

# Asymptote: The Vector Graphics Language

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`http://asymptote.sf.net`

# History

- T<sub>E</sub>X and METAFONT (Knuth, 1979)
- MetaPost (Hobby, 1989)
- Asymptote (Hammerlindl, Bowman, Prince, 2004)

# Statistics (as of April, 2007)

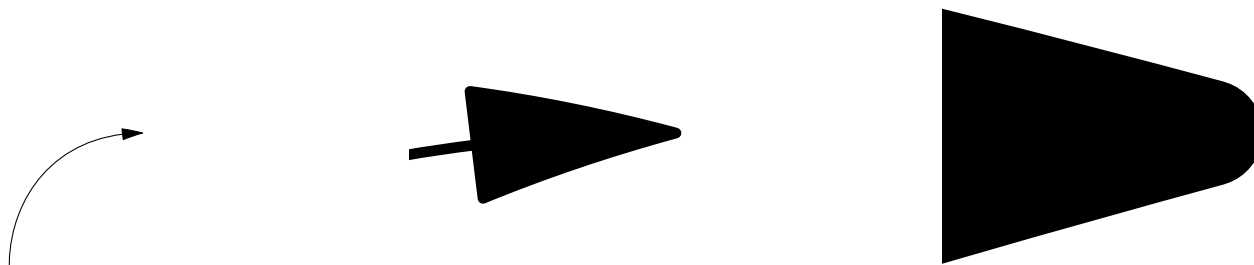
- Runs on Windows, Mac OS X, Linux, etc.
- 1800 downloads a month from `asymptote.sourceforge.net`.
- 33 000 lines of C++ code.
- 18 000 lines of Asymptote code.

# Vector Graphics

- Raster graphics assign colors to a grid of pixels.

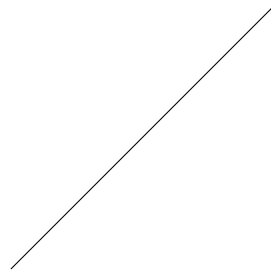


- Vector graphics are graphics which still maintain their look when inspected at arbitrarily small scales.



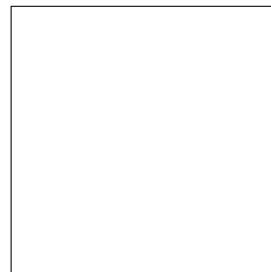
# Cartesian Coordinates

```
draw((0,0)--(100,100));
```



- units are **PostScript** *big points* (1 bp = 1/72 inch)
- -- means join the points with a linear segment to create a *path*
- cyclic path:

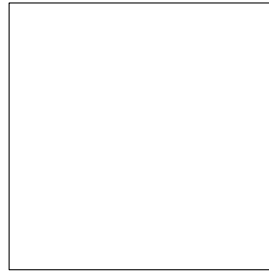
```
draw((0,0)--(100,0)--(100,100)--(0,100)--cycle);
```



# Scaling to a Given Size

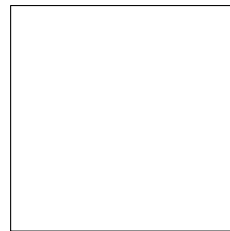
- PostScript units are often inconvenient.
- Instead, scale user coordinates to a specified final size:

```
size(101,101);  
draw((0,0)--(1,0)--(1,1)--(0,1)--cycle);
```



- One can also specify the size in **cm**:

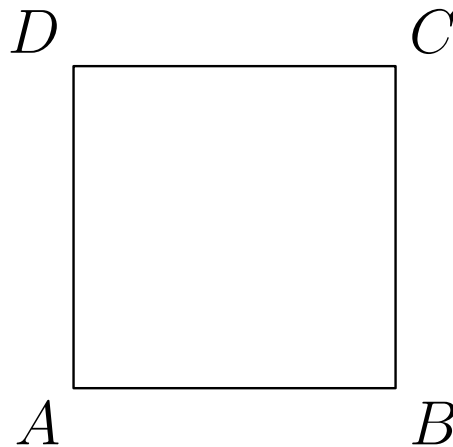
```
size(0,3cm);  
draw(unitsquare);
```



# Labels

- Adding and aligning L<sup>A</sup>T<sub>E</sub>X labels is easy:

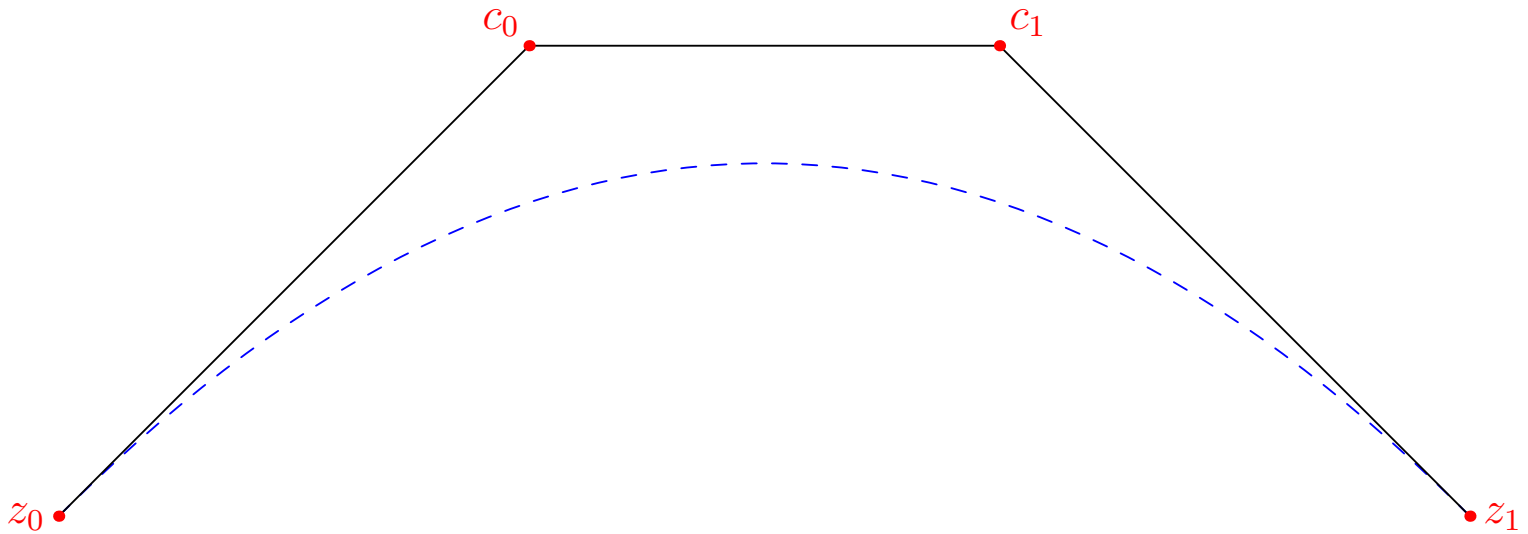
```
size(0,3cm);  
draw(unitsquare);  
label("$A$", (0,0), SW);  
label("$B$", (1,0), SE);  
label("$C$", (1,1), NE);  
label("$D$", (0,1), NW);
```



# Bezier Splines

- Using `..` instead of `--` specifies a *Bezier cubic spline*:

```
draw(z0 .. controls c0 and c1 .. z1,blue+dashed);
```



$$(1 - t)^3 z_0 + 3t(1 - t)^2 c_0 + 3t^2(1 - t) c_1 + t^3 z_1, \quad t \in [0, 1].$$

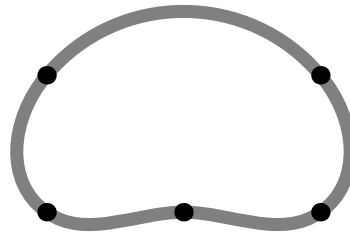


# Smooth Paths

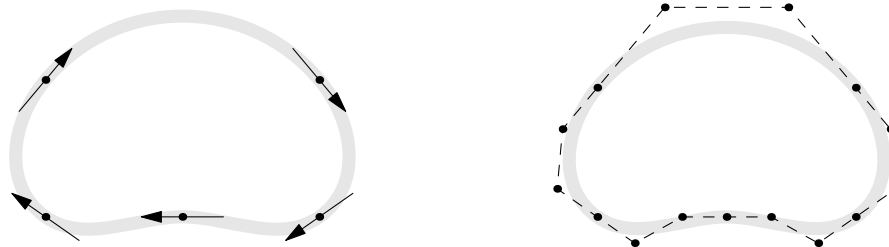
- Asymptote can choose control points for you.

```
pair[] z={ (0,0), (0,1), (2,1), (2,0), (1,0) };
```

```
draw(z[0]..z[1]..z[2]..z[3]..z[4]..cycle,  
     grey+linewidth(5));  
dot(z,linewidth(7));
```



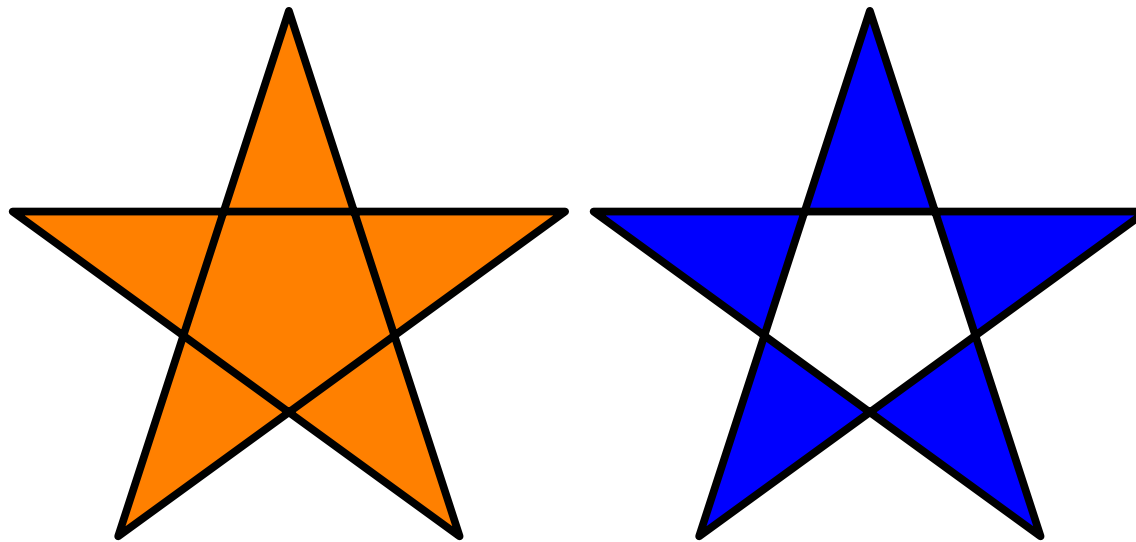
- First, linear equations involving the curvature are solved to find the direction through each knot. Then, control points along those directions are chosen [Hob86, Knu86]:



# Filling

- Use `fill` to fill the inside of a path:

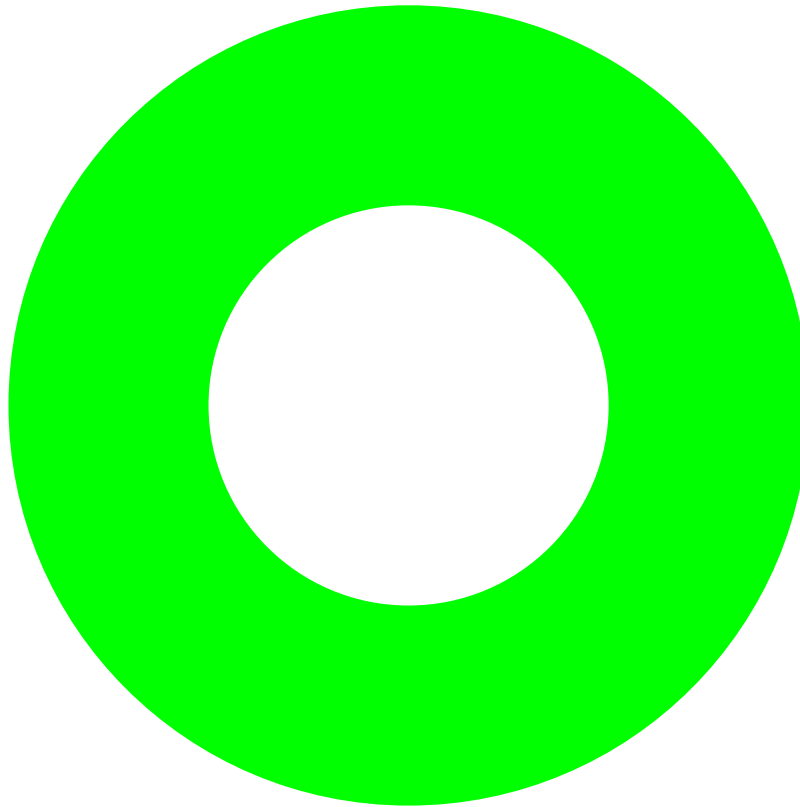
```
path star;  
for (int i=0; i<5; ++i)  
    star=star--dir(90+144i);  
star=star--cycle;  
fill(shift(-1,0)*star,orange+zerowinding);  
draw(shift(-1,0)*star,linewidth(3));  
fill(shift(1,0)*star,blue+evenodd);  
draw(shift(1,0)*star,linewidth(3));
```



# Filling

- Use a list of paths to fill a region with holes:

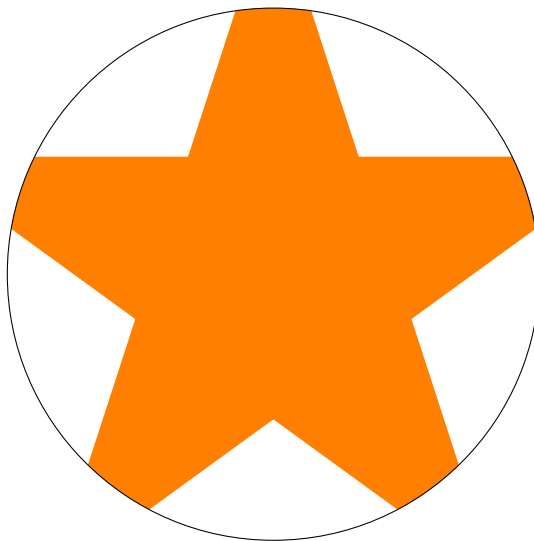
```
path[] p={scale(2)*unitcircle, reverse(unitcircle)};  
fill(p,green+zerowinding);
```



# Clipping

- Pictures can be clipped to lie inside a path:

```
fill(star,orange+zerowinding);  
clip(scale(0.7)*unitcircle);  
draw(scale(0.7)*unitcircle);
```



- All of Asymptote's graphical capabilities are based on four primitive commands: **draw**, **fill**, **clip**, and **label**.

# Transforms

- Affine transformations include shifts, rotations, and scalings.

```
transform t=rotate(90);  
write(t*(1,0)); // Writes (0,1).
```

- Pairs, paths, pens, and whole pictures can be transformed.

```
fill(P,blue);  
fill(shift(2,0)*reflect((0,0),(0,1))*P, red);  
fill(shift(4,0)*rotate(30)*P, yellow);  
fill(shift(6,0)*yscale(0.7)*xscale(2)*P, green);
```



# C++/Java-like Programming Syntax

```
// Declaration: Declare x to be real:
```

```
real x;
```

```
// Assignment: Assign x the value 1.
```

```
x=1.0;
```

```
// Conditional: Test if x equals 1 or not.
```

```
if(x == 1.0) {  
    write("x equals 1.0");  
} else {  
    write("x is not equal to 1.0");  
}
```

```
// Loop: iterate 10 times
```

```
for(int i=0; i < 10; ++i) {  
    write(i);  
}
```

# Helpful Math Notation

- Integer division returns a **real**. Use **quotient** for an integer result:

$$3/4 == 0.75 \qquad \text{quotient}(3,4) == 0$$

- Caret for real and integer exponentiation:

$$2^3 \quad 2.7^3 \quad 2.7^{3.2}$$

- Many expressions can be implicitly scaled by a numeric constant:

$$2\pi \quad 10\text{cm} \quad 2x^2 \quad 3\sin(x) \quad 2(a+b)$$

- Pairs are complex numbers:

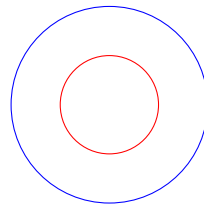
$$(0,1) * (0,1) == (-1,0)$$

# Function Calls

- Functions can take default arguments in any position. Arguments are matched to the first possible location:

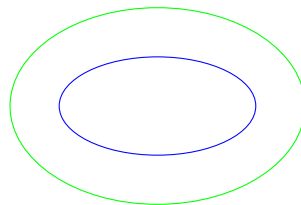
```
void drawEllipse(real xsize=1, real ysize=xsize, pen p=blue) {  
    draw(xscale(xsize)*yscale(ysize)*unitcircle, p);  
}
```

```
drawEllipse(2);  
drawEllipse(red);
```



- Arguments can be given by name:

```
drawEllipse(xsize=2, ysize=1);  
drawEllipse(ysize=2, xsize=3, green);
```





# Rest Arguments

- Rest arguments allow one to write a function that takes an arbitrary number of arguments:

```
int sum(... int[] nums) {  
    int total=0;  
    for (int i=0; i < nums.length; ++i)  
        total += nums[i];  
    return total;  
}
```

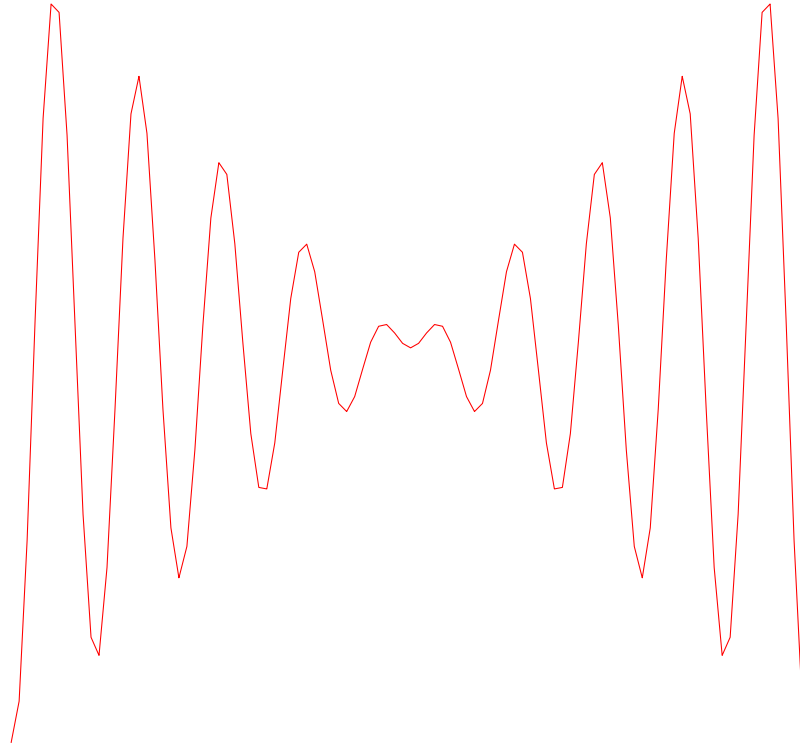
```
sum(1,2,3,4);           // returns 10  
sum();                  // returns 0  
sum(1,2,3 ... new int[] {4,5,6}); // returns 21
```

```
int subtract(int start ... int[] subs) {  
    return start - sum(... subs);  
}
```

# Higher-Order Functions

- Functions are first-class values. They can be passed to other functions:

```
real f(real x) {  
    return x*sin(10x);  
}  
draw(graph(f,-3,3),red);
```



# Higher-Order Functions

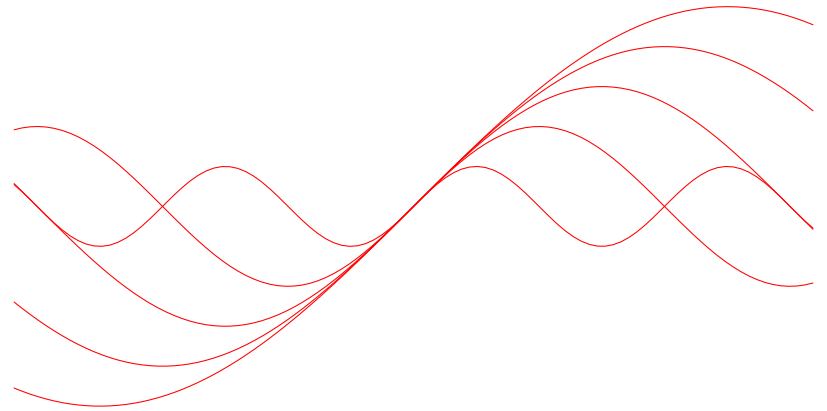
- Functions can return functions:

$$f_n(x) = n \sin\left(\frac{x}{n}\right).$$

```
typedef real func(real);  
func f(int n) {  
    real fn(real x) {  
        return n*sin(x/n);  
    }  
    return fn;  
}
```

```
func f1=f(1);  
real y=f1(pi);
```

```
for (int i=1; i<=5; ++i)  
    draw(graph(f(i),-10,10),red);
```



# Anonymous Functions

- Create new functions with **new**:

```
path p=graph(new real (real x) { return x*sin(10x); },-3,3,red);
```

```
func f(int n) {  
    return new real (real x) { return n*sin(x/n); };  
}
```

- Function definitions are just syntactic sugar for assigning function objects to variables.

```
real square(real x) {  
    return x^2;  
}
```

is equivalent to

```
real square(real x);  
square=new real (real x) {  
    return x^2;  
};
```

# Structures

- As in other languages, structures group together data.

```
struct Person {  
    string firstname, lastname;  
    int age;  
}  
Person bob=new Person;  
bob.firstname="Bob";  
bob.lastname="Chesterton";  
bob.age=24;
```

- Any code in the structure body will be executed every time a new structure is allocated...

```
struct Person {  
    write("Making a person.");  
    string firstname, lastname;  
    int age=18;  
}  
Person eve=new Person;    // Writes "Making a person."  
write(eve.age);           // Writes 18.
```

# Object-Oriented Programming

- Functions are defined for each instance of a structure.

```
struct Quadratic {  
    real a,b,c;  
    real discriminant() {  
        return b^2-4*a*c;  
    }  
    real eval(real x) {  
        return a*x^2 + b*x + c;  
    }  
}
```

- This allows us to construct “methods” which are just normal functions declared in the environment of a particular object:

```
Quadratic poly=new Quadratic;  
poly.a=-1; poly.b=1; poly.c=2;
```

```
real f(real x)=poly.eval;  
real y=f(2);  
draw(graph(poly.eval, -5, 5));
```

# Specialization

- Can create specialized objects just by redefining methods:

```
struct Shape {  
    void draw();  
    real area();  
}
```

```
Shape rectangle(real w, real h) {  
    Shape s=new Shape;  
    s.draw = new void () {  
        fill((0,0)--(w,0)--(w,h)--(0,h)--cycle);  
    };  
    s.area = new real () { return w*h; };  
    return s;  
}
```

```
Shape circle(real radius) {  
    Shape s=new Shape;  
    s.draw = new void () { fill(scale(radius)*unitcircle); };  
    s.area = new real () { return pi*radius^2; };  
    return s;  
}
```

# Overloading

- Consider the code:

```
int x1=2;  
int x2() {  
    return 7;  
}  
int x3(int y) {  
    return 2y;  
}
```

```
write(x1+x2()); // Writes 9.  
write(x3(x1)+x2()); // Writes 11.
```



# Overloading

- **x1**, **x2**, and **x3** are never used in the same context, so they can all be renamed **x** without ambiguity:

```
int x=2;
int x() {
    return 7;
}
int x(int y) {
    return 2y;
}
```

```
write(x+x()); // Writes 9.
write(x(x)+x()); // Writes 11.
```

- Functions definitions are just variable definitions, but variables are distinguished by their signatures to allow overloading.

# Operators

- Operators are just syntactic sugar for functions, and can be addressed or defined as functions with the **operator** keyword.

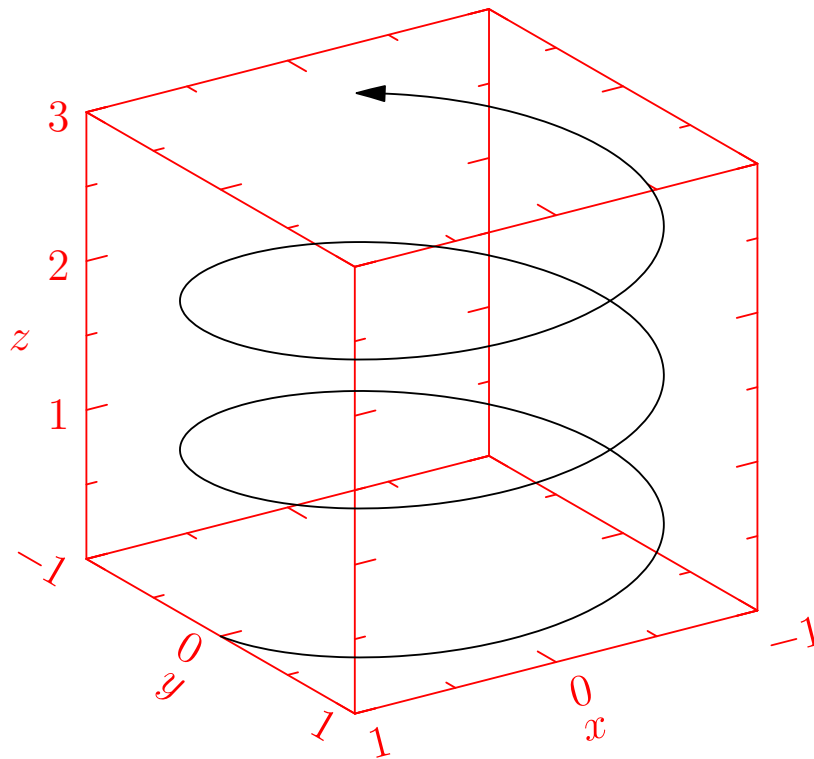
```
int add(int x, int y)=operator +;
write(add(2,3)); // Writes 5.
```

```
// Don't try this at home.
int operator +(int x, int y) {
    return add(2x,y);
}
write(2+3); // Writes 7.
```

- This allows operators to be defined for new types.

# Operators

- Operators for constructing paths are also functions:  
`a.. controls b and c .. d--e`  
is equivalent to  
`operator --(operator ..(a, operator controls(b,c), d), e)`
- This allowed us to redefine all of the path operators for 3D paths.



# Packages

- Function and structure definitions can be grouped into packages:

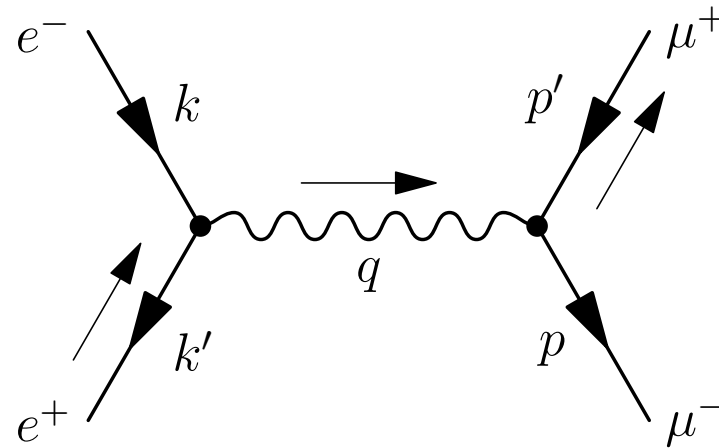
```
// powers.asy  
real square(real x) { return x^2; }  
real cube(real x) { return x^3; }
```

and imported:

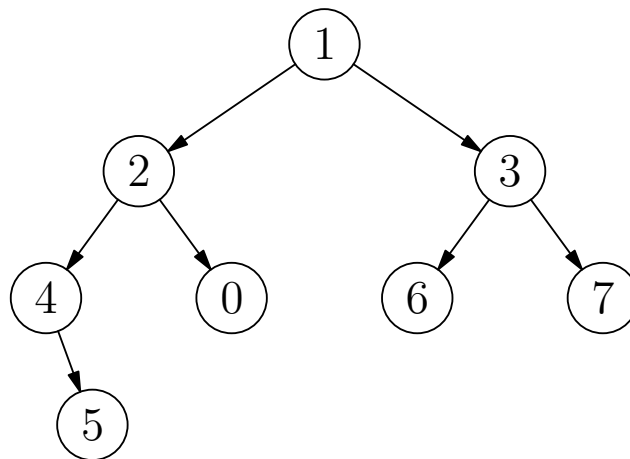
```
import powers;  
real eight=cube(2.0);  
draw(graph(powers.square, -1, 1));
```

# Packages

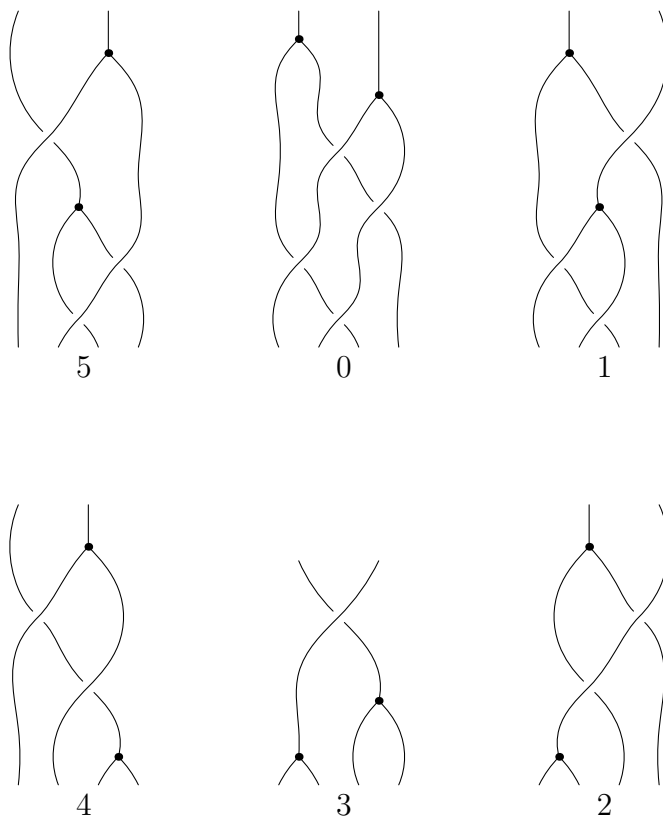
- There are packages for Feynman diagrams,



data structures,



and algebraic knot theory:



$$\begin{aligned}
 \Phi\Phi(x_1, x_2, x_3, x_4, x_5) = & \rho_{4b}(x_1 + x_4, x_2, x_3, x_5) + \rho_{4b}(x_1, x_2, x_3, x_4) \\
 & + \rho_{4a}(x_1, x_2 + x_3, x_4, x_5) - \rho_{4b}(x_1, x_2, x_3, x_4 + x_5) \\
 & - \rho_{4a}(x_1 + x_2, x_3, x_4, x_5) - \rho_{4a}(x_1, x_2, x_4, x_5).
 \end{aligned}$$

# Textbook Graph

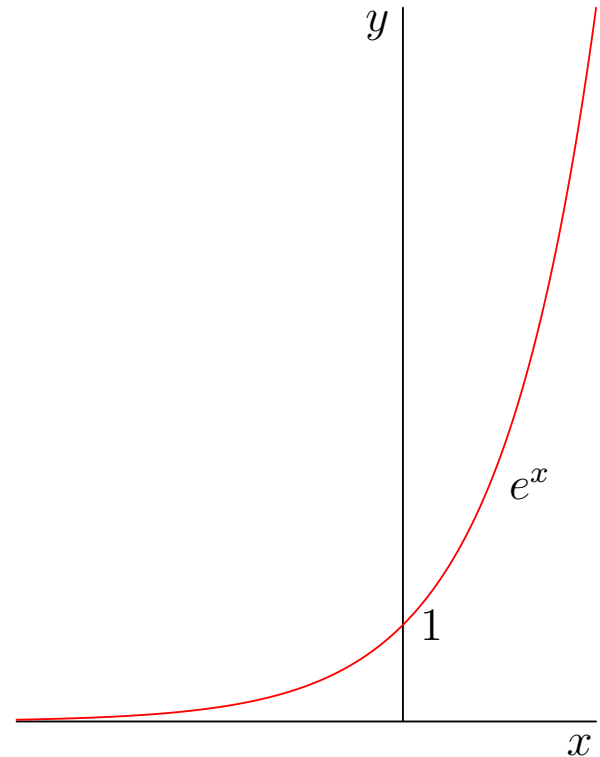
```
import graph;
size(150,0);

real f(real x) {return exp(x);}
pair F(real x) {return (x,f(x));}

xaxis("$x$");
yaxis("$y$",0);

draw(graph(f,-4,2,operator ..),red);

lately(1,E);
label("$e^x$",F(1),SE);
```



# Scientific Graph

```
import graph;

size(250,200,IgnoreAspect);

real Sin(real t) {return sin(2pi*t);}
real Cos(real t) {return cos(2pi*t);}

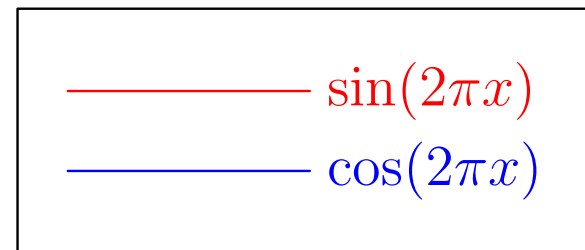
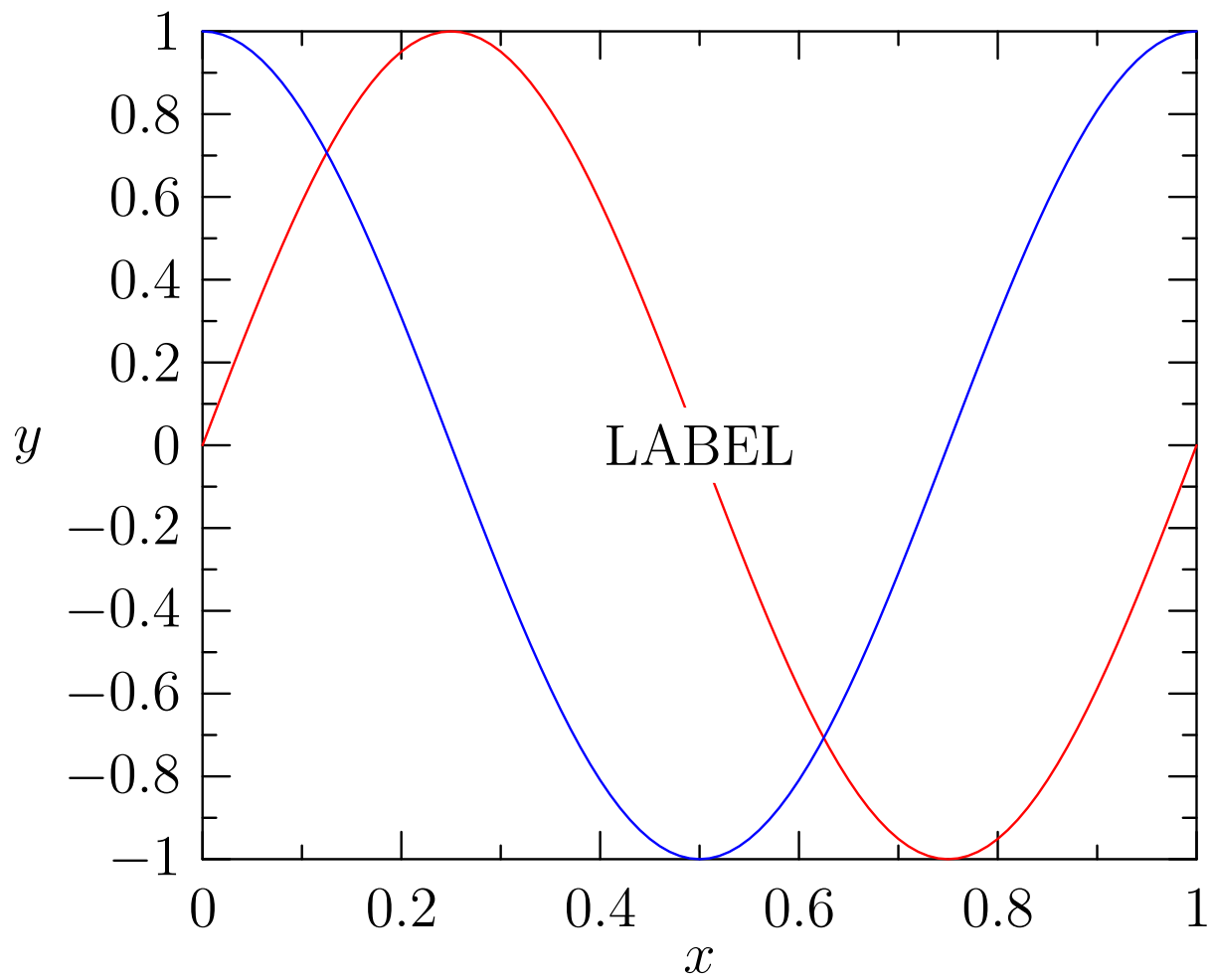
draw(graph(Sin,0,1),red,"$\sin(2\pi x)$");
draw(graph(Cos,0,1),blue,"$\cos(2\pi x)$");

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks);

label("LABEL",point(0),UnFill(1mm));

attach(legend(),point(E),20E,UnFill);
```





# Data Graph

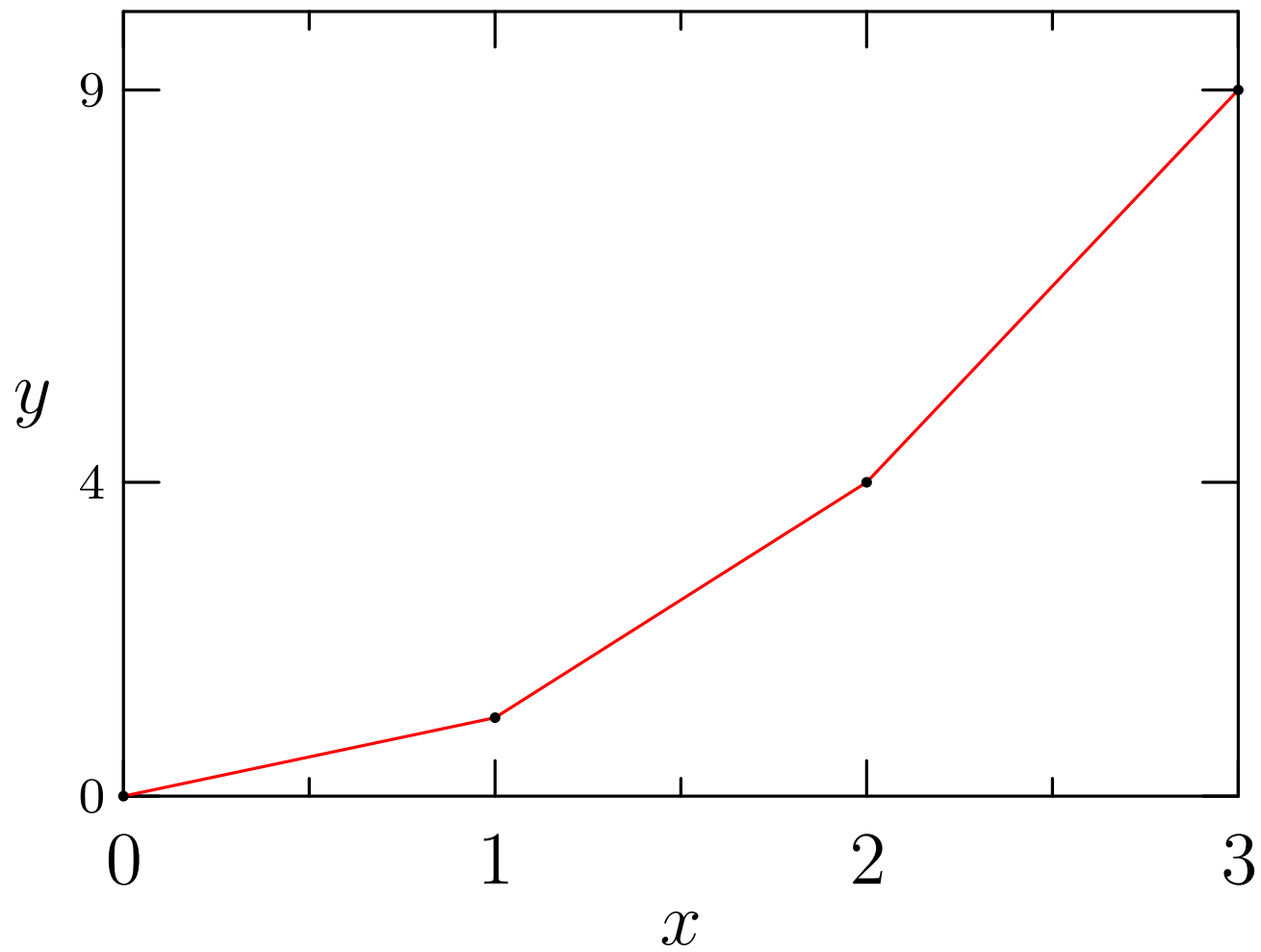
```
import graph;

size(200,150,IgnoreAspect);

real[] x={0,1,2,3};
real[] y=x^2;

draw(graph(x,y),red,MarkFill[0]);

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,
      RightTicks(Label(fontsize(8)),new real[]{0,4,9}));
```



# Imported Data Graph

```
import graph;

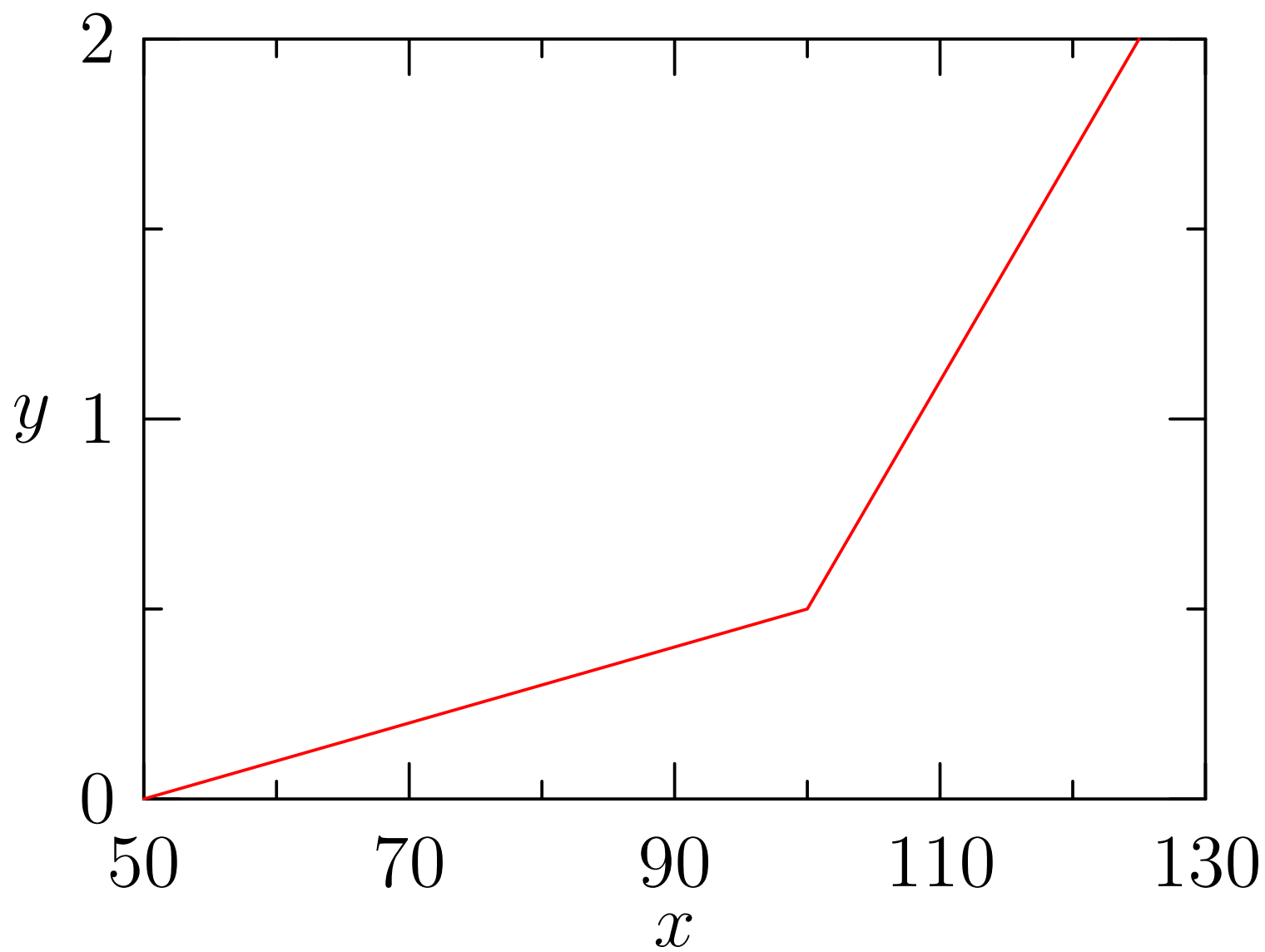
size(200,150,IgnoreAspect);

file in=line(input("filegraph.dat"));
real[] [] a=transpose(dimension(in,0,0));

real[] x=a[0];
real[] y=a[1];

draw(graph(x,y),red);

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks);
```



# Logarithmic Graph

```
import graph;

size(200,200,IgnoreAspect);

real f(real t) {return 1/t;}

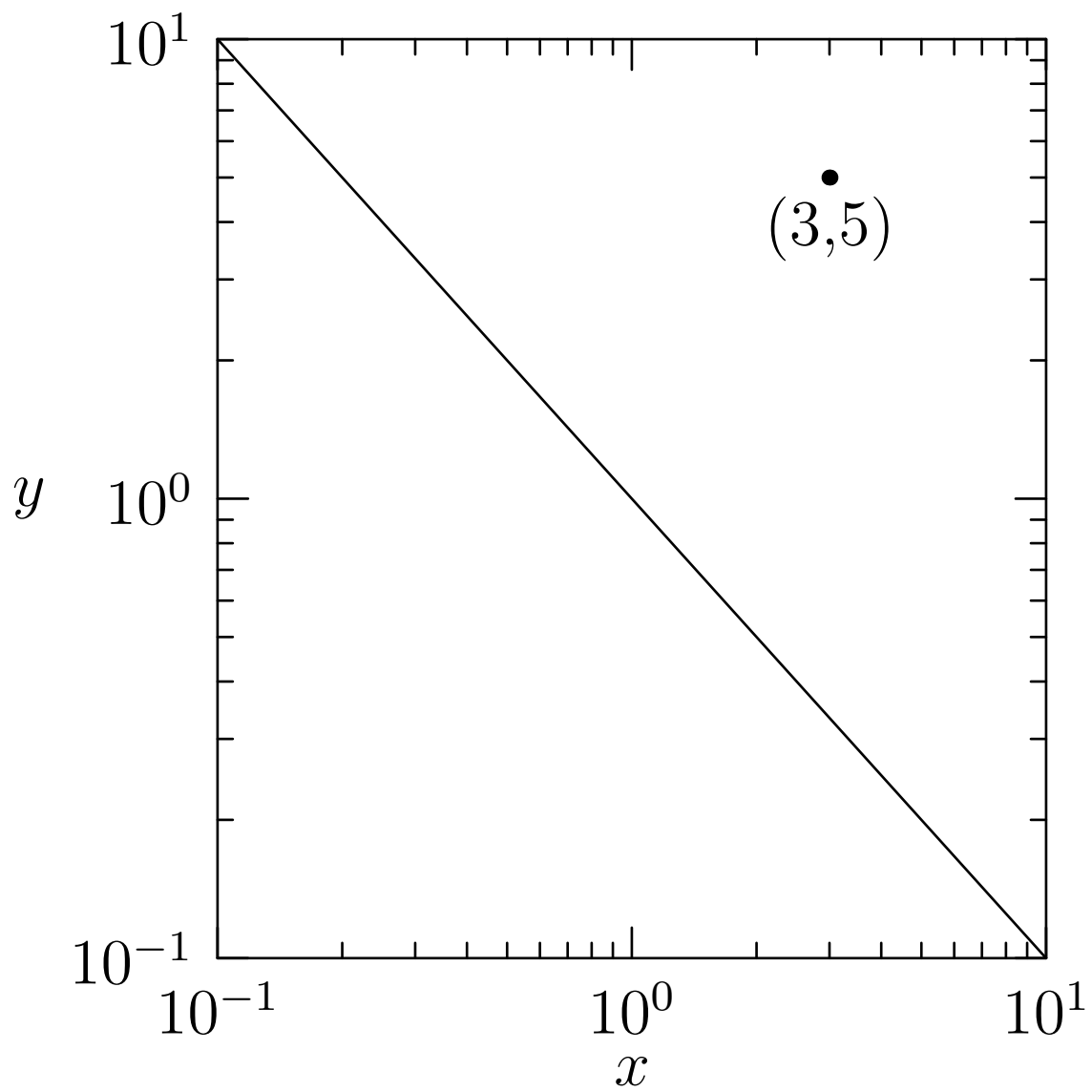
scale(Log,Log);

draw(graph(f,0.1,10));

//xlimits(1,10);
//ylimits(0.1,1);

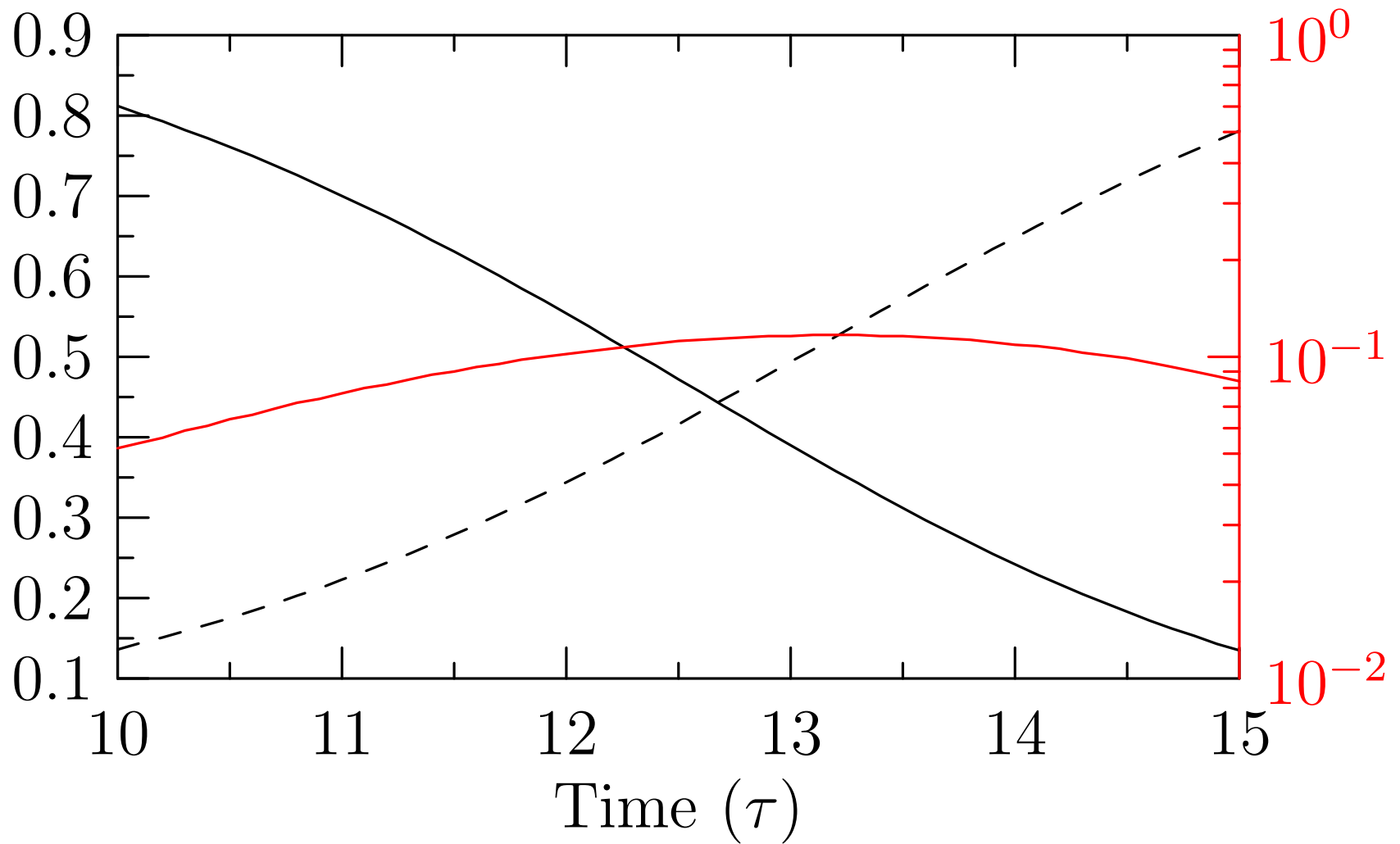
dot(Label("(3,5)",align=S),Scale((3,5)));

xaxis("$x$",BottomTop,LeftTicks);
yaxis("$y$",LeftRight,RightTicks);
```



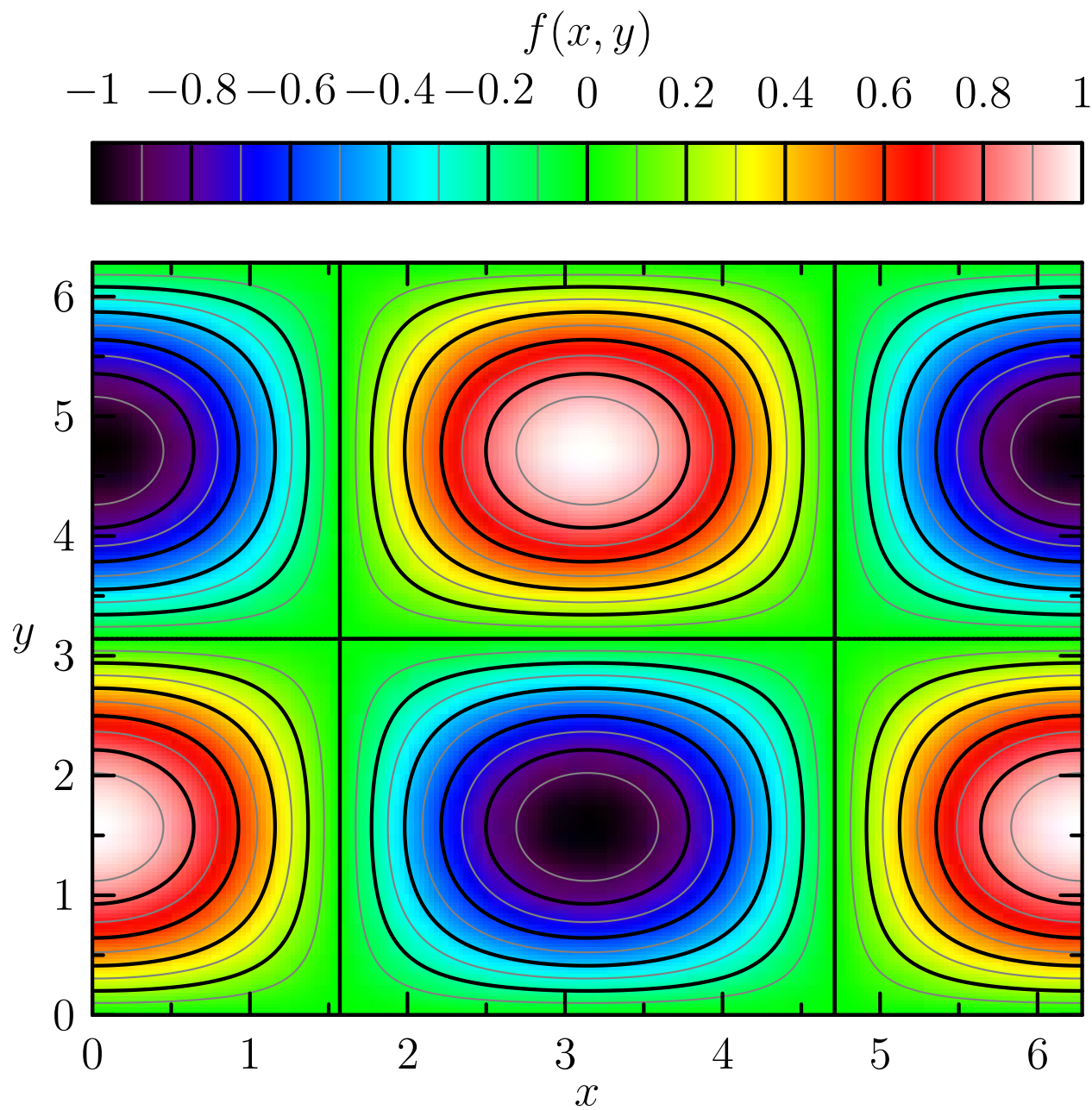
# Secondary Axis

Proportion of crows

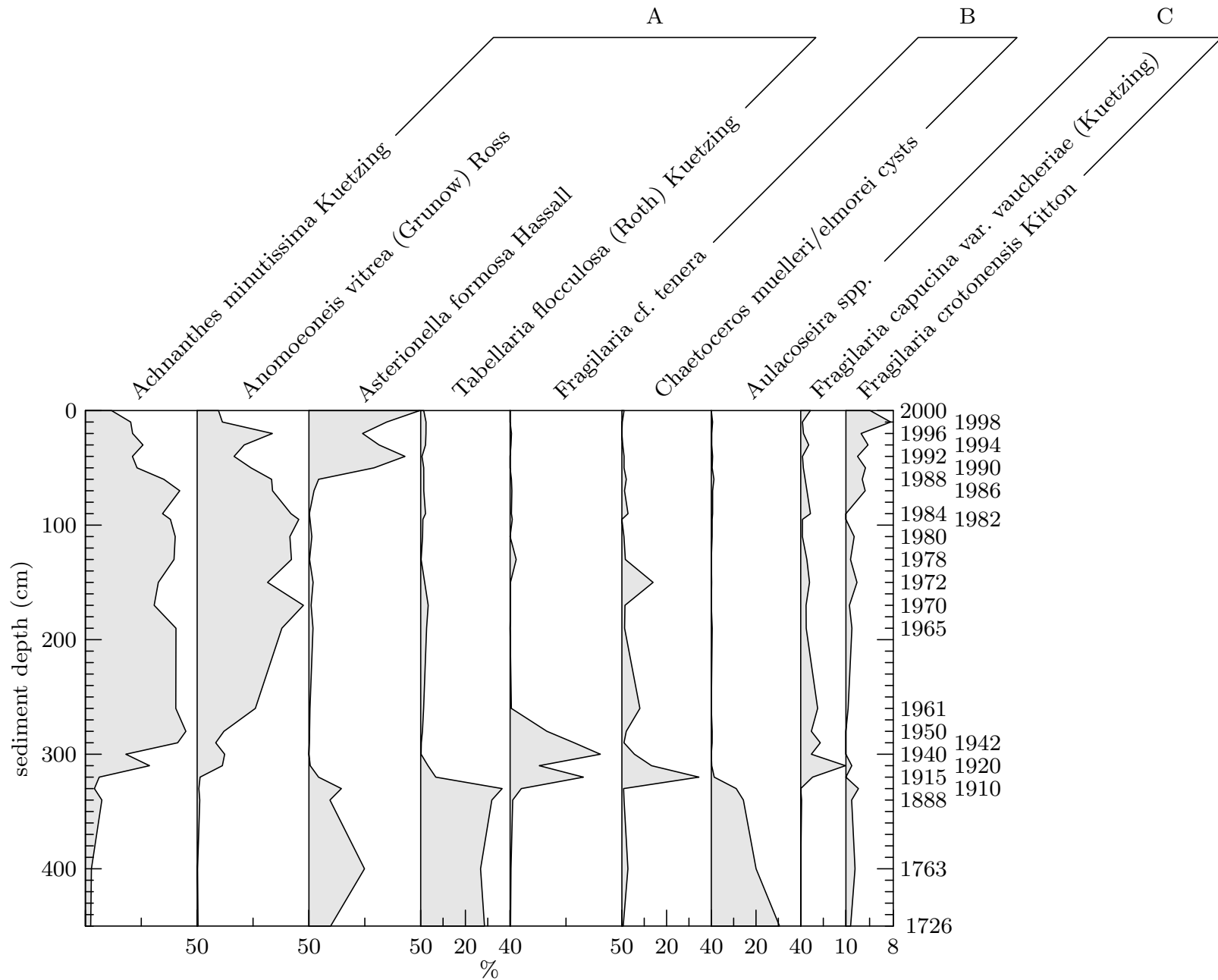




# Images



# Multiple Graphs



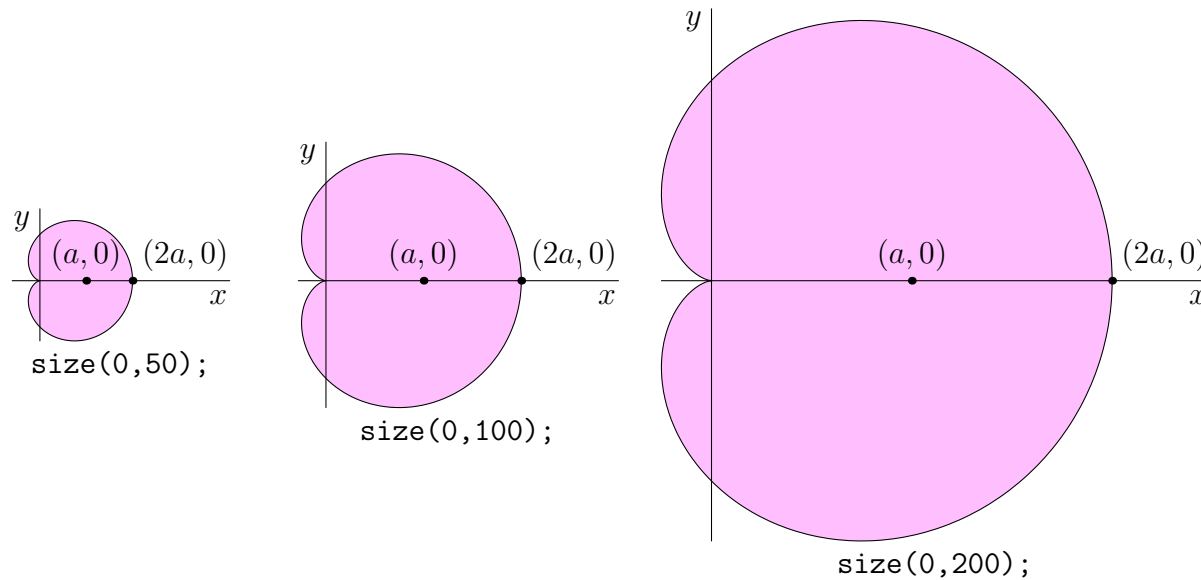
# Slide Presentations

- Asymptote has a package for preparing slides.

```
title("Slides");  
item("Asymptote has a package for preparing slides.");  
...
```

# Automatic Sizing

- Recall that figures can be specified in user coordinates, then automatically scaled to the final size.



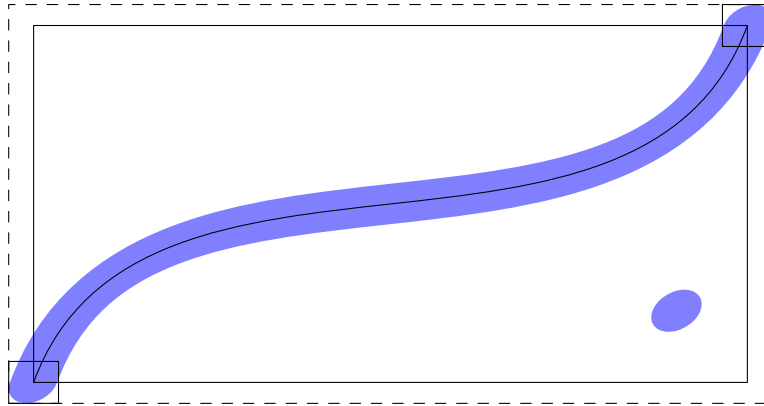
# Deferred Drawing

- We can't draw a graphical object until we know the scaling factors for the user coordinates.
- Instead, store a function that when given the scaling information, draws the scaled object.

```
void draw(picture pic=currentpicture, path g, pen p=currentpen) {  
    pic.add(new void(frame f, transform t) {  
        draw(f,t*g,p);  
    });  
    pic.addPoint(min(g),min(p));  
    pic.addPoint(max(g),max(p));  
}
```

# Coordinates

- Store bounding box information as a sum of user and true-size coordinates:



```
pic.addPoint(min(g),min(p));  
pic.addPoint(max(g),max(p));
```

- Filling ignores the pen width:

```
pic.addPoint(min(g),(0,0));  
pic.addPoint(max(g),(0,0));
```

- Communicate with L<sup>A</sup>T<sub>E</sub>X to determine label sizes:

$$E = mc^2$$

# Sizing

- When scaling the final figure to a given size  $S$ , we first need to determine a scaling factor  $a > 0$  and a shift  $b$  so that all of the coordinates when transformed will lie in the interval  $[0, S]$ . That is, if  $u$  and  $t$  are the user and truesize components:

$$0 \leq au + t + b \leq S.$$

- We are maximizing the variable  $a$  subject to a number of inequalities. This is a linear programming problem that can be solved by the simplex method.

# Sizing

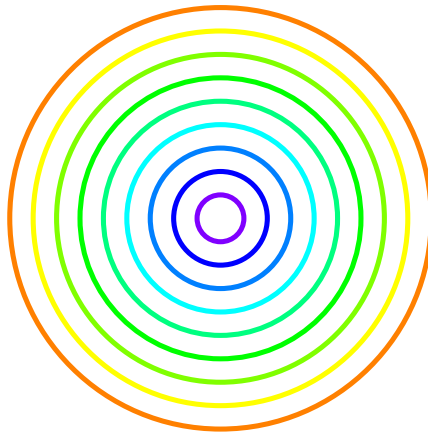
- Every addition of a coordinate  $(t, u)$  adds two restrictions

$$au + t + b \geq 0,$$

$$au + t + b \leq S,$$

and each drawing component adds two coordinates.

- A figure could easily produce thousands of restrictions, making the simplex method impractical.
- Most of these restrictions are redundant, however. For instance, with concentric circles, only the largest circle needs to be accounted for.





# Redundant Restrictions

- In general, if  $u \leq u'$  and  $t \leq t'$  then

$$au + t + b \leq au' + t' + b$$

for all choices of  $a > 0$  and  $b$ , so

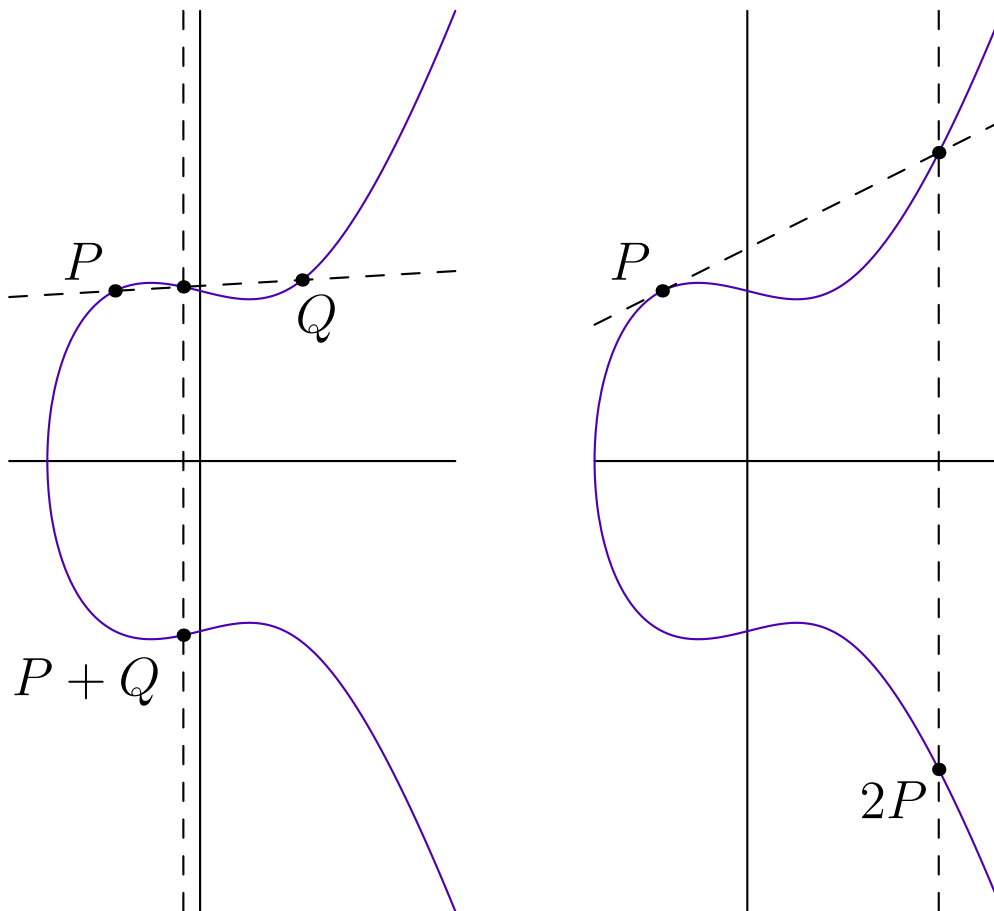
$$0 \leq au + t + b \leq au' + t' + b \leq S.$$

- This defines a partial ordering on coordinates. When sizing a picture, the program first computes which coordinates are maximal (or minimal) and only sends effective restraints to the simplex algorithm.
- In practice, the linear programming problem will have less than a dozen restraints.
- All picture sizing is implemented in Asymptote code.

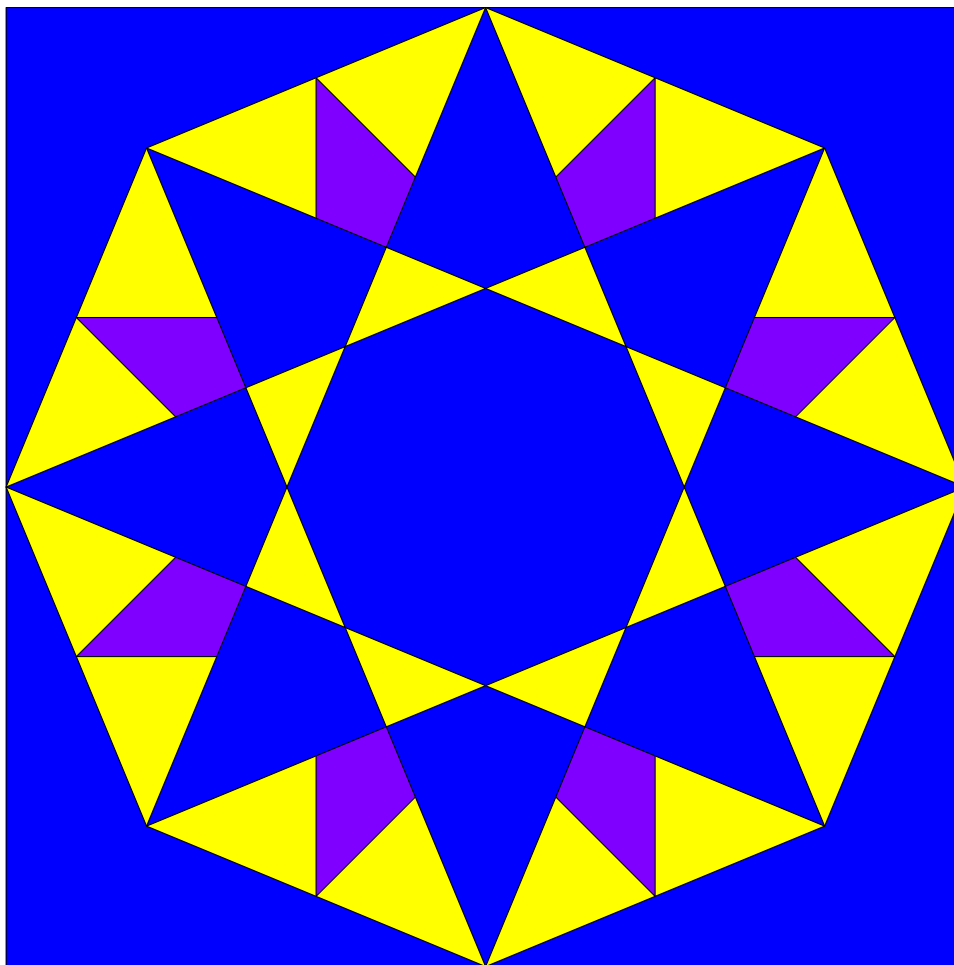
# Infinite Lines

- Deferred drawing allows us to draw infinite lines.

```
drawline(P, Q);
```



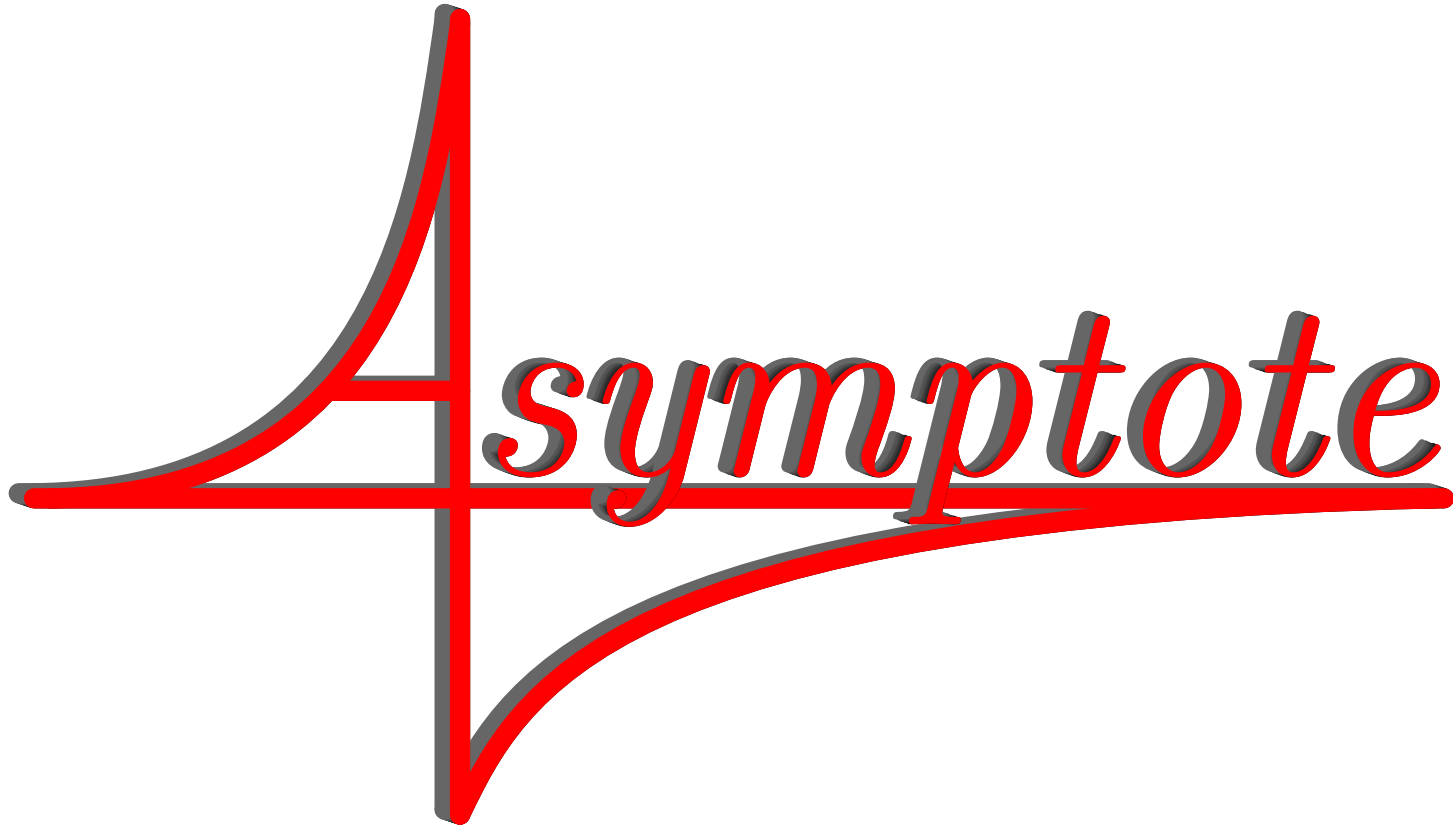
# A Final Example: Quilting



# References

- [Hob86] John D. Hobby. Smooth, easy to compute interpolating splines.  
*Discrete Comput. Geom.*, 1:123–140, 1986.
- [Knu86] Donald E. Knuth. *The METAFONTbook*. Addison-Wesley,  
Reading, Massachusetts, 1986.

# Asymptote: The Vector Graphics Language



<http://asymptote.sf.net>

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