal:a:. fvar statDeclC stat fn a ..; ?R}
  proof (rule MGFnD′ [THEN conseq12],clarsimp)
    fix s0 s1 s2 E a
    let ?fvar = fvar statDeclC stat fn a s2
    assume eval-init: G⊢Norm s0 −Init statDeclC→ s1
    assume eval-e: G⊢s1 −e−≻a→ s2
    assume conf-s1: s1::≼(G, L)
    assume da-e: (|prg=G,cls=accC′,lcl=L|)⊢ dom (locals (store s1)) »⟨e⟩ₑ» E
    show G⊢Norm s0 −{accC,statDeclC,stat}e..fn=≻fst ?fvar→ snd ?fvar
    proof −
      obtain v s2′ where
        v: v=fst ?fvar and s2′: s2′=snd ?fvar
        by simp
      obtain s3 where
        s3: s3= check-field-access G accC′ statDeclC fn stat a s2′
        by simp
      have eq-s3-s2′: s3=s2′
      proof −
        from eval-e conf-s1 wt-e da-e wf obtain
          conf-s2: s2::≼(G, L)  and
          conf-a: normal s2 ⟹ G,store s2⊢a::≼Class statC
          by (rule eval-type-soundE) simp
        from accfield wt-e eval-init eval-e conf-s2 conf-a - wf
        show ?thesis
          by (rule  error-free-field-access
                 [where ?v=v and ?s2′=s2′,elim-format])
```

```
            (simp add: s3 v s2′ stat)+
        qed
        from eval-init eval-e
        show ?thesis
          apply (rule eval.FVar [where ?s2′=s2′])
          apply  (simp add: s2′)
          apply  (simp add: s3 [symmetric]   eq-s3-s2′ eq-accC s2′ [symmetric])
          done
      qed
    qed
  qed
qed


lemma MGFn-Fin:
  assumes wf: wf-prog G
  and     mgf-c1: G,A⊢{=:n} ⟨c1⟩ₛ≻ {G→}
  and     mgf-c2: G,A⊢{=:n} ⟨c2⟩ₛ≻ {G→}
  shows G,(A::state triple set)⊢{=:n} ⟨c1 Finally c2⟩ₛ≻ {G→}
proof (rule MGFn-free-wt-da-NormalConformI [rule-format],clarsimp)
  fix T L accC C
  assume wt: (|prg=G,cls=accC,lcl=L|)⊢In1r (c1 Finally c2)::T
  then obtain
    wt-c1: (|prg=G,cls=accC,lcl=L|)⊢c1::√ and
    wt-c2: (|prg=G,cls=accC,lcl=L|)⊢c2::√
    by cases simp
  let  ?Q = (λY′ s′ s. normal s ∧ G⊢s −c1→ s′ ∧
              (∃ C1. (|prg=G,cls=accC,lcl=L|)⊢dom (locals (store s)) »⟨c1⟩ₛ» C1)
              ∧ s::≼(G, L))
             ∧. G⊢init≤n
  show G,A⊢{Normal
             ((λY′ s′ s. s′ = s ∧ abrupt s = None) ∧. G⊢init≤n ∧.
             (λs. s::≼(G, L)) ∧.
             (λs. (|prg=G,cls=accC,lcl =L|)
                  ⊢dom (locals (store s)) »⟨c1 Finally c2⟩ₛ» C))}
             .c1 Finally c2.
             {λY s′ s. Y = ◇ ∧ G⊢s −c1 Finally c2→ s′}
    (is G,A⊢{Normal ?P} .c1 Finally c2. {?R})
  proof (rule ax-derivs.Fin [where ?Q=?Q])
    from mgf-c1
    show G,A⊢{Normal ?P} .c1. {?Q}
    proof (rule MGFnD′ [THEN conseq12],clarsimp)
      fix s0
      assume (|prg=G,cls=accC,lcl=L|)⊢ dom (locals s0) »⟨c1 Finally c2⟩ₛ» C
      thus ∃C1. (|prg=G,cls=accC,lcl=L|)⊢ dom (locals s0) »⟨c1⟩ₛ» C1
        by cases (auto simp add: inj-term-simps)
    qed
  next
    from mgf-c2
    show ∀abr. G,A⊢{?Q ∧. (λs. abr = abrupt s) ;. abupd (λabr. None)} .c2.
         {abupd (abrupt-if (abr ≠ None) abr) .; ?R}
    proof (rule MGFnD′ [THEN conseq12, THEN allI],clarsimp)
      fix s0 s1 s2 C1
      assume da-c1:(|prg=G,cls=accC,lcl=L|)⊢ dom (locals s0) »⟨c1⟩ₛ» C1
      assume conf-s0: Norm s0::≼(G, L)
      assume eval-c1: G⊢Norm s0 −c1→ s1
      assume eval-c2: G⊢abupd (λabr. None) s1 −c2→ s2
      show G⊢Norm s0 −c1 Finally c2
```

```
          → abupd (abrupt-if (∃y. abrupt s1 = Some y) (abrupt s1)) s2
    proof -
      obtain abr1 str1 where s1: s1=(abr1,str1)
        by (cases s1)
      with eval-c1 eval-c2 obtain
        eval-c1': G⊢Norm s0 −c1→ (abr1,str1) and
        eval-c2': G⊢Norm str1 −c2→ s2
        by simp
      obtain s3 where
        s3: s3 = (if ∃err. abr1 = Some (Error err)
                    then (abr1, str1)
                    else abupd (abrupt-if (abr1 ≠ None) abr1) s2)
        by simp
      from eval-c1' conf-s0 wt-c1 - wf
      have error-free (abr1,str1)
        by (rule eval-type-soundE) (insert da-c1,auto)
      with s3 have eq-s3: s3=abupd (abrupt-if (abr1 ≠ None) abr1) s2
        by (simp add: error-free-def)
      from eval-c1' eval-c2' s3
      show ?thesis
        by (rule eval.Fin [elim-format]) (simp add: s1 eq-s3)
    qed
  qed
 qed
qed


lemma Body-no-break:
 assumes eval-init: G⊢Norm s0 −Init D→ s1
   and      eval-c: G⊢s1 −c→ s2
   and       jmpOk: jumpNestingOkS {Ret} c
   and        wt-c: (|prg=G, cls=C, lcl=L|)⊢c::√
   and        clsD: class G D=Some d
   and          wf: wf-prog G
  shows ∀ l. abrupt s2 ≠ Some (Jump (Break l)) ∧
             abrupt s2 ≠ Some (Jump (Cont l))
proof
  fix l show abrupt s2 ≠ Some (Jump (Break l)) ∧
             abrupt s2 ≠ Some (Jump (Cont l))
  proof -
    from clsD have wt-init: (|prg=G, cls=accC, lcl=L|)⊢(Init D)::√
      by auto
    from eval-init wf
    have s1-no-jmp: ⋀ j. abrupt s1 ≠ Some (Jump j)
      by - (rule eval-statement-no-jump [OF - - - wt-init],auto)
    from eval-c - wt-c wf
    show ?thesis
      apply (rule jumpNestingOk-eval [THEN conjE, elim-format])
      using jmpOk s1-no-jmp
      apply auto
      done
  qed
qed


lemma MGFn-Body:
  assumes wf: wf-prog G
  and     mgf-init: G,A⊢{=:n} ⟨Init D⟩ₛ≻ {G→}
  and     mgf-c: G,A⊢{=:n} ⟨c⟩ₛ≻ {G→}
```

```
  shows  G,(A::state triple set)⊢{=:n} ⟨Body D c⟩ₑ≻ {G→}
proof (rule MGFn-free-wt-da-NormalConformI [rule-format],clarsimp)
  fix T L accC E
  assume wt: ⦇prg=G, cls=accC,lcl=L⦈⊢⟨Body D c⟩ₑ::T
  let ?Q=(λY' s' s. normal s ∧ G⊢s −Init D→ s' ∧ jumpNestingOkS {Ret} c)
        ∧. G⊢init≤n
  show G,A⊢{Normal
            ((λY' s' s. s' = s ∧ fst s = None) ∧. G⊢init≤n ∧.
             (λs. s::≼(G, L)) ∧.
             (λs. ⦇prg=G,cls=accC,lcl=L⦈
                  ⊢ dom (locals (store s)) »⟨Body D c⟩ₑ» E))}
           Body D c−≻
           {λY s' s. ∃ v. Y = In1 v ∧ G⊢s −Body D c−≻v→ s'}
    (is G,A⊢{Normal ?P} Body D c−≻ {?R})
  proof (rule ax-derivs.Body [where ?Q=?Q])
    from mgf-init
    show G,A⊢{Normal ?P} .Init D. {?Q}
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s0
      assume da: ⦇prg=G,cls=accC,lcl=L⦈⊢ dom (locals s0) »⟨Body D c⟩ₑ» E
      thus jumpNestingOkS {Ret} c
        by cases simp
    qed
  next
    from mgf-c
    show G,A⊢{?Q}.c.{λs.. abupd (absorb Ret) .; ?R←⌊the (locals s Result)⌋ₑ}
    proof (rule MGFnD' [THEN conseq12],clarsimp)
      fix s0 s1 s2
      assume eval-init: G⊢Norm s0 −Init D→ s1
      assume eval-c: G⊢s1 −c→ s2
      assume nestingOk: jumpNestingOkS {Ret} c
      show G⊢Norm s0 −Body D c−≻the (locals (store s2) Result)
              → abupd (absorb Ret) s2
      proof −
        from wt obtain d where
          d: class G D=Some d and
          wt-c: ⦇prg = G, cls = accC, lcl = L⦈⊢c::√
          by cases auto
        obtain s3 where
          s3: s3= (if ∃ l. fst s2 = Some (Jump (Break l)) ∨
                          fst s2 = Some (Jump (Cont l))
                    then abupd (λx. Some (Error CrossMethodJump)) s2
                    else s2)
          by simp
        from eval-init eval-c nestingOk wt-c d wf
        have eq-s3-s2: s3=s2
          by (rule Body-no-break [elim-format]) (simp add: s3)
        from eval-init eval-c s3
        show ?thesis
          by (rule eval.Body [elim-format]) (simp add: eq-s3-s2)
      qed
    qed
  qed
qed


lemma MGFn-lemma:
  assumes mgf-methds:
          ⋀ n. ∀ C sig. G,(A::state triple set)⊢{=:n} ⟨Methd C sig⟩ₑ≻ {G→}
```

```
  and wf: wf-prog G
  shows ⋀ t. G,A⊢{=:n} t≻ {G→}
proof (induct rule: full-nat-induct)
  fix n t
  assume hyp: ∀ m. Suc m ≤ n ⟶ (∀ t. G,A⊢{=:m} t≻ {G→})
  show G,A⊢{=:n} t≻ {G→}
  proof −
  {
    fix v e c es
    have G,A⊢{=:n} ⟨v⟩ᵥ≻ {G→} and
      G,A⊢{=:n} ⟨e⟩ₑ≻ {G→} and
      G,A⊢{=:n} ⟨c⟩ₛ≻ {G→} and
      G,A⊢{=:n} ⟨es⟩ₗ≻ {G→}
    proof (induct rule: var-expr-stmt.inducts)
      case (LVar v)
      show G,A⊢{=:n} ⟨LVar v⟩ᵥ≻ {G→}
        apply (rule MGFn-NormalI)
        apply (rule ax-derivs.LVar [THEN conseq1])
        apply (clarsimp)
        apply (rule eval.LVar)
        done
    next
      case (FVar accC statDeclC stat e fn)
      from MGFn-Init [OF hyp] and ‹G,A⊢{=:n} ⟨e⟩ₑ≻ {G→}› and wf
      show ?case
        by (rule MGFn-FVar)
    next
      case (AVar e1 e2)
      note mgf-e1 = ‹G,A⊢{=:n} ⟨e1⟩ₑ≻ {G→}›
      note mgf-e2 = ‹G,A⊢{=:n} ⟨e2⟩ₑ≻ {G→}›
      show G,A⊢{=:n} ⟨e1.[e2]⟩ᵥ≻ {G→}
        apply (rule MGFn-NormalI)
        apply (rule ax-derivs.AVar)
        apply  (rule MGFnD [OF mgf-e1, THEN ax-NormalD])
        apply (rule allI)
        apply (rule MGFnD' [OF mgf-e2, THEN conseq12])
        apply (fastsimp intro: eval.AVar)
        done
    next
      case (InsInitV c v)
      show ?case
        by (rule MGFn-NormalI) (rule ax-derivs.InsInitV)
    next
      case (NewC C)
      show ?case
        apply (rule MGFn-NormalI)
        apply (rule ax-derivs.NewC)
        apply (rule MGFn-InitD [OF hyp, THEN conseq2])
        apply (fastsimp intro: eval.NewC)
        done
    next
      case (NewA T e)
      thus ?case
        apply −
        apply (rule MGFn-NormalI)
        apply (rule ax-derivs.NewA
               [where ?Q = (λY' s' s. normal s ∧ G⊢s −In1r (init-comp-ty T)
                         ≻→ (Y',s')) ∧. G⊢init≤n])
        apply  (simp add: init-comp-ty-def split add: split-if)
```

```
      apply (rule conjI, clarsimp)
      apply (rule MGFn-InitD [OF hyp, THEN conseq2])
      apply (clarsimp intro: eval.Init)
      apply clarsimp
      apply (rule ax-derivs.Skip [THEN conseq1])
      apply (clarsimp intro: eval.Skip)
      apply (erule MGFnD' [THEN conseq12])
      apply (fastsimp intro: eval.NewA)
      done
  next
    case (Cast C e)
    thus ?case
      apply -
      apply (rule MGFn-NormalI)
      apply (erule MGFnD'[THEN conseq12,THEN ax-derivs.Cast])
      apply (fastsimp intro: eval.Cast)
      done
  next
    case (Inst e C)
    thus ?case
      apply -
      apply (rule MGFn-NormalI)
      apply (erule MGFnD'[THEN conseq12,THEN ax-derivs.Inst])
      apply (fastsimp intro: eval.Inst)
      done
  next
    case (Lit v)
    show ?case
      apply -
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.Lit [THEN conseq1])
      apply (fastsimp intro: eval.Lit)
      done
  next
    case (UnOp unop e)
    thus ?case
      apply -
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.UnOp)
      apply (erule MGFnD' [THEN conseq12])
      apply (fastsimp intro: eval.UnOp)
      done
  next
    case (BinOp binop e1 e2)
    thus ?case
      apply -
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.BinOp)
      apply (erule MGFnD [THEN ax-NormalD])
      apply (rule allI)
      apply (case-tac need-second-arg binop v1)
      apply simp
      apply (erule MGFnD' [THEN conseq12])
      apply (fastsimp intro: eval.BinOp)
      apply simp
      apply (rule ax-Normal-cases)
      apply (rule ax-derivs.Skip [THEN conseq1])
      apply clarsimp
      apply (rule eval-BinOp-arg2-indepI)
```

apply *simp*
apply *simp*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*], *clarsimp simp add*: *Let-def*)
apply (*fastsimp intro*: *eval.BinOp*)
done
next
case *Super*
show *?case*
apply –
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Super* [*THEN conseq1*])
apply (*fastsimp intro*: *eval.Super*)
done
next
case (*Acc v*)
thus *?case*
apply –
apply (*rule MGFn-NormalI*)
apply (*erule MGFnD'*[*THEN conseq12*,*THEN ax-derivs.Acc*])
apply (*fastsimp intro*: *eval.Acc simp add*: *split-paired-all*)
done
next
case (*Ass v e*)
thus $G,A\vdash\{=:n\}\ \langle v{:=}e\rangle_e\succ\ \{G\to\}$
apply –
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Ass*)
apply (*erule MGFnD* [*THEN ax-NormalD*])
apply (*rule allI*)
apply (*erule MGFnD'*[*THEN conseq12*])
apply (*fastsimp intro*: *eval.Ass simp add*: *split-paired-all*)
done
next
case (*Cond e1 e2 e3*)
thus $G,A\vdash\{=:n\}\ \langle e1\ ?\ e2\ :\ e3\rangle_e\succ\ \{G\to\}$
apply –
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Cond*)
apply (*erule MGFnD* [*THEN ax-NormalD*])
apply (*rule allI*)
apply (*rule ax-Normal-cases*)
prefer *2*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*],*clarsimp simp add*: *Let-def*)
apply (*fastsimp intro*: *eval.Cond*)
apply (*case-tac b*)
apply *simp*
apply (*erule MGFnD'*[*THEN conseq12*])
apply (*fastsimp intro*: *eval.Cond*)
apply *simp*
apply (*erule MGFnD'*[*THEN conseq12*])
apply (*fastsimp intro*: *eval.Cond*)
done
next
case (*Call accC statT mode e mn pTs' ps*)
note *mgf-e* = ‹$G,A\vdash\{=:n\}\ \langle e\rangle_e\succ\ \{G\to\}$›
note *mgf-ps* = ‹$G,A\vdash\{=:n\}\ \langle ps\rangle_l\succ\ \{G\to\}$›
from *mgf-methds mgf-e mgf-ps wf*
show $G,A\vdash\{=:n\}\ \langle\{accC,statT,mode\}e{\cdot}mn(\{pTs'\}ps)\rangle_e\succ\ \{G\to\}$
by (*rule MGFn-Call*)

next
  case (*Methd D mn*)
  from *mgf-methds*
  show $G,A\vdash\{=:n\}\ \langle Methd\ D\ mn\rangle_e\succ\ \{G\rightarrow\}$
    by *simp*
next
  case (*Body D c*)
  note *mgf-c* = ‹$G,A\vdash\{=:n\}\ \langle c\rangle_s\succ\ \{G\rightarrow\}$›
  from *wf MGFn-Init* [*OF hyp*] *mgf-c*
  show $G,A\vdash\{=:n\}\ \langle Body\ D\ c\rangle_e\succ\ \{G\rightarrow\}$
    by (*rule MGFn-Body*)
next
  case (*InsInitE c e*)
  show *?case*
    by (*rule MGFn-NormalI*) (*rule ax-derivs.InsInitE*)
next
  case (*Callee l e*)
  show *?case*
    by (*rule MGFn-NormalI*) (*rule ax-derivs.Callee*)
next
  case *Skip*
  show *?case*
    apply −
    apply (*rule MGFn-NormalI*)
    apply (*rule ax-derivs.Skip* [*THEN conseq1*])
    apply (*fastsimp intro*: *eval.Skip*)
    done
next
  case (*Expr e*)
  thus *?case*
    apply −
    apply (*rule MGFn-NormalI*)
    apply (*erule MGFnD′*[*THEN conseq12*,*THEN ax-derivs.Expr*])
    apply (*fastsimp intro*: *eval.Expr*)
    done
next
  case (*Lab l c*)
  thus $G,A\vdash\{=:n\}\ \langle l\cdot\ c\rangle_s\succ\ \{G\rightarrow\}$
    apply −
    apply (*rule MGFn-NormalI*)
    apply (*erule MGFnD′* [*THEN conseq12*, *THEN ax-derivs.Lab*])
    apply (*fastsimp intro*: *eval.Lab*)
    done
next
  case (*Comp c1 c2*)
  thus $G,A\vdash\{=:n\}\ \langle c1;;\ c2\rangle_s\succ\ \{G\rightarrow\}$
    apply −
    apply (*rule MGFn-NormalI*)
    apply (*rule ax-derivs.Comp*)
    apply (*erule MGFnD* [*THEN ax-NormalD*])
    apply (*erule MGFnD′* [*THEN conseq12*])
    apply (*fastsimp intro*: *eval.Comp*)
    done
next
  case (*If′ e c1 c2*)
  thus $G,A\vdash\{=:n\}\ \langle If(e)\ c1\ Else\ c2\rangle_s\succ\ \{G\rightarrow\}$
    apply −
    apply (*rule MGFn-NormalI*)
    apply (*rule ax-derivs.If*)

```
      apply  (erule MGFnD [THEN ax-NormalD])
      apply (rule allI)
      apply (rule ax-Normal-cases)
      prefer 2
      apply  (rule ax-derivs.Abrupt [THEN conseq1],clarsimp simp add: Let-def)
      apply  (fastsimp intro: eval.If)
      apply (case-tac b)
      apply  simp
      apply  (erule MGFnD' [THEN conseq12])
      apply  (fastsimp intro: eval.If)
      apply simp
      apply (erule MGFnD' [THEN conseq12])
      apply (fastsimp intro: eval.If)
      done
  next
    case (Loop l e c)
    note mgf-e = ‹G,A⊢{=:n} ⟨e⟩e≻ {G→}›
    note mgf-c = ‹G,A⊢{=:n} ⟨c⟩s≻ {G→}›
    from mgf-e mgf-c wf
    show G,A⊢{=:n} ⟨l• While(e) c⟩s≻ {G→}
      by (rule MGFn-Loop)
  next
    case (Jmp j)
    thus ?case
      apply −
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.Jmp [THEN conseq1])
      apply (auto intro: eval.Jmp simp add: abupd-def2)
      done
  next
    case (Throw e)
    thus ?case
      apply −
      apply (rule MGFn-NormalI)
      apply (erule MGFnD' [THEN conseq12, THEN ax-derivs.Throw])
      apply (fastsimp intro: eval.Throw)
      done
  next
    case (TryC c1 C vn c2)
    thus G,A⊢{=:n} ⟨Try c1 Catch(C vn) c2⟩s≻ {G→}
      apply −
      apply (rule MGFn-NormalI)
      apply (rule ax-derivs.Try [where
        ?Q = (λY' s' s. normal s ∧ (∃s''. G⊢s −⟨c1⟩s≻→ (Y',s'') ∧
                        G⊢s'' −sxalloc→ s')) ∧. G⊢init≤n])
      apply   (erule MGFnD [THEN ax-NormalD, THEN conseq2])
      apply   (fastsimp elim: sxalloc-gext [THEN card-nyinitcls-gext])
      apply  (erule MGFnD'[THEN conseq12])
      apply  (fastsimp intro: eval.Try)
      apply (fastsimp intro: eval.Try)
      done
  next
    case (Fin c1 c2)
    note mgf-c1 = ‹G,A⊢{=:n} ⟨c1⟩s≻ {G→}›
    note mgf-c2 = ‹G,A⊢{=:n} ⟨c2⟩s≻ {G→}›
    from wf mgf-c1 mgf-c2
    show G,A⊢{=:n} ⟨c1 Finally c2⟩s≻ {G→}
      by (rule MGFn-Fin)
  next
```

case (*FinA abr c*)
show *?case*
by (*rule MGFn-NormalI*) (*rule ax-derivs.FinA*)
next
case (*Init C*)
from *hyp*
show $G,A\vdash\{=:n\}\ \langle Init\ C\rangle_s\succ\ \{G\rightarrow\}$
by (*rule MGFn-Init*)
next
case *Nil-expr*
show $G,A\vdash\{=:n\}\ \langle[]\rangle_l\succ\ \{G\rightarrow\}$
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Nil* [*THEN conseq1*])
apply (*fastsimp intro*: *eval.Nil*)
done
next
case (*Cons-expr e es*)
thus $G,A\vdash\{=:n\}\ \langle e\#\ es\rangle_l\succ\ \{G\rightarrow\}$
apply −
apply (*rule MGFn-NormalI*)
apply (*rule ax-derivs.Cons*)
apply (*erule MGFnD* [*THEN ax-NormalD*])
apply (*rule allI*)
apply (*erule MGFnD'*[*THEN conseq12*])
apply (*fastsimp intro*: *eval.Cons*)
done
qed
}
thus *?thesis*
by (*cases rule*: *term-cases*) *auto*
qed
qed

lemma *MGF-asm*:
$[\![\forall\ C\ sig.\ is\text{-}methd\ G\ C\ sig \longrightarrow G,A\vdash\{\doteq\}\ In1l\ (Methd\ C\ sig)\succ\ \{G\rightarrow\};\ wf\text{-}prog\ G]\!]$
$\Longrightarrow G,(A::state\ triple\ set)\vdash\{\doteq\}\ t\succ\ \{G\rightarrow\}$
apply (*simp* (*no-asm-use*) *add*: *MGF-MGFn-iff*)
apply (*rule allI*)
apply (*rule MGFn-lemma*)
apply (*intro strip*)
apply (*rule MGFn-free-wt*)
apply (*force dest*: *wt-Methd-is-methd*)
apply *assumption*
done

nested version

lemma *nesting-lemma'* [*rule-format* (*no-asm*)]:
assumes *ax-derivs-asm*: $\bigwedge A\ ts.\ ts \subseteq A \Longrightarrow P\ A\ ts$
and *MGF-nested-Methd*: $\bigwedge A\ pn.\ \forall\, b{\in}bdy\ pn.\ P\ (insert\ (mgf\text{-}call\ pn)\ A)\ \{mgf\ b\}$
$\Longrightarrow P\ A\ \{mgf\text{-}call\ pn\}$
and *MGF-asm*: $\bigwedge A\ t.\ \forall\, pn{\in}U.\ P\ A\ \{mgf\text{-}call\ pn\} \Longrightarrow P\ A\ \{mgf\ t\}$
and *finU*: *finite U*
and *uA*: *uA* = *mgf-call‘U*
shows $\forall\, A.\ A \subseteq uA \longrightarrow n \leq card\ uA \longrightarrow card\ A = card\ uA - n$
$\longrightarrow (\forall\, t.\ P\ A\ \{mgf\ t\})$
using *finU uA*

apply –
apply (*induct-tac n*)
apply (*tactic ALLGOALS* (*clarsimp-tac* @{*clasimpset*}))
apply (*tactic* ⟪ *dtac* (*permute-prems 0 1* (*thm card-seteq*)) *1* ⟫)
apply *simp*
apply (*erule finite-imageI*)
apply (*simp add*: *MGF-asm ax-derivs-asm*)
apply (*rule MGF-asm*)
apply (*rule ballI*)
apply (*case-tac mgf-call pn* : *A*)
apply (*fast intro*: *ax-derivs-asm*)
apply (*rule MGF-nested-Methd*)
apply (*rule ballI*)
apply (*drule spec*, *erule impE*, *erule-tac* [*2*] *impE*, *erule-tac* [*3*] *spec*)
apply *fast*
apply (*drule finite-subset*)
apply (*erule finite-imageI*)
apply *auto*
done

lemma *nesting-lemma* [*rule-format* (*no-asm*)]:
assumes *ax-derivs-asm*: $\bigwedge A\ ts.\ ts \subseteq A \Longrightarrow P\ A\ ts$
and *MGF-nested-Methd*: $\bigwedge A\ pn.\ \forall b{\in}bdy\ pn.\ P\ (insert\ (mgf\ (f\ pn))\ A)\ \{mgf\ b\}$
$\Longrightarrow P\ A\ \{mgf\ (f\ pn)\}$
and *MGF-asm*: $\bigwedge A\ t.\ \forall pn{\in}U.\ P\ A\ \{mgf\ (f\ pn)\} \Longrightarrow P\ A\ \{mgf\ t\}$
and *finU*: *finite U*
shows $P\ \{\}\ \{mgf\ t\}$
using *ax-derivs-asm MGF-nested-Methd MGF-asm finU*
by (*rule nesting-lemma′*) (*auto intro*!: *le-refl*)

lemma *MGF-nested-Methd*: ⟦
$G, insert\ (\{Normal\ \doteq\}\ \langle Methd\ \ C\ sig\rangle_e \succ \{G{\rightarrow}\})\ A$
$\vdash \{Normal\ \doteq\}\ \langle body\ G\ C\ sig\rangle_e \succ \{G{\rightarrow}\}$
⟧ $\Longrightarrow\ G, A \vdash \{Normal\ \doteq\}\ \langle Methd\ \ C\ sig\rangle_e \succ \{G{\rightarrow}\}$
apply (*unfold MGF-def*)
apply (*rule ax-MethdN*)
apply (*erule conseq2*)
apply *clarsimp*
apply (*erule MethdI*)
done

lemma *MGF-deriv*: $wf\text{-}prog\ G \Longrightarrow G,(\{\}{::}state\ triple\ set) \vdash \{\doteq\}\ t\succ\ \{G{\rightarrow}\}$
apply (*rule MGFNormalI*)
apply (*rule-tac* $mgf = \lambda t.\ \{Normal\ \doteq\}\ t\succ\ \{G{\rightarrow}\}$ **and**
$bdy = \lambda\ (C,sig)\ .\{\langle body\ G\ C\ sig\rangle_e\ \}$ **and**
$f = \lambda\ (C,sig)\ .\ \langle Methd\ C\ sig\rangle_e$ **in** *nesting-lemma*)
apply (*erule ax-derivs.asm*)
apply (*clarsimp simp add*: *split-tupled-all*)
apply (*erule MGF-nested-Methd*)
apply (*erule-tac* [*2*] *finite-is-methd* [*OF wf-ws-prog*])
apply (*rule MGF-asm* [*THEN MGFNormalD*])
apply (*auto intro*: *MGFNormalI*)
done

simultaneous version

lemma *MGF-simult-Methd-lemma*: *finite ms* $\Longrightarrow$
$G,A \cup (\lambda(C,sig).\ \{Normal\ \doteq\}\ \langle Methd\ \ C\ sig\rangle_e\succ\ \{G\rightarrow\})$ ' *ms*
$\quad|\!\vdash(\lambda(C,sig).\ \{Normal\ \doteq\}\ \langle body\ G\ C\ sig\rangle_e\succ\ \{G\rightarrow\})$ ' *ms* $\Longrightarrow$
$G,A|\!\vdash(\lambda(C,sig).\ \{Normal\ \doteq\}\ \langle Methd\ \ C\ sig\rangle_e\succ\ \{G\rightarrow\})$ ' *ms*
apply (*unfold MGF-def*)
apply (*rule ax-derivs.Methd* [*unfolded mtriples-def*])
apply (*erule ax-finite-pointwise*)
prefer *2*
apply (*rule ax-derivs.asm*)
apply *fast*
apply *clarsimp*
apply (*rule conseq2*)
apply (*erule* (*1*) *ax-methods-spec*)
apply *clarsimp*
apply (*erule eval-Methd*)
done

lemma *MGF-simult-Methd*: *wf-prog G* $\Longrightarrow$
$G,(\{\}::state\ triple\ set)|\!\vdash(\lambda(C,sig).\ \{Normal\ \doteq\}\ \langle Methd\ C\ sig\rangle_e\succ\ \{G\rightarrow\})$
' *Collect* (*split* (*is-methd G*))
apply (*frule finite-is-methd* [*OF wf-ws-prog*])
apply (*rule MGF-simult-Methd-lemma*)
apply *assumption*
apply (*erule ax-finite-pointwise*)
prefer *2*
apply (*rule ax-derivs.asm*)
apply *blast*
apply *clarsimp*
apply (*rule MGF-asm* [*THEN MGFNormalD*])
apply (*auto intro*: *MGFNormalI*)
done

corollaries

lemma *eval-to-evaln*: $[\![G\vdash s\ -t\succ\rightarrow\ (Y',\ s');type\text{-}ok\ G\ t\ s;\ wf\text{-}prog\ G]\!]$
$\Longrightarrow\ \ \exists\, n.\ G\vdash s\ -t\succ-n\rightarrow\ (Y',\ s')$
apply (*cases normal s*)
apply (*force simp add*: *type-ok-def intro*: *eval-evaln*)
apply (*force intro*: *evaln.Abrupt*)
done

lemma *MGF-complete*:
assumes *valid*: $G,\{\}\models\{P\}\ t\succ\ \{Q\}$
and *mgf*: $G,(\{\}::state\ triple\ set)\vdash\{\doteq\}\ t\succ\ \{G\rightarrow\}$
and *wf*: *wf-prog G*
shows $G,(\{\}::state\ triple\ set)\vdash\{P::state\ assn\}\ t\succ\ \{Q\}$
proof (*rule ax-no-hazard*)
from *mgf*
have $G,(\{\}::state\ triple\ set)\vdash\{\doteq\}\ t\succ\ \{\lambda Y\ s'\ s.\ G\vdash s\ -t\succ\rightarrow\ (Y,\ s')\}$
by (*unfold MGF-def*)
thus $G,(\{\}::state\ triple\ set)\vdash\{P\ \wedge.\ type\text{-}ok\ G\ t\}\ t\succ\ \{Q\}$
proof (*rule conseq12*,*clarsimp*)
fix $Y\ s\ Z\ Y'\ s'$
assume *P*: $P\ Y\ s\ Z$
assume *type-ok*: *type-ok G t s*

assume *eval-t*: $G \vdash s \ -t\succ\rightarrow \ (Y', s')$
show $Q \ Y' \ s' \ Z$
proof −
from *eval-t type-ok wf*
obtain n **where** *evaln*: $G \vdash s \ -t\succ-n\rightarrow \ (Y', s')$
by (*rule eval-to-evaln* [*elim-format*]) *iprover*
from *valid* **have**
valid-expanded:
$\forall n \ Y \ s \ Z. \ P \ Y \ s \ Z \longrightarrow$ *type-ok* $G \ t \ s$
$\longrightarrow (\forall Y' \ s'. \ G \vdash s \ -t\succ-n\rightarrow \ (Y', s') \longrightarrow Q \ Y' \ s' \ Z)$
by (*simp add*: *ax-valids-def triple-valid-def*)
from P *type-ok evaln*
show $Q \ Y' \ s' \ Z$
by (*rule valid-expanded* [*rule-format*])
qed
qed
qed

theorem *ax-complete*:
assumes *wf*: *wf-prog* G
and *valid*: $G,\{\} \models \{P{::}state \ assn\} \ t\succ \ \{Q\}$
shows $G,(\{\}{::}state \ triple \ set) \vdash \{P\} \ t\succ \ \{Q\}$
proof −
from *wf* **have** $G,(\{\}{::}state \ triple \ set) \vdash \{\doteq\} \ t\succ \ \{G\rightarrow\}$
by (*rule MGF-deriv*)
from *valid this wf*
show *?thesis*
by (*rule MGF-complete*)
qed

end

Chapter 25

AxExample

64 Example of a proof based on the Bali axiomatic semantics

theory *AxExample* **imports** *AxSem Example* **begin**

constdefs
$arr\text{-}inv :: st \Rightarrow bool$
$arr\text{-}inv \equiv \lambda s.\ \exists\, obj\ a\ T\ el.\ globs\ s\ (Stat\ Base) = Some\ obj\ \wedge$
$values\ obj\ (Inl\ (arr,\ Base)) = Some\ (Addr\ a)\ \wedge$
$heap\ s\ a = Some\ (\!|tag{=}Arr\ T\ 2, values{=}el|\!)$

lemma *arr-inv-new-obj*:
$\bigwedge a.\ [\![arr\text{-}inv\ s;\ new\text{-}Addr\ (heap\ s){=}Some\ a]\!] \Longrightarrow arr\text{-}inv\ (gupd(Inl\ a{\mapsto}x)\ s)$
apply (*unfold arr-inv-def*)
apply (*force dest*!: *new-AddrD2*)
done

lemma *arr-inv-set-locals* [*simp*]: $arr\text{-}inv\ (set\text{-}locals\ l\ s) = arr\text{-}inv\ s$
apply (*unfold arr-inv-def*)
apply (*simp* (*no-asm*))
done

lemma *arr-inv-gupd-Stat* [*simp*]:
$Base \neq C \Longrightarrow arr\text{-}inv\ (gupd(Stat\ C{\mapsto}obj)\ s) = arr\text{-}inv\ s$
apply (*unfold arr-inv-def*)
apply (*simp* (*no-asm-simp*))
done

lemma *ax-inv-lupd* [*simp*]: $arr\text{-}inv\ (lupd(x{\mapsto}y)\ s) = arr\text{-}inv\ s$
apply (*unfold arr-inv-def*)
apply (*simp* (*no-asm*))
done

declare *split-if-asm* [*split del*]
declare *lvar-def* [*simp*]

ML ⟪
```
local
  val ax-Skip = thm ax-Skip;
  val ax-StatRef = thm ax-StatRef;
  val ax-MethdN = thm ax-MethdN;
  val ax-Alloc = thm ax-Alloc;
  val ax-Alloc-Arr = thm ax-Alloc-Arr;
  val ax-SXAlloc-Normal = thm ax-SXAlloc-Normal;
  val ax-derivs-intros = funpow 7 tl (thms ax-derivs.intros);
in

fun inst1-tac s t st =
  case AList.lookup (op =) (rev (Term.add-varnames (prop-of st) [])) s of
  SOME i => Tactic.instantiate-tac' [((s, i), t)] st | NONE => Seq.empty;

val ax-tac =
  REPEAT o rtac allI THEN'
  resolve-tac (ax-Skip :: ax-StatRef :: ax-MethdN :: ax-Alloc ::
    ax-Alloc-Arr :: ax-SXAlloc-Normal :: ax-derivs-intros);
```

end;
⟫

theorem *ax-test*: *tprg*,({}::*'a triple set*)⊢
{*Normal* (λ*Y s Z*::*'a. heap-free four s* ∧ ¬*initd Base s* ∧ ¬ *initd Ext s*)}
.*test* [*Class Base*].
{λ*Y s Z. abrupt s* = *Some* (*Xcpt* (*Std IndOutBound*))}
apply (*unfold test-def arr-viewed-from-def*)
apply (*tactic ax-tac 1*)
defer
apply (*tactic ax-tac 1*)
defer
apply (*tactic* ⟪ *inst1-tac Q*
λ*Y s Z. arr-inv* (*snd s*) ∧ *tprg*,*s*⊢*catch SXcpt NullPointer* ⟫)
prefer *2*
apply *simp*
apply (*rule-tac P'* = *Normal* (λ*Y s Z. arr-inv* (*snd s*)) **in** *conseq1*)
prefer *2*
apply *clarsimp*
apply (*rule-tac Q'* = (λ*Y s Z. ?Q Y s Z*)←=*False*↓=◇ **in** *conseq2*)
prefer *2*
apply *simp*
apply (*tactic ax-tac 1*)
prefer *2*
apply (*rule ax-impossible* [*THEN conseq1*], *clarsimp*)
apply (*rule-tac P'* = *Normal ?P* **in** *conseq1*)
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
prefer *2*
apply (*rule ax-subst-Val-allI*)
apply (*tactic* ⟪ *inst1-tac P'* λ*u a. Normal* (*?PP a*←*?x*) *u* ⟫)
apply (*simp del*: *avar-def2 peek-and-def2*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)

apply (*rule-tac Q'* = *Normal* (λ*Var*:(*v*, *f*) *u ua. fst* (*snd* (*avar tprg* (*Intg 2*) *v u*)) = *Some* (*Xcpt* (*Std IndOutBound*))) **in** *conseq2*)
prefer *2*
apply (*clarsimp simp add*: *split-beta*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 2*)
apply (*rule ax-derivs.Done* [*THEN conseq1*])
apply (*clarsimp simp add*: *arr-inv-def inited-def in-bounds-def*)
defer
apply (*rule ax-SXAlloc-catch-SXcpt*)
apply (*rule-tac Q'* = (λ*Y* (*x*, *s*) *Z. x* = *Some* (*Xcpt* (*Std NullPointer*)) ∧ *arr-inv s*) ∧. *heap-free two* **in** *conseq2*)
prefer *2*
apply (*simp add*: *arr-inv-new-obj*)
apply (*tactic ax-tac 1*)
apply (*rule-tac C* = *Ext* **in** *ax-Call-known-DynT*)
apply (*unfold DynT-prop-def*)
apply (*simp* (*no-asm*))
apply (*intro strip*)
apply (*rule-tac P'* = *Normal ?P* **in** *conseq1*)
apply (*tactic ax-tac 1*)

apply (*rule ax-thin* [*OF - empty-subsetI*])
apply (*simp* (*no-asm*) *add*: *body-def2*)
apply (*tactic ax-tac 1*)

defer
apply (*simp* (*no-asm*))
apply (*tactic ax-tac 1*)

apply (*rule-tac* [*2*] *ax-derivs.Abrupt*)

apply (*rule ax-derivs.Expr*)
apply (*tactic ax-tac 1*)
prefer *2*
apply (*rule ax-subst-Var-allI*)
apply (*tactic* ⟪ *inst1-tac P′ λa vs l vf. ?PP a vs l vf←?x* ∧. *?p* ⟫)
apply (*rule allI*)
apply (*tactic* ⟪ *simp-tac* (*simpset*() *delloop split-all-tac delsimps* [*thm peek-and-def2*]) *1* ⟫)
apply (*rule ax-derivs.Abrupt*)
apply (*simp* (*no-asm*))
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 2*, *tactic ax-tac 2*, *tactic ax-tac 2*)
apply (*tactic ax-tac 1*)
apply (*tactic* ⟪ *inst1-tac R λa′. Normal* ((*λVals:vs* (*x*, *s*) *Z*. *arr-inv s* ∧ *inited Ext* (*globs s*) ∧ *a′* ≠ *Null* ∧ *vs* = [*Null*]) ∧. *heap-free two*) ⟫)
apply *fastsimp*
prefer *4*
apply (*rule ax-derivs.Done* [*THEN conseq1*],*force*)
apply (*rule ax-subst-Val-allI*)
apply (*tactic* ⟪ *inst1-tac P′ λu a. Normal* (*?PP a←?x*) *u* ⟫)
apply (*simp* (*no-asm*) *del*: *peek-and-def2*)
apply (*tactic ax-tac 1*)
prefer *2*
apply (*rule ax-subst-Val-allI*)
apply (*tactic* ⟪ *inst1-tac P′ λaa v. Normal* (*?QQ aa v←?y*) ⟫)
apply (*simp del*: *peek-and-def2*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)

apply (*simp* (*no-asm*))

apply (*rule-tac Q′* = *Normal* ((*λY* (*x*, *s*) *Z*. *arr-inv s* ∧ (∃ *a*. *the* (*locals s* (*VName e*)) = *Addr a* ∧ *obj-class* (*the* (*globs s* (*Inl a*))) = *Ext* ∧
invocation-declclass tprg IntVir s (*the* (*locals s* (*VName e*))) (*ClassT Base*)
⦇*name* = *foo*, *parTs* = [*Class Base*]⦈ = *Ext*)) ∧. *initd Ext* ∧. *heap-free two*)
in *conseq2*)
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
defer
apply (*rule ax-subst-Var-allI*)
apply (*tactic* ⟪ *inst1-tac P′ λu vf. Normal* (*?PP vf* ∧. *?p*) *u* ⟫)
apply (*simp* (*no-asm*) *del*: *split-paired-All peek-and-def2*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)

apply (*rule-tac Q′* = *Normal* ((*λY s Z*. *arr-inv* (*store s*) ∧ *vf*=*lvar* (*VName e*) (*store s*)) ∧. *heap-free tree*

$\wedge$. *initd Ext*) **in** *conseq2*)
prefer *2*
apply (*simp add*: *invocation-declclass-def dynmethd-def*)
apply (*unfold dynlookup-def*)
apply (*simp add*: *dynmethd-Ext-foo*)
apply (*force elim*!: *arr-inv-new-obj atleast-free-SucD atleast-free-weaken*)

apply (*rule ax-InitS*)
apply *force*
apply (*simp* (*no-asm*))
apply (*tactic* ⟪ *simp-tac* (*simpset*() *delloop split-all-tac*) *1* ⟫)
apply (*rule ax-Init-Skip-lemma*)
apply (*tactic* ⟪ *simp-tac* (*simpset*() *delloop split-all-tac*) *1* ⟫)
apply (*rule ax-InitS* [*THEN conseq1*])
apply *force*
apply (*simp* (*no-asm*))
apply (*unfold arr-viewed-from-def*)
apply (*rule allI*)
apply (*rule-tac* P' = *Normal ?P* **in** *conseq1*)
apply (*tactic* ⟪ *simp-tac* (*simpset*() *delloop split-all-tac*) *1* ⟫)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
apply (*rule-tac* [*2*] *ax-subst-Var-allI*)
apply (*tactic* ⟪ *inst1-tac* P' λ*vf l vfa*. *Normal* (*?P vf l vfa*) ⟫)
apply (*tactic* ⟪ *simp-tac* (*simpset*() *delloop split-all-tac delsimps* [*split-paired-All*, *thm peek-and-def2*]) *2* ⟫)
apply (*tactic ax-tac 2*)
apply (*tactic ax-tac 3*)
apply (*tactic ax-tac 3*)
apply (*tactic* ⟪ *inst1-tac P* λ*vf l vfa*. *Normal* (*?P vf l vfa*$\leftarrow\diamondsuit$) ⟫)
apply (*tactic* ⟪ *simp-tac* (*simpset*() *delloop split-all-tac*) *2* ⟫)
apply (*tactic ax-tac 2*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 2*)
apply (*rule ax-derivs.Done* [*THEN conseq1*])
apply (*tactic* ⟪ *inst1-tac Q* λ*vf*. *Normal* ((λ*Y s Z*. *vf*=*lvar* (*VName e*) (*snd s*)) $\wedge$. *heap-free four* $\wedge$. *initd Base* $\wedge$. *initd Ext*) ⟫)
apply (*clarsimp split del*: *split-if*)
apply (*frule atleast-free-weaken* [*THEN atleast-free-weaken*])
apply (*drule initedD*)
apply (*clarsimp elim*!: *atleast-free-SucD simp add*: *arr-inv-def*)
apply *force*
apply (*tactic* ⟪ *simp-tac* (*simpset*() *delloop split-all-tac*) *1* ⟫)
apply (*rule ax-triv-Init-Object* [*THEN peek-and-forget2*, *THEN conseq1*])
apply (*rule wf-tprg*)
apply *clarsimp*
apply (*tactic* ⟪ *inst1-tac P* λ*vf*. *Normal* ((λ*Y s Z*. *vf* = *lvar* (*VName e*) (*snd s*)) $\wedge$. *heap-free four* $\wedge$. *initd Ext*) ⟫)
apply *clarsimp*
apply (*tactic* ⟪ *inst1-tac PP* λ*vf*. *Normal* ((λ*Y s Z*. *vf* = *lvar* (*VName e*) (*snd s*)) $\wedge$. *heap-free four* $\wedge$. *Not* $\circ$ *initd Base*) ⟫)
apply *clarsimp*

apply (*rule conseq1*)
apply (*tactic ax-tac 1*)
apply *clarsimp*
done

lemma *Loop-Xcpt-benchmark*:
$Q = (\lambda Y\ (x,s)\ Z.\ x \neq \mathit{None} \longrightarrow \mathit{the\text{-}Bool}\ (\mathit{the}\ (\mathit{locals}\ s\ i))) \Longrightarrow$
$G,(\{\}::'a\ \mathit{triple\ set})\vdash\{\mathit{Normal}\ (\lambda Y\ s\ Z::'a.\ \mathit{True})\}$
$.\mathit{lab1}\cdot\ \mathit{While}(\mathit{Lit}\ (\mathit{Bool\ True}))\ (\mathit{If}(\mathit{Acc}\ (\mathit{LVar}\ i))\ (\mathit{Throw}\ (\mathit{Acc}\ (\mathit{LVar\ xcpt})))\ \mathit{Else}$
$(\mathit{Expr}\ (\mathit{Ass}\ (\mathit{LVar}\ i)\ (\mathit{Acc}\ (\mathit{LVar}\ j))))).\ \{Q\}$
apply (*rule-tac* $P' = Q$ **and** $Q' = Q{\leftarrow}{=}\mathit{False}{\downarrow}{=}\diamondsuit$ **in** *conseq12*)
apply *safe*
apply (*tactic ax-tac 1*)
apply (*rule ax-Normal-cases*)
prefer *2*
apply (*rule ax-derivs.Abrupt* [*THEN conseq1*], *clarsimp simp add*: *Let-def*)
apply (*rule conseq1*)
apply (*tactic ax-tac 1*)
apply *clarsimp*
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac 1*)
apply (*tactic*
⟪ *inst1-tac* P' *Normal* $(\lambda s..\ (\lambda Y\ s\ Z.\ \mathit{True}){\downarrow}{=}\mathit{Val}\ (\mathit{the}\ (\mathit{locals}\ s\ i)))$ ⟫)
apply (*tactic ax-tac 1*)
apply (*rule conseq1*)
apply (*tactic ax-tac 1*)
apply *clarsimp*
apply (*rule allI*)
apply (*rule ax-escape*)
apply *auto*
apply (*rule conseq1*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
apply *clarsimp*
apply (*rule-tac* $Q' = \mathit{Normal}\ (\lambda Y\ s\ Z.\ \mathit{True})$ **in** *conseq2*)
prefer *2*
apply *clarsimp*
apply (*rule conseq1*)
apply (*tactic ax-tac 1*)
apply (*tactic ax-tac 1*)
prefer *2*
apply (*rule ax-subst-Var-allI*)
apply (*tactic* ⟪ *inst1-tac* P' $\lambda b\ Y\ ba\ Z\ vf.\ \lambda Y\ (x,s)\ Z.\ x{=}\mathit{None} \wedge \mathit{snd}\ vf = \mathit{snd}\ (\mathit{lvar}\ i\ s)$ ⟫)
apply (*rule allI*)
apply (*rule-tac* $P' = \mathit{Normal}\ ?P$ **in** *conseq1*)
prefer *2*
apply *clarsimp*
apply (*tactic ax-tac 1*)
apply (*rule conseq1*)
apply (*tactic ax-tac 1*)
apply *clarsimp*
apply (*tactic ax-tac 1*)
apply *clarsimp*
done

end